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Département de finance

FINANCE RESEARCH SEMINAR

"Asset Prices and Trading Volume with Delegations"

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Friday, November 30, 2018, 10:30-12:00
Room 126, Extranef building at the University of Lausanne

Asset Pricing and Trading Volume with Delegation *

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Abstract

I build a dynamic general equilibrium model of delegation where the fund manager faces an equity constraint in the spirit of He and Krishnamurthy (2013) and trades against a trading desk. When the constraint binds, the model delivers a lower interest rate, a higher risk premium, and a larger stock trading volume. When the constraint does not bind, trading fund shares allows agents to achieve first-best allocations with low trading volume. Therefore, in an economy with costly trade, liquidity is likely to increase. The effect is expected to weaken when the constraint binds, as all trading is through the stock market.

JEL classification: G11, G12, G23

Key words: delegation, asset pricing, trading volume, incomplete markets, general equilibrium.

*I am grateful to Yuki Sato and Norman Schürhoff, my advisors, and Paul Ehling and Theodosios Dimopoulos for support and valuable advice. I would like to also thank Bernard Dumas, Chunyu Yang, and participants at the UNIL Brown Bag seminar for helpful discussions and suggestions. All errors and omissions are my own.

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I. Introduction

In this paper, I develop a dynamic general equilibrium model of delegation. Agents with heterogeneous preferences and differential access to financial markets trade to share risk. Experts—a fund manager and a trading desk—can trade the risky asset directly. While the latter trades in its own name, the fund manager acts as an agent for fund investors: herself and a household whose access to the risky security is restricted. The presence of the trading desk allows to investigate the impact of delegation on trading volume. In particular, the risky asset trades only indirectly, through fund shares, absent the second expert. I incorporate agency frictions in reduced form: the manager must hold a minimum number of fund shares to align her incentives to those of fund investors.

I start by showing that when the constraint binds, it can increase the prices of both the risky and the risk-free assets. When the manager is relatively poor, despite the increased exposure to the stock, the fund cannot accommodate the household's stock demand. Instead, the trading desk absorbs the residual stock supply. Relative to an unconstrained economy, experts hold riskier portfolios, their consumption growth volatility is higher, and the precautionary savings effect increases their demand for bonds. At the same time, the household substitutes bonds for stocks and the overall higher demand increases the equilibrium bond price by lowering the interest rate. For the stock, the experts' increased consumption growth volatility has two opposing effects. On the one hand, as in the case of bonds, the precautionary savings effect raises its price. On the other hand, the consumption growth volatility increases the state price volatility. This translates into a larger negative covariance between state prices and dividends, which reduces the price. Generally, the first effect dominates, and delegation yields a higher stock price relative to a frictionless economy. As the strength of the binding constraint dampens, the larger covariance becomes more important and the price can decrease. Turning to the risk premium, the constraint has an amplification effect: as the economy moves further into the constrained region, the risk premium increases faster.

Varying the risk aversion of the trading desk helps to assess its role in dampening the effects of the equity constraint. Compared to the benchmark parametrization, I find that a less risk averse trading desk reduces the probability of being in a constrained state. A notable impact on the interest rate and the risk premium is evident when the binding of the constraint is more severe. However, the simulation results show that the median impact is comparable to that of the benchmark case. Performing a similar exercise for the fund manager's risk aversion reveals her relatively larger role in determining equilibrium quantities in the constrained region. In particular, in this case there is a much higher probability for the equity constraint to bind and a significant increase in price distortions.

Focusing on trades, simulations of the model illustrate the potential benefits of fund investing. Delegation allows investors to substitute trading fund shares for trading assets directly. In the unconstrained region, this implies that agents can achieve the same asset allocation as in a frictionless economy, but with a lower trading volume. However, when the constraint binds, it increases trading between the fund and the trading desk. This implies that in a world with trading fees and an unconstrained manager, delegated investment lowers transaction costs and increases welfare. When the manager is constrained, we can expect fees to amplify price distortions and reduce the likelihood of an improvement in the fund's risk capacity.

In the second part of the paper, I characterize asset prices in the presence of both delegation and trading fees. Because the fund's transaction costs are shared among investors proportional to their holdings of fund shares, the manager faces an endogenous, stochastic, proportional transaction fee. I show that it leads to a direct price effect for both current and expected future transaction costs. This is in contrast to the model of Buss and Dumas (2017), where future trading fees have an indirect effect only, through deviations from first-best consumption paths.

My paper contributes to the literature on the role of delegation in financial markets. Kaniel and Kondor (2013) and Basak and Pavlova (2013), among others, investigate the asset pricing effects of delegation starting from salient features such as the convex flow-performance relationship and the relative performance concerns of asset managers. In models of reputation concerns, DiMaggio (2015) and Malliaris and Yan (2015) study the risk taking behavior of fund managers under fixed investment strategies. Malliaris and Yan (2015), similar to He and Krishnamurthy (2013) or He and Xiong (2013), also show how slow-moving capital can arise in models with intermediation. With the exception of Kaniel and Kondor (2013), these models do not investigate trading volume. Furthermore, my framework allows for the introduction of asset illiquidity stemming from costly trade.

Dasgupta and Prat (2006) analyze a trading model where fund managers have career concerns. With low-power incentives they obtain a churning equilibrium where fund managers trade even in the absence of information, thus providing liquidity to the market. Dasgupta and Prat (2008) also model career concerns with a focus on the implications for price informativeness. They show that career concerns can lead to herding while increasing liquidity, as measured by bid-ask spreads. My model provides a richer framework with the potential to analyze the welfare implications of delegation in the presence of illiquidity.

The version of my model without trading fees can be seen as a generalization of the He and Krishnamurthy (2013) model, being less restrictive in terms of preferences and investment decisions, and allowing for another type of agent, outside the intermediation relationship.

Similar to theirs, my model produces lower interest rates, higher risk premia, and a positive relationship between risk premia and leverage.

Schumacher (2012) studies an economy in the spirit of He and Krishnamurthy (2012) where the equity constraint on the fund manager is endogenized. He uses a numerical procedure introduced in Dumas and Lyasoff (2012) to solve a model with two agents who invest in an intermediary. He extends the He and Krishnamurthy (2012) framework to include multiple risky assets and various degrees of accessibility for households. His focus is on the fragility versus the resilience of markets, depending on the concentration of investor base, and unlike mine, his model cannot deliver trading volume results for intermediated assets.

The equilibrium effects of trading fees have been recently investigated in Buss and Dumas (2017). They show that the fee-induced tradeoff between smoothing consumption and smoothing asset holdings gives rise to a stochastic liquidity process: an agent's decision not to trade reduces the probability of trading by depriving other agents of a counterparty. They assess analytically and numerically the impact of trading fees on asset prices and derive empirical predictions about liquidity premia and slow moving capital. Studying an extension of their model with three symmetrical traders, they show that the effects of fees are not mitigated by the introduction of more traders. Delegation in my model can be seen as a means to curb the effects of trading fees. The downside is the existence of the equity constraint, which, as I show, has similar effects to trading fees. However, these arise mostly when the constraint is binding.

The paper proceeds as follows. Section II describes the setup of a general model, able to incorporate asset illiquidity as trading fees. In Section III, I define the equilibrium and describe the solution method. I analyze the model without trading fees in Section IV and in Section V I present analytical results on the impact of trading fees on asset prices in the presence of delegation. Section VI concludes and the Appendices contain technical details.

II. Model

I model a finite horizon economy, where time t is discrete and runs from zero to T. There is one consumption good, the *dollar*. There are also $N+1$ (financial) assets, N of which are risky, the *stocks*, and one that is risk-free, the *bond*. Stocks, indexed by $i = 1, \ldots, N$, are in unit supply and pay dividends $D_{i,t}$ at each point in time. In contrast, the zero-supply bond is reissued every period after having paid off one dollar. There are three classes of risk-averse and competitive agents in the economy: the household, h, and two experts, a trading desk, d, and a fund manager, m. The experts can access all assets directly, while the household has direct access only to the risk-free one. I present a general version of the model that allows for asset illiquidity stemming from trading fees. Throughout the numerical application in Section IV I assume no trading costs. In Section V, I present analytical results for asset prices *with* trading fees.

A. Risky Assets

The dividends of the risky assets, $D_{i,t}$, are placed on a recombining binomial tree. To allow for trading risky assets to entail a cost, I adopt the framework in Buss and Dumas (2017), BD 2017 henceforth. In particular, I assume that trader j pays proportional trading fees, $\lambda^j_{i,t}$, for trading asset i, with $i = 1, \ldots, N$ and $j \in \{d, m\}$.

Each period, trading fees are collected and redistributed to agents as a single transfer. This assumption simplifies the implementation of the numerical procedure, while at the same time keeping agents competitive. [1]

I consider a recursive Walrasian market for assets, as in BD 2017. Traders submit their demands conditional on asset prices and the auctioneer sets prices such that markets clear. With transaction costs, clearing may involve no trade, in which case the prices set by the auctioneer are just *posted* prices and not transaction prices.

B. Agents

All agents in the economy maximize lifetime utility over consumption. I assume they have heterogenous CRRA preferences and, therefore, trade to share consumption risk.[2] They are initially endowed with asset holdings $\tilde{\theta}^j_i$, expressed as number of shares, and each period t they choose their consumption, $c_{j,t}$, and their investment policies. The trading desk trades

[1] BD 2017 find that assuming instead that trading fees are a deadweight cost does not alter the conclusions of their model.

[2] An alternative would be to assume that the agents have homogenous preferences, but receive stochastic endowments, as in BD 2017.

in its own name (proprietary trading) and I denote its period-t *after-trade* asset holdings by $\theta^d_{i,t}$, with $i = 0, \ldots, N$, and $t = 0, \ldots, T$. The fund manager launches a fund and invests in the name of fund investors. She is not allowed to trade privately any of the assets, but invests herself in the fund. The after-trade fund portfolio is given by $\theta^f_{i,t}$, with $i = 0, \ldots, N$, and $t = 0, \ldots, T$. The household invests in the risky assets through the fund and can directly trade the risk-free asset. Therefore, the stocks are traded by the trading desk and fund manager, whereas the risk-free asset by all agents. I model both the manager and the household as fund investors to capture the sharing of transaction costs in a fund, as described below.

C. *Investment in the Fund*

The fund manager issues shares in the fund. For simplicity, I assume a unit supply. Fund investors indirectly trade assets by trading fund shares. Denoting by $x_{k,t}$ the *after-trade* fund shares held by investor k, with $k \in \{h, m\}$, the total indirect holdings of asset i are given by $x_{k,t}\theta^f_{i,t}$.

The fund portfolio is observable and all fund investors take the fund's holdings into account when trading shares. Moreover, if trading risky assets is costly, they also account for the reduction in fund value due to trading fees. Costs are shared by fund investors proportionally to their after-trade holdings of fund shares, such that investor k incurs costs of

$$x_{k,t} \times \sum_{i=1}^{N} \left| \theta^f_{i,t} - \theta^f_{i,t-1} \right| P_{i,t} \lambda^f_{i,t}. \tag{1}$$

Allowing the fund manager to invest in her own fund is a simple way to model a counterparty to the household's flows and capture the (transaction) cost sharing benefits of a fund.[3] To better understand the cost sharing benefits of delegation, consider the following example: the household wants to increase its exposure to the stock, while the fund manager wants to decrease it; the trade of the household is given by $\Delta z_h \equiv z'_h - z_h$, while that of the fund manager is $\Delta z_m \equiv z'_m - z_m$; assume that $\Delta z_h > -\Delta z_m$, that is, the trades do not completely cancel out, and that the agents pay the same proportional fee, λ. If agents trade on their own account, the trading costs they incur are $\Delta z_h \times \lambda \times P$ for the household and $|\Delta z_m| \times \lambda \times P$ for the manager; if, however, they trade stocks indirectly by trading fund shares, the fund increases its holdings to $\theta^{f\prime} \equiv z'_h + z'_m$ from $\theta^f \equiv z_h + z_m$ and the total trading cost are $\Delta\theta^f \times \lambda \times P$, with $\Delta\theta^f \equiv \theta^{f\prime} - \theta^f$. Both agents face lower trading costs as

[3] Vayanos and Woolley (2013) make a similar assumption to accommodate flows between an index and an active fund.

fund investors, since they trade in opposite directions: $|\Delta z_m + \Delta z_h| < \Delta z_k$, $\forall k \in \{h, m\}$.[4]

As in He and Krishnamurthy (2013), I assume that the fund is subject to an equity constraint. In particular, the fund manager needs to hold at least m shares in the fund. I model neither an agency problem, nor an informational asymmetry explicitly. The role of the "skin in the game" constraint is to align the incentives of the fund manager and fund investors. It can be interpreted as an incentive contract in the hedge fund industry, that is, m can be seen as the performance fee earned by managers.

D. *Optimization Problems*

The fund manager acts as both fund investor and securities trader. She chooses her consumption stream, $\{c_{m,t}\}$, the investment in the active fund, $\{x_{m,t}\}$, and the fund portfolio, $\{\theta^f_{i,t}\}_{i=0}^N$, to maximize lifetime utility, subject to the equity constraint. If trading is costly, given the assumption of the redistribution of trading fees, the fund manager receives every period a lump-sum transfer

$$\zeta_{m,t} \equiv x_{m,t} \sum_{i=1}^{N} \left|\theta^d_{i,t} - \theta^d_{i,t-1}\right| P_{i,t} \lambda^d_{i,t}. \tag{2}$$

This transfer appears in the fund manager's budget constraint but not in the other first order conditions: when choosing her fund exposure and the fund portfolio, the manager does not take into account the effect of her choice on the size of the redistribution; she also doesn't account for the trades of the other agents. The fund manager's problem is

$$\max_{c_m, x_m, \theta^f} \ \mathbb{E}\left[\sum_{t=0}^{T} u_m(c_{m,t}, t)\right], \tag{3}$$

$$\text{s.t.} \quad c_{m,t} + x_{m,t} \sum_{i=0}^{N} \theta^f_{i,t} P_{i,t} + x_{m,t} \sum_{i=0}^{N} \left|\theta^f_{i,t} - \theta^f_{i,t-1}\right| P_{i,t} \lambda^f_{i,t}$$

$$= x_{m,t-1} \sum_{i=0}^{N} \theta^f_{i,t-1} \left(P_{i,t} + D_{i,t}\right) + \zeta_{m,t}, \quad t = 0, \ldots, T, \tag{4}$$

$$x_{m,t} \geq m, \quad t = 0, \ldots, T, \quad \theta^f_{-1,i} = \tilde{\theta}^f_i, \quad x^m_{-1} = \tilde{x}^m.$$

[4] There can be situations in which a fund investor pays higher trading costs than if trading individually; for example, for large enough trades of the other investors when he holds a high fraction of the fund and everyone (investing through the fund) wants to trade in the same direction; the higher the share of his holdings, the lower the trades of the other(s) have to be for fund investing to be currently disadvantageous.

Following BD 2017, I rewrite the manager's optimization problem as one with inequality constraints. To do so, I define purchases and sales of risky assets as

$$\hat{\theta}^f_{i,t} \equiv \max\left[0, \theta^f_{i,t} - \theta^f_{i,t-1}\right] \qquad \text{and} \qquad \check{\theta}^f_{i,t} \equiv -\min\left[0, \theta^f_{i,t} - \theta^f_{i,t-1}\right]. \tag{5}$$

Stock holdings become $\theta^f_{i,t} = \theta^f_{i,t-1} + \hat{\theta}^f_{i,t} - \check{\theta}^f_{i,t}$ and purchases and sales satisfy the following non-negativity constraints:

$$\hat{\theta}^f_{i,t} \geq 0 \quad \text{and} \quad \check{\theta}^f_{i,t} \geq 0. \tag{6}$$

Let $J_{m,t}(\cdot)$ be the value function of the fund manager. The Bellman equation is:

$$J_{m,t}(x_{m,t-1}, \{\theta^f_{i,t-1}\}) = \sup_{c_m, x_m, \theta^f} u_m(c_{m,t}, t) + \mathbb{E}_t\left[J_{m,t+1}(x_{m,t}, \{\theta^f_{i,t}\})\right], \tag{7}$$

subject to the time-t only budget constraint (4) and the portfolio constraint. Defining recursively the number of fund shares as $x_{k,t} = x_{k,t-1} + \Delta x_{k,t}$, the manager's problem is to choose $c_{m,t}$, $\Delta x_{m,t}$, $\hat{\theta}^f_{i,t}$ and $\check{\theta}^f_{i,t}$ to maximize

$$\begin{aligned}
\mathcal{L}_m(\cdot, t) \equiv \mathcal{L}_{m,t} &= u_m(c_{m,t}, t) + \mathbb{E}_t\left[J_{m,t+1}(\cdot)\right] \\
&\quad + \phi_{m,t}\Bigg[x_{m,t-1}\sum_{i=0}^{N}\theta^f_{i,t-1}(P_{i,t} + D_{i,t}) + \zeta_{m,t} - c_{m,t} \\
&\qquad - x_{m,t}\sum_{i=0}^{N}\theta^f_{i,t}P_{i,t} - x_{m,t}\sum_{i=1}^{N}(\hat{\theta}^f_{i,t} + \check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t}\Bigg] \\
&\quad + \phi_{m,t}\mu_{m,t}(x_{m,t} - m) + \sum_{i=1}^{N}\hat{\nu}^f_{i,t}\hat{\theta}^f_{i,t} + \sum_{i=1}^{N}\check{\nu}^f_{i,t}\check{\theta}^f_{1,t},
\end{aligned} \tag{8}$$

with

$$\zeta_{m,t} = x_{m,t}\sum_{i=1}^{N}(\hat{\theta}^d_{i,t} + \check{\theta}^d_{i,t})P_{i,t}\lambda^d_{i,t}. \tag{9}$$

In Appendix B, I derive the Karush-Kuhn-Tucker (KKT) conditions corresponding to the manager's problem.

The optimization problems of the other two agents, the household and the trading desk, are similar and are, therefore, relegated to the Appendix A. Specifically, the household chooses its consumption stream, $\{c_{h,t}\}$, the investment in the active fund, $\{x_{h,t}\}$, and the number of bonds to hold directly, $\{y_{h,t}\}$, to maximize lifetime utility. The trading desk on the other hand, being able to invest directly in all securities, chooses its consumption stream, $\{c_{d,t}\}$, and its optimal portfolio, $\{\theta^d_{i,t}\}_{i=0}^N$.

III. Equilibrium

The first order conditions and the market clearing conditions give the system of equations that must hold in equilibrium, as defined below.

DEFINITION 1: *An equilibrium consists of the consumption processes of all agents,* $\{c_{k,t}\}$, $k \in \{d, h, m\}$, *the trading decisions processes* $\{\hat{\theta}^j_{i,t}, \check{\theta}^j_{i,t}\}$, $j \in \{d, f\}$, $\{\Delta x_{k,t}\}$, $k \in \{h, m\}$, *and* $\{y_{h,t}\}$, *the posted prices processes* $\{P_{i,t}\}$, *the state prices processes* $\{\phi_{k,t}\}$, $k \in \{h, d, m\}$, *and the shadow prices processes* $\{\mu_{m,t}\}$ *and* $\{R^j_{i,t}\}$, $j \in \{d, f\}$ *that solve* (10)–(19) *for all* t*:*

$$u'_k(c_{k,t}, t) = \phi_{k,t},\ k \in \{m, h, d\}, \tag{10}$$

$$\begin{aligned}\mathbb{E}_t &\left[\frac{\phi_{k,t+1}\sum_{i=0}^N \theta^f_{i,t}(P_{i,t+1}+D_{i,t+1})}{\phi_{k,t}} + \mu_{k,t}\right] \\ &= \sum_{i=0}^N \left(\theta^f_{i,t}P_{i,t} + (\hat{\theta}^f_{i,t} + \check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t}\right),\ k \in \{m, h\},\end{aligned} \tag{11}$$

$$\mathbb{E}_t\left[\phi_{k,t+1}\right] = \phi_{k,t}P_{0,t},\ k \in \{m, h, d\} \tag{12}$$

$$\begin{aligned}\mathbb{E}_t &\left[\frac{\phi_{k,t+1}}{\phi_{k,t}}\frac{1}{R^j_{i,t}}\left(\left(R^j_{i,t+1} - (1-R^j_{i,t+1})\frac{\Delta x_{k,t+1}}{x_{k,t}}\right)P_{i,t+1} + D_{i,t+1}\right)\right] \\ &= P_{i,t},\ (k, j) \in \{(m, f), (d, d)\}\end{aligned} \tag{13}$$

$$\begin{aligned}c_{k,t} &+ x_{k,t}\sum_{i=0}^N \theta^j_{i,t}P_{i,t} + x_{k,t}\sum_{i=0}^N(\hat{\theta}^j_{i,t} + \check{\theta}^j_{i,t})P_{i,t}\lambda^j_{i,t} + y_{k,t}P_{0,t} \\ &= x_{k,t-1}\sum_{i=0}^N \theta^j_{i,t-1}(P_{i,t}+D_{i,t}) + y_{k,t-1} + \zeta_{k,t}, \\ \textit{with}\ \ \zeta_{k,t} &= x_{k,t}\sum_{i=0}^N(\hat{\theta}^{j'}_{i,t} + \check{\theta}^{j'}_{i,t})P_{i,t}\lambda^{j'}_{i,t},\ (k, j, j') \in \{(m, f, d), (h, f, d), (d, d, f)\},\end{aligned} \tag{14}$$

$$\theta^j_{i,t} = \theta^j_{i,t-1} + \hat{\theta}^j_{i,t} - \check{\theta}^j_{i,t},\ j \in \{f, d\},\ i = 1, \ldots, N, \tag{15}$$

$$x_{k,t} = x_{k,t-1} + \Delta x_{k,t},\ k \in \{m, h\}, \tag{16}$$

$$\mu_{m,t} \geq 0, \quad (x_{m,t} - m) \geq 0, \quad \mu_{m,t}(x_{m,t} - m) = 0, \tag{17}$$

$$\begin{aligned}&1 - \lambda^j_{i,t} \leq R^j_{i,t} \leq 1 + \lambda^j_{i,t} \quad \hat{\theta}^j_{i,t} \geq 0,\ \check{\theta}^j_{i,t} \geq 0, \\ &\left((1+\lambda^j_{i,t}) - R^j_{i,t}\right)\hat{\theta}^j_{i,t} = 0, \quad \left(R^j_{i,t} - (1-\lambda^j_{i,t})\right)\check{\theta}^j_{i,t} = 0, \quad j \in \{f, d\},\ i = 1, \ldots, N,\end{aligned} \tag{18}$$

$$\begin{aligned}\theta^f_{i,t} + \theta^d_{i,t} &= 1,\ i = 1, \ldots, N, \\ x_{h,t} + x_{m,t} &= 1, \\ y_{h,t} + \theta^f_{0,t} + \theta^d_{0,t} &= 0,\end{aligned} \tag{19}$$

$$\textit{where}\ \ \mu_{h,t} = 0,\ x_{d,t} = 1,\ \Delta x_{d,t} = 0,\ \textit{and}\ y_{k,t} = 0,\ \textit{for}\ \ k \in \{d, m\}. \tag{20}$$

A. *Recursive Solution*

To solve the system of equations recursively, I follow Dumas and Lyasoff (2012), DL 2012 henceforth. They propose a shift of the timing of the equations that leaves the system backward only. In the present model, I shift the following equations one period ahead: the state price equations (10), the budget constraints (14), the complementary slackness conditions associated with stock purchases and sales (18), the asset holdings definitions (15) and (16), and the market clearing conditions (19).

In the resulting system of equations (presented in Appendix C) the exogenous state variables are the dividends while the endogenous ones are the current state prices, $\phi_{k,t+1}$, with $k \in \{d, h, m\}$, and the current shadow prices of securities, $R^j_{t,i}$, $i = 1, \ldots, N$, with $j \in \{f, d\}$.[5] To simplify solving the system numerically, the state prices can be reduced to the consumption shares of two of the agents, which are naturally bounded.[6] Similarly, the second set of endogenous variables can be reduced to the bounded ratios $\frac{R^d_{i,t}}{R^f_{i,t}}$: $\frac{1-\lambda^d_{i,t}}{1+\lambda^f_{i,t}} \leq \frac{R^d_{i,t}}{R^f_{i,t}} \leq \frac{1+\lambda^d_{i,t}}{1-\lambda^f_{i,t}}$.

In Appendix B, I derive the stock price equations as

$$\begin{aligned} P_{i,t} &= \mathbb{E}_t \left[\frac{\phi_{d,t+1}(R^d_{i,t+1} P_{i,t+1} + D_{i,t+1})}{\phi_{d,t} R^d_{i,t}} \right], \\ P_{i,T} &= 0. \end{aligned} \tag{21}$$

Since the state variables are the ratios $\frac{R^d_{i,t}}{R^f_{i,t}}$ and not the individual shadow prices, I compute at each node the trading desk's private valuation of stock i:

$$\begin{aligned} R^d_{i,t} P_{i,t} &= \mathbb{E}_t \left[\frac{\phi_{d,t+1}(R^d_{i,t+1} P_{i,t+1} + D_{i,t+1})}{\phi_{d,t}} \right], \\ R^d_{i,T} P_{i,T} &= 0. \end{aligned} \tag{22}$$

Letting K_t denote the number of nodes $(t+1, \eta)$ that come out of the current one, (t, ξ), the unknowns we solve for at each point in time and for each state are: future consumption, $c_{k,t+1,\eta}$, $k \in \{m, h, d\}$, future state prices $\phi_{k,t+1,\eta}$, $k \in \{m, h, d\}$, current number of fund shares $x_{k,t}$, $k \in \{m, h\}$, current bond holdings, $y_{h,t}$ and $\theta^j_{0,t}$, $j \in \{d, m\}$, current holdings of risky asset(s), $\theta^j_{i,t}$, future trades, $\Delta x_{m,t+1,\eta}$, $\hat{\theta}^j_{i,t+1,\eta}$ and $\check{\theta}^j_{i,t+1,\eta}$, future shadow prices of securities, $R^j_{i,t+1,\eta}$, and the current shadow price of the equity constraint, $\mu_{m,t}$. The future

[5]The shadow prices of securities are defined in Appendix B, equation (B12).

[6]State prices are a one to one mapping of consumption. Assuming trading fees are redistributed, aggregate consumption equals the aggregate dividend, such that the consumption shares of two of the tree agents contain all the relevant information. Since consumption and output cannot be negative, and since no agent can consume more than the aggregate output/dividend, consumption shares are in the interval $[0, 1]$.

private valuations, asset holdings, and holdings of fund shares, $P_{0,t+1\eta}$, $R^d_{i,t+1\eta}P_{i,t+1\eta}$, $\theta^j_{i,t+1,\eta}$, $y_{h,t+1,\eta}$, and $x_{k,t+1,\eta}$, are carried backward from the solution of the system at the previous, time $t+1$, nodes through interpolation.

The system of equations is solved for each time-t node, with $t = T-1,\ldots,0$, giving all the unknowns, except for the time $t = 0$ consumption, security shadow prices, and trades. We obtain these by solving a slightly modified system: we remove the kernel equations, the complementary slackness condition for the manager's equity constraint, and the market clearing conditions, since we already solved for time $t = 0$ prices and holdings. Once we solve for the rest of time $t = 0$ unknowns given initial portfolios, we can make the final step in solving the model: we move forward in the tree and compute consumption, holdings, and prices at each node.

In what follows I solve a simplified version of the model where there is only one risky asset, that is, $N = 1$. Further, I show in Appendix D that with this setup we obtain a scale invariance property that allows solving the system of equations only once for each time period. The solution can be rescaled to obtain the quantities for every time-t node, (t,ξ).

IV. Delegation Model without Trading Fees

In this section I present numerical results for the version of my model where trading fees are absent, which can be seen as a generalization of the model of He and Krishnamurthy (2013). They study an economy with two classes of agents that form bilateral intermediation relationships, households and specialists. They assume overlapping generations of households with log utility, a subset of which invests only in debt (the short-term risk free asset). Specialists are long-lived and manage the portfolios of the intermediaries in which they themselves and the households hold equity. My framework features long-lived agents only, less restrictive preference assumptions, and endogenous investment decisions by all agents. The model with three agents allows for the study of the stock trading volume while also highlighting the roles of different types of experts in the model. Throughout this section, I benchmark the delegation economy against one with no frictions.

I make the following assumptions regarding the stock dividend process and preferences. I model the dividend on a recombining binomial tree that matches the Basak and Cuoco (1998) illustration in DL 2012. In particular, the dividend growth rate and its volatility are $\mu_D = 1.83\%$ and $\sigma_D = 3.57\%$, respectively, the dividend having equal probability to move up or down next period. The preference parameters are as follows: the risk aversion coefficient of the trading desk is $\gamma_d = -3$, that of the household, $\gamma_h = -5$, and that of the manager, $\gamma_m = -2$, while the time preference is $\rho = 0.999$ for all agents. I set the risk tolerance of the trading desk in between that of the other two agents such that the fund manager is a trading counterparty to the household. In this way, delegation is expected to yield savings in terms of transaction costs. For the equity constraint I follow He and Krishnamurthy (2013) and assume that the fund manager must hold a minimum of 20 percent of the fund's shares. I reproduce the model parameters in Table I. The horizon of the economy is $T = 29$, or 30 periods.[7]

In this section I focus the analysis on the constrained region, that is, the region of the state space where the equity constraint binds. The unconstrained region does not differ in material ways from the economy with no frictions, as in Schumacher (2012).[8] In his extension of He and Krishnamurthy (2012) he shows numerically that the constraint produces no significant effects when not binding, while in the one period model in which he endogenizes the constraint, he obtains the result analytically.

A. *The Equity Constraint and Portfolio Holdings*

What happens to agents' holdings when the equity constraint binds? That is, what is the impact on portfolio holdings when the manager is relatively poor and her first-best holdings of the stock (absent frictions) are too low compared to those of the household? Since delegating enough to cover stock demand leaves the manager with too little skin-in-the game, the household substitutes bonds for stocks.

Panel (a) of Figure 1 shows the stock holdings of the household after trade at time $t = 0$. In the unconstrained region, depicted in the figure by light blue circles, the holdings $\theta_{1,h}$ increase as the consumption share of the manager decreases. The same applies for the otherwise constrained states in the economy without frictions. In contrast, in the delegation economy of my model, stock holdings in the constrained region, depicted by dark blue asterisks, decrease sharply for the household. This result also differs from what He and Krishnamurthy (2013) obtain in the model with two agents, where the intermediary (the fund) holds the entire stock supply. In my model, the trading desk captures a significant fraction of the disintermediation supply, that is, the difference between the household's stock holdings in the no frictions versus the delegation economy. This can be seen in Panel (b) of Figure 1, which plots this fraction against the level of disintermediation, $\Delta\theta_{1,h}$.

[7] The reason for choosing a short horizon is that the algorithm fails to deliver a solution (sometimes just within the required precision) for states in which the consumption share of the fund manager is relatively small. As we move backward in time, this happens more often. The main focus of this model being the behavior of the economy when the equity constraint binds, which is when the problems mostly occur, a short horizon seems more appropriate. Appendix E illustrates the issue of the untrusted domain.

[8] The interest rate in the delegation economy always deviates by less than 7×10^{-10} percent from the one in the frictionless economy; for the risk premium the deviation is never above 1 percent and only rarely above 0.1 percent (in 1.16 percent of the unconstrained states on the grid).

[Figure 1 about here]

To absorb the disintermediation supply, the experts borrow more. I illustrate the bond holdings of the household, the trading desk, and the manager in Figure 2, Panels (a), (b), and (c), respectively. The fund manager, having the highest risk tolerance is always a borrower, whereas the household, the most risk averse, always a lender. The trading desk assumes both roles, being a borrower in states where the household dominates the manager (in terms of consumption shares). As the figure shows, for most of the constrained states the trading desk's risk capacity is larger: it absorbs more of the bond demand coming from the household. Although the differences between the experts' holdings are significant in absolute terms, as percentage of their no frictions allocations they are less farther apart. When its consumption share is relatively large, the trading desk's holdings relative to the first-best are larger than for the manager, but as it decreases, it is the manager who borrows more relative to its first best.

[Figure 2 about here]

B. *Initial Asset Prices*

When the fund manager is relatively poor and the constraint binds, the bond price increases. Panel (a) of Figure 3 illustrates the corresponding decrease in the interest rate, showing further that in the constrained region the interest rate decreases with the consumption share of the fund manager, ω_m. Panel (b) illustrates that the opposite holds true absent frictions: as the more risk averse agents dominate the economy, the interest rate increases. He and Krishnamurthy (2013) also find that the interest rate decreases with delegation and identify two channels through which this comes about. First, the higher consumption growth volatility of the specialist (fund manager) increases his demand for the bond—the precautionary savings effect. Second, the household demands more bonds as it withdraws funds from the intermediary. For the bond market to clear, the price has to go up. I illustrated the second effect for my model in Section IV.A, and I report the consumption growth volatility of the experts in Figure 4: Panel (a) for the fund manager, and Panel (b) for the trading desk. The figure shows that as disintermediation occurs, the consumption growth volatility of the experts increases.

[Place Figures 3 and 4 about here]

The initial stock price depends on expected future state prices and their covariances with future dividends. As in BD 2017, I decompose the stock price into the *perpetual expected value* and the *perpetual risk premium*, using the trading desk's state prices:

$$
\begin{aligned}
P_{1,0} &= \sum_{\tau=1}^{T} \mathbb{E}_0\left[\frac{\phi_{d,\tau}}{\phi_{d,0}}\right]\mathbb{E}_0\left[D_{1,\tau}\right] + \sum_{\tau=1}^{T} \mathrm{cov}_0\left[\frac{\phi_{d,\tau}}{\phi_{d,0}}, D_{1,\tau}\right] \\
&= \sum_{\tau=1}^{T} \mathbb{E}_0\left[\frac{\phi_{d,\tau}}{\phi_{d,0}}\right]\mathbb{E}_0\left[D_{1,\tau}\right] + \sum_{\tau=1}^{T} \mathrm{stdev}_0\left[\frac{\phi_{d,\tau}}{\phi_{d,0}}\right]\mathrm{corr}_0\left[\frac{\phi_{d,\tau}}{\phi_{d,0}}, D_{1,\tau}\right]\mathrm{stdev}_0\left[D_{1,\tau}\right]. \qquad (23)
\end{aligned}
$$

Focusing on the constrained region, Panels (a) and (b) of Figure 5 illustrate the first term in (23), the perpetual expected value, for the initial stock price in the economy with delegation and for that without frictions, respectively. The higher value obtained in the delegation economy reflects the precautionary savings effect, namely that the increased consumption growth volatility tends to increase the price of the stock.[9]

The second component, the perpetual risk premium, works in the opposite direction: the negative correlation between state prices and dividends tends to reduce the price, and the variance of state prices determines the strength of this dampening effect. With larger consumption growth volatility, the variance of state prices is larger, which is why we observe a larger perpetual risk premium (in absolute terms) in Panel (c) versus Panel (d) of Figure 5. BD 2017 obtain similar results for the effect of trading fees on stock prices. Untabulated results show that, similar to trading fees, the equity constraint has the effect of reducing the correlation between state prices and dividends, but that this effect is dominated by the increase in the variance of state prices.

[Place Figure 5 about here]

I illustrate the overall effect of the constraint on the stock price in Panel (a) of Figure 6, where I plot the stock price relative to the no frictions economy. Dark blue asterisks represent states where the price is higher in the delegation economy, while light blue circles, states where it is lower. As the figure shows, for most of the states in the constrained region the overall effect of the constraint is to increase the stock price, that is, the impact through the perpetual expected value dominates, despite the more striking differences in the perpetual risk premium. In states where the price decreases, it does so by less than 1 percent.

[Place Figure 6 about here]

[9] I obtain the decomposition of the stock price for each state (ω_m, ω_d) illustrated in Figure 5 by assuming that it is the equilibrium outcome at $t = 0$. I then compute the future state variables and state prices by moving forward in the dividend tree.

Panel (c) shows that the initial risk premium tends to be higher in the constrained region relative to the no frictions economy. In particular, it can increase by approximately 45 percent, reaching 100 basis points (as shown in Panel (a)). As in He and Krishnamurthy (2013), who relate the risk premium to the economy's risk capacity, high risk premia are associated with high levels of leverage. Figure 7 plots the experts' time $t = 0$ portfolio weight in the stock. Recall that both the trading desk and the fund manager absorb the supply resulting from disintermediation by borrowing more. The manager takes on more leverage than the dealer in the constrained region, but it is the trading desk that has a higher relative leverage compared to the no frictions economy, in general.

[Place Figure 7 about here]

C. *Sensitivity to Experts' Risk Aversion*

I investigate the role of each expert in the equilibrium of the delegation economy by solving the model for two additional parametrizations. For the trading desk, I compare the benchmark case, $\gamma_m = -2$ and $\gamma_d = -3$, against an economy where the trading desk is less risk averse, $\gamma_m = \gamma_d = -2$. For isolating the role of the manager, I compare the latter case with one in which the manager is more risk averse, $\gamma_m = -3$ and $\gamma_d = -2$. The conclusions of the previous sections regarding policy functions and asset pricing quantities as functions of the endogenous state variables still hold: to absorb the excess stock supply resulting from agency frictions, experts hold riskier portfolios and, as a consequence, the interest rate decreases while the risk premium increases.[10]

Focusing on states where the constraint binds, Figures 8 and 9 compare the changes in the interest rate and the risk premium for the above parametrizations. In each panel, I fix the initial consumption share of the manager and plot the relative interest rate (Figure 8) and the relative risk premium (Figure 9) as a function of the trading desk's consumption share. I start from a very low consumption share for the manager, $\omega_m = 0.01\%$ in Panel (a) of both figures, and increase it until $\omega_m = 7.14\%$ in Panel (d). Comparing the $\gamma_m = -2$ and $\gamma_d = -3$ case (asterisks) against the $\gamma_m = \gamma_d = -2$ one (dots), shows that a more risk averse trading desk leads to both a more significant reduction in the interest rate and a more pronounced increase in the risk premium when the manager's consumption is very low. Varying the risk aversion of the manager has similar effects, but at higher manager consumption shares: with a more risk averse manager (the $\gamma_m = -3$ and $\gamma_d = -2$, depicted by circles, against the $\gamma_m = \gamma_d = -2$ case, depicted by asterisks), the decrease in the interest

[10] The comparison is against the frictionless economy corresponding to each parametrization.

rate and the increase in the risk premium are higher as the manager's consumption share increases. The results are intuitive: the higher the relative consumption share captured by an expert, the larger its role in determining equilibrium quantities.

Thus far, I compared the results for different parametrizations at time $t = 0$, on a state-by-state basis. However, the economies will likely exhibit different dynamics, leading to different steady state distributions. I simulate the model and focus on its dynamics in the next section.

[**Place Figures 8 and 9 about here**]

D. Simulation Results

This section presents results from a simulation of 10 000 dividend paths. I start the economy from a state where the fund owns 87.5 percent of the stock supply and sells approximately 6.17 units of bonds, the manager holds 20 percent of fund shares, and the household directly buys a little under 6.42 units of bonds.[11] In what follows, unless otherwise stated, I compute median values from pooling quantities together both over time and paths. For example, with 10 000 paths and 30 periods, we have 300 000 *states*.[12]

For the benchmark parametrization, $\gamma_m = -2$ and $\gamma_d = -3$, I obtain that the average probability for the constraint to bind over the lifetime of the economy is equal to 35.8 percent. This means that, on average, the constraint binds in approximately 10 out of the 29 periods.[13] Furthermore, there is a 54 percent probability that the manager is constrained for at least 8 periods (more than 25 percent of the time) and an almost 10 percent probability that she is constrained for at least 22 periods (more than 75 percent of the time).

The simulation of the economy where the trading desk is less risk averse, $\gamma_d = \gamma_m = -2$, shows a lower probability for the constraint to bind: on average 5.7 periods out of 29, or a 19 percent probability. The manager is constrained for at least 8 periods with 21 percent probability and for at least 22 periods with approximately 3 percent probability. Intuitively, with a less risk averse trading desk that wants to hold more stocks, the equilibrium demand of the household is lower, thus alleviating the severity of the agency friction.

Comparing this same economy, $\gamma_d = \gamma_m = -2$, against the one where the manager's risk aversion is higher, $\gamma_m = -3$ and $\gamma_d = -2$, paints a rather different picture: the average

[11] The initial allocation implies that the trading desk holds 12.5 percent of the stock supply and sells 0.25 units of the bond. The no frictions economy starts from an equivalent initial state.

[12] The initial allocations is chosen such that we obtain a high proportion of constrained states, while at the same time being able to gauge the potential costs and benefits of delegation in the presence of trading fees.

[13] The last period corresponds to final consumption and is, therefore, not included.

probability that the manager is constrained is almost 97 percent, or 28 out of the 29 periods. In fact, the probability that the constraint always binds is almost 80 percent.

[Place Table II about here]

In Table II, I report the median interest rate and the median risk premium for constrained states. Consider the case where experts are equally risk averse against the cases where I increase the risk aversion of one of the experts. The simulation results show that a more risk averse trading desk (column (1) versus column (2)) has a rather limited impact on both the median change in the interest rate and the median change in the risk premium (relative to the frictionless economy). On the contrary, a more risk averse manager (column (3) versus column (2)) not only increases the probability of the constraint to bind, but it also produces more significant distortions in asset prices. Specifically, the median decrease in the interest rate goes up from a mere 0.46 percent to a bit above 2 percent, while the median increase in the risk premium reaches almost 25 percent (relative to the 16.5 percent increase with lower manager risk aversion).

D.1. Trading Volume

Focusing on trading volume, I show that delegation has an impact on the trading of both assets. The number of bonds traded is higher than in the economy with no frictions, while that of stocks is lower.

[Place Table III about here]

Column (1) of Table III reports cumulative trading volumes corresponding to the short-term bond.[14] The median volumes of 2.25 and 5.92 units in the no frictions and delegation economies translate into a 2.62 state by state increase in cumulative volume. The result is confirmed when conditioning on the binding of the equity constraint. Columns (3) and (6) show that delegation bond volumes are almost 1.55 times higher in constrained states, and almost 3 times higher in unconstrained ones. To understand the increase, consider the fund portfolio, where the manager levers the position in the stock. The household, being a natural lender, has to de-lever the position held through fund shares to achieve its optimal allocation. This is the first effect of delegation, the second being that of disintermediation:

[14] I compute cumulative trading volume by summing up per-period trading volume over a simulated path, leaving out the initial and final trades. Relative cumulative trading volume is defined as the cumulative trading volume over a dividend path in the delegation economy versus the same path in the economy without frictions.

bond volumes increase as the household substitutes bonds for stocks and as the experts take on debt to finance higher stock holdings.

By replacing stock trades between the fund manager and the household with trades in fund shares, delegation reduces volume in the stock. For the initial allocation considered in this simulation, if there are no frictions, stock trading occurs between the household on the one side, and the experts, on the other. Among the two experts, it is the fund manager that accommodates the lion's share of the household's trading needs. This is shown in Panel A of Table IV, where I report bilateral trades as fraction of total trading volume. The median share accounted for by household-manager trades in otherwise constrained and unconstrained states is 75 and 78 percent, respectively. Table III, column (2) quantifies the reduction in cumulative trading volume: from a median of 0.14 units absent frictions to 0.04 units under delegation, and on a path by path basis, a median decrease of about 72 percent.

Columns (4) and (6) in Table IV show that the two economies exhibit a different behavior across constrained and unconstrained states: whereas median volume is higher for unconstrained states in the no frictions economy, the reverse is true for delegation, translating into a higher relative stock volume in constrained states, at 54 percent, versus 21 percent for unconstrained ones. Panel B of Table IV shows that actual trade is just 22 percent of the change in the exposure to the risky asset in the unconstrained region, the rest being achieved by indirect trades in fund shares. In the constrained region, however, all trading is between the fund and the trading desk, since the "skin in the game" constraint prevents the manager and household from trading the stock bilaterally through the fund, and achieve their first-best. These two-way trade results explain why we obtain a higher relative volume in constrained states. They also suggest that, in a world with trading fees, the cost sharing benefits of delegation might be weaker in the constrained region.

[Place Table IV about here]

V. Delegation in the Presence of Trading Fees

In this section, I analytically study asset prices in the presence of both delegation and trading fees and compare the results to those in BD 2017. The posted prices in my model are given by

$$P_{i,t} = \mathbb{E}_t \left[\frac{\phi_{d,t+1}}{\phi_{d,t}} \frac{1}{R^d_{i,t}} \left(R^d_{i,t+1} P_{i,t+1} + D_{i,t+1} \right) \right] \tag{24}$$

$$= \mathbb{E}_t \left[\frac{\phi_{m,t+1}}{\phi_{m,t}} \frac{1}{R^f_{i,t}} \left(\left(R^f_{i,t+1} - \left(1 - R^f_{i,t+1}\right) \frac{\Delta x_{m,t+1}}{x_{m,t}} \right) P_{i,t+1} + D_{i,t+1} \right) \right]. \quad (25)$$

Equations (24) and (25) are written from the perspectives of the trading desk and of the fund manager, respectively. When a trade takes place, the prices above are actual *transaction prices*. The difference with the result in BD 2017 stems from the endogenous effective proportional trading fee paid by the fund manager. Since the fund's trading costs are divided between fund investors proportional to their holdings of fund shares, there is the possibility for the fund manager to privately incur a proportional trading fee different than λ_f.

With the private valuation of trader k given by $\hat{P}_{k,i,t} = \mathbb{E}_t \left[\frac{\phi_{k,t+1}}{\phi_{k,t}} \left(\hat{P}_{k,i,t+1} + D_{i,t+1} \right) \right]$, I obtain that the result in BD 2017 holds for the trading desk: the largest possible difference between the private valuation of a stock's dividends and the posted price is given by the amount of the current trading fee only, i.e.,

$$R^d_{i,t} P_{i,t} = \hat{P}_{d,i,t}.$$

However, in my model, both current and future trading fees may contribute to the difference between the *manager*'s private valuation and the posted price:

$$\begin{aligned} R^f_{i,t} P_{i,t} &= \hat{P}_{m,i,t} \\ &- \mathbb{E}_t \left[\sum_{\tau=t}^{T-1} \frac{\phi_{m,\tau+1}}{\phi_{m,t}} \frac{1 - R^f_{i,\tau+1}}{R^f_{i,\tau+1}} \frac{\Delta x_{m,\tau+1}}{x_{m,\tau}} \prod_{s=t}^{\tau-1} \left(1 - \frac{1 - R^f_{i,s+1}}{R^f_{i,s+1}} \frac{\Delta x_{m,s+1}}{x_{m,s}} \right) \hat{P}_{m,i,\tau+1} \right]. \end{aligned} \quad (26)$$

Comparing prices in the economy with and that without transaction costs, unlike BD 2017, I obtain that future expected fees have a direct effect, above and beyond their impact through consumption. With $P^*_{i,t}$ denoting the price in the economy without fees, the following relationship between equilibrium prices holds:

$$\begin{aligned} R^f_{i,t} P_{i,t} = P^*_{i,t} &+ \mathbb{E}_t \left[\sum_{\tau=t}^{T-1} \frac{\phi_{m,\tau}}{\phi_{m,t}} \Delta\phi_{m,\tau} (D_{i,\tau+1} + P^*_{i,\tau+1}) \prod_{s=t}^{\tau-1} \left(1 - \frac{1 - R^f_{i,s+1}}{R^f_{i,s+1}} \frac{\Delta x_{m,s+1}}{x_{m,s}} \right) \right] \\ &- \mathbb{E}_t \left[\sum_{\tau=t+1}^{T} \frac{\phi_{m,\tau}}{\phi_{m,t}} \frac{1 - R^f_{i,\tau}}{R^f_{i,\tau}} \frac{\Delta x_{m,\tau}}{x_{m,\tau-1}} \prod_{s=t+1}^{\tau-1} \left(1 - \frac{1 - R^f_{i,s}}{R^f_{i,s}} \frac{\Delta x_{m,s}}{x_{m,s-1}} \right) P^*_{i,\tau} \right], \end{aligned} \quad (27)$$

with $\Delta\phi_{m,t} \equiv \frac{\phi_{m,t}}{\phi_{m,t-1}} - \frac{\phi^*_{m,t}}{\phi^*_{m,t-1}}$.

VI. Conclusion

In this paper, I build a general equilibrium model of delegation that is able to accommodate trading fees, starting from the works of Buss and Dumas (2017) and He and Krishnamurthy (2013).

In a first step, I study an economy where trading fees are absent. It can be seen as a generalization of the model of He and Krishnamurthy (2013) with three classes of agents. I add a second expert to my model, a trading desk that acts as a trading counterparty to the fund, and in this way I generate trading volume in the stock. Even in the absence of trading fees, the market is incomplete: unmodeled agency frictions are assumed to result in an equity constraint on the fund manager. The model is then solved using a simplified version of the numerical procedure in Buss and Dumas (2017). When the equity constraint binds the interest rate falls and risk premia rise, as in He and Krishnamurthy (2013). I show that the effects of the constraint can be likened to those of trading fees in the model of Buss and Dumas (2017). In particular, the constraint can increase the prices of all assets. Both the risk-free and risky assets are affected positively through the increased volatility of consumption. For the stock, however, this effect is dampened by the larger (negative) covariance between state prices and dividends.

To isolate the role of the two experts when the constraint binds, I vary their risk aversion. I show that whether this has a significant impact depends on the consumption share of the fund manager: the higher risk aversion of the trading desk worsens the impact of the constraint on initial prices for very low manager consumption shares; in contrast, a higher manager risk aversion leads to worse outcomes when the consumption of the manager is relatively higher.

I also illustrate the effects of delegation on trading patterns. The indirect trading of stocks through fund shares can mitigate the effects of trading fees on security prices, especially in the unconstrained region. In the constrained region, trading fees are expected to worsen price distortions. First, the constraint already pushes agents away from their first best allocation, generating only direct trades in the stock, between the fund and the trading desk. Paying fees would reduce the fund value and thus the manager's wealth, lowering the likelihood of emerging from the constrained region. Second, transaction costs can lead to less trading, as shown in Buss and Dumas (2017), such that portfolios and consumption can deviate farther from the no frictions case, with an impact on prices.

The model with trading fees can be solved numerically using an approach similar to that put forward in Buss and Dumas (2017). The framework could allow for liquidity management restrictions on funds, such as limits on illiquid investments, and the analysis of the impact on asset prices and welfare. With an increase in the computational burden, multiple assets with different degrees of liquidity can be introduced, as well as the possibility for both experts to launch funds. In such a framework one could study fund flows, or the effects of asset managers' specialization.

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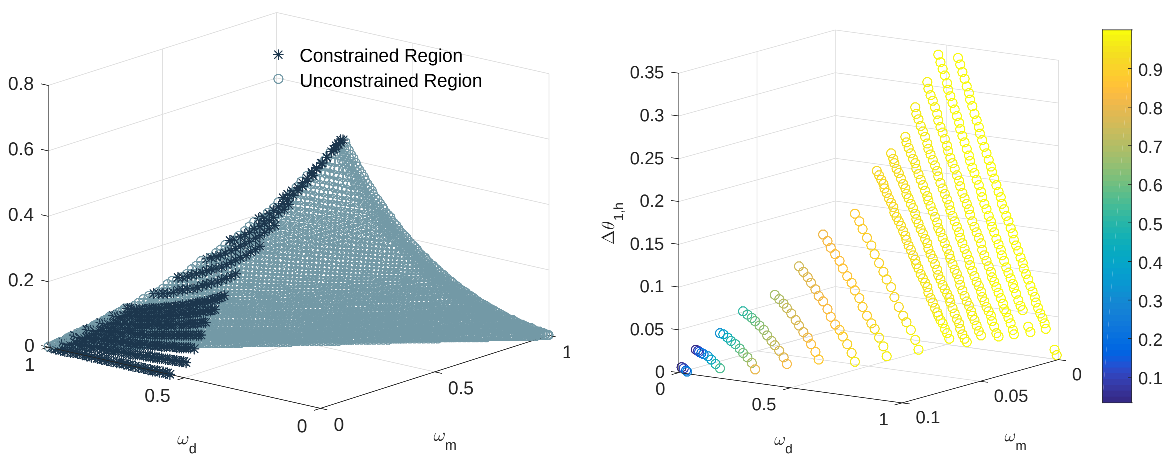
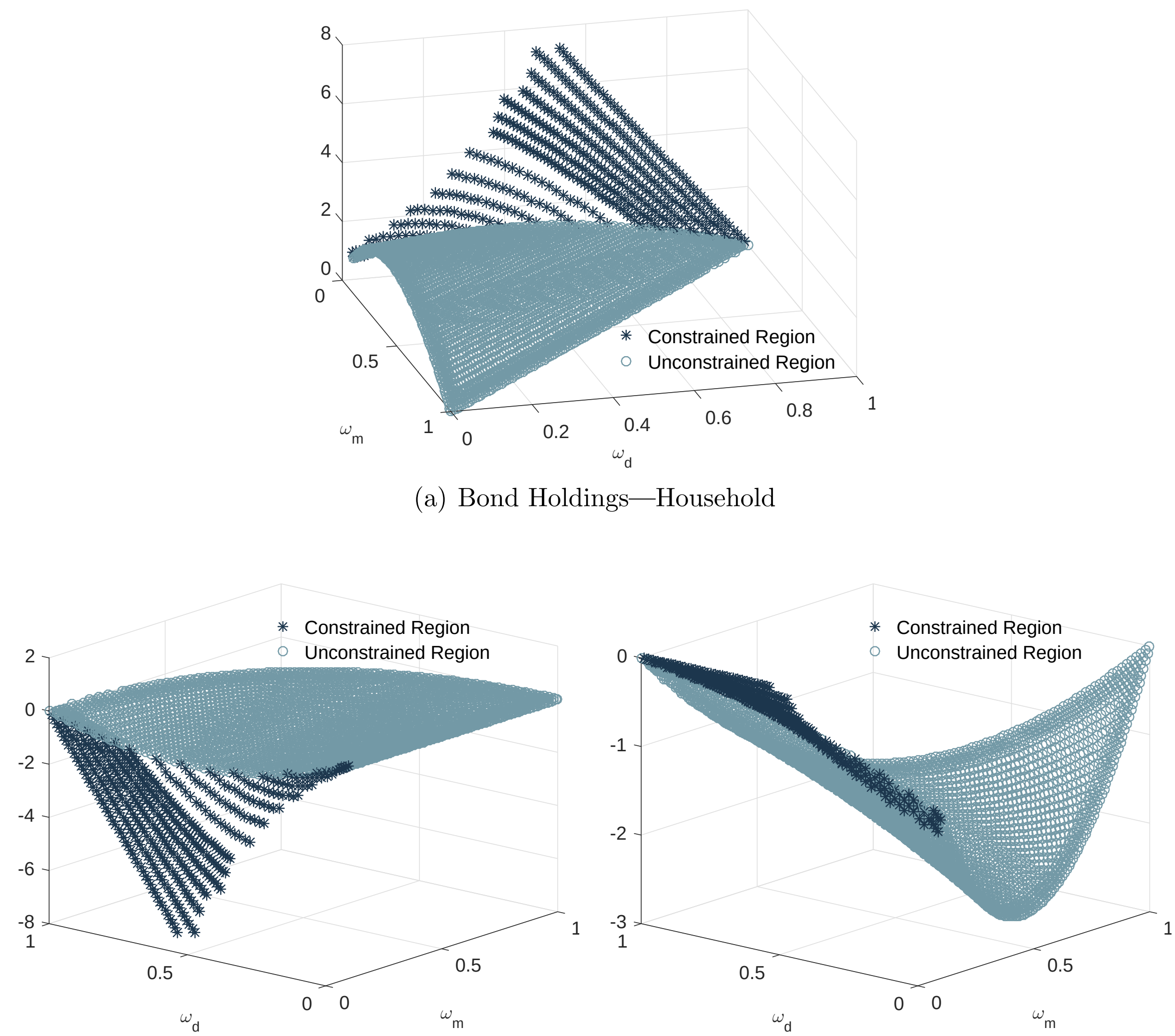


(a) Stock Holdings—Household

(b) Trading desk's share of the Disintermediation Supply

Figure 1. Disintermediation in the Delegation Economy. Panel (a) reports the household's stock holdings in the delegation economy, while Panel (b) reports the fraction of the disintermediation supply that is absorbed by the trading desk in the constrained region. In Panel (a), the light blue circles represent unconstrained states, while the dark blue asterisks show constrained ones. Holdings are plotted against the endogenous state variables, the consumption share of the fund manager, ω_m, and that of the trading desk, ω_d. The disintermediation supply, $\Delta\theta_{1,h}$, in Panel (b) is defined as the deviation in the household's stock holdings from the economy without frictions. For each state (ω_m, ω_d) the fraction of the supply captured by the trading desk is plotted against the corresponding $\Delta\theta_{1,h}$. Parameters are as in Table I.

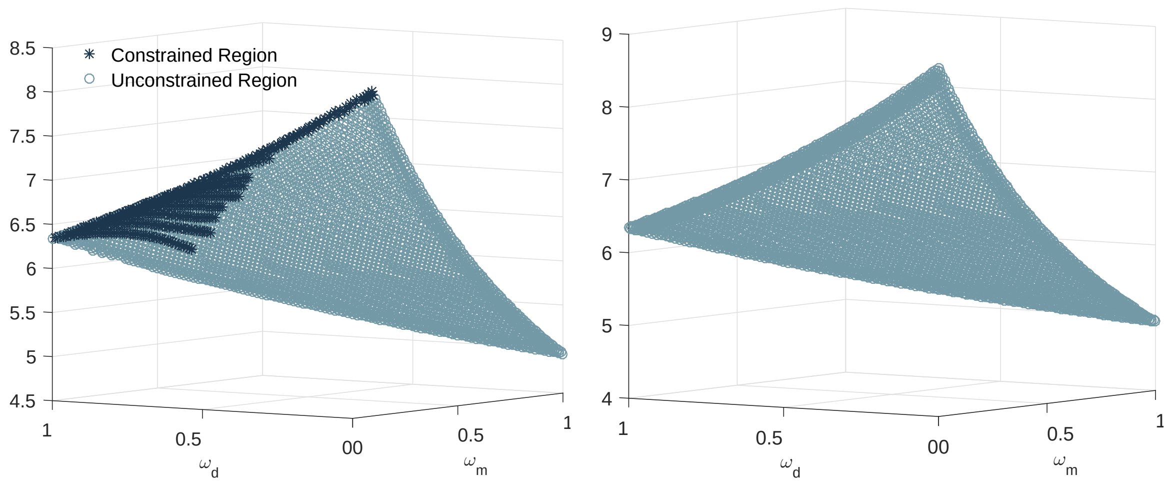


(a) Bond Holdings—Household

(b) Bond Holdings—Trading Desk

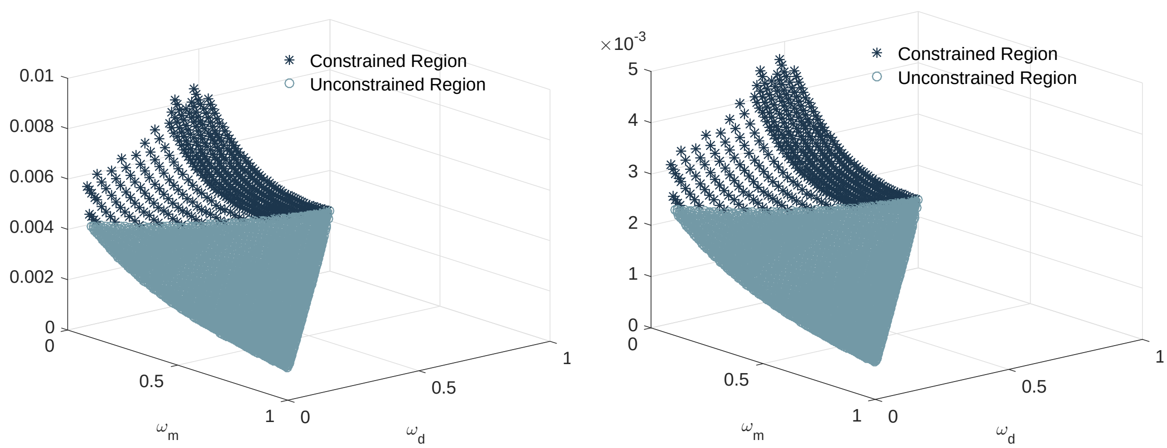
(c) Bond Holdings—Manager

Figure 2. Bond Holdings in the Delegation Economy. Panel (a) plots the household's time-0 bond holdings (number of units), while Panels (b) and (c) plot the time-0 consumption growth volatility (cgv) for the manager and the trading desk, respectively. The light blue circles represent unconstrained states, while the dark blue asterisks show constrained ones. All quantities are plotted against the endogenous state variables, the consumption share of the fund manager, ω_m, and that of the trading desk, ω_d. Parameters are as in Table I.



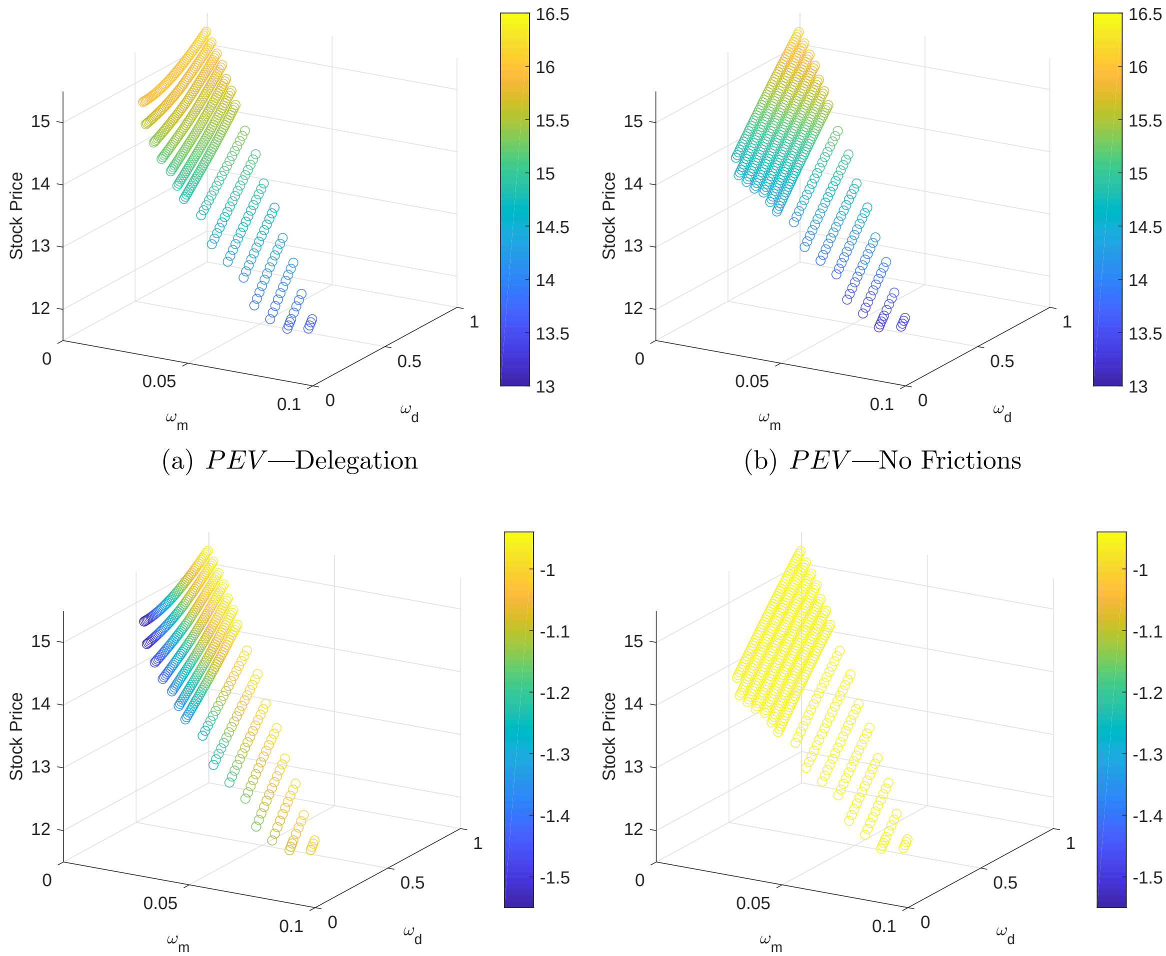
(a) Delegation (b) No Frictions

Figure 3. Initial Interest Rate (%). Panel (a) reports the level of the interest rate in the delegation economy, while Panel (b) reports the same for the no frictions economy. In the delegation economy in Panel (a) the light blue circles represent unconstrained states, while the dark blue asterisks show constrained ones. The rates are plotted against the endogenous state variables, the consumption share of the fund manager, ω_m, and that of the trading desk, ω_d. Parameters are as in Table I.



(a) *cgv*—Manager (b) *cgv*—Dealer

Figure 4. Consumption Growth Volatility. Panels (a) and (b) plot the time-0 consumption growth volatility (cgv) for the manager and the trading desk, respectively. The light blue circles represent unconstrained states, while the dark blue asterisks show constrained ones. All quantities are plotted against the endogenous state variables, the consumption share of the fund manager, ω_m, and that of the trading desk, ω_d. Parameters are as in Table I.



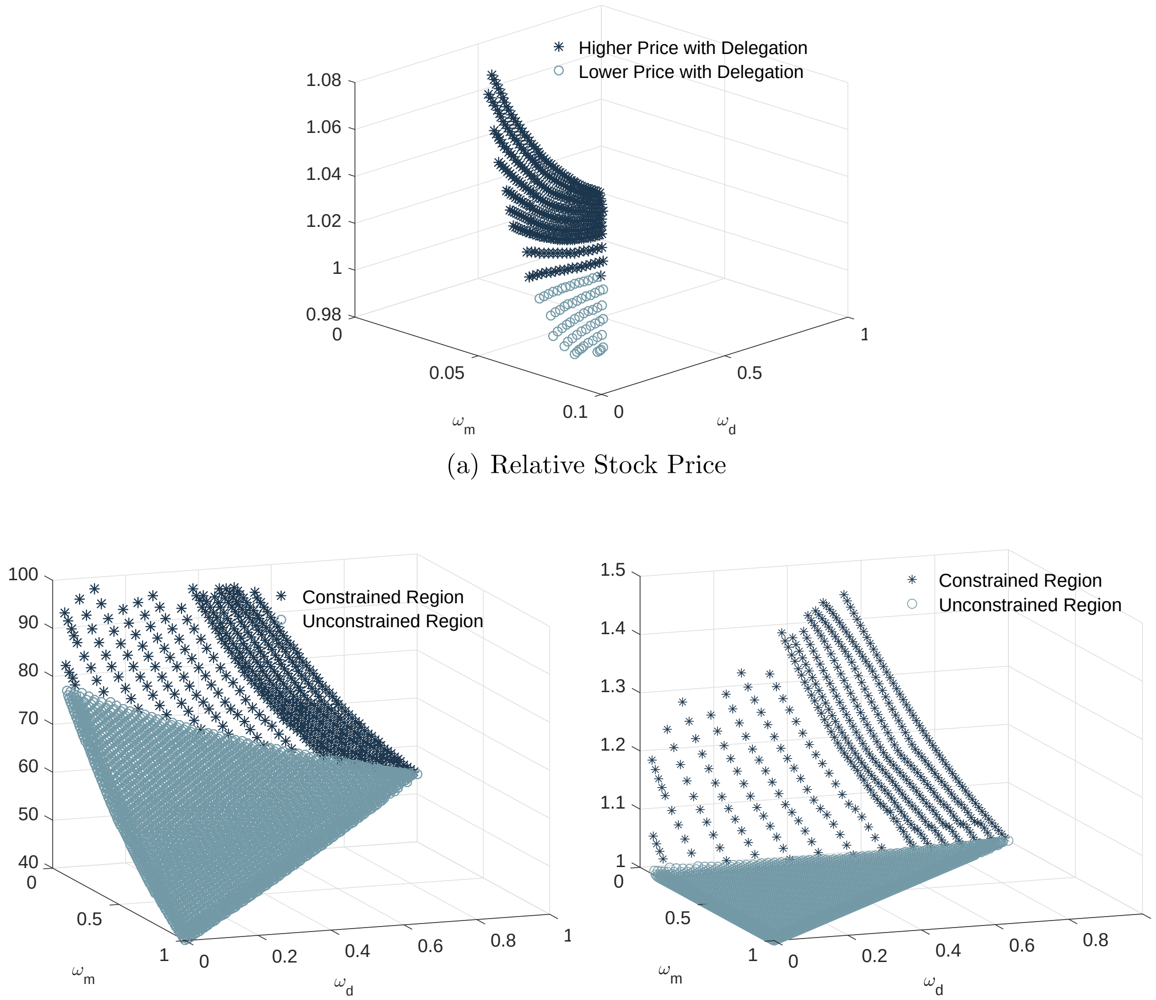
(a) PEV—Delegation

(b) PEV—No Frictions

(c) PRP—Delegation

(d) PRP—No Frictions

Figure 5. Initial Price Decomposition. The figure plots the components of the stock price decomposition in equation (23) against the stock price (on the $z-$axis) for states in the constrained region: the perpetual expected value (PEV) in Panels (a) and (b) and the perpetual risk premium (PRP) in Panels (c) and (d). Parameters are as in Table I.

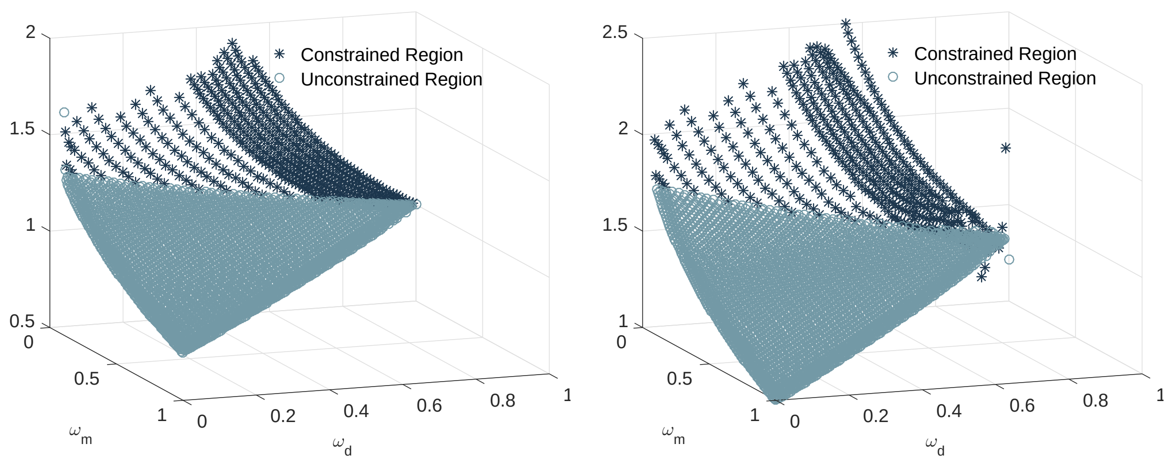


(a) Relative Stock Price

(b) Risk Premium (bp)—Delegation

(c) Relative Risk Premium

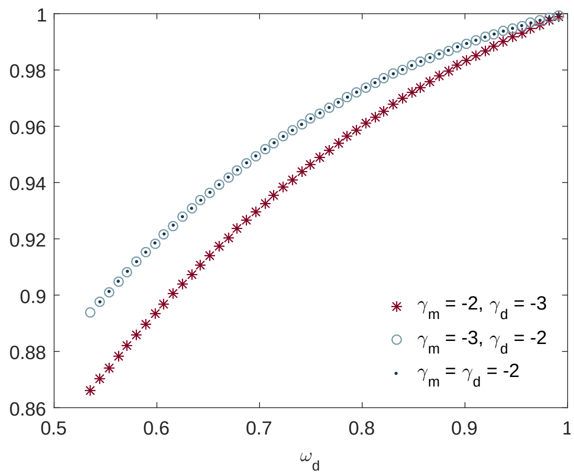
Figure 6. Initial Stock Price and Risk Premium. Panel (a) plots the time $t = 0$ stock price in the constrained region of the delegation economy versus the one in the economy without frictions. The light blue circles represent relative prices lower than 1, while the dark blue asterisks show relative prices that are larger than 1. Panel (b) shows the time $t = 0$ risk premium in the delegation economy (in basis points), while Panel (c) illustrates the same risk premium relative to the one in the no frictions economy. The light blue circles in Panels (b) and (c) represent states in the constrained region, while the dark blue asterisks show unconstrained states. All quantities are plotted against the endogenous state variables, the consumption share of the fund manager, ω_m, and that of the trading desk, ω_d. Parameters are as in Table I.



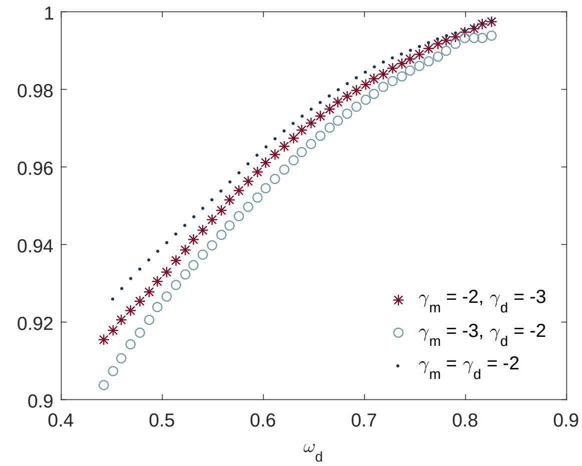
(a) Stock Portfolio Weight, Trading Desk

(b) Stock Portfolio Weight, Manager

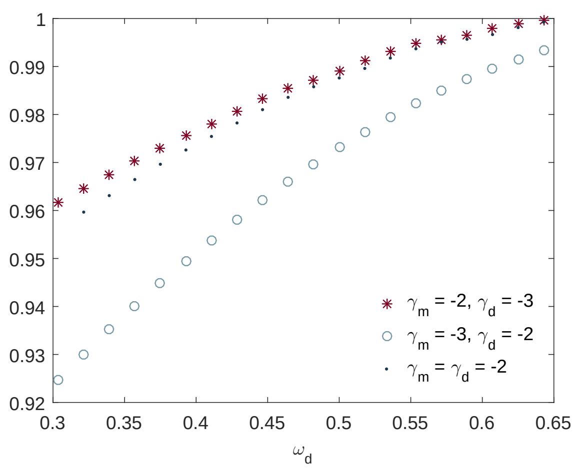
Figure 7. Leverage in the Constrained Region. The figure shows the experts' portfolio weight in the stock. Panel (a) reports leverage for the trading desk, while Pnel (b) for the manager. The light blue circles represent states in the constrained region, while the dark blue asterisks show unconstrained states. All quantities are plotted against the consumption shares of the three agents, ω_m for the fund manager, ω_d for the trading desk, and ω_h for the household. Parameters are as in Table I.



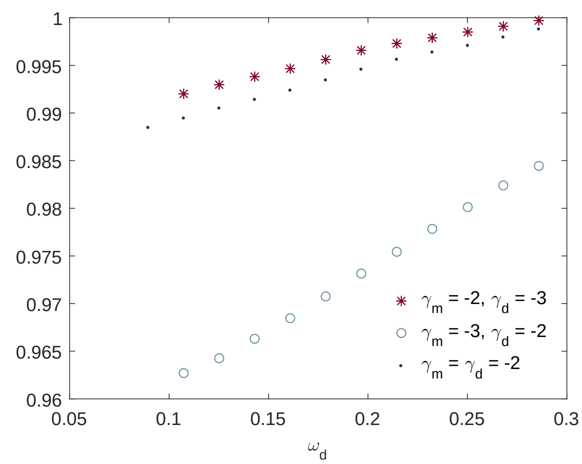
(a) $\omega_m = 0.01\%$



(b) $\omega_m = 1.34\%$

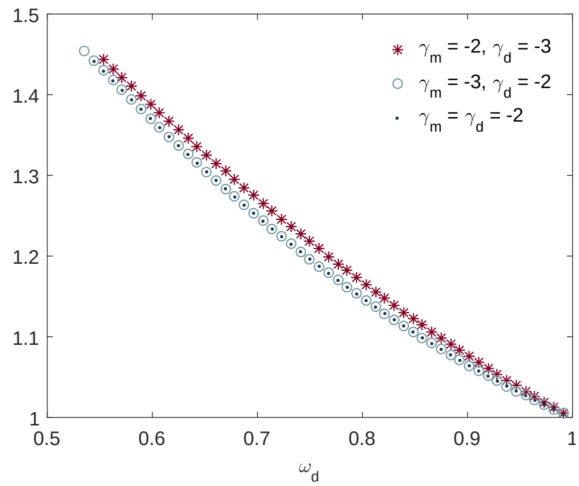


(c) $\omega_m = 3.57\%$

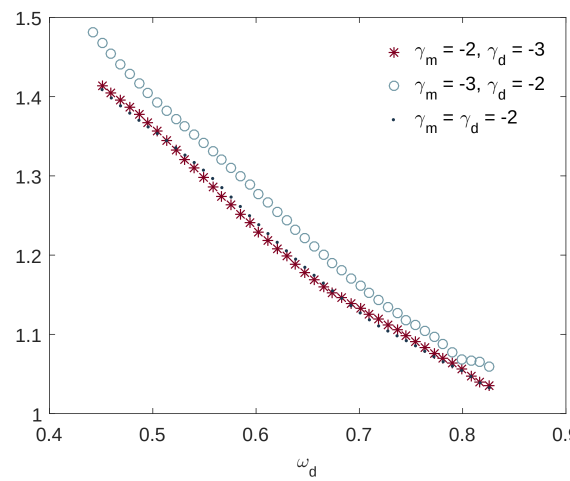


(d) $\omega_m = 7.14\%$

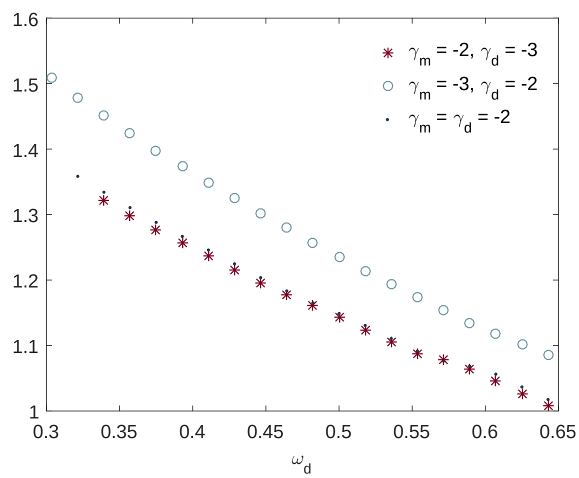
Figure 8. Relative Interest Rate, Sensitivity to Risk Aversion. Each panel depicts the relative interest rate as a function of the consumption share of the trading desk (ω_d), for a fixed consumption share of the manager (ω_m). The relative interest rate is defined as the interest rate corresponding to the delegation economy as a percentage of the one prevailing absent frictions. Parameters are as in Table I.



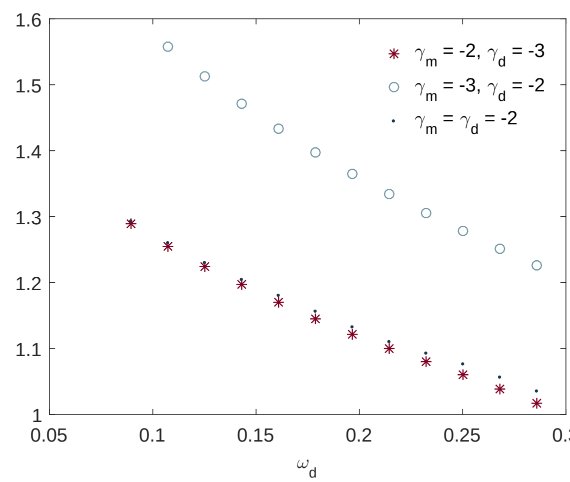
(a) $\omega_m = 0.01\%$



(b) $\omega_m = 1.34\%$



(c) $\omega_m = 3.57\%$



(d) $\omega_m = 7.14\%$

Figure 9. Relative Risk Premium, Sensitivity to Risk Aversion. Each panel depicts the relative risk premium as a function of the consumption share of the trading desk (ω_d), for a fixed consumption share of the manager (ω_m). The relative risk premium is defined as the risk premium corresponding to the delegation economy as a percentage of the one prevailing absent frictions. Parameters are as in Table I.

Table I. Model Parameters.

Panel A: Dividend Process		
Drift	μ_D	0.0183
Standard Deviation	σ_D	0.0357
Probability up move	π	0.5
Panel B: Preferences		
Time-preference, all agents	ρ	0.999
Risk aversion coefficient, trading desk	γ_d	-3
Risk aversion coefficient, household	γ_h	-5
Risk aversion coefficient, fund manager	γ_m	-2
Panel C: Equity Constraint		
Threshold fund shares	m	0.2

Table II. The Interest Rate and the Risk Premium in Constrained States. I report median values for the interest rate (Panel A) and the risk premium (Panel B) for the tree risk aversion parametrizations I consider in the paper. Constrained states are periods where the constraint binds along simulated dividend paths in the delegation economy. I report results for the same states in the economy without frictions (NF, second row in each panel). The third row reports the median difference (state by state) as a percentage of the NF value: a decrease for the risk free rate and an increase for the risk premium. All parameters except the experts' risk aversion coefficients are as in Table I.

	$\gamma_m = -2$, $\gamma_d = -3$ (1)	$\gamma_m = \gamma_d = -2$ (2)	$\gamma_m = -3$, $\gamma_d = -2$ (3)
Panel A: Interest Rate (%)			
Delegation	7.92	7.66	7.66
No Frictions (NF)	7.70	7.69	7.82
Decrease (Vs. NF)	0.45%	0.51%	2.06%
Panel B: Risk Premium (bp)			
Delegation	87.42	86.60	91.32
No Frictions (NF)	75.43	73.82	73.02
Increase (Vs. NF)	16.23%	16.45%	24.83%

Table III. Trading Volume (units). Columns (1) and (2) report the average cumulative trading volume in the short-term bond and the stock. Columns (3)–(6) report per-period relative trading volume by pooling together all periods across all paths. Relative volume is defined as volume in the delegation economy compared to the volume in the no frictions one. Constrained states are periods in which the equity constraint binds, and represent 33 percent of all states (93 707 states out of 280 000 across all paths). The number of states is different from the number of periods × number of paths because I do not account for trades at the initial and terminal dates. Parameters are as in Table I.

	Cumulative Trading Volume		Constrained States		Unconstrained States	
	bond	stock	bond	stock	bond	stock
	(1)	(2)	(3)	(4)	(5)	(6)
No Frictions	2.25	0.14	0.06	0.0030	0.08	0.0070
Delegation	5.92	0.04	0.09	0.0018	0.23	0.0009
Relative	2.62	0.28	1.55	0.54	2.95	0.21

Table IV. Two-way Trades in the Stock (units). Panel A shows median, high (95th percentile), and low (5th percentile) values for bilateral trades as percentage of total stock trading volume in the no frictions economy. A dash reports the absence of bilateral trades between the agents. Panel B shows the same for bilateral trades as percentage of *direct and indirect* stock trading volume in the delegation economy. Indirect trading volume is trade in fund shares between fund investors—Intra-Fund. Direct trades are between the fund and the trading desk. Constrained states are periods in which the equity constraint binds, and represent 33 percent of all states (93 707 states out of 280 000 across all paths). The number of states is different from the number of periods × number of paths because I do not account for trades at the initial and terminal dates. Parameters are as in Table I.

	constrained states			unconstrained states		
	median	high	low	median	high	low
Panel A: No frictions						
Household-Manager	0.75	0.77	0.74	0.78	0.82	0.76
Household-TradingDesk	0.25	0.26	0.23	0.22	0.24	0.18
Manager-TradingDesk	–	–	–	–	–	–
Panel B: Delegation						
Intra-Fund	–	–	–	0.78	0.94	–
Fund-TradingDesk	1	1	1	0.22	1	0.06

Appendix A. Optimization problems: household and trading desk

The Lagrangian corresponding to the household's problem is given by:

$$
\begin{aligned}
\mathcal{L}_h(\cdot,t) \equiv \mathcal{L}_{h,t} = {} & u_h(c_{h,t},t) + \mathbb{E}_t\left[J_{h,t+1}(\cdot)\right] \\
& + \phi_{h,t}\Bigg[x_{h,t-1}\sum_{i=0}^{N}\theta^f_{i,t-1}(P_{i,t}+D_{i,t}) + y_{h,t-1} + \zeta_{h,t} - c_{h,t} \\
& \quad - (x_{h,t-1}+\Delta x_{h,t})\sum_{i=0}^{N}(\theta^f_{i,t-1}+\hat{\theta}^f_{i,t}-\check{\theta}^f_{i,t})P_{i,t} - (x_{h,t-1}+\Delta x_{h,t})\sum_{i=0}^{N}(\hat{\theta}^f_{i,t}+\check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t} \\
& \quad - (y_{h,t-1}+\Delta y_{h,t})P_{0,t}\Bigg],
\end{aligned}
\tag{A1}
$$

with

$$
\zeta_{h,t} = x_{h,t}\sum_{i=1}^{N}(\hat{\theta}^d_{i,t}+\check{\theta}^d_{i,t})P_{i,t}\lambda^d_{i,t}. \tag{A2}
$$

For the trading desk, the Lagrangian is

$$
\begin{aligned}
\mathcal{L}_d(\cdot,t) \equiv \mathcal{L}_{d,t} = {} & u_d(c_{d,t},t) + \mathbb{E}_t\left[J_{d,t+1}(\cdot)\right] \\
& + \phi_{d,t}\Bigg[\sum_{i=0}^{N}\theta^d_{i,t-1}(P_{i,t}+D_{i,t}) + \zeta_{d,t} - c_{d,t} \\
& \quad - \sum_{i=0}^{N}(\theta^d_{i,t-1}+\hat{\theta}^d_{i,t}-\check{\theta}^d_{i,t})P_{i,t} - \sum_{i=0}^{N}(\hat{\theta}^d_{i,t}+\check{\theta}^d_{i,t})P_{i,t}\lambda^d_{i,t}\Bigg] \\
& + \sum_{i=0}^{N}\hat{\nu}^d_{i,t}\hat{\theta}^d_{i,t} + \sum_{i=0}^{N}\check{\nu}^d_{i,t}\check{\theta}^d_{i,t},
\end{aligned}
\tag{A3}
$$

with

$$
\zeta_{d,t} = \sum_{i=1}^{N}(\hat{\theta}^f_{i,t}+\check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t}. \tag{A4}
$$

Appendix B. KKT conditions—fund manager

The Lagrangian:

$$
\begin{aligned}
\mathcal{L}_m(\cdot,t) \equiv \mathcal{L}_{m,t} = {} & u_m(c_{m,t},t) + \mathbb{E}_t\left[J_{m,t+1}(\cdot)\right] \\
& + \phi_{m,t}\Bigg[x_{m,t-1}\sum_{i=0}^{N}\theta^f_{i,t-1}(P_{i,t}+D_{i,t}) + \zeta_{m,t} - c_{m,t} \\
& \quad - (x_{m,t-1}+\Delta x_{m,t})\sum_{i=0}^{N}(\theta^f_{i,t-1}+\hat{\theta}^f_{i,t}-\check{\theta}^f_{i,t})P_{i,t} \\
& \quad - (x_{m,t-1}+\Delta x_{m,t})\sum_{i=0}^{N}(\hat{\theta}^f_{i,t}+\check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t}\Bigg] \\
& + \phi_{m,t}\mu_{m,t}(x_{m,t-1}+\Delta x_{m,t}-m) + \sum_{i=0}^{N}\hat{\nu}^f_{i,t}\hat{\theta}^f_{i,t} + \sum_{i=0}^{N}\check{\nu}^f_{i,t}\check{\theta}^f_{i,t}. && \text{(B1)}
\end{aligned}
$$

The KKT conditions are:

$$
\begin{aligned}
&\frac{\partial\mathcal{L}_{m,t}}{\partial c_{m,t}} : u'_m(c_{m,t},t) = \phi_{m,t}, && \text{(B2)} \\
&\frac{\partial\mathcal{L}_{m,t}}{\partial\Delta x_{m,t}} : \mathbb{E}_t\left[\frac{\partial J_{m,t+1}(\cdot)}{\partial x_{m,t}}\right] = \phi_{m,t}\sum_{i=0}^{N}\theta^f_{i,t}P_{i,t} + \phi_{m,t}\sum_{i=0}^{N}(\hat{\theta}^f_{i,t}+\check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t} \\
&\qquad\qquad - \phi_{m,t}\mu_{m,t}, && \text{(B3)} \\
&\frac{\partial\mathcal{L}_{m,t}}{\partial\hat{\theta}^f_{i,t}} : \mathbb{E}_t\left[\frac{\partial J_{m,t+1}(\cdot)}{\partial\theta^f_{i,t}}\right] = \phi_{m,t}x_{m,t}P_{i,t} + \phi_{m,t}x_{m,t}P_{i,t}\lambda^f_{i,t} - \hat{\nu}^f_{i,t}, && \text{(B4)} \\
&\frac{\partial\mathcal{L}_{m,t}}{\partial\check{\theta}^f_{i,t}} : \mathbb{E}_t\left[\frac{\partial J_{m,t+1}(\cdot)}{\partial\theta^f_{i,t}}\right] = \phi_{m,t}x_{m,t}P_{i,t} - \phi_{m,t}x_{m,t}P_{i,t}\lambda^f_{i,t} + \check{\nu}^f_{i,t}, && \text{(B5)} \\
&\frac{\partial\mathcal{L}}{\partial\phi_{m,t}} : x_{m,t-1}\sum_{i=0}^{N}\theta^f_{i,t-1}(P_{i,t}+D_{i,t}) + \zeta_{m,t} - c_{m,t} \\
&\qquad\qquad - x_{m,t}\sum_{i=0}^{N}\theta^f_{i,t}P_{i,t} - x_{m,t}\sum_{i=0}^{N}(\hat{\theta}^f_{i,t}+\check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t} = 0, && \text{(B6)}
\end{aligned}
$$

$$
\begin{aligned}
&\mu_{m,t} \geq 0, \quad x_{m,t}-m \geq 0, \quad \mu_{m,t}(x_{m,t}-m) = 0, && \text{(B7)} \\
&\hat{\nu}^f_{i,t} \geq 0, \quad \hat{\theta}^f_{i,t} \geq 0, \qquad\qquad \hat{\nu}^f_{i,t}\hat{\theta}^f_{i,t} = 0, && \text{(B8)} \\
&\check{\nu}^f_{i,t} \geq 0, \quad \check{\theta}^f_{i,t} \geq 0, \qquad\qquad \check{\nu}^f_{i,t}\check{\theta}^f_{i,t} = 0. && \text{(B9)}
\end{aligned}
$$

We can re-write (B4) and (B5) as

$$\mathbb{E}_t\left[\frac{\partial J_{m,t+1}(\cdot)}{\partial \theta^f_{i,t}}\right] = x_{m,t}\phi_{m,t}P_{i,t}\left((1+\lambda^f_{i,t}) - \frac{\hat{\nu}^f_{i,t}}{x_{m,t}\phi_{m,t}P_{i,t}}\right) = x_{m,t}\phi_{m,t}R^f_{i,t}P_{i,t}, \tag{B10}$$

$$\mathbb{E}_t\left[\frac{\partial J_{m,t+1}(\cdot)}{\partial \theta^f_{i,t}}\right] = x_{m,t}\phi_{m,t}P_{i,t}\left((1-\lambda^f_{i,t}) + \frac{\check{\nu}^f_{i,t}}{x_{m,t}\phi_{m,t}P_{i,t}}\right) = x_{m,t}\phi_{m,t}R^f_{i,t}P_{i,t}, \tag{B11}$$

with $R^f_{i,t}$ defined by

$$x_{m,t}\phi_{m,t}P_{i,t}R^f_{i,t} = (1+\lambda^f_{i,t})x_{m,t}\phi_{m,t}P_{i,t} - \hat{\nu}^f_{i,t} = (1-\lambda^f_{i,t})x_{m,t}\phi_{m,t}P_{i,t} + \check{\nu}^f_{i,t}. \tag{B12}$$

From the Envelope Theorem, we obtain:

$$\begin{aligned}\frac{\partial J_{m,t}(\cdot)}{\partial x_{m,t-1}} = \frac{\partial \mathcal{L}_{m,t}}{\partial x_{m,t-1}} &= \mathbb{E}_t\left[\frac{\partial J_{m,t+1}}{\partial x_{m,t}}\right] + \phi_{m,t}\sum_{i=0}^{N}\left(\theta^f_{i,t-1}(P_{i,t}+D_{i,t}) - \theta^f_{i,t}P_{i,t}\right)\\ &- \phi_{m,t}\sum_{i=0}^{N}(\hat{\theta}^f_{i,t}+\check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t} + \phi_{m,t}\mu_{m,t},\end{aligned} \tag{B13}$$

$$\begin{aligned}\frac{\partial J_{m,t}(\cdot)}{\partial \theta^f_{i,t-1}} = \frac{\partial \mathcal{L}_m(\cdot,t)}{\partial \theta^f_{i,t-1}} &= \mathbb{E}_t\left[\frac{\partial J_{m,t+1}(\cdot)}{\partial \theta^f_{i,t}}\right] + \phi_{m,t}x_{m,t-1}(P_{i,t}+D_{i,t})\\ &- \phi_{m,t}(x_{m,t-1}+\Delta x_{m,t})P_{i,t}.\end{aligned} \tag{B14}$$

Using (B3) in (B13) we obtain

$$\frac{\partial J_{m,t}(\cdot)}{\partial x_{m,t-1}} = \phi_{m,t}\sum_{i=0}^{N}\theta^f_{i,t-1}(P_{i,t}+D_{i,t}). \tag{B15}$$

Finally, from (B10) (or, equivalently (B11)) and (B14)

$$\frac{\partial J_{m,t}(\cdot)}{\partial \theta^f_{i,t-1}} = \phi_{m,t}\left[(x_{m,t}R^f_{i,t} - \Delta x_{m,t})P_{i,t} + x_{m,t-1}D_{i,t}\right]. \tag{B16}$$

Consider the fund's investment in the bond. Substituting $\lambda^f_{0,t} = 0$ in (B12) we obtain

$$R^f_{0,t} = 1 - \frac{\hat{\nu}^f_{0,t}}{x_{m,t}\phi_{m,t}P_{0,t}} = 1 + \frac{\check{\nu}^f_{0,t}}{x_{m,t}\phi_{m,t}P_{0,t}} \iff R^f_{0,t} = 1 \quad \text{and} \quad \hat{\nu}^f_{0,t} = \check{\nu}^f_{0,t} = 0. \tag{B17}$$

As a consequence, $\hat{\theta}^f_{0,t} > 0$, $\check{\theta}^f_{0,t} > 0$, and the KKT conditions (B10) and (B11) for the bond can be written as:

$$\frac{\partial \mathcal{L}_{m,t}}{\partial \hat{\theta}^f_{0,t}} : \mathbb{E}_t \left[\frac{\partial J_{m,t+1}(\cdot)}{\partial \theta^f_{0,t}} \right] = x_{m,t}\phi_{m,t}P_{0,t}, \tag{B18}$$

$$\frac{\partial \mathcal{L}_{m,t}}{\partial \check{\theta}^f_{0,t}} : \mathbb{E}_t \left[\frac{\partial J_{m,t+1}(\cdot)}{\partial \theta^f_{0,t}} \right] = x_{m,t}\phi_{m,t}P_{0,t}. \tag{B19}$$

Given that the bond is a short-term asset that pays one unit of consumption good at maturity, (B16) for the bond becomes:

$$\begin{aligned}\frac{\partial J_{m,t}(\cdot)}{\partial \theta^f_{0,t-1}} = \frac{\partial \mathcal{L}_m(\cdot,t)}{\partial \theta^f_{0,t-1}} &= \mathbb{E}_t \left[\frac{\partial J_{m,t+1}(\cdot)}{\partial \theta^f_{0,t}} \right] + \phi_{m,t}x_{m,t-1} - \phi_{m,t}(x_{m,t-1} + \Delta x_{m,t})P_{0,t} \\ &= x_{m,t-1}\phi_{m,t}.\end{aligned} \tag{B20}$$

We can now re-write the fund manager's KKT conditions wrt asset holdings:

$$\mathbb{E}_t \left[\phi_{m,t+1} \sum_{i=0}^{N} \theta^f_{i,t}(P_{i,t+1} + D_{i,t+1}) \right] = \phi_{m,t} \sum_{i=0}^{N} \left(\theta^f_{i,t}P_{i,t} + (\hat{\theta}^f_{i,t} + \check{\theta}^f_{i,t})P_{i,t}\lambda^f_{i,t} \right) - \phi_{m,t}\mu_{m,t}, \tag{B21}$$

$$\mathbb{E}_t \left[\phi_{m,t+1} \right] = \phi_{m,t}P_{0,t}, \tag{B22}$$

$$\mathbb{E}_t \left[\phi_{m,t+1} \left((x_{m,t+1}R^f_{i,t+1} - \Delta x_{m,t+1})P_{i,t+1} + x_{m,t}D_{i,t+1} \right) \right] = x_{m,t}\phi_{m,t}R^f_{i,t}P_{i,t}, \tag{B23}$$

keeping in mind that $P_{0,t+1} + D_{0,t+1} = 1$.

Appendix C. Recursive System

Let K_t denote the number of nodes η that immediately succeed the current node (ξ, t): $\eta = 1, \ldots, K_t$. The system of equations for node (ξ, t) (unless otherwise specified, written for $k \in \{h, m\}$ and $j \in \{d, f\}$) is given by:

1. time $t+1$ consumption and state prices ($3 \times K_t$ equations, 3 being the number of agents):
$$u'_k(c_{k,t+1,\eta}, t) = \phi_{k,t+1,\eta}, \quad k \in \{m, h, d\} \tag{C1}$$
2. time $t+1$ flow budget constraints ($3 \times K_t$ equations):

$$c_{m,t+1,\eta} + x_{m,t+1,\eta} \sum_{i=0}^{N} (\theta^f_{i,t+1,\eta} - \theta^f_{i,t})P_{i,t+1,\eta}R^f_{i,t+1,\eta} + \Delta x_{m,t+1,\eta} \sum_{i=0}^{N} \theta^f_{i,t}P_{i,t+1,\eta},$$

$$= x_{m,t} \sum_{i=0}^{N} \theta^f_{i,t} D_{i,t+1,\eta} + \zeta_{m,t+1,\eta}, \tag{C2}$$

$$c_{h,t+1,\eta} + x_{h,t+1,\eta} \sum_{i=0}^{N} (\theta^f_{i,t+1,\eta} - \theta^f_{i,t}) P_{i,t+1,\eta} R^f_{i,t+1,\eta} + \Delta x_{h,t+1,\eta} \sum_{i=0}^{N} \theta^f_{i,t} P_{i,t+1,\eta}$$

$$+ y_{h,t+1,\eta} P_{0,t+1,\eta} = x_{h,t} \sum_{i=0}^{N} \theta^f_{i,t} D_{i,t+1,\eta} + y_{h,t} + \zeta_{h,t+1,\eta}, \tag{C3}$$

$$c_{d,t+1,\eta} + \sum_{i=1}^{N} \left(\theta^d_{i,t+1,\eta} - \theta^d_{i,t}\right) P_{i,t+1,\eta} R^d_{i,t+1,\eta} + \theta^d_{0,t+1,\eta} P_{0,t+1,\eta}$$

$$= \sum_{i=1}^{N} \theta^d_{i,t} D_{i,t+1,\eta} + \theta^d_{0,t} + \zeta_{d,t+1,\eta}, \tag{C4}$$

with

$$\zeta_{k,t+1,\eta} = x_{k,t+1,\eta} \sum_{i=1}^{N} (\hat{\theta}^d_{i,t+1,\eta} + \check{\theta}^d_{i,t+1,\eta}) P_{i,t+1,\eta} \lambda^d_{i,t+1,\eta}, \quad k \in \{h, m\}, \tag{C5}$$

$$\zeta_{d,t+1,\eta} = \sum_{i=1}^{N} (\hat{\theta}^f_{i,t+1,\eta} + \check{\theta}^f_{i,t+1,\eta}) P_{i,t+1,\eta} \lambda^f_{i,t+1,\eta}. \tag{C6}$$

3. time t kernel conditions ($1 + N + 2$ equations):

$$\mathbb{E}_t \left[\frac{\phi_{m,t+1}}{\phi_{m,t}} \left(\sum_{i=0}^{N} \theta^f_{i,t} (P_{i,t+1} + D_{i,t+1}) \right) + \mu_{m,t} \right]$$

$$= \mathbb{E}_t \left[\frac{\phi_{h,t+1}}{\phi_{h,t}} \sum_{i=0}^{N} \theta^f_{i,t} (P_{i,t+1} + D_{i,t+1}) \right], \tag{C7}$$

$$\mathbb{E}_t \left[\frac{\phi_{m,t+1}}{\phi_{m,t}} \frac{1}{R^f_{i,t}} \left(\left(R^f_{i,t+1} - (1 - R^f_{i,t+1}) \frac{\Delta x_{m,t+1}}{x_{m,t}} \right) P_{i,t+1} + D_{i,t+1} \right) \right]$$

$$= \mathbb{E}_t \left[\frac{\phi_{d,t+1}}{\phi_{d,t}} \frac{1}{R^d_{i,t}} \left(R^d_{i,t+1} P_{i,t+1} + D_{i,t+1} \right) \right], \tag{C8}$$

$$\mathbb{E}_t \left[\frac{\phi_{k,t+1}}{\phi_{k,t}} \right] = \mathbb{E}_{d,t} \left[\frac{\phi_{d,t+1}}{\phi_{d,t}} \right]. \tag{C9}$$

4. Definitions ($K_t + 2 \times N \times K_t$ equations):[15]

$$x_{m,t+1,\eta} = x_{m,t} + \Delta x_{m,t+1,\eta}, \tag{C10}$$

$$\theta^j_{i,t+1,\eta} = \theta^j_{i,t} + \hat{\theta}^j_{i,t+1,\eta} - \check{\theta}^j_{i,t+1,\eta}, \quad i = 1, \ldots, N. \tag{C11}$$

5. Complementary-slackness conditions ($1 + 2 \times (2 \times N \times K_t)$ equations):

$$\mu_{m,t}(x_{m,t} - m) = 0, \tag{C12}$$

$$\left((1 + \lambda^j_{i,t+1,\eta}) - R^j_{i,t+1,\eta}\right) \hat{\theta}^j_{i,t+1,\eta} = 0, \tag{C13}$$

$$\left(R^j_{i,t+1,\eta} - (1 - \lambda^j_{i,t+1,\eta})\right) \check{\theta}^j_{i,t+1,\eta} = 0. \tag{C14}$$

6. market clearing restrictions ($N + 1 + 1 + K_t$ equations):

$$\theta^f_{i,t} + \theta^d_{i,t} = 1, \tag{C15}$$

$$x_{h,t} + x_{m,t} = 1, \tag{C16}$$

$$y_{h,t} + y_{m,t} + \theta^d_{0,t} = 0, \tag{C17}$$

7. Inequalities:

$$\mu_{m,t} \geq 0, \tag{C18}$$

$$x_{m,t} - m \geq 0 \tag{C19}$$

$$1 - \lambda^j_{i,t+1,\eta} \leq R^j_{i,t+1,\eta} \leq 1 + \lambda^j_{i,t+1,\eta}, \quad \hat{\theta}^j_{i,t+1,\eta} \geq 0, \quad \check{\theta}^j_{i,t+1,\eta} \geq 0. \tag{C20}$$

The unknowns for the above system of equations are:

Unknown(s)	Notation	#
future consumption	$c_{k,t+1,\eta},\ k \in \{m,h,d\}$	$3 \times K_t$
future state prices	$\phi_{k,t+1,\eta},\ k \in \{m,h,d\}$	$3 \times K_t$
current holdings of fund shares	$x_{k,t},\ k \in \{m,h\}$	2
current holdings of bond	$y_{h,t};\ \theta^j_{0,t},\ j \in \{d,m\}$	3
current holdings of risky asset(s)	$\theta^j_{i,t}$	$2 \times N$
future trades in fund shares	$\Delta x_{m,t+1,\eta}$	K_t
future trades in risky assets	$\hat{\theta}^j_{i,t+1,\eta}$ & $\check{\theta}^j_{i,t+1,\eta}$	$2 \times 2 \times N \times K_t$
future shadow prices of securities	$R^j_{i,t+1,\eta}$	$2 \times N \times K_t$
current shadow price of the equity constraint	$\mu_{m,t}$	1

[15] We can eliminate $y_{h,t+1,\eta} = y_{h,t} + \Delta y_{h,t+1,\eta}$ and $x_{h,t+1,\eta} = x_{h,t} + \Delta x_{h,t+1,\eta}$, since Δy_h and Δx_h do not appear separately anywhere else in the system.

The future prices/private valuations, asset holdings, and holdings of fund shares ($P_{0,t+1\eta}$, $R^d_{1,t+1\eta}P_{1,t+1\eta}$, $\theta^j_{i,t+1,\eta}$, $y_{k,t+1,\eta}$, and $x_{k,t+1,\eta}$) are carried backward, from the solution of the system at the previous nodes, time $t+1$, through interpolation.

Appendix D. Scale Invariance

At time T asset holdings and prices are equal to zero. The *terminal conditions* are:

$$\begin{aligned} x_{k,T} &= 0, \quad k \in \{h, m\}, \\ y_{h,T} &= 0, \\ \theta^j_{1,T} &= 0, \quad j \in \{d, f\}, \\ \theta^d_{0,T} &= 0, \\ P_{i,T} &= 0, \quad i = 0, 1. \end{aligned} \tag{D1}$$

I show now that with $N = 1$ and a fund portfolio composed of both the risk-free and the risky asset the model satisfies the scale invariance property (i.e., the system of equations can be solved independently of the value of the current dividend—the exogenous state variable).

I make the simplifying assumption that the proportional trading fees are time and state invariant: λ^j_1, with $j \in \{d, f\}$. I also assume that there are two future states of nature, in that the dividend can either go up (u or 1) or go down (d or 2) and I use the following notation for dividend *growth*: $r_\eta \equiv \frac{D_{1,t+1,\eta}}{D_{1,t}}$, with $r_\eta \in \{u, d\}$. Letting $\omega_{k,t} \equiv \frac{c_{k,t}}{D_{1,t}}$ be the consumption share of agent k, with $k \in \{d, h, m\}$, I re-write consumption as:

$$c_{k,t} = \omega_{k,t} D_{1,t}. \tag{D2}$$

With CRRA preferences the individual state prices are

$$\phi_{k,t} = u'(c_{k,t}, t) = \rho^t c^{\gamma_k - 1}_{k,t} = \rho^t (\omega_{k,t} D_{1,t})^{\gamma_k - 1}. \tag{D3}$$

Time $T-1$

To derive the system of equations independent of the dividend value, we start by substituting for consumption using consumption shares. As a result, we eliminate the first equation in the system of Appendix C.

We can write the $T-1$ flow budget constraints for fund investor k as:

$$\omega_{k,T,1} \times D_{1,T-1,1} \times u = x_{k,T-1}\left(\theta^f_{1,T-1} \times D_{1,T-1,1} \times u + \theta^f_{0,T-1}\right) + y_{k,T-1}, \tag{D4}$$

$$\omega_{k,T,2} \times D_{1,T-1,2} \times d = x_{k,T-1}\left(\theta^f_{1,T-1} \times D_{1,T-1,2} \times d + \theta^f_{0,T-1}\right) + y_{k,T-1}, \tag{D5}$$

keeping in mind that $y_{m,t} = 0,\ \forall t$.
Taking the difference for each investor k we get

$$x_{k,T-1}\theta^f_{1,T-1} = \frac{1}{u-d}(u \times \omega_{k,T,1} - d \times \omega_{k,T,2}). \tag{D6}$$

Summing over $k \in \{m, h\}$ and using the market clearing condition for fund shares we obtain the fund's holdings of the risky asset,

$$\theta^f_{1,T-1} = \frac{1}{u-d}\left(u \times (\omega_{m,T,1} + \omega_{h,T,1}) - d \times (\omega_{m,T,2} + \omega_{h,T,2})\right). \tag{D7}$$

Further, knowing the manager's holdings of the stock, (D6) for $k = m$, we can obtain an expression for the fund's scaled bond holdings from (D5):

$$\frac{\theta^f_{0,T-1}}{D_{1,T-1}} \equiv \bar{\theta}^f_{0,T-1} = \frac{d}{x_{m,T-1}}(\omega_{m,T,2} - x_{m,T-1}\theta^f_{1,T-1}). \tag{D8}$$

Finally, from (D5), using the above results, we obtain the household's *direct* scaled bond holdings:

$$\bar{y}_{h,T-1} \equiv \frac{y_{h,T-1}}{D_{1,T-1}} = d \times (\omega_{h,T,2} - x_{h,T-1}\theta^f_{1,T-1}) - x_{h,T-1}\bar{\theta}^f_{0,T-1}, \tag{D9}$$

with

$$x_{h,T-1} = 1 - x_{m,T-1}. \tag{D10}$$

Following similar steps, from the trading desk's flow budget constraints we obtain:

$$\theta^d_{1,T-1} = \frac{1}{u-d}\left(u \times \omega_{d,T,1} - d \times \omega_{d,T,2}\right), \tag{D11}$$

and

$$\bar{\theta}^d_{0,T-1} \equiv \frac{\theta^d_{0,T-1}}{D_{1,T-1}} = \frac{u \times d}{u-d}(\omega_{d,T,2} - \omega_{d,T,1}). \tag{D12}$$

Before moving on to the kernel conditions, let us fix some notation. We denote agent k's probability of future state η, $\eta = 1, 2$, by $\pi^k_{t,t+1,\eta}$, and the scaled shadow price of the portfolio constraint by $\bar{\mu}_{m,t} \equiv \mu_{m,t}/\left(\rho \times D_{1,t}\right)$.

Knowing $\theta^f_{1,T-1}$ (equation (D7)), substituting for future consumption and for the shadow prices μ_k, and writing future dividends in terms of current ones and growth rates, we can re-write the **fund kernel condition** as

$$\sum_{\eta=1}^{2}\left(\pi^m_{T-1,T,\eta}\left(\frac{\omega_{m,T,\eta}}{\omega_{m,T-1}}\right)^{\gamma_m-1} r_\eta^{\gamma_m-1} - \pi^h_{T-1,T,\eta}\left(\frac{\omega_{h,T,\eta}}{\omega_{h,T-1}}\right)^{\gamma_h-1} r_\eta^{\gamma_h-1}\right)$$
$$\times\left(\theta^f_{1,T-1}\times D_{1,T-1}\times r_\eta + \bar{\theta}^f_{0,T-1}\right) + \bar{\mu}_{m,T-1} = 0. \tag{D13}$$

The kernel conditions for the **risky asset** and the **bond** are

$$\sum_{\eta=1}^{2}\pi^m_{T-1,T,\eta}\frac{R^d_{1,T-1}}{R^f_{1,T-1}}\left(\frac{\omega_{m,T,\eta}}{\omega_{m,T-1}}\right)^{\gamma_m-1} r_\eta^{\gamma_m} - \pi^d_{T-1,T,\eta}\left(\frac{\omega_{d,T,\eta}}{\omega_{d,T-1}}\right)^{\gamma_d-1} r_\eta^{\gamma_d} = 0 \tag{D14}$$

and

$$\sum_{\eta=1}^{2}\pi^k_{T-1,T,\eta}\left(\frac{\omega_{k,T,\eta}}{\omega_{k,T-1}}\right)^{\gamma_k-1} r_\eta^{\gamma_k-1} - \pi^d_{T-1,T,\eta}\left(\frac{\omega_{d,T,\eta}}{\omega_{d,T-1}}\right)^{\gamma_d-1} r_\eta^{\gamma_d-1} = 0, \quad k\in\{m,h\}, \tag{D15}$$

respectively.

The system of equations corresponding to the $T-1$ node is reduced to (D6) for $k=m$ (1 equation) and the kernel conditions (4 equations) written above and is completed by the market clearing conditions for the risky and the risk-free assets, the complementary slackness condition associated with the fund manager's portfolio constraint, and the associated inequalities:

$$\theta^f_{1,T-1} + \theta^d_{1,T-1} = 1, \tag{D16}$$
$$\bar{\theta}^f_{0,T-1} + \bar{\theta}^d_{1,T-1} + \bar{y}_{h,T-1} = 0, \tag{D17}$$
$$\bar{\mu}_{m,T-1}(x_{m,T-1} - m) = 0, \tag{D18}$$
$$\bar{\mu}_{m,t} \geq 0,$$
$$x_{m,t} \geq m. \tag{D19}$$

The state variables are the current consumption shares $w_{k,T-1}$, $k\in\{d,m\}$ and the ratio of current shadow prices, $\dfrac{R^d_{1,T-1}}{R^f_{1,T-1}}$. The unknowns that we solve for are

- future consumption shares, $\omega_{k,T,\eta}$, $k\in\{d,h,m\}$, $\eta=1,2$;
- manager's current holdings of fund shares, $x_{m,T-1}$;
- the scaled multiplier associated with the portfolio constraint, $\bar{\mu}_{m,T-1}$.

Once we solve the system we can compute for each current node, $(T-1,\xi)$, (1) the fund's and trading desk's holdings of the risky assets (from (D7) and (D11)), and using x_m and x_h, those of individual fund investors, (2) the agents' bond holdings, and (3) the agents' future consumption. Bond holdings and future consumption depend on the dividend. Bond holdings are obtained from (D8) and (D9) using x_m and x_h for the fund investors, and from (D12) for the trading desk, by multiplying by the current dividend $D_{1,T-1,\xi}$. Similarly, future consumption is obtained by multiplying consumption shares by the future dividends.

The current bond price is independent of current and future dividends and is given by:

$$P_{0,T-1} = \rho \sum_{\eta=1}^{2} \pi^{k}_{T-1,T,\eta} \left(\frac{\omega_{k,T,\eta}}{\omega_{k,T-1}} \right)^{\gamma_k - 1} r_\eta^{\gamma_k - 1}. \tag{D20}$$

The stock price depends on the current dividend, but we can work with scaled prices, or in the case of non-zero trading fees, with scaled private valuations of the dividends. The price is

$$P_{1,T-1} = \rho \frac{1}{R^d_{1,T-1}} \sum_{\eta=1}^{2} \pi^{d}_{T-1,T,\eta} \left(\frac{\omega_{d,T,\eta}}{\omega_{d,T-1}} \right)^{\gamma_d - 1} r_\eta^{\gamma_d} D_{1,T-1}. \tag{D21}$$

Denoting by $\bar{P}_{1,T-1} \equiv \frac{P_{1,T-1}}{D_{1,T-1}}$ the scaled stock price, we obtain at each node the trading desk's scaled private valuation of dividends, $\breve{P}_{1,T-1} \equiv R^d_{1,T-1}\bar{P}_{1,T-1}$:

$$\breve{P}_{1,T-1} = \rho \sum_{\eta=1}^{2} \pi^{d}_{T-1,T,\eta} \left(\frac{\omega_{d,T,\eta}}{\omega_{d,T-1}} \right)^{\gamma_d - 1} r_\eta^{\gamma_d}. \tag{D22}$$

Time $t < T - 1$

From the $t+1$ node, solved at the previous step, we carry backward:

- the trading desk's scaled future private valuation of the stock, $\breve{P}_{1,t+1,\eta} \equiv \frac{P_{1,t+1,\eta} R^d_{1,t+1,\eta}}{D_{1,t} \times r_\eta}$
- the future bond price, $P_{0,t+1,\eta}$
- future stock holdings, $\theta^j_{1,t+1,\eta}$, $j \in \{d, f\}$
- future scaled bond holdings, $\bar{y}_{h,t+1,\eta}$ and $\bar{\theta}^j_{0,t+1,\eta}$, $j \in \{d, f\}$, and
- future holdings of fund shares, $x_{k,t+1,\eta}$, $k \in \{h, m\}$.

That is, we interpolate all of the above from the point by point quantities we solved for at the previous step. In fact we can drop the trading desk's holdings and the households number of fund shares, since they can be recovered from market clearing conditions.

In what follows we substitute for future stock prices, future bond holdings, current stock holdings, current bond holdings, and no-short-sales shadow prices in the system of equations from Appendix C using:

$$
\begin{aligned}
P_{1,t+1,\eta} &= \frac{\breve{P}_{1,t+1,\eta} D_{1,t} r_\eta}{R^d_{1,t+1,\eta}},\\
y_{h,t+1,\eta} &= \bar{y}_{h,t+1,\eta} D_{1,t} r_\eta,\\
\theta^j_{0,t+1,\eta} &= \bar{\theta}^j_{0,t+1,\eta} D_{1,t} r_\eta,\\
\theta^j_{0,t} &= \bar{\theta}^j_{0,t} D_{1,t},\\
y_{h,t} &= \bar{y}_{h,t} D_{1,t},\\
\mu_{m,t} &= \rho D_{1,t} \bar{\mu}_{m,t}.
\end{aligned}
$$

From the flow budget constraints and using the market clearing condition for fund shares, after some algebra, we obtain:

$$
x_{m,t}\theta^f_{1,t}\left(u\left(\frac{\breve{P}_{1,t+1,1}}{R^d_{1,t+1,1}}+1\right)-d\left(\frac{\breve{P}_{1,t+1,2}}{R^d_{1,t+1,2}}+1\right)\right) = u\times\Gamma_{m,t+1,1} - d\times\Gamma_{m,t+1,2} \tag{D23}
$$

and

$$
\begin{aligned}
\theta^f_{1,t} &= \frac{u\left(\Gamma_{h,t+1,1}+\Gamma_{m,t+1,1}\right) - d\left(\Gamma_{h,t+1,2}+\Gamma_{m,t+1,2}\right)}{u\left(\frac{\breve{P}_{1,t+1,1}}{R^d_{1,t+1,1}}+1\right)-d\left(\frac{\breve{P}_{1,t+1,2}}{R^d_{1,t+1,2}}+1\right)}, && \text{(D24)}\\
\theta^d_{1,t} &= \frac{u\times\Gamma_{d,t+1,1} - d\times\Gamma_{h,t+1,2}}{u\left(\frac{\breve{P}_{1,t+1,1}}{R^d_{1,t+1,1}}+1\right)-d\left(\frac{\breve{P}_{1,t+1,2}}{R^d_{1,t+1,2}}+1\right)}, && \text{(D25)}\\
\bar{\theta}^d_{0,t} &= d\times\left(\Gamma_{d,t+1,2} - \left(\frac{\breve{P}_{1,t+1,2}}{R^d_{1,t+1,2}}+1\right)\times\theta^d_{1,t}\right), && \text{(D26)}\\
\bar{\theta}^f_{0,t} &= \frac{d}{x_{m,t}}\times\left(\Gamma_{m,t+1,2} - x_{m,t}\theta^f_{1,t}\left(\frac{\breve{P}_{1,t+1,2}}{R^d_{1,t+1,2}}+1\right)\right), && \text{(D27)}\\
x_{h,t} &= 1 - x_{m,t}, && \text{(D28)}\\
\bar{y}_{h,t} &= d\times\Gamma_{h,t+1,\eta} - x_{h,t}\left(\theta^f_{1,t}\left(\frac{\breve{P}_{1,t+1,2}}{R^d_{1,t+1,2}}+1\right)\times d + \bar{\theta}^f_{0,t}\right), && \text{(D29)}
\end{aligned}
$$

where

$$
\begin{aligned}
\Gamma_{k,t+1,\eta} = {} & \omega_{k,t+1,\eta} + x_{k,t+1,\eta}\bar{\theta}^f_{0,t+1,\eta} P_{0,t+1,\eta}\\
& + x_{k,t+1,\eta}\frac{\breve{P}_{1,t+1,\eta}}{R^d_{1,t+1,\eta}}\left(\theta^f_{1,t+1,\eta} + \lambda^f\left(\hat{\theta}^f_{1,t+1,\eta}+\breve{\theta}^f_{1,t+1,\eta}\right) - \lambda^d\left(\hat{\theta}^d_{1,t+1,\eta}+\breve{\theta}^d_{1,t+1,\eta}\right)\right)\\
& + \bar{y}_{k,t+1,\eta} P_{0,t+1,\eta}, \quad k\in\{h,m\}, \quad y_{m,t+1,\eta}=0, \forall\ \eta,
\end{aligned} \tag{D30}
$$

$$\Gamma_{d,t+1,\eta} = \omega_{d,t+1,\eta} + \frac{\breve{P}_{1,t+1,\eta}}{R^d_{1,t+1,\eta}} \left(\theta^d_{1,t+1,\eta} + \lambda^d \left(\hat{\theta}^d_{1,t+1,\eta} + \breve{\theta}^d_{1,t+1,\eta}\right) - \lambda^f \left(\hat{\theta}^f_{1,t+1,\eta} + \breve{\theta}^f_{1,t+1,\eta}\right)\right)$$
$$+ \bar{\theta}^d_{0,t+1,\eta} P_{0,t+1,\eta}. \tag{D31}$$

The system of equations that we solve at each node is given by

- equation (D23)
- kernel conditions (1+1+2 equations):

$$\sum_{\eta=1}^{2} \left(\theta^f_{1,t} r_\eta \left(\frac{\breve{P}_{1,t+1,\eta}}{R^d_{1,t+1,\eta}} + 1\right) + \bar{\theta}^f_{0,t}\right) \left(\pi^m_{t,t+1,\eta} \left(\frac{\omega_{m,t+1,\eta}}{\omega_{m,t}} r_\eta\right)^{\gamma_m - 1} - \pi^h_{t,t+1,\eta} \left(\frac{\omega_{h,t+1,\eta}}{\omega_{h,t}} r_\eta\right)^{\gamma_h - 1}\right)$$
$$+ \bar{\mu}_{m,t} = 0, \tag{D32}$$

$$\frac{R^d_{1,t}}{R^f_{1,t}} \sum_{\eta=1}^{2} \pi^m_{t,t+1,\eta} \left(\frac{\omega_{m,t+1,\eta}}{\omega_{m,t}}\right)^{\gamma_m - 1} r_\eta^{\gamma_m} \left(\left(R^f_{1,t+1,\eta} - (1 - R^f_{1,t+1,\eta}) \frac{\Delta x_{m,t+1,\eta}}{x_{m,t}}\right) \frac{\breve{P}_{1,t+1,\eta}}{R^d_{1,t+1,\eta}} + 1\right)$$
$$- \sum_{\eta=1}^{2} \pi^d_{t,t+1,\eta} \left(\frac{\omega_{d,t+1,\eta}}{\omega_{d,t}}\right)^{\gamma_d - 1} r_\eta^{\gamma_d} \left(\breve{P}_{1,t+1,\eta} + 1\right) = 0, \tag{D33}$$

$$\sum_{\eta=1}^{2} \left(\pi^k_{t,t+1,\eta} \left(\frac{\omega_{k,t+1,\eta}}{\omega_{k,t}}\right)^{\gamma_k - 1} r_\eta^{\gamma_k - 1} - \pi^d_{t,t+1,\eta} \left(\frac{\omega_{d,t+1,\eta}}{\omega_{d,t}}\right)^{\gamma_d - 1} r_\eta^{\gamma_d - 1}\right) = 0, \quad k \in \{h, m\}. \tag{D34}$$

- definitions (2+4 equations)

$$x_{m,t+1,\eta} = x_{m,t} + \Delta x_{m,t+1,\eta}, \quad \eta = 1, 2, \tag{D35}$$
$$\theta^j_{1,t+1,\eta} = \theta^j_{1,t} + \hat{\theta}^j_{1,t+1,\eta} - \breve{\theta}^j_{1,t+1,\eta}, \quad \eta \times j \in \{(1,2) \times (d,f)\}. \tag{D36}$$

- complementary-slackness conditions $(1 + 4 + 4$ equations):

$$\bar{\mu}_{m,t} (x_{m,t} - m) = 0, \tag{D37}$$
$$\left((1 + \lambda^j_1) - R^j_{1,t+1,\eta}\right) \hat{\theta}^j_{1,t+1,\eta} = 0, \quad \eta \times j \in \{(1,2) \times (d,f)\}, \tag{D38}$$
$$\left(R^j_{1,t+1,\eta} - (1 - \lambda^j_1)\right) \breve{\theta}^j_{1,t+1,\eta} = 0, \quad \eta \times j \in \{(1,2) \times (d,f)\}. \tag{D39}$$

- market clearing (2 equations):

$$\theta^f_{1,t} + \theta^d_{1,t} = 1, \tag{D40}$$
$$\bar{\theta}^f_{0,t} + \bar{\theta}^d_{0,t} + \bar{y}_{h,t} = 0, \tag{D41}$$

- Inequalities:

$$\bar{\mu}_{m,t} \geq 0, \quad x_{m,t} \geq m, \tag{D42}$$

$$1 - \lambda^j_{1,t+1,\eta} \leq R^j_{1,t+1,\eta} \leq 1 + \lambda^j_{1,t+1,\eta}, \quad \eta \times j \in \{(1,2) \times (d,f)\}, \tag{D43}$$

$$\hat{\theta}^j_{1,t+1,\eta} \geq 0, \quad \check{\theta}^j_{1,t+1,\eta} \geq 0, \quad \eta \times j \in \{(1,2) \times (d,f)\}. \tag{D44}$$

The unknowns we solve for are:

- future consumption shares (6): $\omega_{k,t+1,\eta}$, $k \times \eta \in \{d,h,m\} \times \{1,2\}$;
- manager's current holdings of fund shares (1): $x_{m,t}$;
- portfolio restriction shadow price (1): $\bar{\mu}_{m,t}$;
- future fund share trades (2): $\Delta x_{m,t+1,\eta}$, $\eta = 1,2$;
- future stock purchases (4), $\hat{\theta}^j_{1,t+1,\eta}$, $\eta \times j \in \{(1,2) \times (d,f)\}$;
- future stock sales (4), $\check{\theta}^j_{1,t+1,\eta}$, $\eta \times j \in \{(1,2) \times (d,f)\}$;
- securities' shadow prices (4): $R^j_{1,t+1,\eta}$, $\eta \times j \in \{(1,2) \times (d,f)\}$.

Once we solve the system of 22 equations (+ inequalities) for the 22 unknowns we can compute the trading desk's scaled private valuation of the stock and the bond price:[16]

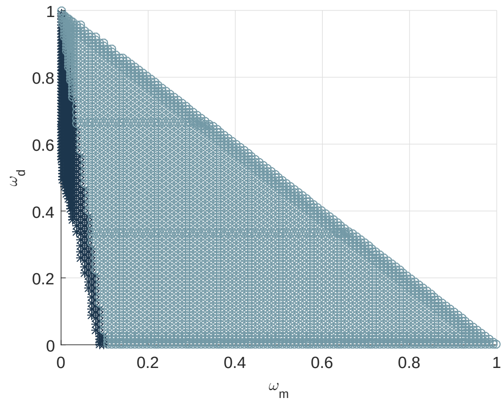
$$\check{P}_{1,t} = \sum_{\eta=1}^{2} \pi_{t,t+1,\eta} \left(\frac{\omega_{d,t+1,\eta}}{\omega_{d,t}} \right)^{\gamma_d - 1} r_\eta^{\gamma_d} \left(\check{P}_{1,t+1,\eta} + 1 \right), \tag{D45}$$

$$P_{0,t} = \sum_{\eta=1}^{2} \pi_{t,t+1,\eta} \left(\frac{\omega_{d,t+1,\eta}}{\omega_{d,t}} \right)^{\gamma_d - 1} r_\eta^{\gamma_d - 1}. \tag{D46}$$

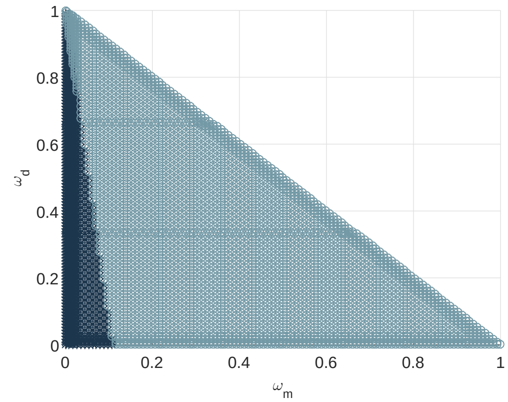
Appendix E. Untrusted Domain

Figure 10 illustrates the size of the untrusted domain in the delegation model without trading fees. I report results for the initial period, $t = 0$, and, for comparison purposes, for the period before last, $t = 28$. As Panel (b) shows, in the latter period the untrusted domain is empty. The unconstrained region, where the equity constraint does not bind, is depicted in light blue, while the constrained region is dark blue. The figure shows that the constrained region is relatively small and tends to shrink slowly over time. The expanding untrusted domain, even though at a relatively small rate, further reduces the constrained region that we are able to analyze.

[16] To solve a system of equations with constraints I call the IPOPT solver through the OPTI toolbox, available at https://www.inverseproblem.co.nz/OPTI/.



(a) $t = 0$



(b) $t = 28$

Figure 10. The Untrusted Domain. The figure characterizes the state space at $t = 0$ and $t = 28$ for $T = 29$, i.e., an economy with 30 periods. The state variables are the manager's consumption share, ω_m, depicted on the x-axis, and the consumption share of the trading desk, ω_d, depicted on the y-axis. The light blue circles represent unconstrained states, while the dark blue asterisks show constrained ones. The missing values in Panel (a) at low ω_m characterize the untrusted domain. Parameters are as in Table I.

To address the issue I follow the approach laid out in DL 2012. In particular, I classify states with no solution as untrusted domain. At the interpolation stage I replace these missing values with quantities interpolated using neighboring states. If at the next step future consumption shares fall in the untrusted domain, I also classify the current state as untrusted domain. I apply a further filter at the forward propagation step: for each solved state at time $t = 0$, I compute the equilibrium quantities along the dividend tree; if the interpolation produces, at any node, consumption shares that lie in the untrusted domain, I discard the time $t = 0$ state.