Group-Managed Real Options*

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Abstract

We study a standard real-option problem where sequential decisions are made through voting by a group of members with heterogeneous beliefs. We show that, when facing both investment and abandonment timing decisions, the group behavior cannot be replicated by that of a representative "median" member. As a result, members' disagreement generates *inertia*—the group delays investment relative to a single-agent case—and *underinvestment*—the group rejects projects that are supported by a majority of members, acting in autarky. These coordination frictions hold in groups of any size, for general voting protocols, and are exacerbated by belief polarization.

Keywords: group decisions, dynamic voting, real investment.

1 Introduction

Many decisions of interest in economics and finance are dynamic *group* decisions. The determination of investment and financing by a corporation, the creation and management of a startup, and the portfolio decisions of households, are just a few examples of the pervasiveness of dynamic group decisions. Despite this pervasiveness, the vast majority of dynamic models in finance abstract from the multi-agent nature of decisions, resorting instead to a setting where a "representative agent", e.g., a manager or an entrepreneur, is the sole decision maker. While this approach is useful for developing economic insights and empirical predictions, it ignores an important aspect of real-world decisions: disagreement within groups. Because heterogeneous groups can behave in a starkly different way from individuals, a deeper understanding of observed group behavior calls for theories of dynamic group decisions.

In this paper we answer this call by studying a group of members with heterogeneous beliefs who manage a real option. The option involves three decisions: (i) whether or not to acquire a license that allows a subsequent investment in a cash flow producing underlying asset (e.g., a patent); (ii) the *timing* of the subsequent investment, if the license is acquired (e.g., start production); and, (iii) the timing of abandonment, if the investment is made (e.g., stop production). Although this is a well-studied problem in the real options literature, prior work has examined this from the point of view of a single decision maker. We add the realistic assumption that the real option is managed by a group whose members hold different beliefs about the growth of the cash flows that form the underlying asset. We assume that, because of the difference in beliefs, group members agree to make decisions through voting. To consider the simplest form of conflict, we ignore asymmetric information, contractual differences, and learning, and focus instead on the coordination frictions that emerge from the existence of *polarized* beliefs among group members.[1]

Viewing the corporation as a vote-based social entity suggests that prior work on voting could provide important insights that apply to corporate finance. A key result in social choice theory, the "median voter theorem" (Black (1958)), predicts that decisions made through voting by group members who disagree are identical to those of the member with median preferences. Applied to a corporate finance context, the median voter theorem suggests that the single manager or

[1]Our focus on voting in a setting with polarized beliefs rules out the information aggregation role of voting. In finance the information aggregation role of voting has been studied widely. See, e.g., Warther (1998), Gillette, Noe, and Rebello (2003), Harris and Raviv (2008), Baranchuk and Dybvig (2008), Maug and Rydqvist (2008), Levit and Malenko (2011), Malenko (2014), and Chemmanur and Fedaseyeu (2017).

entrepreneur can be thought of as the group member with median preferences, that is, the representative agent for the group. Our main result, however, is to demonstrate that in *every* dynamic investment problem that involves *sequential* timing decisions by a group, the median voter theorem does not hold.

In our model, where disagreement is due to differences in beliefs, we show that, although the member with median beliefs is "pivotal" for the initial licensing and final abandonment timing decisions, *any* member of the group may be pivotal for the investment timing decision. The intuition for this result is that, by exercising the investment option, the group acquires both the project's cash flow stream *and* an option to abandon. The option to invest can therefore be thought of as a *compound* call. The incentive to exercise this option, or wait for a later exercise time, depends on the desirability of the project cash flow relative to the abandonment option. Although optimists always value the option to invest *more highly* than pessimists, contrary to common intuition, it is possible that they desire to exercise the option to invest *later*. When this happens, the ranking of members' beliefs does not correspond to the ranking of their desired investment times. This result implies that the group member with median beliefs is not necessarily the pivotal voter for the investment timing decision.

The absence of a representative agent for a group managing a real option has important economic implications which we refer to as *inertia* and *underinvestment*. A group member who is pivotal for investment and who anticipates not being pivotal for abandonment will see the group abandonment decision as suboptimal. We show that, when this happens, the pivotal member for investment delays investment relative to the autarkic case of a single-managed project (inertia). Inertia, in turn, has implications for the initial licensing decision and may lead a group to pass up on projects that are perceived to have a positive NPV by a majority of group members, acting in autarky (underinvestment). Both inertia and underinvestment are *coordination frictions* that occur because, when group members cannot commit to future policies, future pivotal voters may impose negative externalities on current pivotal voters and, as a result, distort group choices.

The changing nature of the pivotal voter over time is key to our results as it does not allow the group problem to be reduced to that of a representative member with median beliefs. Given the parsimonious nature of our model, we are able to show that this violation of the median voter theorem holds for a broad range of empirically plausible parameter values related to cash flow volatility, reversibility of the project, and belief polarization within the group. The coordination frictions that emerge in a group-managed real option are more severe in more polarized groups, where disagreement among pivotal voters is large. These frictions can occur in groups of *any* size, governed by a wide range of non-dictatorial voting rules, such as unanimity, and super-majority. Voting rules that are more stringent than majority produce more polarized pivotal voters and therefore exacerbate the group coordination frictions.

Our results rest on three key assumptions: (i) decisions are collectively made by a group of members with heterogeneous beliefs;[2] (ii) the group faces a *sequence* of decisions over time and settles disagreement through a voting rule such as majority, super-majority, or unanimity; (iii) group members cannot settle their disagreement by trading their shares and/or their control rights or by pre-committing to future decisions.

Our assumption (i), that a group is made up of diverse individuals that must work together to make decisions, seems very natural to us. However, since we identify frictions that heterogeneity may bring about, a simple solution would be to require that groups be composed of members with homogeneous beliefs. Such a solution ignores, however, the much bigger question of how diversity enhances or diminishes group performance. A large literature in psychology and organizational behavior studies the complex relationship between diversity and firm performance.[3] Incorporating other costs and benefits of diversity in order to justify the existence of a diverse group goes beyond the scope of our paper. Assumption (ii) captures the dynamic nature of many real-world investment decisions. This assumption accurately describes, for example, the environment of corporate boards, startups, and young firms where decisions are brought about through formal or informal voting.

Finally, assumption (iii) where group members are not able to sell their shares to each other, is descriptive of many group decisions. For example, startups are often built on the complementary talents of founding partners who usually do not have the capital to buy each other out in case of disagreement. Moreover, while disagreement among members, as in our model, may create potential gains for trading votes, it is not obvious that markets for votes can resolve the coordination frictions in a way that benefits all group members. This is because the trade of votes generates externalities, as it can also affect non-trading members. These externalities make it difficult to achieve efficiency

[2] Other sources of heterogeneity, such as risk aversion, discount rates, or contractual claims, will generate similar coordination frictions. For example, Chen and Lambrecht (2019) discuss how coordination frictions due to heterogeneous risk aversion impact firm's payout policies, capital structure, and internal governance. Jackson and Yariv (2015) show that coordination frictions due to differences in subjective discount rates lead to a time-inconsistent group behavior.

[3] See Roberson, Holmes IV, and Perry (2017) for a survey of the evidence in this literature.

with price mediation in competitive markets for votes.[4] Designing a welfare-improving market for votes that addresses these externalities would require a coordination effort by either some of the group members (e.g. the founding partners) or through regulation. Our assumption of no trading captures instances where trading votes either fails or leads to an inefficient allocation.

Our paper contributes to three strands of literature. First, we contribute to the real options literature. The early stages of this literature (Brennan and Schwartz (1985); McDonald and Siegel (1986) and Dixit and Pindyck (1994)) focus on a single decision maker who exclusively controls and benefits from the investment decision. More recent contributions to this literature extend the basic setting to investigate the effect of informational frictions on the exercise of real options such as moral hazard (Grenadier and Wang (2005), Grenadier, Malenko, and Malenko (2016)), signalling and adverse selection (Morellec and Schürhoff (2011) and Grenadier and Malenko (2011)). We add to this literature by studying the management of a real option by a group of equally-informed members with heterogeneous beliefs who decide through voting. We show that voting leads to frictions that, as with informational frictions, distort timing relative to an autarkic decision maker. The sources of these frictions are, however, fundamentally different. On the one hand, informational frictions result from differential access to and acquisition of information by different parties. On the other hand, the heterogeneity of beliefs in our model results from differences in how information is interpreted. This framework provides a novel and alternative mechanism for investment inertia within a standard real option setting. Unlike existing work, (see, e.g., Grenadier and Wang (2005)) in which inertia is the result of information asymmetry, in our setting investment inertia obtains purely as a consequence of coordination frictions brought about by the heterogeneity of beliefs within a group that decides through voting.

Second, our paper contributes to the literature on disagreement and differences in beliefs in finance. This literature is motivated by the observation that, while the "common prior" assumption is appropriate in a setting where information is plentiful and posterior beliefs have converged, it is less tenable in situations where agents lack information and/or experience (Morris (1995)). In these settings, agents would hold different views and people would agree to disagree. A large literature in finance studies the implications of disagreement on both asset prices and corporate decisions.[5] We

[4]The economics literature has emphasized the difficulties associated with the market for votes, see, e.g. Casella, Llorente-Saguer, and Palfrey (2012) and Casella and Macé (2020)) for a survey.

[5]This literature is too vast to be reviewed here. Lintner (1965), Miller (1977), Harrison and Kreps (1978), Ross (1976), Kandel and Pearson (1995) and, Scheinkman and Xiong (2003) are important contribution to the asset pricing literature that allow for differences in prior beliefs. Theoretical contributions to the corporate finance literature include

contribute to this literature by developing a model to analyze the impact of differences in beliefs within a group on investment dynamics. Allen and Gale (1999) and Thakor and Whited (2011) also study the implication of differences in beliefs on investment. While they focus on the differences in beliefs between the shareholders and the manager, however, we explicitly model voting within a group of members with heterogeneous beliefs and abstract from any other agency or information frictions. To the best of our knowledge, our paper is the first to study the management of a real option through voting.

Third, our paper contributes to the literature on dynamic collective decisions. At a broader level, our paper emphasizes that changes in the identity of the pivotal member over time are a source of coordination friction that distorts current choices. The prior literature on dynamic collective decisions emphasized this point in different contexts. For example, in a political economy model, Roberts (2015) demonstrates that group time inconsistency arises when group members can endogenously include new members in the group.[6] In a model of corporate investment, Garlappi, Giammarino, and Lazrak (2017) show that time inconsistency can arise when group members learn in a Bayesian way from a public signal. In a model of corporate boards, Donaldson, Malenko, and Piacentino (2019) assume that board members have a status-quo bias and show that the coordination friction takes the form of an inefficient deadlock for the board. Our novel insight is to show that important coordination frictions can emerge even in the absence of endogenous group size, learning, or status-quo bias. In fact, they emerge rather naturally in investment decisions involving the acquisition of a project that involves the timing of both investment and abandonment. The interaction between the long- and short-position embedded in such an option creates coordination frictions and, implicitly, a time inconsistency in the group. Given the pervasiveness of dynamic investment/abandonment decisions in practice, our theoretical results are relevant for a wide range of problems in economics and finance.

The rest of the paper proceeds as follows. Section 2 presents the basic model in which a three-member group manages a real option making decisions through majority voting. Section 3 defines

Harris and Raviv (1993), Boot, Gopalan, and Thakor (2006), and more recently, Kakhbod, Loginova, Malenko, and Malenko (2019). Empirically, earlier evidence of revealed differences in beliefs includes Kandel and Pearson (1995), Dittmar and Thakor (2007) among others. More recently, Li, Maug, and Schwartz-Ziv (2020) find empirical support for the difference in beliefs in the voting behavior of mutual funds.

[6] Acemoglu, Egorov, and Sonin (2018) also analyze a voting model with a changing electorate where the time inconsistency issue emerges. More broadly, the change of pivotal voters over time is present in many dynamic bargaining models of political economy, e.g., Strulovici (2010), Dziuda and Loeper (2016), and Chan, Lizzeri, Suen, and Yariv (2017).

group inertia and underinvestment and illustrates the conditions under which these coordination frictions occur. Section 4 extends our results to groups of any size and to alternative voting mechanisms. Section 5 considers the possibility of trading among group members. Section 6 discusses empirical implications and Section 7 concludes. Appendix A contains the proofs of all propositions and Appendix B provides an example of how the market for votes could fail in the context of our model.

2 Model

Consider a group of three members, P (pessimist), M (median) and O (optimist) who face a standard real option problem. We assume that, for exogenous reasons, the group is required to act collectively to make decisions. Failure to acquire the license implies the dissolution of the group. In this sense, the licensing decision can be thought of as essential to the group's existence as an economic entity. Group members agree to make decisions through majority voting.

2.1 Setup

Technology. At time 0 the group faces a take-it-or-leave-it decision of whether to acquire a license for a cost $L > 0$ (*licensing decision*). The acquisition of the license gives the group the right, at any future time, to pay an investment cost $I > 0$ in order to start a project that generates a random cash flow X_t per unit of time (*investment decision*). If the group invests in the project, it subsequently faces the option of abandoning at any time by liquidating for a certain cash recovery amount A such that $0 \leq A < I$ (*abandonment decision*). The quantity A/I can be thought of as a measure of reversibility of the project. Group members share equally the license cost, L, the investment cost I, the project cash flows X_t, and the abandonment value A. In the subsequent analysis, to simplify the notation, we interpret the variables L, I, A, and X_t as representing per-capita quantities.

Beliefs. P, M and O disagree on the growth rate of the cash flow X_t. Specifically, each group member $n \in \{P, M, O\}$ believes that the cash flow X_t of the project is governed by a geometric Brownian motion with drift μ_n and volatility σ, that is,

$$dX_t = \mu_n X_t dt + \sigma X_t dB_{n,t}, \quad X_0 = x > 0, \quad n \in \{P, M, O\}, \tag{1}$$

where $\mu_P < \mu_M < \mu_O$ and $B_{n,t}$ is a standard Brownian motion under member n's belief. Group members do not update their beliefs about the growth rate of cash flows after observing a realization of X_t. In this sense, we refer to their beliefs as being "polarized." In realistic settings, such as the case of investment in startups and new technologies, learning about the mean growth rate of a project cash flow can take a long time. Therefore, our assumption of members with polarized beliefs about cash flow growth rates captures the persistence of disagreement in these settings.

Governance. Each member has one vote in every decision that the group faces. Members cast their votes to maximize self-interest, rationally anticipating the group's future decisions given the common knowledge of all the model parameters. The licensing, investment, and abandonment decisions are each determined by separate votes, each subject to a simple majority rule: a proposal is successful if it is supported by at least two votes. The licensing vote occurs at time 0; the investment vote can happen at any subsequent time; and, similarly, the abandonment vote can happen at any time following investment.

Individual valuations. We assume that group members are risk-neutral and denote by r the discount rate. Member n's subjective valuation of the perpetual stream of cash flow X_t is

$$\mathbb{E}_n\left[\int_0^\infty e^{-rt}X_t dt \,\middle|\, X_0 = x\right] = \frac{x}{r-\mu_n}, \quad n \in \{P, M, O\}, \tag{2}$$

where $X_0 = x$ denotes the initial cash flow level and $\mathbb{E}_n$ denotes the expectation under member n's belief. Since $0 \leq \mu_P < \mu_M < \mu_O$, to insure that subjective valuations are properly defined for each member, we impose that $\mu_O < r$.

2.2 Individual decisions

Before studying the equilibrium group decisions, we examine the optimal individual (autarkic) decisions. If acting individually, each group member $n \in \{P, M, O\}$ would face an optimal stopping problem involving the choice of when to pay I and invest in the project, and when to abandon a project in exchange for the liquidation value A. Formally, member n solves the following optimal stopping problem:

$$V_n^*(x) = \sup_{\tau \leq \nu} \mathbb{E}_n\left[-Ie^{-r\tau} + \int_\tau^\nu X_t e^{-rt}dt + Ae^{-r\nu} \,\middle|\, X_0 = x\right], \tag{3}$$

where we denote by τ and ν the random investment and abandonment times, $\tau \leq \nu$. We refer to $V_n^*(x)$ as the value of the license. Note that this value is the value of a *compound option*, that is, the option value to invest depends on the cash flow and on the value of the option to abandon. The constraint $\tau \leq \nu$ captures the assumption that abandonment can only occur after the investment has been undertaken. Member n will buy the license if and only if $V_n^*(x) \geq L$.

The solution of problem (3) is standard (see, e.g., Dixit and Pindyck (1994)) and involves finding two cash flow thresholds $X_n^{A,*} < X_n^{I,*}$ such that the optimal strategy for member n is to (i) invest the first time X_t hits $X_n^{I,*}$ from below, and, after investment, (ii) abandon the first time X_t hits $X_n^{A,*}$ from above. By standard results from optimal stopping theory (e.g., Peskir and Shiryaev (2006), Pham (2009), and Øksendal (2013)), the stopping rule based on the thresholds $X_n^{I,*}$ and $X_n^{A,*}$ is optimal among *all* stopping rules, including those that depend on the whole path of $(X_t)_{t\geq 0}$.

To characterize the individual optimal investment and abandonment strategies, we consider member n's time-0 subjective value $V_n(x, X^I, X^A)$ of an investment/abandonment strategy characterized by an arbitrary pair of thresholds, $X^A < X^I$, that is,

$$V_n(x, X^I, X^A) \equiv \mathbb{E}_n \left[-Ie^{-r\tau_{X^I}} + \int_{\tau_{X^I}}^{\nu_{X^A}} X_t e^{-rt} dt + Ae^{-r\nu_{X^A}} \middle| X_0 = x \right], \tag{4}$$

where τ_{X^I} is the hitting time of the threshold X^I from below and ν_{X^A} is the hitting time of the threshold X^A from above. Formally,

$$\tau_{X^I} = \inf \left\{ t \geq 0 \ : \ X_t \geq X^I \right\}, \quad \nu_{X^A} = \inf \left\{ t \geq \tau_{X^I} \ : \ X_t \leq X^A \right\}. \tag{5}$$

Similarly, we consider member n's subjective value $W_n(x, X^A)$ of operating the project until the abandonment date:

$$W_n(x, X^A) \equiv \mathbb{E}_n \left[\int_0^{\nu_{X^A}} X_t e^{-rt} dt + Ae^{-r\nu_{X^A}} \middle| X_0 = x \right], \tag{6}$$

where ν_{X^A} is now defined as $\nu_{X^A} = \inf \left\{ t \geq 0 \ : \ X_t \leq X^A \right\}$. The following proposition provides closed-form expressions of the values in equation (4) and (6).

Proposition 1. *Let X^I and X^A be arbitrary investment and abandonment thresholds satisfying $0 < X^A < X^I$. Member n's subjective value of the operating project $W_n(x, X^A)$, defined in equation (6),* *is given by*

$$W_n(x, X^A) = \begin{cases} \frac{x}{r-\mu_n} + \left(A - \frac{X^A}{r-\mu_n}\right)\left(\frac{x}{X^A}\right)^{m_n} & \text{if } x \geq X^A \\ A & \text{if } x < X^A \end{cases}, \tag{7}$$

where $m_n < 0$ *is the constant*

$$m_n = \frac{-(\mu_n - \frac{\sigma^2}{2}) - \sqrt{(\mu_n - \frac{\sigma^2}{2})^2 + 2\sigma^2 r}}{\sigma^2} < 0. \tag{8}$$

Member n's subjective value of the license, defined in equation (4) is given by

$$V_n(x, X^I, X^A) \quad = \quad \begin{cases} W_n(x, X^A) - I, & \text{if } x \geq X^I \\ \left(W_n(X^I, X^A) - I\right)\left(\frac{x}{X^I}\right)^{q_n}, & \text{if } x < X^I \end{cases}, \tag{9}$$

where $W_n(x, X^A)$ *is given in equation (7) and* $q_n > 1$ *is the constant*

$$q_n = \frac{-(\mu_n - \frac{\sigma^2}{2}) + \sqrt{(\mu_n - \frac{\sigma^2}{2})^2 + 2\sigma^2 r}}{\sigma^2} > 1. \tag{10}$$

The solution of the optimal stopping problem (3) can then be obtained by finding the thresholds $X^A = X_n^{A,*}$ and $X^I = X_n^{I,*}$ that maximize equation (9). The following proposition characterizes the solution of the optimal individual investment and abandonment problem.

Proposition 2. *Consider member n's optimal investment and abandonment timing problem described in equation (3).*

1. *The optimal abandonment threshold* $X_n^{A,*}$ *is given by*

$$X_n^{A,*} = A\left(r + m_n \frac{\sigma^2}{2}\right) \equiv \frac{m_n}{m_n - 1} A(r - \mu_n), \quad n \in \{P, M, O\}, \tag{11}$$

with m_n *given in equation (8).*

2. *The optimal investment threshold* $X_n^{I,*}$ *is the largest root of the following equation*

$$\frac{X_n^{I,*}}{r - \mu_n}(q_n - 1) + \left(A - \frac{X_n^{A,*}}{r - \mu_n}\right)\left(\frac{X_n^{I,*}}{X_n^{A,*}}\right)^{m_n}(q_n - m_n) = q_n I, \quad n \in \{P, M, O\}, \tag{12}$$

with $X_n^{A,*}$ *given in equation (11) and* m_n *and* q_n *given in equations (8) and* (10).

For any cash flow $x > 0$, member n's subjective time-0 valuation of the license under the optimal policy is given by

$$V_n^*(x) = V_n(x, X_n^{I,*}, X_n^{A,*}), \quad n \in \{P, M, O\}, \tag{13}$$

with the function $V_n(\cdot)$ defined in equation (9). The individual license values $V_n^*(x)$ are monotonic in members' beliefs, that is,

$$0 < V_P^*(x) < V_M^*(x) < V_O^*(x), \text{ for all } x > 0. \tag{14}$$

Proposition 2 allows us to study how member n's investment and abandonment decisions depend on the belief parameter μ_n. Note that, from the definition of the constant $m_n < 0$ in equation (8), m_n is decreasing in μ_n. This implies that the abandonment threshold $X_n^{A,*}$ in equation (11) is decreasing in the belief parameter μ_n, that is,

$$X_O^{A,*} < X_M^{A,*} < X_P^{A,*}. \tag{15}$$

Therefore, the optimal abandonment times are ranked according to the members' beliefs. Intuitively, an optimist abandons later (and thus uses a lower abandonment threshold) than a pessimist.

This one-to-one mapping between the rankings of beliefs and abandonment thresholds *does not* hold for the investment thresholds $X_n^{I,*}$. Even if, by condition (14) of Proposition 2, a pessimist would always value the project less than an optimist, it is possible for the pessimist's investment threshold to be *lower* than the optimist's. This implies that the ranking of members' beliefs does not translate into a unique ranking of optimal investment thresholds. This discrepancy between the ranking of *values* and *thresholds* is key for understanding equilibrium group decisions in Section 2.3. To illustrate why a ranking of beliefs does not imply a unique ranking of investment thresholds, it is useful to re-express the optimal investment condition (12) as the following "break-even" condition:[7]

$$\frac{x}{r - \mu_n} + A^{1-m_n} \mathcal{O}_n \, x^{m_n} = I + \frac{1}{q_n - 1} I. \tag{16}$$

where the term $\mathcal{O}_n$ depends on member n's beliefs, μ_n, the cash flow volatility σ, and the risk-free rate r.[8] The left-hand side of the break-even condition (16) represents the benefit from investing,

[7]Equation (16) is obtained by substituting the expression for the optimal abandonment threshold $X_n^{A,*}$ from equation (11) in the condition that defines the optimal investment threshold in equation (12).

[8]Formally, $\mathcal{O}_n \equiv \frac{(q_n - m_n)}{(1-m_n)(q_n-1)} \left(\frac{m_n - 1}{m_n(r-\mu_n)}\right)^{m_n} > 0$, where $m_n < 0$ and $q_n > 1$ are given in equations (8) and (10).

consisting of the perpetuity value $x/(r-\mu_n)$ and the option value to abandon, $A^{1-m_n}\mathcal{O}_n\, x^{m_n}$. The right-hand side is the cost of investing which consists of both the direct sunk cost I *and* the indirect opportunity cost of giving up the option to wait, captured by the term $I + I/(q_n - 1)$. Consider the special case in which the abandonment option is valueless, i.e., $A = 0$. In this case, equation (16) implies that the optimal investment threshold $X_n^{I,*}$ is given by

$$\frac{X_n^{I,*}}{r-\mu_n} = \frac{q_n}{q_n - 1} I, \tag{17}$$

which is a well-known property of the optimal exercise of a perpetual investment option. Because of the option to wait, exercise occurs only when the value of the cash flow annuity $x/(r-\mu_n)$ is sufficiently larger (by the factor $q_n/(q_n-1) > 1$) than the investment cost I. In this case the optimal investment thresholds are ranked according to beliefs, that is, $X_O^{I,*} < X_M^{I,*} < X_P^{I,*}$.[9] If we further let $\sigma \to 0$ we obtain $q_n \to r/\mu_n$ and the breakeven condition (17) collapses to $X_n^{I,*} = rI$, that is, in the absence of uncertainty, beliefs heterogeneity is irrelevant and we recover the standard NPV rule of investing as long as the cash flow is larger than the rental cost of capital rI.

In general, however, when $A > 0$ and $\sigma > 0$, the benefit from investing in equation (16) depends in a non-monotonic way on the members' beliefs, μ_n. All else being equal, when the abandonment value A and cash flow volatility are high, the desirability of the abandonment option to P can overcome the desirability of the cash flow stream, thus leading to a *lower* threshold for P than for O. This happens because investing is a necessary requirement for the pessimist to acquire an attractive abandonment option.

Condition (16) does not deliver an analytical expression for the investment threshold $X_n^{I,*}$, which makes it difficult to study the sensitivity of the optimal investment threshold to the underlying parameters. To gain some intuition, we rely on a numerical illustration for the case of a three-member group. Figure 1 shows that *any* ranking of the optimal investment thresholds $X_n^{I,*}$, $n \in \{P, M, O\}$ is possible. In the figure, we fix the belief of the median member $\mu_M = r/2$ and define the beliefs of O and P as $\mu_O = \mu_M + \varepsilon$ and $\mu_P = \mu_M - \varepsilon$. We refer to the spread ε as *polarization* and report the optimal investment thresholds $X_n^{I,*}$, $n \in \{P, M, O\}$ as a function of ε and for different configurations of the abandonment value A and cash flow volatility σ. As the figure shows, for low

[9] To see this, note that, because $q_n > 1$ is a solution of the quadratic equation (A.4), it follows that the threshold $X_n^{I,*}$ from equation (17) can be written as $X_n^{I,*} = r + \frac{\sigma^2}{2} q_n$. Differentiating equation (10) with respect to μ_n shows that q_n is decreasing in μ_n. Hence, it follows that $X_n^{I,*}$ is decreasing in μ_n.

values of A and σ (Panel A) the investment thresholds are ranked according to beliefs, consistent with intuition: the most pessimistic member (blue line) being more reluctant to invest, that is, $X_O^{I,*} < X_M^{I,*} < X_P^{I,*}$. However for high values of A and/or σ, (Panels B and C) the pessimist's investment threshold can be lower than the optimist's (green line). In summary, the compound nature of the investment option implies that, in contrast to the abandonment thresholds $X_n^{A,*}$, the ranking of optimal individual investment thresholds $X_n^{I,*}$ does not in general correspond to the ranking of beliefs. In fact, Figure 1 illustrates that any of the possible six rankings of the optimal investment thresholds $X_n^{I,*}$, $n \in \{P, M, O\}$ is possible, depending on the model parameters A and σ, and belief polarization ε. In the next section, we show that this property holds also for the ranking of the equilibrium group investment thresholds.

2.3 Group decisions

The individual timing decisions and project values derived in Propositions 1 and 2 allow us to study the group's licensing, investment, and abandonment choices. We examine the equilibrium group strategy recursively, starting with the abandonment decision, assuming the group has acquired the license and invested in the project. Given the group abandonment decision, we then determine the investment timing and initial licensing decision. Group members vote on the basis of their own preferences while correctly anticipating the outcomes of future group decisions.

Group abandonment timing decision. As shown in equation (15), the individual abandonment thresholds are ranked according to members' beliefs, that is, $X_O^{A,*} < X_M^{A,*} < X_P^{A,*}$. Given a level of cash flow $x > X_P^{A,*}$, no group member would propose abandonment. As the cash flow drops to the level $x = X_P^{A,*}$, P would like to abandon. However, a proposal to abandon will be rejected by the majority, since M and O would vote against it: at this cash flow level, their individual valuations under M's optimal abandonment threshold are larger than the value of immediate abandonment, that is, $W_n(X_P^{A,*}, X_M^{A,*}) > A$, $n \in \{O, M\}$. Therefore, at the cash flow level $x = X_P^{A,*}$, both O and M agree to delay abandonment leading the group to also delay abandonment. Moreover, M knows that, by rejecting abandonment at $x = X_P^{A,*}$ and waiting until the cash flow drops to the lower threshold $x = X_M^{A,*}$, P will support M's abandonment proposal, since P would support abandonment at any cash flow $x < X_P^{A,*}$. This implies that P would vote for abandonment as soon as M proposes such a vote, that is, when cash flow hits the threshold $X_M^{A,*}$. At this point, a P-M

majority votes for abandonment, and, despite O's opposition, the group abandons the project. Hence the abandonment policy of the group, X^{A}_{G}, is the same as M's optimal abandonment policy, that is,

$$X^{A}_{\mathrm{G}} = X^{A,*}_{M}, \tag{18}$$

and the group abandons at the (random) time $\nu_{X^{A,*}_{M}} \equiv \inf\{t \geq 0 \;:\; X_t \leq X^{A,*}_{M}\}$. We refer to M as the *pivotal voter* for the abandonment decision: having both median beliefs, μ_M, and abandonment threshold, $X^{A,*}_{M}$, member M is the median abandonment *timer* who casts the pivotal vote. For notational convenience, we denote member n's value of the group-managed operating project at the cash flow level x by $W_{n,G}(x) := W_n(x, X^{A}_{\mathrm{G}})$, where X^{A}_{G} denotes the equilibrium group abandonment threshold defined in equation (18).

Group investment timing decision. By the same logic describing the group abandonment decision, the pivotal group member for the investment timing decision is the member with the median investment threshold or, equivalently, the median investment time. All group members determine their investment timing strategy with the expectation that M would be pivotal for the abandonment decision. Accordingly, M's preferred investment threshold corresponds to the optimal individual threshold $X^{I,*}_{M}$ defined in equation (12). In contrast, for P and O, the preferred investment threshold is determined by solving the following stopping time problem

$$\sup_{\tau} \mathbb{E}_n \left[e^{-r\tau}(W_{n,G}(X_\tau) - I) \,\middle|\, X_0 = x \right], \text{ for } n \in \{P, O\}, \tag{19}$$

where $W_{n,G}(X_\tau) \equiv W_n(X_\tau, X^{A,*}_{M})$, defined in equation (7), is the abandonment value to member $n \in \{P, O\}$ at the cash flow level X_τ *when the group abandonment timing decision is controlled by* M. We denote by $\tau_{X^{I,\mathrm{SB}}_{n}} = \inf\{t \geq 0 : X_t \geq X^{I,\mathrm{SB}}_{n}\}$ the solution to the stopping time defined in equation (19) where the label 'SB' indicates that for $n \in \{P, O\}$, the investment threshold $X^{I,\mathrm{SB}}_{n}$ is *second best*, relative to the decisions they would make in autarky. This threshold is given by the largest root of a breakeven condition similar to the one defining the optimal individual investment threshold in equation (12), that is,

$$\frac{X^{I,\mathrm{SB}}_{n}}{r - \mu_n}(q_n - 1) + \left(A - \frac{X^{A,*}_{M}}{r - \mu_n}\right)\left(\frac{X^{I,\mathrm{SB}}_{n}}{X^{A,*}_{M}}\right)^{m_n}(q_n - m_n) = q_n I, \text{ for } n \in \{P, O\}, \tag{20}$$

As in the case of the individually optimal investment thresholds $X_n^{I,*}$, different rankings among the second best investment thresholds $X_n^{I,\mathrm{SB}}$ are possible, depending on the values of the exogenous parameters of the model. In sum, denoting by n_I the group member with median second best investment threshold, the equilibrium group thresholds, X_{G}^I and X_{G}^A, determining the investment and abandonment decisions are[10]

$$X_{\mathrm{G}}^I = X_{n_I}^{I,\mathrm{SB}} = \mathrm{Median}\{X_P^{I,\mathrm{SB}}, X_M^{I,*}, X_O^{I,\mathrm{SB}}\}, \quad \text{and} \quad X_{\mathrm{G}}^A = X_M^{A,*}. \tag{21}$$

Note that, in the absence of the abandonment option, $A = 0$, the second-best investment thresholds correspond to the individual optimal investment thresholds. It follows from equation (17) that these thresholds are ranked according to members' beliefs. In this case, $X_{\mathrm{G}}^I = X_M^{I,*}$, implying that M is always pivotal for the investment timing decision.

In contrast, when the project contains an abandonment option, $A > 0$, any group member can be pivotal for the investment timing decision. To illustrate this property, Figure 2 partitions the space of parameters A/I, polarization ε, and cash volatility σ into three regions:[11] the gray area represents parameter combinations for which $n_I = M$, that is, the member with median beliefs is pivotal for investment timing. The dark blue area denotes parameters for which $n_I = O$ and the light blue area denotes parameters for which $n_I = P$.

The top panel of Figure 2 shows that, when cash flow volatility is low, O and P can only be pivotal for high levels of polarization and in projects that have a high degree of reversibility, that is, large A/I (North-East corner of Panel A). This is broadly consistent with the intuition that high reversibility makes the put option to abandon relatively more valuable to a pessimist member. When this effect is sufficiently strong, a pessimistic member invests at an earlier date than a more optimistic member. Panels B and C show the effect of cash flow volatility. In general, higher volatility lowers the range of values A/I for which the pivotal voter is either P or O. Intuitively, high reversibility and high volatility can be thought of as substitutes in affecting the value of the abandonment put option: the option to abandon is more valuable the higher is the

[10]There are knife-edge parameterizations of the model in which the second-best thresholds coincide for multiple members. In these cases there are multiple pivotal members for the investmet decision. The analysis that follows ignores these knife-edge cases and assumes that the pivotal member for investment is unique. Including these cases would complicate the exposition without changing any of our results. In contrast, the pivotal members for licensing and abandonment are always unique in our model.

[11]The investment and abandonment thresholds are homogeneous of degree one in I and A. Therefore the ratio A/I is sufficient to identify the pivotal member for investment.

abandonment value, A, and/or the higher is the probability of abandoning, i.e., the higher is the cash flow volatility σ. Panels B and C show that, starting with a high level of polarization and a low level of reversibility, A/I (South-East corner), the second best thresholds have the intuitive ranking $X_O^{I,\mathrm{SB}} < X_M^{I,*} < X_P^{I,\mathrm{SB}}$, implying that M is pivotal for investment (lower gray area). As A/I increases, M becomes more eager to invest than O, that is, $X_M^{I,*} < X_O^{I,\mathrm{SB}} < X_P^{I,\mathrm{SB}}$, and O is then pivotal for investment (dark-blue area). As A/I increases further, P becomes more eager to invest than O, but less than M, that is, $X_M^{I,*} < X_P^{I,\mathrm{SB}} < X_O^{I,\mathrm{SB}}$, and becomes pivotal for investment (light blue area). Finally, for very high values of A/I, P becomes more eager to invest than both M and O, that is, $X_P^{I,\mathrm{SB}} < X_M^{I,*} < X_O^{I,\mathrm{SB}}$, resulting in M becoming again pivotal for investment (upper gray area). Figure 2 further shows that, all else being equal, high group polarization typically implies that the pivotal member for investment is different from member M, that is, $n_I \neq M$. Finally, the darker shaded areas in Figure 2 indicate that if the member with median beliefs is not pivotal for investment, $n_I \neq M$, he remains non-pivotal for any higher level of polarization.

Group licensing decision. Internalizing the future investment/abandonment thresholds of the group, member n's project valuation at time 0 when cash flow is x, is given by

$$V_{n,\mathrm{G}}(x) := V_n(x, X_{\mathrm{G}}^I, X_{\mathrm{G}}^A) \equiv V_n(x, X_{n_I}^{I,\mathrm{SB}}, X_M^{A,*}),\ n \in \{P, M, O\}, \tag{22}$$

where the function $V_n(\cdot)$ is defined in equation (9). Therefore, given a licensing cost L, member n supports licensing if and only if $V_{n,\mathrm{G}}(x) \geq L$.

The following proposition shows that, unlike investment thresholds, the abandonment and investment options *values* are ranked according to the group members' beliefs.

Proposition 3 (**Ranking of the group-managed option values**). *Consider a three-member group, P, M, and O, governed by the majority rule and assume that P's value of the group-managed operating project upon exercise of the investment option is non-negative, that is,*

$$I < W_{P,G}(X_G^I), \tag{23}$$

where X_G^I denotes the equilibrium group investment threshold defined in equation (21). Then, the values of the group-managed abandonment and investment options, $W_{n,G}(x) \equiv W_n(x, X_G^A)$ and

$V_{n,G}(x) \equiv V_n(x, X_G^I, X_G^A)$, *are monotonic in the group members' beliefs, that is, for all* $x > X_G^A$*,*

$$I < W_{P,G}(x) < W_{M,G}(x) < W_{O,G}(x), \tag{24}$$

and for all $x > 0$*,*

$$0 < V_{P,G}(x) < V_{M,G}(x) < V_{O,G}(x). \tag{25}$$

The condition in equation (23) requires that the pessimist payoff at the time of group investment is positive and guarantees that his license value is positive, that is, $V_{P,G}(x) > 0$ for all $x > 0$. Because all group members have perfect information on each other's beliefs, a violation of condition (23) would imply that P is not willing to acquire the license, even for a zero license fee L.[12] By the inequalities in equation (25) we have that $L \leq V_{M,G}(x)$ implies $L \leq V_{O,G}(x)$ and, consequently, an M-O majority would lead the group to license. Reciprocally, $V_{M,G}(x) < L$ implies $V_{P,G}(x) < L$ and consequently an M-P majority would lead the group to forgo licensing. Therefore, M is pivotal for the licensing decision.

In sum, the analysis of this section shows that in a three-member group (P, M, and O), with beliefs $\mu_P < \mu_M < \mu_O$, facing a one-time licensing decision and a sequence of investment and abandonment timing decisions: (i) the member with median beliefs, M, is always pivotal for *both* the licensing and abandonment timing decisions; (ii) *any* member, P, M, or O, can be pivotal for the investment timing decision.

3 Group coordination frictions

In this section we show that the change in the identity of the pivotal member across group decisions gives rise to coordination frictions that affect the value of group-managed real options. To highlight these coordination frictions, we compare the group to individual decisions. Specifically, we consider the licensing/investment/abandonment decisions of the three-member group introduced in Section 2 and contrast them to those of the pivotal member acting in autarky.

We highlight two main coordination frictions. The first refers to the timing of group investment. In general, the group tends to delay investment relative to the timing that the pivotal member

[12]Intuitively, condition (23) constrains the negative externality that the more optimistic group members, M and O, can impose on the most pessimistic one, P. Imposing condition (23) rules out cases in which P is forced to acquire a license with a negative intrinsic value. Although our model is silent on why groups form, condition (23) would hold in an equilibrium model of group formation where group members have outside options with non-negative values.

would select if acting individually. We refer to this phenomenon as *investment inertia*. The second coordination friction we identify refers to the group acquisition of positive NPV projects. We show that a heterogeneous group may decide to forgo the acquisition of a license even though the majority would acquire the license if they could operate the project under autarky. We refer to this phenomenon as *underinvestment*.

Group investment inertia The following definition formalizes the concept of investment inertia in a three-member group.

Definition 1 (**Investment Inertia**). *Consider a three-member group governed by majority and denote by $n_I \in \{P, M, O\}$ the pivotal member for investment. The group exhibits investment inertia if the group investment threshold $X_G^I \equiv X_{n_I}^{I,SB}$ is larger than the optimal individual investment threshold $X_{n_I}^{I,*}$ of the pivotal member n_I, that is,*

$$X_G^I > X_{n_I}^{I,*}. \tag{26}$$

Equivalently, the group invests at a later time relative to member n_I acting individually, that is, $\tau_{X_G^I} > \tau_{X_{n_I}^{I,*}}$, a.s.

When M is pivotal for investment, i.e., $n_I = M$, the group's timing decision coincides with M's optimal timing decisions. We therefore have that $X_{\mathrm{G}}^I = X_M^{I,*} \equiv X_{n_I}^{I,*}$, that is, there is no inertia, as condition (26) is violated. When M is not pivotal for investment, i.e., $n_I \neq M$, the pivotal member n_I views the abandonment decision as suboptimal. In this case, the anticipation of M being pivotal in the future abandonment decision lowers the attractiveness of investment to n_I. As we show in Proposition 4 below, n_I responds to this perceived suboptimal abandonment by using the pivotal power to delay investment relative to the optimal autarkic timing. In this case the group exhibits inertia, since $X_G^I \equiv X_{n_I}^{I,\mathrm{SB}} > X_{n_I}^{I,*}$ and therefore condition (26) holds. Notice that, when inertia occurs, condition (26) also implies that the group delays investment relative to the autarkic investment time of a *majority* of group members.[13] To the extent that a majority

[13]To see this, note that inertia implies $n_I \neq M$. By equation (21), $X_{\mathrm{G}}^I \equiv X_{n_I}^{I,\mathrm{SB}} = \mathrm{Median}\{X_P^{I,\mathrm{SB}}, X_M^{I,*}, X_O^{I,\mathrm{SB}}\}$. As we assumed that there is a single pivotal member for investment, there must exist a group member $n \neq n_I$, such that $X_{\mathrm{G}}^I > X_n^{I,\mathrm{SB}}$. Proposition 4 below shows that $X_n^{I,\mathrm{SB}} \geq X_n^{I,*}$ and therefore $X_{\mathrm{G}}^I > X_n^{I,*}$. Hence, there exist a member $n \neq n_I$ such that $X_{\mathrm{G}}^I > \mathrm{Max}\{X_n^{I,*}, X_{n_I}^{I,*}\}$, implying that the group delays the investment relative to the autarkic investment time of a majority of group members.

of group members is unsatisfied with the group's investment timing, this result shows that inertia represents a cost of group membership.

Our model provides a novel and alternative mechanism for investment inertia within a standard real option setting. Existing work (see, e.g., Grenadier and Wang (2005)) shows that, in a dynamic model where investment decisions are delegated to managers, moral hazard leads to greater inertia, as the manager holds a more valuable option to wait than the owner. In our setting investment inertia obtains without asymmetric information and moral hazard, but emerges purely as a consequence of coordination frictions brought about by the heterogeneity of beliefs within a group.

Group underinvestment. The following definition formalizes the concept of underinvestment in a three-member group.

Definition 2 (**Underinvestment**). *Consider a three-member group governed by majority. Underinvestment occurs if there exists a license fee L and cash flow x for which the license is rejected by the group and yet it is accepted under autarky by a majority of group members, that is,*

$$V_{M,G}(x) < L < V_M^*(x). \tag{27}$$

When M is pivotal for investment, i.e., $n_I = M$, we have that $V_{M,\mathrm{G}}(x) = V_M^*(x)$ for all $x > 0$. Therefore, condition (27) is violated and underinvestment does not occur. However, when $n_I \neq M$, the investment timing policy adopted by the group is suboptimal from M's perspective and therefore, $V_{M,\mathrm{G}}(x) < V_M^*(x)$. The left inequality of equation (27) implies that the group rejects the license when its cost is L: Because, by equation (25) the license value is monotonic in beliefs, $V_{P,\mathrm{G}}(x) < V_{M,\mathrm{G}}(x)$, it follows that both P and M vote against licensing. Therefore, if condition (27) holds, a majority formed by P and M will reject the license. Furthermore, because, by equation (14), the individual license value is also monotonic in beliefs, it follows from the underinvestment condition (27) that $L < V_M^*(x) < V_O^*(x)$. Therefore, both M and O would acquire the individually-managed license and hence, implying that, under autarky, a majority would support licensing. In sum, underinvestment occurs because of the distortion caused by the expected change in pivotal members from the investment to abandonment votes. This causes M—who, along with O, would individually choose licensing—to withdraw support for the group-managed license.

The next proposition formalizes that a necessary and sufficient condition for inertia and underinvestment to occur is that the pivotal member for investment differs from the member with median beliefs, M.

Proposition 4 (Necessary and sufficient condition for inertia and underinvestment). *Consider a three-member group governed by majority. For any member $n \in \{P, O\}$, the second best investment threshold is strictly larger than the optimal investment threshold:*

$$X_{n}^{I,*} < X_{n}^{I,SB}. \tag{28}$$

Therefore, group inertia and underinvestment occur if and only if $n_I \neq M$.

Because, by Proposition 3, member M is always pivotal for both the licensing and abandonment decisions, Proposition 4 implies that a necessary and sufficient condition for inertia and underinvestment is that there is a change in the pivotal member across the decisions a group faces.[14] As discussed earlier, without an abandonment option, the pivotal member for investment timing is also pivotal for licensing, so there would be no underinvestment (or inertia).

The result of proposition 4 shows that, in our dynamic setting, the prescriptions of the "median voter theorem" of static voting models (e.g., Downs (1957)) are violated in the presence of coordination frictions. According to this theorem, with the exception of a "pivotal member," every other group member has no real voting power, despite having explicit voting rights.[15] An implication of the median voter theorem is that small changes in the beliefs of non-pivotal members would not affect the group's behavior, provided that these small changes do not affect the ranking of the pivotal members in the distribution of group beliefs. In our *dynamic* voting problem, these conditions are clearly not satisfied. To the extent that any member can be pivotal for the investment timing decision, individual preferences *do* influence the group behavior and the median voter theorem fails.

The result of Proposition 4 further highlights the potential drawbacks of majority voting in a dynamic setting. A well-known drawback of majority voting in a static context is that it can produce

[14] If P, the member more eager to abandon, could threaten to separate from the group in order to force abandonment, he would be pivotal for the abandonment decision. This alternative collective decision rule would still produce a change in the identity of the pivotal member over time from licensing, M, to abandonment, P.

[15] The conditions for the median voter theorem to hold require that agents vote along a single dimension and preferences are single-peaked. Because in our model group members vote over binary choices, both conditions are satisfied in our model for each single vote. However, in our setting voting is sequential. The interaction between consecutive votes creates the change of pivotal voters over time that ultimately precludes a representation of the group behavior as that of a single-member or even as the behavior of a subset of group members.

outcomes that are unsatisfactory from the perspective of minorities. The coordination frictions that we highlight further show that, in a dynamic context, majority voting can be unsatisfactory not only to minorities but also to majorities. Our result thus indicates that the drawbacks of majority voting are exacerbated in a dynamic setting.

3.1 Group coordination frictions and belief polarization

As proposition 4 shows, a necessary and sufficient condition for inertia or underinvestment is that the pivotal member for the investment timing decision is different from the member with median beliefs. We now explore how these group coordination frictions depend on three primitive elements of our model: (i) members' belief polarization, captured by the symmetric spread $\varepsilon > 0$ around the median belief μ_M, $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$; (ii) project reversibility, captured by the ratio of the abandonment value to investment cost, A/I; and (iii) project riskiness, captured by the cash flow volatility σ.

Figure 3 shows the investment threshold X_{G}^I in a three-member group, P, M, O, as a function of polarization. Lines marked with '+' refer to the baseline case in which the abandonment value is 50% of the investment cost ($A/I = 0.5$). Lines marked with '$\circ$' refer to the case of an irreversible project ($A/I = 0.05$) and lines marked with '$*$' refer to reversible projects ($A/I = 0.9$). In each line, the identity of the pivotal member for the investment decision, n_I, is highlighted by a different color: blue denotes $n_I = P$, red denotes $n_I = M$, and green denotes $n_I = O$. Finally, Panel A refers to projects with low cash flow volatility ($\sigma = 0.3$) and Panel B refers to projects with high cash flow volatility ($\sigma = 0.5$).

The figure highlights two main properties of the group investment threshold X_{G}^I. First, more polarization implies stronger group inertia. In all panels, the group investment threshold X_{G}^I is (weakly) increasing in group polarization, implying that the more diverse P and O's beliefs are, the later the group invests. Second, any group member can be pivotal for investment, as implied by Proposition 3. When $n_I = M$ (red markers) the group investment strategy corresponds to the individual optimal strategy of the median member, that is $X_{\mathrm{G}}^I = X_M^{I,*}$. In this case there is no investment inertia. In all other cases where $n_I \neq M$ (green and blue markers) the group investment threshold is larger than the individually optimal threshold of the median, $X_{G,n_I}^I > X_{n_I}^{I,*}$ for $n_I \neq M$, and increases in polarization.

The effect of project irreversibility ($\circ$-marked lines vs. $*$-marked lines) and cash flow volatility (Panel A vs Panel B) on the group investment thresholds are broadly consistent with standard intuition from the real option literature. In particular, when irreversibility is high ($\circ$-marked line), the option to invest is less attractive and, the investment threshold is larger. Similarly, when cash flow volatility is high (Panel B) the option to wait is more valuable and the group delays investment relative to the case of a low volatility project (Panel A). However, the top panel of Figure 3 shows that polarization can counterbalance the effect of irreversibility on the timing of investment. When polarization is sufficiently large, the group investment threshold for the baseline case of $A/I = 0.5$ ($+$-marked line) is *higher* than the group investment threshold for an irreversible project with $A/I = 0.05$ ($\circ$-marked line). Therefore, when beliefs are sufficiently polarized, contrary to standard intuition from single-managed real options, a group may delay investment in a more reversible project. This result highlights that belief heterogeneity and the resulting coordination friction within a group acts as an alternative channel to project characteristics in the determination of the timing of investment.

The condition for underinvestment in equation (27) provides an intuitive way to quantify the cost of group coordination frictions. Given the license values and $V_{M,G}(x) < V_M^*(x)$ we define the *coordination premium* $\delta > 0$ as the percentage of the cash flow x that member M would require as compensation for joining the group, i.e., $V_M^*(x) = V_{M,\mathrm{G}}(x\,(1+\delta))$. In other words, M is indifferent between being the sole manager managing a project with initial cash flow x or managing, as part of a group, a project with initial cash flow $x(1+\delta)$. Using the license value in equation (9), we obtain the following expression for the coordination premium:

$$\delta = \Gamma \cdot \frac{X_{\mathrm{G}}^I}{X_M^{I,*}} - 1, \quad \text{where} \quad \Gamma \equiv \left(\frac{W_M^*(X_M^{I,*}) - I}{W_{M,G}(X_{\mathrm{G}}^I) - I} \right)^{1/q_M}, \tag{29}$$

where X_{G}^I is the group investment threshold defined in equation (21); $X_M^{I,*}$ is member M's optimal investment threshold defined in equation (12), respectively; $W_M^*(X_M^{I,*}) \equiv W_M(X_M^{I,*}, X_M^{A,*})$ is member $M's$ optimal value of the operating project defined in equation (7); and the constant $q_M > 1$ is given in equation (10). As the expression in equation (29) shows, the coordination premium δ is zero in the absence of inertia, that is $X_{\mathrm{G}}^I = X_M^{I,*}$ and is directly related to the ratio of group and individual investment thresholds.

Figure 4 shows the coordination premium δ from equation (29) as a function of polarization. As in Figure 3, we consider the effect of irreversibility ($\circ$-marked lines vs. $*$-marked lines) and cash flow volatility (Panel A vs Panel B) on the coordination premium. Consistent with the analysis of group inertia in Figure 3, the two panels of Figure 4 shows that the group coordination premium δ is (i) positive only when the pivotal member for investment is different from the member with the median belief, $n_I \neq M$ and (ii) weakly increases with polarization. Moreover, Panel A of Figure 4 shows that, for large levels of polarization, the coordination premium required by M can be larger in the baseline case (+-marked lines) than in both the reversible and irreversible cases ($*$- and $\circ$-marked lines). These properties are not surprising, in light of the tight connection between the coordination premium δ and the group investment thresholds presented in equation (29).

4 Real options in a general group setting

In this section, we broaden the scope of our results and consider groups of any size, and more general voting protocols. In Section 4.1, we show that, under this general framework, the group behavior cannot be represented by a single group member or even a subgroup of members. In Section 4.2, we characterize underinvestment and inertia in this general setting and discuss whether different voting protocols can mitigate or exacerbate group coordination frictions.

4.1 Larger groups and more general voting rules

Consider a group of N members, $\mathcal{N} = \{1, \ldots, N\}$, with $N \geq 3$ an arbitrary integer. Each member n believes that the cash flow process $(X_t)_{t\geq 0}$ follows the dynamics described in equation (1), with $\mu_1 < \mu_2 < \ldots < \mu_N < r$. For each member n, we denote by $X_n^{A,*}$ and $X_n^{I,*}$ the optimal abandonment and investment thresholds defined in equations (11) and (12), respectively.

A *voting rule* refers to a set of instructions that determines group decisions based on voting outcomes. We consider the class of *quota rules* with majority requirement k, with k an integer in the interval $[N/2, N]$. According to a quota rule with majority requirement k, the group accepts a proposal if at least k group members vote in favor.[16] The quota governance rule nests as special cases: (i) unanimity, in which the quota is $k = N$; (ii) simple majority, in which the quota is

[16] All the results of this section remain valid if the group gives special veto power to a subset of members, such as the chair of a board of directors or the founders of a company.

$k = N/2 + 1$, if N is even, and $k = (N+1)/2$, if N is odd; and (iii) super-majority, in which the quota k satisfies $N/2 + 1 < k < N$.

Following a similar argument as in the three-member group of Section 2, we note that, because the abandonment thresholds are ranked, it follows that the majority $\{1, \ldots, k\}$ of group members supports abandonment only when the cash flow process hits $X_k^{A,*}$. Therefore member k is the pivotal member for abandonment, that is $n_A = k$. It follows that each group member determines his/her second best investment threshold anticipating that the group will abandon at the threshold $X_k^{A,*}$. As in the three-member group of Section 2, the pivotal member for investment n_I is the member with the k-th highest second best investment thresholds, that is,

$$\mid \{n : X_n^{I,\mathrm{SB}} \leq X_{n_I}^{I,\mathrm{SB}}\} \mid = k,$$

where, $\mid \cdot \mid$ denotes the count of the elements in a set.[17]

The following proposition characterizes the pivotal members for an N-member group governed by any quota voting rule.

Proposition 5 (**Ranking of license values**). *Consider an N-member group governed by a quota voting rule with a majority requirement k. Assume that the values of the group-managed operating project upon exercise to members $1, 2, \ldots, k-1$ are monotonic in the group members' beliefs, that is,*

$$I < W_{1,G}(X_G^I) < W_{2,G}(X_G^I) < \ldots < W_{k-1,G}(X_G^I). \tag{30}$$

where $X_G^I = X_{n_I}^{I,SB}$ and $X_G^A = X_k^{A,*}$ denote the equilibrium group investment and abandonment thresholds and $W_{n,G}(X_G^I) \equiv W_n(X_G^I, X_G^A)$, $n = 1, \ldots, k-1$. Then the individual valuations of the investment option are ranked for all $x > 0$, that is,

$$0 < V_{1,G}(x) < V_{2,G}(x) < \ldots < V_{N,G}(x), \tag{31}$$

and the pivotal member for licensing is $n_L = N - k + 1$.

Equation (30) requires that the intrinsic values of the group-managed investment options $W_{n,G}(X_{\mathrm{G}}^I) - I$, be non-negative and ranked for all group members $n = 1, \ldots, k-1$. This condition implies that the incentives to join the project is larger for the member with more optimistic

[17] As in the three-member group case, without loss of generality, we rule out the knife-edge cases with multiple pivotal members for investment.

beliefs.[18] By the inequalities in equation (31) the values of the license $V_{n,G}(x)$ under the group timing policy (X_G^I, X_G^A) are non-negative and uniformly ranked according to the group members' beliefs, with optimists valuing the license more than pessimists for every level of cash flow. The group will invest if the license fee L is smaller than the valuation of a subset of group members with cardinality k. Equation (31) implies that the smallest coalition that can lead the group to license is $\{N, N-1, .., N-k+1\}$. The least eager member to license in this coalition is member $n_L = N - k + 1$, that is, the pivotal member for licensing. Because the abandonment thresholds are monotonic in beliefs, the pivotal voter for abandonment is member $n_A = k$ and the smallest coalition that can lead the group to abandon is $\{1, 2, \dots, k\}$.

As in the three-member group in Section 2.3, in groups of any size, the second-best investment thresholds are not ranked monotonically with beliefs, implying that any member can be pivotal for investment. This plurality of pivotal members makes it difficult to represent the group behavior by that of a subset of members governed by a fictitious voting rule for all decisions.[19] In an odd-numbered group governed by majority, as in the case of Section 2, the quota is $k = (N+1)/2$. In this case, $N - k + 1 = k$ and therefore the pivotal member for licensing, $n_L = N - k - 1$, is also the pivotal member for abandonment, $n_A = k$, and corresponds to the member with median beliefs, $\mu_{(N+1)/2}$. If investment must happen at time zero, instead of being optimally timed, the group is *de-facto* ruled by the member with median beliefs who determines both the initial investment decision and the abandonment time. In contrast, in an even-numbered group, the quota is $k = N/2 + 1$ and therefore $n_L = N - k + 1 < k = n_A$, that is, the pivotal member for abandonment is more optimistic than the pivotal member for licensing. In this case, belief heterogeneity leads to a violation of the median voter theorem and to group coordination frictions *even* in the absence of an investment timing decision.

[18] An alternative to condition (30) would be to require only $I < W_{1,G}(X_G^I)$ and to impose a restriction on the dispersion of beliefs $\mu_1, \dots, \mu_n$. Under this alternative condition, we can prove that the value of the abandonment options are uniformly ranked, that is, for all $x > X_G^A$, $W_{1,G}(x) < W_{2,G}(x) < \dots < W_{k-1,G}(x)$. This property is sufficient to prove that the license value $V_{n,G}(x)$ is positive and monotonic in beliefs, for all $x > 0$, and therefore inequalities (31) hold.

[19] In the absence of the investment timing decision, it can be shown that the decisions of any N-member group governed by a quota rule with majority requirement k can be replicated by those of a fictitious group of two members, k and $N - k + 1$, governed by unanimity. This result is similar to that in the collective search problem studied by Compte and Jehiel (2010).

4.2 Inertia and underinvestment in a general framework

The concepts of group inertia and underinvestment introduced in Section 3 for the case of a three-member group generalize to a group of any size and to majority voting rules with general quota requirements. Following Definition 1, in an N-member group governed by a majority rule with k quota requirement, inertia occurs when $X_{\mathrm{G}}^{I} > X_{n_I}^{*}$ where $X_{n_I}^{*}$ is the optimal individual investment threshold of the pivotal member for the investment timing decision n_I, defined in equation (12). Similarly, following Definition 2, underinvestment occurs if there exists a cash flow level x and a licensing fee L for which

$$V_{n_L,\mathrm{G}}(x) < L < V_{n_L}^{*}(x), \tag{32}$$

where $n_L = N - k + 1$ is the pivotal member for the licensing decision and $V_{n_L,\mathrm{G}}(x) \equiv V_{n_L}(x, X_{\mathrm{G}}^{I}, X_{\mathrm{G}}^{A})$. Condition (32) states that n_L is willing to run the project alone but not as a member of the group. Being pivotal for the licensing decision, when L satisfies condition (32), n_L, and hence the group rejects the license.

The following proposition provides the conditions under which group inertia and underinvestment occur in an N-member group.

Proposition 6 (**Inertia and underinvestment**). *Consider a N-member group governed by a quota rule with majority requirement given by the integer $k \in [N/2, N]$. Denote by $n_L = N - k + 1$, n_I, and $n_A = k$ the pivotal members for, respectively, the licensing, investment timing, and abandonment timing decisions. Then:*

1. *The group exhibits inertia, i.e., $X_{G}^{I} > X_{n_I}^{*}$, if and only if $n_I \neq n_A$.*

2. *The group exhibits underinvestment:*

   (a) *If and only if $n_I \neq (N+1)/2$, when N is odd and the group is governed by majority;*

   (b) *Always, in all other cases.*

Proposition 6 shows that even with an arbitrary number of members and under a general class of voting rules, inertia occurs if and only if there is a change in the identity of pivotal member between the investment and abandonment timing decisions. For underinvestment, the result is more nuanced. The proposition shows that, in groups governed by a rule other than majority, the pivotal members for licensing and abandonment always differ, i.e., $n_L \neq n_A$. In particular, even if the pivotal member for investment timing is also pivotal for abandonment timing $n_I = n_A$, inertia does not occur but underinvestment can still occur because $n_L \neq n_A$. This implies that, under a voting rule other than majority or in a group with an even number of members, there is *always* a change in the pivotal members across the sequential decisions of a group. The analysis of this more general framework suggests a new economic force driving the change of the pivotal member. While the change of pivotal member for investment timing is due to the compound nature of the real option, the change of pivotal member between the licensing and abandonment timing decisions is related to the economic nature of these two decisions. The licensing decision implies a long position in an option while the abandonment decision implies a short position in the cash flow of the project.

When a voting rule is more stringent than majority, there are multiple members who belong simultaneously to the coalitions supporting abandonment and licensing. The pivotal members for licensing and abandonment are the "extremal points" of these coalitions.[20] When the intersection of these two coalitions contains a single member, as is the case for an odd-member group ruled by majority, the single member is the median-belief member and is then pivotal for both licensing and abandonment. For voting rules stricter than majority or in even-member groups, the extremal points of the two coalitions are always distinct, with the pivotal member for abandonment being more optimistic than the pivotal member for licensing. This economic force, which is absent in the three-member group of Section 2, is present even in a group-managed real option model with *no* investment timing, as long as the group is even-numbered or is governed by a voting rule more stringent than majority. For example, under the unanimity governance rule $k = N$, $n_L = 1$, and $n_A = N$. The underinvestment condition then becomes

$$V_{1,G}(x) \equiv V_1(x, X_{\mathrm{G}}^I, X_N^A) < L < V_1^*(x). \tag{33}$$

For any given distribution of beliefs among group members, the belief discrepancy between the two pivotal votes n_L and n_A is larger under the unanimity governance rule than under any other quota rule. This observation suggests that the coordination frictions are more severe with more stringent majority rules.

Figure 5 illustrates this point by comparing majority and unanimity on inertia and underinvestment in a three-member group with members P, M, and O. The figure shows the group investment

[20] For example, under unanimity, the coalition supporting licensing is identical to that supporting abandonment and is equal to the set of all members $\mathcal{N}$. The pivotal members for licensing and abandonment are therefore the extremal points of the set $\mathcal{N}$, that is, $n_L = 1$ and $n_A = N$.

threshold X^I_G (top panel) and the coordination premium δ from equation (29) associated with underinvestment (bottom panel), as a function of polarization. As in the previous figures, we define polarization as the spread $\varepsilon > 0$ from the median belief $\mu_M = r/2$ and set $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$. The lines marked with '+' refer to the case of a group governed by majority, while the lines marked with '$\circ$' refer to the case of a group governed by unanimity. The color of the marker in each line indicates the pivotal member n_I: P (blue), M (red), or O (green). As the figure shows, unanimity exacerbates the effect of beliefs heterogeneity on the group coordination friction. Under unanimity, inertia is stronger as the group invests at a much higher threshold. This reflects on the coordination premium δ that, in a group governed by unanimity, is significantly higher than in a group governed by majority.

5 Trading voting rights

An important assumption of our model is that group members cannot trade shares. Our primary motivation for this assumption is that it is descriptive of groups such as startups and small partnerships, where the complementarity of skills among members and financial constraints may limit the ability and the willingness to trade shares. Although in our model group members cannot undo their economic ownership of the project, this does not rule out the possibility of "decoupling," that is, trading votes separately from cash flow rights.[21]

In this section, we explore whether trading in voting rights prior to the licensing decision can eliminate group coordination frictions. In a group-managed real option, coordination frictions arise because a binary vote (license or not) does not allow members to express the intensity of their preferences. When a group makes decisions through voting, members who highly value licensing (e.g., member O) do not have more influence on the decision than members who value licensing less (e.g., M or P). Trading votes would allow members to modulate their influence on group decisions and could result in a voting power reallocation that is beneficial to all group members.

[21] Shareholders of public companies can rely on several strategies to decouple the economic right of a share from its voting right (Hu and Black (2005)). Privately-held companies issue dual class shares to deviate from the one-share-one-vote structure. The one-share-one-vote structure is a desirable arrangement because it aligns voting power with economic incentives and induces a well-functioning market for corporate control (Grossman and Hart (1988), Harris and Raviv (1988)). However, in the presence of asymmetric information, deviations from the one-share-one-vote structure, achievable through a market for votes, can promote efficient takeover activity (At, Burkart, and Lee (2011)).

	Net payoff from real option managed by:			
Member	Group	O	M	P
P	0	-119.733	-117.183	-117.006
M	0	-1.807	0.001	-0.212
O	0	950.220	946.251	943.001

Table 1: Market for votes

The table reports the no-trade allocation and the individual payoffs under m's management $V_n(x, X_m^{I,*}, X_m^{A,*}) - L$, $m, n \in \{P, M, O\}$. Parameter values: $x = 0.9 \times X_M^{I,*} = 14.3494$, $\mu_P = 0.01$, $\mu_M = 0.02$, $\mu_O = 0.04$; $r = 0.05$, $\sigma = 0.50$, $L = 394.24$, $I = 100$, and $A = 65$.

To illustrate this point, we consider the three-member group, P, M, and O governed by majority, introduced in Section 2. We choose model parameters under which the pivotal member for investment is $n_I = P$ and condition (27) for underinvestment holds. Table 1 shows the net payoff to member n when member m is a "dictator" in control of both investment and abandonment timing decisions, that is, $V_n(x, X_m^{I,*}, X_m^{A,*}) - L$, $m, n \in \{P, M, O\}$. In the absence of a market for votes, the group forgoes the license and the net payoff to each member from this group-managed real option is zero as described in the second column of Table 1.

Giving O the power to manage the project would improve the sum of all members' payoffs relative to the voting outcome, that is,

$$\sum_n \left(V_n(x, X_O^{I,*}, X_O^{A,*}) - L\right) = 828.67 > 0.$$

This allocation can be implemented, for example, if O buys both M and P's votes at the unit price $p = 130$. In this case, the after-trading allocation to P, M, and O is $(-119.733 + 130, -1.807 + 130, 950.22 - 2 \times 130) = (19.267, 128.193, 690.22)$, which Pareto dominates the no-trading outcome $(0,\ 0,\ 0)$. This trade would resolve the coordination friction.

The assumption that group members do not trade votes in our setting captures the decision making process of many corporate boards and partnerships. In these settings, group members consider the right to vote as an essential part of the management function of the group, rather than a good that can be traded. Moreover, in large, multi-project organizations, a specific project's control rights cannot be traded independently from shares. Even if one were to assume that group members are willing to trade their right to vote, it is not clear that a market for votes can eliminate group coordination frictions and lead to an efficient outcome. The literature on social choice emphasizes that, since a vote is a "good" that generates an externality. In the context of a three-member group, the trading of votes between O and M makes one of them a dictator, an outcome that may be quite undesirable to the non-trading member P. In more general groups, votes trading can generate both positive and negative externalities on non-trading members. The presence of externalities implies that markets for votes typically lead to inefficient outcomes (see, e.g., Casella and Macé (2020)).

To illustrate this point, in Appendix B we provide an example of a failure in the market for votes, based on the three-member group in Table 1. The example shows that a competitive equilibrium in the market for votes fails to exist since there is no price at which the market can clear.[22]The failure of competitive markets for votes does not imply that group members do not trade vote in practice. It suggests however that self-interest makes it difficult for a market for votes to be welfare improving. Addressing the externalities through a market for votes requires a coordination effort by either some of the group members (e.g., the founding partners) or through regulation. In sum, our results are descriptive of groups where members are either not willing to trade votes, or cannot resolve the externalities associated with votes trading. These assumptions are descriptive for many privately-held companies and board of directors.

6 Empirical implications

The key economic insight of our model is that, in a setting where a group of heterogeneous members faces sequential decisions and disagreements are resolved through voting, coordination frictions can distort both the quantity and timing of investments. Our theory shows that the effect of group decisions on investment distortions depends on (a) the nature of the group and (b) the nature of the investment opportunity. In turn, these provide natural areas of focus for empirical studies.

With respect to the nature of the group, our model shows that the more polarized are the group beliefs, the more reluctant the group is to invest, both initially and in subsequent expansions. Directly measuring polarization is difficult but indirect measures have already been applied in other contexts. For instance, Adams, Akyol, and Verwijmeren (2018) show that more diverse boards are associated with poorer corporate performance. They suggest that the diversity of skill that they

[22]Casella, Llorente-Saguer, and Palfrey (2012) provide a formal analysis of the non existence of a competitive equilibrium in the market for votes and offer experimental evidence. Early work that emphasizes failures in a competitive market for vote include Ferejohn (1974), Philipson and Snyder (1996), Kultti, Salonen, et al. (2005). The survey by Casella and Macé (2020) provides a comprehensive review of the literature on vote trading.

document may proxy for diversity of beliefs. In light of our results, therefore, empirical explorations can go further to relate board diversity to investment quantity and timing. Similarly, Balsmeier, Fleming, and Manso (2017) find that firms with boards having more independent directors tend to be less innovative. If we accept that independent directors bring different beliefs to the table, then our analysis suggests that investment levels and timing are related to the degree of director independence. More broadly, the implications of our model are consistent with the existing literature on the composition of the shareholder base that documents a positive effect of shareholder base cohesiveness on firm valuation (e.g., Kandel, Massa, and Simonov (2011), Schwartz-Ziv and Volkova (2020) and Brav, Jiang, Li, and Pinnington (2018)).

While a corporate board fits our description of a group, venture capital (VC) syndicates are also closely aligned with our theoretical construct. Existing empirical evidence on VCs is consistent with our assumption that heterogeneous groups make investment decisions. For instance, according to Nanda and Rhodes-Kropf (2018), over a recent 15-year period the average startup that received VC funding had 3 investors. Furthermore, Guler (2007) finds that VCs vary considerably in the degree to which they exercise their abandonment option. In addition, when VCs differ in their exit strategy, they are also more likely to have conflicting views on whether to extend financing.

In the context of venture capital syndicates, then, our framework provides a new alternative perspective on decision-making. As emphasized by Nanda and Rhodes-Kropf (2018), while the academic literature has extensively studied the frictions emerging from the asymmetric information between the VC and the entrepreneur, relatively little work has been devoted to the study of the coordination frictions emerging when multiple investors come together to finance a new venture.[23] In the context of our theory, the design of syndicates—e.g., the pervasiveness of "relational contracts," (Baker, Gibbons, and Murphy (2002))—and the use of contractual features observed in practice—e.g., the pervasiveness of dual-class shares in VC-backed companies (Gornall and Strebulaev (2020))—can represent a response to the inefficiencies driven by the dynamic voting structure we study. Similarly, Gompers, Mukharlyamov, and Xuan (2016) show that individual venture capitalists tend to associate with other venture capitalists having common characteristics and backgrounds (e.g., ethnicity, education, past employer, and degree from a top university). Consistent with our theory, the evidence that VC syndicates tend to be formed by investors with

[23]This literature is too vast to be reviewed here. Seminal contributions are, for example, Gompers (1995), Kaplan and Strömberg (2003, 2004), Hellmann (1998), and Cornelli and Yosha (2003).

similar characteristics indicates a desire to mitigate the cost of disagreement via repeated game interactions.

7 Conclusion

We examine the acquisition and subsequent management of a real option by a group of members with heterogeneous beliefs that make licensing and investment/abandonment timing decisions through voting. We show that for this general class of group-managed real options the group behavior cannot be subsumed by the behavior of a representative member, even in a group with an odd number of members. This result is a general property that emerges from the compound nature of the investment/abandonment real option. Specifically, while abandonment timing decisions are ranked according to the group members' beliefs, the same is not necessarily true for investment timing decisions. Depending on belief polarization and on project characteristics such as cash flow volatility and reversibility, more pessimistic members can be more eager to invest even though they place a lower value on the project than optimists. As a consequence, the group member with median beliefs is not necessarily the member with median investment timing. This change of pivotal voter across decisions over time implies a failure of the median voter theorem in groups of any size and under general voting rules. The absence of a representative agent subsuming the behavior of the group results in investment inertia, in that the group delays investment relative to the representative agent case, and underinvestment, in that the group rejects projects that are supported by a majority.

Our model shows that frictions like inertia and underinvestment are the result of differences in beliefs among group members. Obviously, diversity of opinions may also bring benefits to the group such as, for example, better project selection, superior access to information, and better understanding of the product market. These economic channels are outside the scope of our model. In a broader context, our finding that disagreement leads to coordination frictions suggests that, in equilibrium, decision making by groups with heterogeneous beliefs may distort project selections by favoring technologies where coordination frictions are less likely to arise.

While our model is a stylized description of the final decision-making process of groups, in reality, votes are cast in the context of dynamic pre-vote interactions. Developing theories that realistically capture the political economy of corporate decisions is a fascinating subject for future research.

A Appendix: Proofs

Proof of Proposition 1

Given an arbitrary abandonment threshold X^A, the value of the operating project $W_n(x, X^A)$ is

$$W_n(x, X^A) = \begin{cases} \mathbb{E}_n\left[\int_0^{\nu_{X_A}} X_t e^{-rt}dt + Ae^{-r\nu_{X_A}} \,\middle|\, X_0 = x\right] & \text{if} \quad x \geq X^A \\ A & \text{if} \quad x < X^A \end{cases}, \tag{A.1}$$

with $\nu_{X^A} = \inf\left\{t \geq 0 \ : \ X_t \leq X^A\right\}$. The expression of $W_n(x, X^A)$ for $x \geq X^A$ in equation (A.1) can be written as follows

$$\mathbb{E}_n\left[\left[\int_0^{\infty} X_t e^{-rt}dt - \int_{\nu_{X_A}}^{\infty} X_t e^{-rt}dt + Ae^{-r\nu_{X_A}}\right| X_0 = x\right] =$$

$$\frac{x}{r-\mu_n} + \left(A - \frac{X^A}{r-\mu_n}\right)\mathbb{E}_n[e^{-r\nu_{X_A}} \,|\, X_0 = x], \tag{A.2}$$

where, by standard argument (see, e.g., Øksendal (2013)),

$$\mathbb{E}_n\left[e^{-r\nu_{X_A}} \,\middle|\, X_0 = x\right] = \left(\frac{x}{X^A}\right)^{m_n}, \quad x \geq X^A, \tag{A.3}$$

where $m_n < 0$, given in equation (8), is the negative root of the quadratic equation $h(\beta) = 0$, with

$$h(\beta) = \mu_n\beta + \frac{1}{2}\sigma^2\beta(\beta - 1) - r. \tag{A.4}$$

Substituting equation (A.3) in the expression (A.2) shows that equation (7) holds.

Given arbitrary investment and abandonment thresholds X^I and X^A, the value of the license $V_n(x, X^I, x^A)$ is

$$V_n(x, X^A) = \begin{cases} W_n(x, X^A) - I & \text{if} \quad x > X^I \\ \mathbb{E}_n\left[e^{-r\tau_{X^I}}(W_n(X_{\tau_{X^I}}, X^A) - I) \,\middle|\, X_0 = x\right] & \text{if} \quad x \leq X^I \end{cases}, \tag{A.5}$$

with $\tau_{X^I} = \inf\left\{t \geq 0 \ : \ X_t \geq X^I\right\}$. Since $X_{\tau_{X^I}} = X^I$, for $x \leq X^I$, we have

$$V_n(x, X^I, X^A) = (W_n(X^I, X^A) - I)\,\mathbb{E}_n\left[e^{-r\tau_{X^I}} \,\middle|\, X_0 = x\right],$$

where, by a similar argument as above,

$$\mathbb{E}_n\left[e^{-r\tau_{X^I}}\,\middle|\,X_0 = x\right] = \left(\frac{x}{X^I}\right)^{q_n}, \quad x \leq X^I, \tag{A.6}$$

with $q_n > 1$ the positive root of the quadratic equation (A.4) given in equation (10). Substituting (A.6) in equation (A.5) shows that equation (9) holds.

∎

Proof of Proposition 2

Differentiating equation (7) with respect to X^A when $x \geq X^I$, we obtain that the optimal abandonment threshold for member n is

$$X_n^{A,*} = A\frac{m_n}{m_n - 1}(r - \mu_n) = A\left(r + m_n\frac{\sigma^2}{2}\right), \quad n \in \{P, M, O\}, \tag{A.7}$$

where the second equality exploits the fact that $m_n < 0$ is a root of the quadratic equation (A.4). From equation (8), it can be shown that m_n is decreasing in μ_n. Hence $\mu_P < \mu_M < \mu_O$ implies $m_O < m_M < m_P$ and, from equation (A.7), $X_O^{A,*} < X_M^{A,*} < X_P^{A,*}$. This implies that, acting individually, P will abandon the project earlier than O, $\nu_{X_P^{A,*}} < \nu_{X_M^{A,*}} < \nu_{X_O^{A,*}}$, a.s.[24]

Taking $X^A = X_n^{A,*}$ as given and differentiating equation (9) with respect to X^I when $x \leq X^I$ we obtain that the optimal investment threshold $X_n^{I,*}$ is a root of the non-linear equation $g_n(X_n^{I,*}) = 0$ where the function g_n is defined by

$$g_n(x) \equiv \frac{x}{r - \mu_n}(q_n - 1) + \left(A - \frac{X_n^{A,*}}{r - \mu_n}\right)\left(\frac{x}{X_n^{A,*}}\right)^{m_n}(q_n - m_n) - q_n I, \quad n \in \{P, M, O\}. \tag{A.8}$$

Inspection of equation (A.8) shows that the function g_n is convex with $g_n(0) = g_n(+\infty) = +\infty$ and $g_n(X_n^{A,*}) = (A - I)\,q_n < 0$. Therefore, equation $g_n(x) = 0$ has two solutions $x_{n,1}$ and $x_{n,2}$, with $x_{n,1} < X_n^{A,*} < x_{n,2}$. The optimal investment threshold corresponds to the largest of the two roots, that is, $X_n^{I,*} \equiv x_{n,2}$. The individual optimal valuations of the firm at time 0 are then given by

$$V_n^*(x) = V_n(x, X_n^{I,*}, X_n^{A,*}), \quad n \in \{P, M, O\},$$

[24] The a.s. statement is valid under the three subjective probabilities governing members' beliefs, as they are mutually absolutely continuous.

where V_n, $X_n^{A,*}$, and $X_n^{I,*}$ are given in equations (9), (11), and (12).

We now show that $V_P^*(x) < V_M^*(x)$ for all $x > 0$. The proof $V_M^*(x) < V_O^*(x)$ is is similar and will be omitted. For $n \in \{P, M\}$, we denote the optimal stopping times by $\tau_n^{I,*} = \inf\left\{t \geq 0 \ : \ X_t \geq X_n^{I,*}\right\}$ and, $\nu_n^{A,*} = \inf\left\{t \geq \tau_n^{I,*} \ : \ X_t \leq X_n^{A,*}\right\}$. Then,

$$
\begin{aligned}
V_P^*(x) &= \mathbb{E}_P\left[-Ie^{-r\tau_P^{I,*}} + \int_{\tau_P^{I,*}}^{\nu_P^{A,*}} X_t e^{-rt}dt + Ae^{-r\nu_P^{A,*}} \middle| X_0 = x\right] \\
&< \mathbb{E}_P\left[-Ie^{-r\tau_P^{I,*}} + \int_{\tau_P^{I,*}}^{\nu_P^{A,*}} X_t e^{(\mu_M - \mu_P)t} e^{-rt}dt + Ae^{-r\nu_P^{A,*}} \middle| X_0 = x\right] \\
&= \mathbb{E}_P\left[-Ie^{-r\tau_P^{I,*}} + \int_{\tau_P^{I,*}}^{\nu_P^{A,*}} Y_t e^{-rt}dt + Ae^{-r\nu_P^{A,*}} \middle| X_0 = x\right]
\end{aligned}
$$

where the above inequality follows from $\mu_M > \mu_P$ and where we denote $Y_t \equiv X_t e^{(\mu_M - \mu_P)t}$. Under P's subjective probability measure, denoted by $\mathbb{Q}_P$, the process Y_t is a geometric Brownian motion satisfying

$$dY_t = \mu_M Y_t dt + \sigma Y_t dB_{P,t}, \quad Y_0 = x$$

and furthermore, the stopping times $(\tau_P^{I,*}, \nu_P^{A,*})$ can be written as functionals of the path of the process $(Y_t)_{t \geq 0}$ as follows:

$$\tau_P^{I,*} = \inf\left\{t \geq 0 \ : \ Y_t \geq e^{(\mu_M - \mu_P)t} X_P^{I,*}\right\} \quad \text{and} \quad \nu_P^{A,*} = \inf\left\{t \geq \tau_P^{I,*} \ : \ Y_t \leq e^{(\mu_M - \mu_P)t} X_P^{A,*}\right\}.$$

Notice that the law of the triplet $(Y_t, \tau_P^{I,*}, \nu_P^{A,*})$ under P's subjective beliefs $\mathbb{Q}_P$ is identical to the law of the triplet $(X_t, \hat{\tau}_P^{I,*}, \hat{\nu}_P^{A,*})$ under M's subjective beliefs $\mathbb{Q}_M$, where we define the stopping times $(\hat{\tau}_P^{I,*}, \hat{\nu}_P^{A,*})$ as follows:

$$\hat{\tau}_P^{I,*} = \inf\left\{t \geq 0 \ : \ X_t \geq e^{(\mu_M - \mu_P)t} X_P^{I,*}\right\} \quad \text{and} \quad \hat{\nu}_P^{A,*} = \inf\left\{t \geq \hat{\tau}_P^{I,*} \ : \ X_t \leq e^{(\mu_M - \mu_P)t} X_P^{A,*}\right\}.$$

Therefore, for any $x > 0$, we have

$$\begin{aligned} V_P^*(x) &< \mathbb{E}_P\left[-Ie^{-r\tau_P^{I,*}} + \int_{\tau_P^{I,*}}^{\tau_P^{A,*}} Y_t e^{-rt}dt + Ae^{-r\tau_P^{A,*}}\middle| X_0 = x\right] \\ &= \mathbb{E}_M\left[-Ie^{-r\hat{\tau}_P^{I,*}} + \int_{\hat{\tau}_P^{I,*}}^{\hat{\nu}_P^{A,*}} X_t e^{-rt}dt + Ae^{-r\hat{\nu}_P^{A,*}}\middle| X_0 = x\right] \\ &\leq \sup_{\tau\leq\nu}\mathbb{E}_M\left[-Ie^{-r\tau} + \int_{\tau}^{\nu} X_t e^{-rt}dt + Ae^{-r\nu}\middle| X_0 = x\right] \\ &= V_M^*(x) \end{aligned}$$

These inequalities show that $V_P^*(x) < V_M^*(x)$ for all $x > 0$. ∎

Proof of Proposition 3

Ranking of operating project values $W_{n,G}(x)$ in equation (24). Let us start by proving the inequality $W_{M,G}(x) < W_{O,G}(x)$ for $x > X_G^A$.

By definition, $W_{n,G}(x) \equiv W_n(x, X_M^{A,*})$, $n \in \{P, M, O\}$, where $W_n(x, X_M^{A,*})$ is given in equation (7) and $X_M^{A,*}$ is given in equation (11). We denote by $W_n'(x, X_M^{A,*})$ and $W_n''(x, X_M^{A,*})$, $n = M, O$, the first and second derivative of $W_n(x, X_M^{A,*})$ with respect to x. Using the expressions for the optimal abandonment threshold $X_n^{A,*}$ in equation (11) we can express the first derivative of W_n as follows

$$W_n'(X_M^{A,*}, X_M^{A,*}) = \frac{1 - m_n}{X_M^{A,*}(r - \mu_n)}\left(X_M^{A,*} - X_n^{A,*}\right) \text{ for } n = M, O. \tag{A.9}$$

Equation (A.9) implies $W_{M,G}'(X_G^A) \equiv W_M'(X_M^{A,*}, X_M^{A,*}) = 0$ and $W_{O,G}'(X_G^A) \equiv W_O'(X_M^{A,*}, X_M^{A,*}) > 0$, since $m_n < 0$ and $X_M^{A,*} > X_O^{A,*}$. The second derivative of W_O is

$$W_{O,G}''(x) \equiv W_O''(x, X_M^{A,*}) = m_O(m_O - 1)\left(A - \frac{X_M^{A,*}}{r - \mu_O}\right)\frac{x^{m_O - 2}}{(X_M^{A,*})^{m_O}} \text{ for } x \geq X_P^{A,*}. \tag{A.10}$$

Since $m_O < 0$, $W_{O,G}(x)$ is convex in x if μ_O is such that $A - \frac{X_M^{A,*}}{r-\mu_O} > 0$ and concave otherwise. When $W_{O,G}(x)$ is convex, we have for $x \geq X_M^{A,*}$,

$$0 < W_{O,G}'(X_M^{A,*}) < W_{O,G}'(x)$$

where the left inequality follows from (A.9). When $W_{O,G}(x)$ is concave, we have for $x \geq X_M^{A,*}$

$$W'_{O,G}(x) > \lim_{x\to\infty} W'_{O,G}(x) = \frac{1}{r-\mu_O} > 0.$$

Therefore, for all $\mu_O \in (\mu_M, r)$,

$$W'_{O,G}(x) > 0, \quad \text{for all} \quad x \geq X_M^{A,*} = X_{\mathrm{G}}^{A} \tag{A.11}$$

Let us define the operator

$$\mathcal{L}_n = \mu_n x \frac{\partial}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2} \quad \text{for} \quad n = M, O. \tag{A.12}$$

Standard arguments relying on Itô's lemma show that, for $n = M, O$, the function $W_{n,G}(x)$ is a solution to the following boundary value problem

$$\begin{aligned} rW_{n,G}(x) &= \mathcal{L}_n W_{n,G}(x) + x, && x > X_M^{A,*}, \\ W_{n,G}(x) &= A, && x \leq X_M^{A,*} \end{aligned}$$

Note that, by definition of the operator $\mathcal{L}$ in equation (A.12), we can write $\mathcal{L}_O = \mathcal{L}_M + \xi\, x\, \frac{\partial}{\partial x}$ where $\xi \equiv \mu_O - \mu_M > 0$. Therefore we have that the function $w(x) \equiv W_{O,G}(x) - W_{M,G}(x)$ is a solution to the following boundary value problem

$$rw(x) = \mathcal{L}_M w(x) + \xi\, x\, W'_{O,G}(x), \qquad x > X_M^{A,*}, \tag{A.13}$$

$$w(x) = 0, \qquad x \leq X_M^{A,*}. \tag{A.14}$$

Denoting by $\nu \equiv \nu_{X_M^{A,*}} = \inf\left\{t \geq 0 \ : \ X_t \leq X_M^{A,*}\right\}$ and by $t \wedge \nu = \inf\{t, \nu\}$, equation (A.13) implies that the process

$$N_t = e^{-r\,t\wedge\nu} w(X_{t\wedge\nu}) + \int_0^{t\wedge\nu} e^{-rs} \xi X_s W'_{O,G}(X_s) ds$$

is a local martingale under the probability measure $\mathbb{Q}_M$. It follows that there exists a sequence of stopping times $\theta_n \uparrow \infty$ such that the process $(N_{\theta_n})_n$ is a martingale. Therefore,

$$w(x) = N_0 = \mathbb{E}_M\left[N_{\theta_n} \middle| X_0 = x\right] = \mathbb{E}_M\left[e^{-r\theta_n\wedge\nu} w(X_{\theta_n\wedge\nu}) + \int_0^{\theta_n\wedge\nu} e^{-rs}\xi X_s W'_{O,G}(X_s) ds \middle| X_0 = x\right]. \tag{A.15}$$

We now show that $w(x) > 0$ and, thererore, $W_{O,G}(x) > W_{M,G}(x)$, for all $x > 0$. We first consider the integral term in equation (A.15). Since $W'_{O,G}(X_s) > 0$, the integrand in equation (A.15) is non-negative. By the monotone convergence theorem we therefore have

$$\lim_{n\to\infty} \mathbb{E}_M \left[\int_0^{\theta_n \wedge \nu} e^{-rs} \xi X_s W'_{O,G}(X_s) ds \middle| X_0 = x \right] = \mathbb{E}_M \left[\int_0^{\nu} e^{-rs} \xi X_s W'_{O,G}(X_s) ds \middle| X_0 = x \right] > 0. \tag{A.16}$$

Next, consider the first term in the expectation on the right hand side of equation (A.15). Using the expression of W_n in equation (7), we can show that both W_M and W_O satisfy $W_n(x, X_M^{A,*})| \leq a_n + b_n|x|$ for some $a_n, b_n > 0$ and for $n = M, O$. Therefore, since $w(x) \equiv W_{O,G}(x) - W_{M,G}(x) \equiv W_O(x, X_M^{A,*}) - W_M(x, X_M^{A,*})$, we have $|w(x)| \leq a + b|x|$ for some $a, b > 0$. Thus we can write

$$e^{-r\theta_n \wedge \nu} w(X_{\theta_n \wedge \nu}) \leq a + b e^{-r\theta_n \wedge \nu} X_{\theta_n \wedge \nu} \leq a + b \sup_{t \geq 0} |e^{-rt} X_t|.$$

Let $B_{M,t}$ denote a Brownian motion under the probability measure $\mathbb{Q}_M$ and let $\alpha > 0$ be a constant. It is known (see, e.g., equation (4.2) in Doob (1949)) that the distribution of the lifetime maximum of a Brownian motion with negative drift, $\left(B_{M,t} - \alpha t\right)_{t \geq 0}$, is given by

$$\mathbb{Q}_M \left(\sup_{t \geq 0} \left(B_{M,t} - \alpha t\right) \geq \beta \right) = e^{-2\alpha\beta} \text{ for all } \beta \geq 0.$$

Because $r > \mu_M$, using the fact that X_t follows the geometric Brownian motion under $\mathbb{Q}_M$given in equation (1), we can choose $\alpha = \frac{r - \mu_M}{\sigma} + \frac{\sigma}{2} > 0$ and $\beta = \sigma^{-1} \ln\left(\frac{y}{x}\right) \geq 0$ for $y \geq x = X_0$ and obtain

$$\mathbb{Q}_M \left(\sup_{t \geq 0} \left(B_{M,t} - \alpha t\right) \geq \beta \right) = \mathbb{Q}_M \left(\sup_{t \geq 0} \left(e^{-rt} X_t\right) \geq y \right) = \left(\frac{y}{x}\right)^{-2\left(\frac{r-\mu_M}{\sigma^2} + \frac{1}{2}\right)}. \tag{A.17}$$

From equation (A.17), the cumulative density function of the random variable $\sup_{t \geq 0}\left(e^{-rt} X_t\right)$ is $F(y) = 1 - \left(\frac{y}{x}\right)^{-2\left(\frac{r-\mu_M}{\sigma^2} + \frac{1}{2}\right)}$ and hence the density is

$$f(y) = 2\left(\frac{r - \mu_P}{\sigma^2} + \frac{1}{2}\right) y^{-1} \left(\frac{y}{x}\right)^{-2\left(\frac{r-\mu_M}{\sigma^2} + \frac{1}{2}\right)}, \text{ for all } y \geq x.$$

Therefore,

$$\mathbb{E}_M \left[\sup_{t \geq 0} |e^{-rt} X_t| \right] = \int_x^\infty y f(y) dy = x \left(1 + \frac{\sigma^2}{2(r - \mu_M)}\right),$$

and, by the dominated convergence theorem, we have

$$\lim_{n\to\infty} \mathbb{E}_{M}\left[e^{-r\theta_n\wedge\nu} w(X_{\theta_n\wedge\nu})\right] = \mathbb{E}_{M}\left[e^{-r\nu} w(X_\nu)\right] = 0, \tag{A.18}$$

where the last equality follows from the boundary condition (A.14), noting that $\nu \equiv \tau_{X_M^{A,*}}$ and therefore $X_\nu = X_M^{A,*}$. Substituting equations (A.16) and (A.18) in equation (A.15), we obtain

$$w(x) = \mathbb{E}_{M}\left[\int_0^{\tau_{X_M^{A,*}}} e^{-rs}\xi X_s W'_{O,G}(X_s)ds\right] \text{ for all } x > X_M^{A,*}. \tag{A.19}$$

Since equation (A.11) shows that $W'_{O,G}(X_s) > 0$ for all $x \geq X_M^{A,*}$ we have $w(x) \equiv W_{O,G}(x) - W_{M,G}(x) > 0$, or, $W_{M,G}(x) < W_{O,G}(x)$, for all $x > X_M^{A,*}$. A similar proof also shows that $W_{P,G}(x) < W_{M,G}(x)$ for all $x > X_M^{A,*}$.

Ranking of license values $V_{n,G}(x)$ in equation (25). By definition, we have

$$V_{n,G}(x) = V_n(x, X_{\mathrm{G}}^I, X_{\mathrm{G}}^A) \;=\; \begin{cases} W_n(x, X_{\mathrm{G}}^A) - I, & \text{if } x \geq X_{\mathrm{G}}^I \\ \left(W_n(X_{\mathrm{G}}^I, X_{\mathrm{G}}^A) - I\right)\left(\frac{x}{X_{\mathrm{G}}^I}\right)^{q_n}, & \text{if } x < X_{\mathrm{G}}^I \end{cases}. \tag{A.20}$$

We prove first the inequality $V_{P,G}(x) > 0$ for all $x > 0$. From equation (A.20), condition (23) implies $V_{P,G}(x) > 0$, when $x < X_{\mathrm{G}}^I$. When $x \geq X_{\mathrm{G}}^I$, $V_{P,G}(x) = W_P(x, X_{\mathrm{G}}^A) - I$. To show that $V_{P,G}(x) > 0$ for $x > X_{\mathrm{G}}^I$, note first that $X_{\mathrm{G}}^A < X_{\mathrm{G}}^I$. This is because if $X_{\mathrm{G}}^I \leq X_{\mathrm{G}}^A$, then $V_{n_I,G}(X_{\mathrm{G}}^I) = A - I < 0$. This contradicts the fact that member n_I solves the (second best) optimal stopping problem (19). This problem must yield a non negative value because it is always possible for member n_I to never invest by choosing the stopping time $\tau = \infty$. For $x > X_G^A$, we have

$$\begin{aligned} W''_P(x, X_G^A) &= m_P(m_P - 1)\left(A - \frac{X_G^A}{r-\mu_P}\right)\frac{x^{m_P-2}}{(X_G^A)^{m_P}} \\ &= m_P(m_P-1)\frac{1}{r-\mu_P}\left(X_P^{A,*} - X_G^A - \frac{X_P^{A,*}}{m_P}\right)\left(\frac{x}{X_G^A}\right)^{m_P} > 0, \end{aligned} \tag{A.21}$$

where the second equality follows from the definition of the optimal abandonment threshold $X_P^{A,*}$ in equation (11). Therefore the function $W_P(\cdot, X_G^A)$ is convex on the region $(X_{\mathrm{G}}^A, \infty)$. From equation (A.9), using $X_{\mathrm{G}}^{A} \equiv X_{M}^{A,*}$, we have

$$W_P'(X_G^A, X_G^A) = \frac{1 - m_P}{X_G^A(r - \mu_P)} \left(X_G^A - X_P^{A,*}\right) < 0,$$

since $m_P < 0$ and $X_{\mathrm{G}}^{A} \equiv X_{M}^{A,*} < X_{P}^{A,*}$. Moreover, by direct calculation, we have $\lim_{x\to\infty} W_P'(x, X_G^A) = \frac{1}{r-\mu_P}$. The convexity of W_P implies that the function $W_P'(x, X_G^A)$ increases from $W_P'(x = X_G^A, X_G^A) < 0$ to $\frac{1}{r-\mu_P} > 0$ as $x \to \infty$. Because, by condition (23), $W_P(X_{\mathrm{G}}^I, X_{\mathrm{G}}^A) > I > A = W_P(X_{\mathrm{G}}^A, X_{\mathrm{G}}^A)$, the function $W_P(\cdot, X_G^A)$ must be increasing in a neighborhood of $x = X_{\mathrm{G}}^I$ and therefore $W_P'(x, X_{\mathrm{G}}^A) > 0$ for all $x \geq X_{\mathrm{G}}^I$. The monotonicity of the function $W_P(\cdot, X_G^A)$ in the interval $[X_{\mathrm{G}}^I, \infty)$ implies that

$$W_{P,G}(x) \equiv W_P(x, X_G^A) > W_P(X_G^A, X_G^A) > I, \quad \text{for} \quad x > X_{\mathrm{G}}^I, \tag{A.22}$$

and hence $V_{P,G}(x) = W_P(x, X_G^A) - I > 0$ for all $x > X_G^I$. In sum, we have $V_{P,G}(x) > 0$ for all $x > 0$.

We next prove the inequality $V_{P,\mathrm{G}}(x) < V_{M,\mathrm{G}}(x)$ from equation (25). When $x > X_{\mathrm{G}}^I$, we have

$$V_{M,\mathrm{G}}(x) - V_{P,\mathrm{G}}(x) = (W_M(x, X_G^A) - I) - (W_P(x, X_G^A) - I) = W_M(x, X_G^A) - W_P(x, X_G^A) > 0$$

where the inequality follows from the operating project ranking in equation (24). Hence, $V_{M,\mathrm{G}}(x) > V_{P,\mathrm{G}}(x)$ for $x > X_{\mathrm{G}}^I$. For $x \leq X_{\mathrm{G}}^I$, we have

$$\begin{aligned} V_{P,G}(x) &= \left(W_P(X_G^I, X_G^A) - I\right)\left(\frac{x}{X_G^I}\right)^{q_P} < \left(W_M(X_G^I, X_G^A) - I\right)\left(\frac{x}{X_G^I}\right)^{q_P} \\ &\leq \left(W_M(X_G^I, X_G^A) - I\right)\left(\frac{x}{X_G^I}\right)^{q_M} = V_M(x, X_G^I, X_G^I) \equiv V_{M,G}(x) \end{aligned}$$

where the first equality follows from equation (A.20); the first (strict) inequality follows from (24), and the second inequality follows from the fact that $q_M < q_P$[25] and that $x/X_{\mathrm{G}}^I < 1$. This concludes the proof that for $x > 0$, we have $0 < V_{P,\mathrm{G}}(x) < V_{M,\mathrm{G}}(x)$, as claimed in equation (25). The proof of the inequality $V_{M,\mathrm{G}}(x) < V_{O,\mathrm{G}}(x)$ in equation (25) follows similar steps and is omitted. ∎

[25]Differentiating q_P with respect to μ_P we obtain $\frac{\partial q_P}{\partial \mu_P} = -\frac{q_P}{\sqrt{(\mu_n - \frac{\sigma^2}{2})^2 + 2\sigma^2 r}} < 0$. Therefore, since $\mu_P < \mu_M$, it follows that $q_M < q_P$.

Proof of Proposition 4

By Proposition 2, the optimal investment threshold $X_n^{I,*}$ is the largest root of the non-linear equation $g_n(x) = 0$ where the function g_n is defined in equation (A.8). Using the definition of $W_n(x, X_n^{A,*})$ for $x \geq X_n^{A,*}$ in equation (7), we can rewrite condition (12) determining the optimal investment threshold $X_n^{I,*}$ as follows

$$f_n(X_n^{I,*}) = W_n(X_n^{I,*}, X_n^{A,*}), \quad \text{where} \;\; f_n(x) \equiv \frac{1-m_n}{q_n - m_n}\frac{1}{r-\mu_n}x + \frac{q_n}{q_n - m_n}I. \tag{A.23}$$

Equation (A.23) shows that the optimal investment threshold is characterized by the intersection of the graph of the function $W_n(\cdot, X_P^{A,*})$ with the line defined by the linear equation $y = f_n(x)$. We now show that this intersection is unique and occurs at a point larger than $X_n^{A,*}$. To see this, note that, using the expression for $X_n^{A,*}$ from equation (11), we have

$$f_n(X_n^{A,*}) = A + \frac{q_n}{q_n - m_n}(I - A) > A = W_n(X_n^{A,*}, X_n^{A,*}).$$

Direct inspection of W_n in equation (7) shows that the function $W_n(\cdot, X_n^{A,*})$ is non decreasing with an asymptote of slope $\frac{1}{r-\mu_n}$ at $x \to \infty$. Moreover, because $q_n > 1$, the function $f_n(x)$ has the slope $f_n'(x) = \frac{1-m_n}{q_n-m_n}\frac{1}{r-\mu_n} < \frac{1}{r-\mu_P}$, since $q_n > 1$ and $m_n < 0$. Therefore, there is a unique intersection between the graph of the function $W_P(\cdot, X_P^{A,*})$ and the line defined by the equation $y = f_n(x)$. This intersection characterizes the investment threshold $X_n^{I,*}$.

Following similar steps, we obtain a characterization of the second-best investment threshold $X_n^{I,\mathrm{SB}}$ equivalent to equation (A.23), that is,

$$f_n(X_n^{I,\mathrm{SB}}) = W_n(X_n^{I,\mathrm{SB}}, X_M^{A,*}), \tag{A.24}$$

The function $W_n(\cdot, X_n^{A,*})$ also has an asymptote of slope $\frac{1}{r-\mu_n}$ as $x \to \infty$. Furthermore, because $X_n^{A,*}$ is the optimal abandonment threshold for member n, we have that, if $n \neq M$,

$$W_n(x, X_M^{A,*}) \leq W_n(x, X_n^{A,*}) \text{ for all } x > 0.$$

Therefore, if $n \neq M$, the line defined by the equation $y = f_n(x)$ intersects the graph of the function $W_n(\cdot, X_n^{A,*})$ at a smaller value of x than that at which it intersects the graph of the function $W_n(\cdot, X_M^{A,*})$. This implies that $X_n^{I,*} < X_n^{I,\mathrm{SB}}$ if $n \neq M$.

We now prove that inertia is equivalent to $n_I \neq M$. If the pivotal voter for investment is $n_I = M$, then $X^I_{\mathrm{G}} = X^{I,\mathrm{SB}}_M = X^{I,*}_M$ and therefore inertia cannot occur. If the pivotal voter for investment is $n_I = P$, then $X^I_{\mathrm{G}} = X^{I,\mathrm{SB}}_P$ and, since $X^{I,*}_P < X^{I,\mathrm{SB}}_P$, we have $X^{I,*}_P < X^I_{\mathrm{G}}$ and hence inertia occurs. Similarly if $n_I = O$ we have $X^{I,*}_O < X^I_{\mathrm{G}}$. Hence, inertia occurs if and only if $n_I \in \{P, O\}$.

Finally, we prove that underinvestment is equivalent to $n_I \neq M$. If $n_I = M$, $V_{M,G}(x) = V^*_M(x)$ for all $x > 0$ and therefore the condition (27) for underinvestment cannot be satisfied. If, instead, $n_I \in \{P, O\}$, then $V_{M,G}(x) < V^*_M(x)$ and therefore there always exists a choice of L that satisfies condition (27) for underinvestment. ∎

Proof of Proposition 5

Ranking of operating project values $W_{n,G}(x)$. We first show that condition (30) implies that

$$W_{1,G}(x) \leq W_{2,G}(x) \leq \cdots \leq W_{N,G}(x), \quad \text{for all } x \geq X^I_{\mathrm{G}}. \tag{A.25}$$

For any $n = 1, \ldots, N$, following the steps used to derive equation (A.9) in Proposition 3 we can write,

$$W'_{n,G}(X^A_{\mathrm{G}}) \equiv W'_n(X^A_{\mathrm{G}}, X^A_{\mathrm{G}}) \equiv W'_n(X^{A,*}_k, X^{A,*}_k) = \frac{1 - m_n}{X^{A,*}_k (r - \mu_n)} (X^{A,*}_k - X^{A,*}_n). \tag{A.26}$$

Using the fact that m_n and q_n are the roots of the quadratic equation (A.4) we obtain

$$W'_{n,G}(X^A_{\mathrm{G}}) = \frac{2}{\sigma^2} \frac{1}{q_n - 1} \frac{1}{X^{A,*}_k} (X^{A,*}_k - X^{A,*}_n). \tag{A.27}$$

Since $X^{A,*}_n$ and q_n are declining sequences in n, equation (A.27) implies

$$W'_{k-1,G}(X^A_{\mathrm{G}}) < 0 = W'_{k,G}(X^A_{\mathrm{G}}) < W'_{k+1,G}(X^A_{\mathrm{G}}) < \cdots < W'_{N,G}(X^A_{\mathrm{G}}).$$

Using arguments similar to those in the proof of equation (A.11) in Proposition 3 we obtain that, for all $x > X^A_{\mathrm{G}}$, $W'_{n,G}(x) > 0$, $n = k, k+1, \ldots, N$ and

$$W_{n+1,G}(x) - W_{n,G}(x) = \mathbb{E}_n \left[\int_0^{\nu_{X^{A,*}_k}} e^{-rs} \xi_n X_s W'_{n+1,G}(X_s) ds \mid X_0 = x \right] > 0, \quad n = k-1, \ldots, N-1,$$

where $\nu_{X_k^{A,*}} = \inf\{t \geq 0 \ : \ X_t \leq X_k^{A,*}\}$ and $\xi_n \equiv \mu_{n+1} - \mu_n > 0$. Therefore,

$$W_{k-1,G}(x) < W_{k,G}(x) < \cdots < W_{N,G}(x), \quad \text{for all } x > X_G^A. \tag{A.28}$$

Since $X_G^A < X_G^I$, the inequalities in equation (A.28) hold *a fortiori* for $x \geq X_G^I$.

We next show that the ranking of $W_{n,G}(x)$ for $x \geq X_G^I$ also holds for $n = 1, \ldots, k-1$. To prove this claim, we show the graph of the functions $W_n(\cdot, X_G^A)$ and $W_{n+1}(\cdot, X_G^A)$, $n = 1, \ldots, k-2$, cannot intersect in the interval $[X_G^I, \infty)$. To see this, it is sufficient to consider the case of $n = 1$, as the proof for $n = 2, \ldots, k-2$ is identical. Note that, by the definition of W_n in equation (7), $W_{1,G}(X_G^I) \equiv W_1(X_G^I, X_G^A) < W_2(X_G^I, X_G^A) \equiv W_{2,G}(X_G^I)$. Moreover, because $W_n(x, X_G^A) \sim_{x\to\infty} \frac{x}{r-\mu_n}$ for $n = 1, 2$, we have $W_{1,G}(x) < W_{2,G}(x)$ for x sufficiently large. Therefore, if the functions $W_{1,G}(x)$ and $W_{2,G}(x)$ intersect in the interval $[X_G^I, \infty)$ they must intersect at least twice. Suppose that there exist two level of cash flows $\hat{x}$ and $\tilde{x}$ satisfying $X_G^I < \hat{x} < \tilde{x}$ such that $W_{1,G}(\hat{x}) = W_{2,G}(\hat{x})$, $W_{1,G}(\tilde{x}) = W_{2,G}(\tilde{x})$, and $W_{1,G}(x) > W_{2,G}(x)$ for $x \in (\hat{x}, \tilde{x})$.

The function $W_{2,G}(x)$ is strictly increasing for $x \in [X_G^I, \infty)$. To see this, notice that, from equation (A.26) we have

$$W'_{2,G}(X_G^A) = \frac{1 - m_2}{X_k^{A,*}(r - \mu_2)}(X_k^{A,*} - X_2^{A,*}) < 0, \tag{A.29}$$

where the inequality follows because $X_k^{A,*} < X_2^{A,*}$ for $k > 2$ and $m_2 < 0$. Moreover, from equation (A.21), the function $W_{2,G}(x)$ is convex on the interval $[X_G^A, \infty)$. Hence, the function $W'_{2,G}(x)$ increases from $W'_{2,G}(X_G^A) < 0$ at $x = X_G^A$ to $\frac{1}{r-\mu_2} > 0$ for $x \to \infty$. Because, by condition (30), $W_{2,G}(X_G^I) > I > A = W_{2,G}(X_G^A)$, the function $W_{2,G}(x)$ must be increasing in a neighborhood of $x = X_G^I$ and therefore $W'_{2,G}(x) > 0$ for all $x \geq X_G^I$.

Following similar arguments to those used in deriving equation (A.19) in the proof of Proposition 3, it can be shown that

$$W_{2,G}(x) - W_{1,G}(x) = \mathbb{E}_1\left[\int_0^{\nu_{\hat{x}} \wedge \tau_{\tilde{x}}} e^{-rs}\xi_1 X_s W'_{2,G}(X_s)ds \mid X_0 = x\right], \quad \text{for } \hat{x} \leq x \leq \tilde{x}, \tag{A.30}$$

where $\xi_1 \equiv \mu_2 - \mu_1 > 0$, $\nu_{\hat{x}}$ is the hitting time of $\hat{x}$ from above, and $\tau_{\tilde{x}}$ is the hitting time of $\tilde{x}$ from below. Because $X_G^I < \hat{x} \leq X_s$ almost surely, we have $W'_{2,G}(X_s) > 0$ almost surely. Therefore the right hand side of (A.30) is positive, implying that $W_{2,G}(x) > W_{1,G}(x)$ for $\hat{x} \leq x \leq \tilde{x}$. This contradicts the assumption that $W_{1,G}(x,) > W_{2,G}(x)$ for $x \in (\hat{x}, \tilde{x})$ and therefore the functions $W_{1,G}(x)$ and $W_{2,G}(x)$ cannot intersect in the interval $[X^I_{\mathrm{G}}, \infty)$. Extending this argument to all members $n = 1, 2, \ldots, k-1$ we obtain

$$W_{1,G}(x) < W_{2,G}(x) < \cdots < W_{k-1,G}(x), \quad \text{for all } x \geq X^I_{\mathrm{G}}. \tag{A.31}$$

Combining equations (A.28) and (A.31), we conclude that

$$W_{1,G}(x) < W_{2,G}(x) < \cdots < W_{N,G}(x), \quad \text{for all } x \geq X^I_{\mathrm{G}}. \tag{A.32}$$

The left inequality in equation (30) further implies that

$$I < W_{1,G}(x) < W_{2,G}(x) < \ldots < W_{N,G}(x), \quad \text{for all } x \geq X^I_{\mathrm{G}}. \tag{A.33}$$

Ranking of license values $V_{n,G}(x)$. When $x \geq X^I_G$, from equation (9) in Proposition 1, we have that, $V_{n,G}(x) = W_{n,G}(x) - I$ and condition (A.33) implies the ranking (31).

When $x < X^I_G$, for $n = 1, \ldots, N-1$ we have

$$\begin{aligned} V_{n,\mathrm{G}}(x) \equiv V_n(x, X^I_{\mathrm{G}}, X^A_{\mathrm{G}}) &= \mathbb{E}_n \left[\left(W_n(X_{\tau_{X^I_G}}, X^A_G) - I \right) e^{-r\tau_{X^I_G}} \,\middle|\, X_0 = x \right] \\ &= \left(W_n(X^I_G, X^A_G) - I \right) \left(\frac{x}{X^I_G} \right)^{q_n} < \left(W_{n+1}(X^I_G, X^A_G) - I \right) \left(\frac{x}{X^I_G} \right)^{q_n} \\ &< \left(W_{n+1}(X^I_G, X^A_G) - I \right) \left(\frac{x}{X^I_G} \right)^{q_{n+1}} = V_{n+1}(x, X^I_{\mathrm{G}}, X^A_{\mathrm{G}}), \end{aligned}$$

where the first inequality follows from equation (A.33) and the second inequality follows from the fact that $q_{n+1} < q_n$. Therefore the inequalities (31) hold for all $x > 0$. ∎

Proof of Proposition 6

1. Group inertia, that is, $X^I_{\mathrm{G}} > X^*_{n_I}$, follows from inequality (28) in Proposition 4, as the proof of this result generalizes to groups of any size. If $n_I = n_A$, then $X^{I,\mathrm{SB}}_{n_I} = X^I_{\mathrm{G}} = X^*_{n_I}$ and there is no inertia. If $n_I \neq n_A$, then inequality (28) shows that $X^{I,\mathrm{SB}}_{n_I} = X^I_{\mathrm{G}} > X^*_{n_I}$ and inertia occurs.

2. Underinvestment

(a) If N is odd and the group is ruled by majority, then the discussion following Proposition 5 shows that $n_L = n_A = (N+1)/2$. In this case, as in a three-member group, a change of pivotal voter over time occurs if and only if $n_I \neq (N+1)/2$. Therefore underinvestment occurs if and only if $n_I \neq (N+1)/2$.

(b) In all other cases, we have $n_L \neq n_A$ and hence for all $x > 0$, $V_{n_L,G}(x) < V^*_{n_L}(x)$. Therefore, underinvestment occurs regardless of the identity of the pivotal member n_I for the investment timing decision. ∎

B Trading voting rights: An example

Suppose that a competitive market for votes exists at time $t = 0$ where all group members described in Table 1 (reproduced below for convenience) are price-takers and are allowed to trade with each other at a price p. A vote acquired at time $t = 0$ gives the right to vote for the three group decisions: license, invest, and abandon. Any member $n \in \{P, M, O\}$ who acquires a vote becomes a dictator and receives the net payoff $V_n(x, X_n^{I,*}, X_n^{A,*}) - L - p$, if n acquires the license or $-p$ otherwise. For an equilibrium to exist in this market we need that (i) any pair of trading members have mutual gains from trade, relative to the no-trade voting outcome, and (ii) the remaining member finds it optimal not to trade.

Using the values in Table 1, note that any price in the range $(950.22, +\infty]$ cannot clear the market because the three members are only willing to sell at that price. If $p \in (119.733, 950.22]$, both M and P are willing to sell their votes and O is willing to buy. However, because of the majority rule, O only needs *one* vote to become the dictator of the group. Therefore, supply would not equal demand for any price $p \in (119.733, 950.22]$.

	Net payoff from real option managed by:			
Member	Group	O	M	P
P	0	−119.733	−117.183	−117.006
M	0	−1.807	0.001	−0.212
O	0	950.220	946.251	943.001

Table 1: Market for votes

The table reports the no-trade allocation and the individual payoffs under m's management $V_n(x, X_m^{I,*}, X_m^{A,*}) - L$, $m, n \in \{P, M, O\}$. Parameter values: $x = 0.9 \times X_M^{I,*} = 14.3494$, $\mu_P = 0.01$, $\mu_M = 0.02$, $\mu_O = 0.04$; $r = 0.05$, $\sigma = 0.50$, $L = 394.24$, $I = 100$, and $A = 65$.

When $p \in (0, 119.733]$, P cannot be a trading member in equilibrium. In fact, relative to the zero-payoff voting outcome, there are no mutual gains from trade between P and any of the two other members. To see this, note that if P buys a vote, his payoff is $-p < 0$, since it is never optimal for P to run the project.[26] If P sells his vote to O, his payoff is $p - 119.733 < 0$. Hence there are no gains from trade between P and O. If P sells his vote to M, his payoff is $p - 117.183$ and M's payoff if $-p + 0.001$. Since for all prices $p \in [117.183, 119.733]$, M's payoff from buying is negative, $-p + 0.001 < 0$, and for prices $p < 117.183$, P's payoff from selling is negative, $p - 117.183 < 0$, it follows that there are also no mutual gains from trade between M and P. This implies that, when $p \in (0, 119.733]$, P cannot find a group member with whom to benefit from trade. He may, however, be willing to trade to alleviate the negative externality that a trade between M and O would impose on him. These observations imply that the only possible trading equilibrium is one where O and M have gains from trade and P optimally chooses not to trade.

Following similar analysis, it can be shown that if $p \in (0, 0.001] \cup [1.807, 119.733)$, there are gains from trade between O and M. This trade, however, generates an undesirable outcome for the non-trading member P who ends up with a negative payoff -119.733 if O is the buyer, or -117.183, if M is the buyer. To minimize the negative impact of this trade, P will have the incentive to either buy or sell his vote. Since in both cases it is optimal for P to trade, the market for votes cannot clear due to the imbalance between supply and demand in this price range. Similar analysis shows that, for prices $p \in (0.001, 1.807)$, there are no mutual gains from trade between any pair of group members and therefore there will be no trade also in this price range.

The above analysis shows that no price can clear a competitive market for votes and therefore the group coordination frictions we highlight in Section 3 cannot be resolved by trading votes in a competitive market.

[26]The net payoff to P in Table 1 are all negative, therefore in equilibrium, P will never acquire the license. This means that we can ignore the fifth column of Table 1 in our analysis.

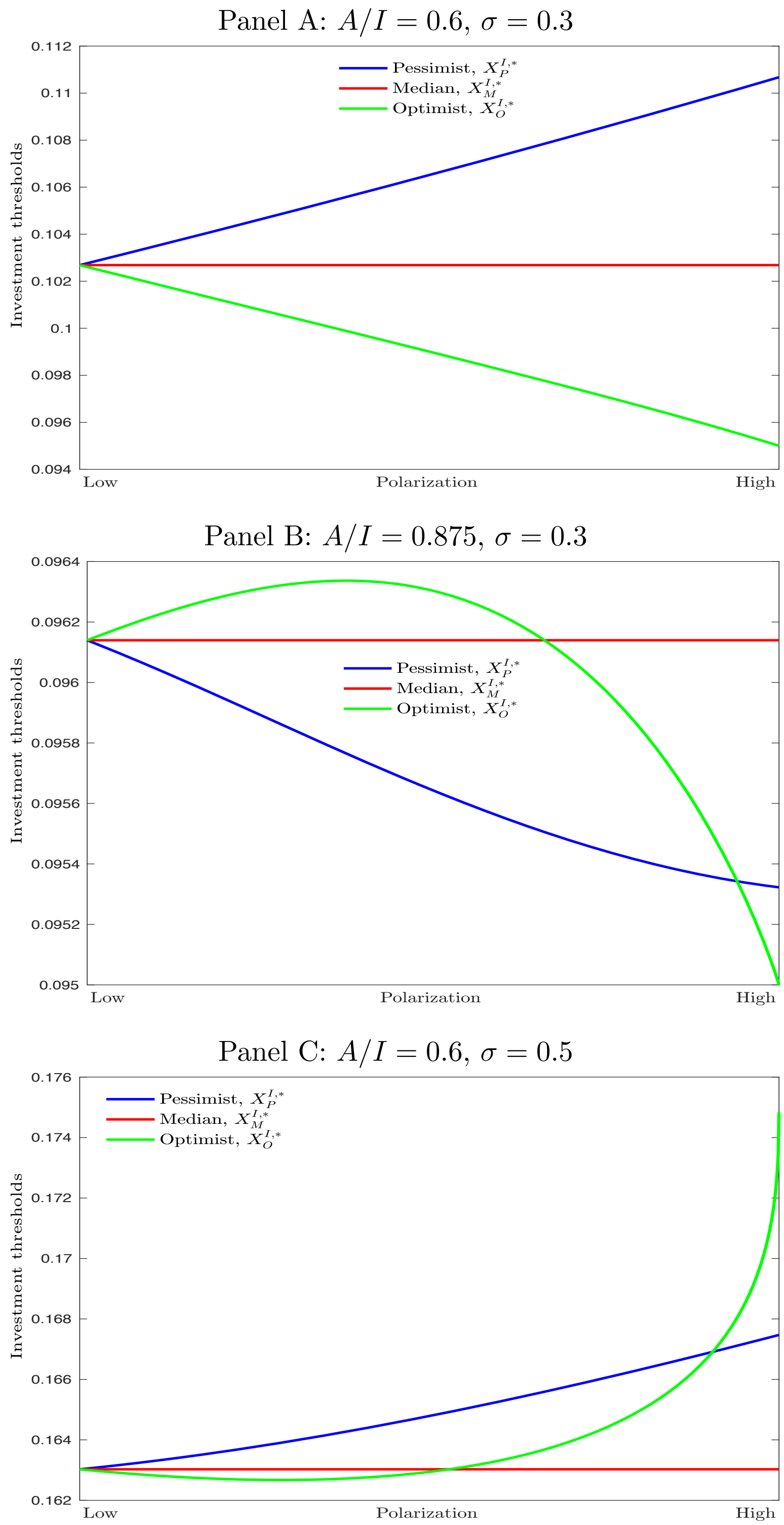
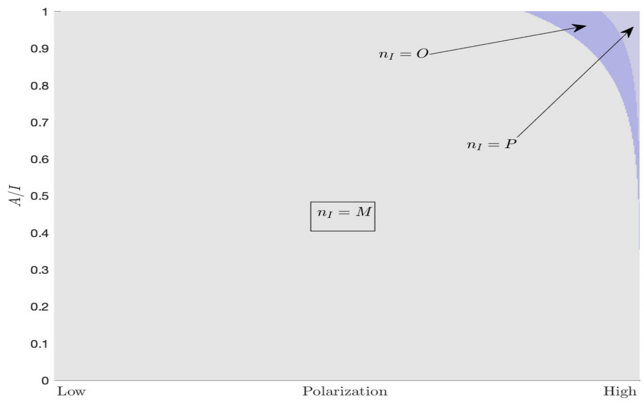


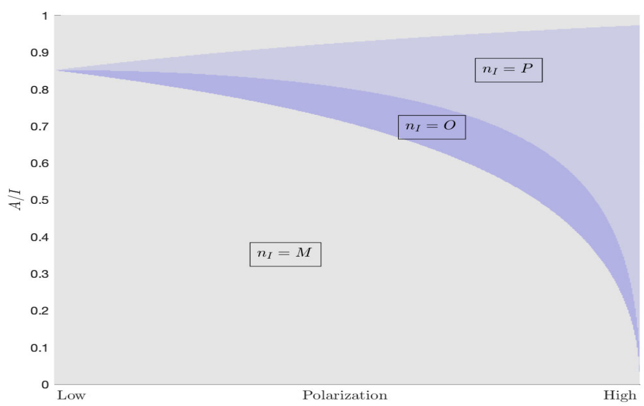
Figure 1: Optimal Individual Investment Thresholds.

The figures reports the optimal individual investment thresholds $X_n^{I,*}$ from equation (12) as a function of polarization. We set $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$ and define polarization as the spread $\varepsilon > 0$. We set $I = 1$, $\mu_M = r/2$ with $r = 0.05$, and choose $\varepsilon \in (0, r/2)$.

Panel A: $\sigma = 0.15$



Panel B: $\sigma = 0.3$



Panel C: $\sigma = 0.5$

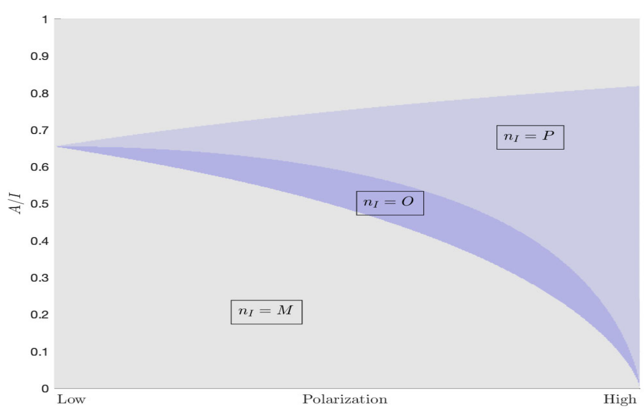


Figure 2: Identity of investment timing pivotal voter.

The figures shows the identity of the investment timing pivotal voter under the majority rule, for different value of the parameters $A/I \in [0, 1]$, polarization, and cash flow volatility σ. We set $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$ and define polarization as the spread $\varepsilon > 0$. For parameters $(\varepsilon, A/I)$ lying in the gray area M is pivotal. In the dark blue area O is pivotal and in the light blue area P is pivotal. We set $I = 1$, $\mu_M = r/2$ with $r = 0.05$, and choose $\varepsilon \in (0, r/2)$.

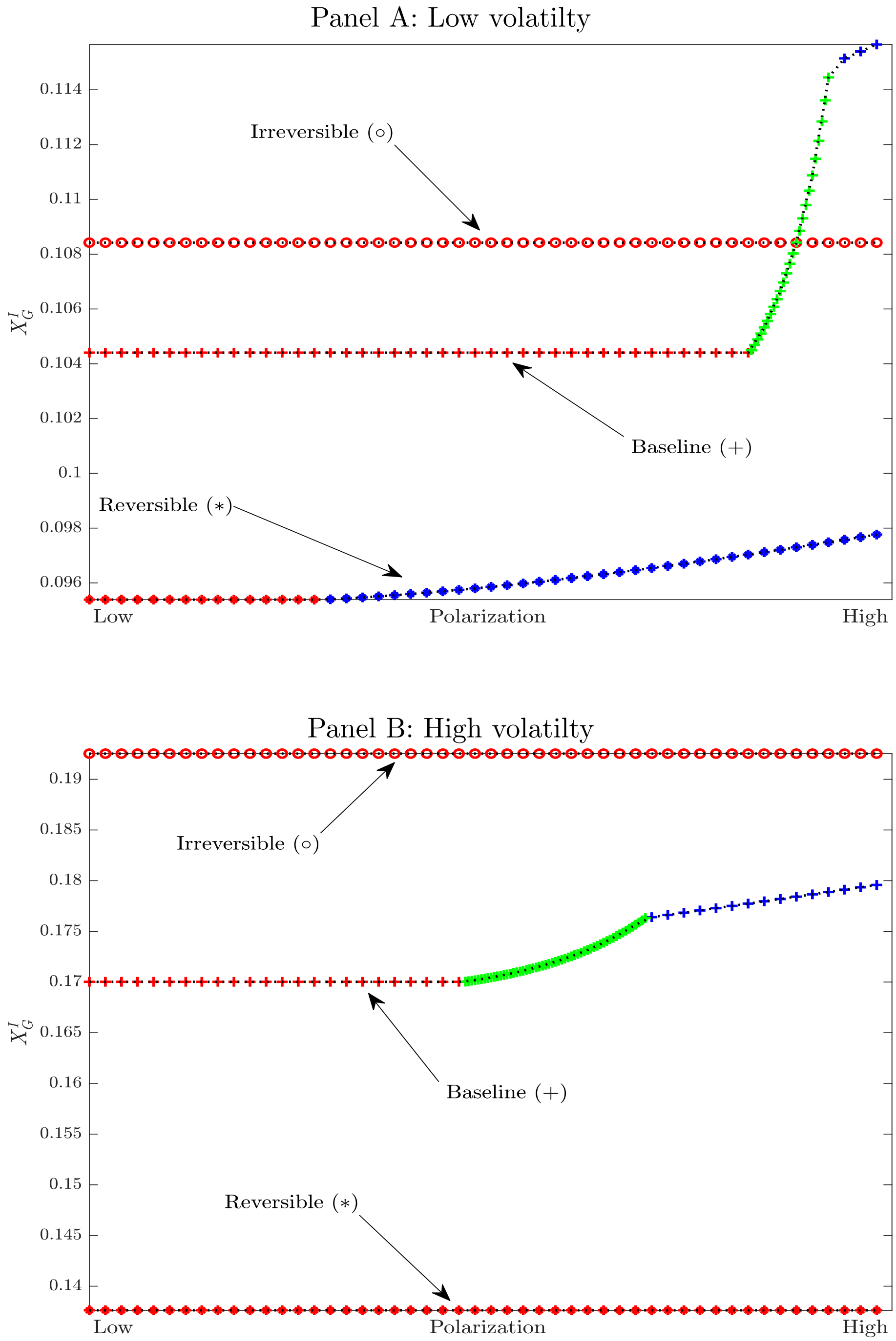


Figure 3: Inertia.

The figure shows the investment threshold X_I^G in a three-member group (P, M, O), as a function of polarization. We set $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$ and define polarization as the spread $\varepsilon > 0$. In Panel A cash flow volatility is set to $\sigma = 0.3$ and, in Panel B, $\sigma = 0.5$. In each panel the lines marked with '+' refer to the investment thresholds for the baseline case of $A/I = 0.5$, the lines marked with '$*$' refer to the case of reversible ($A/I = 0.9$), while the lines marked with $\circ$ refer to the case of irreversible ($A/I = 0.05$) projects. The color of the marker in each line indicates the pivotal member: P (blue), M (red), or O (green). We set $I = 1$, $\mu_M = r/2$ with $r = 0.05$, and choose $\varepsilon \in (0, r/2)$.

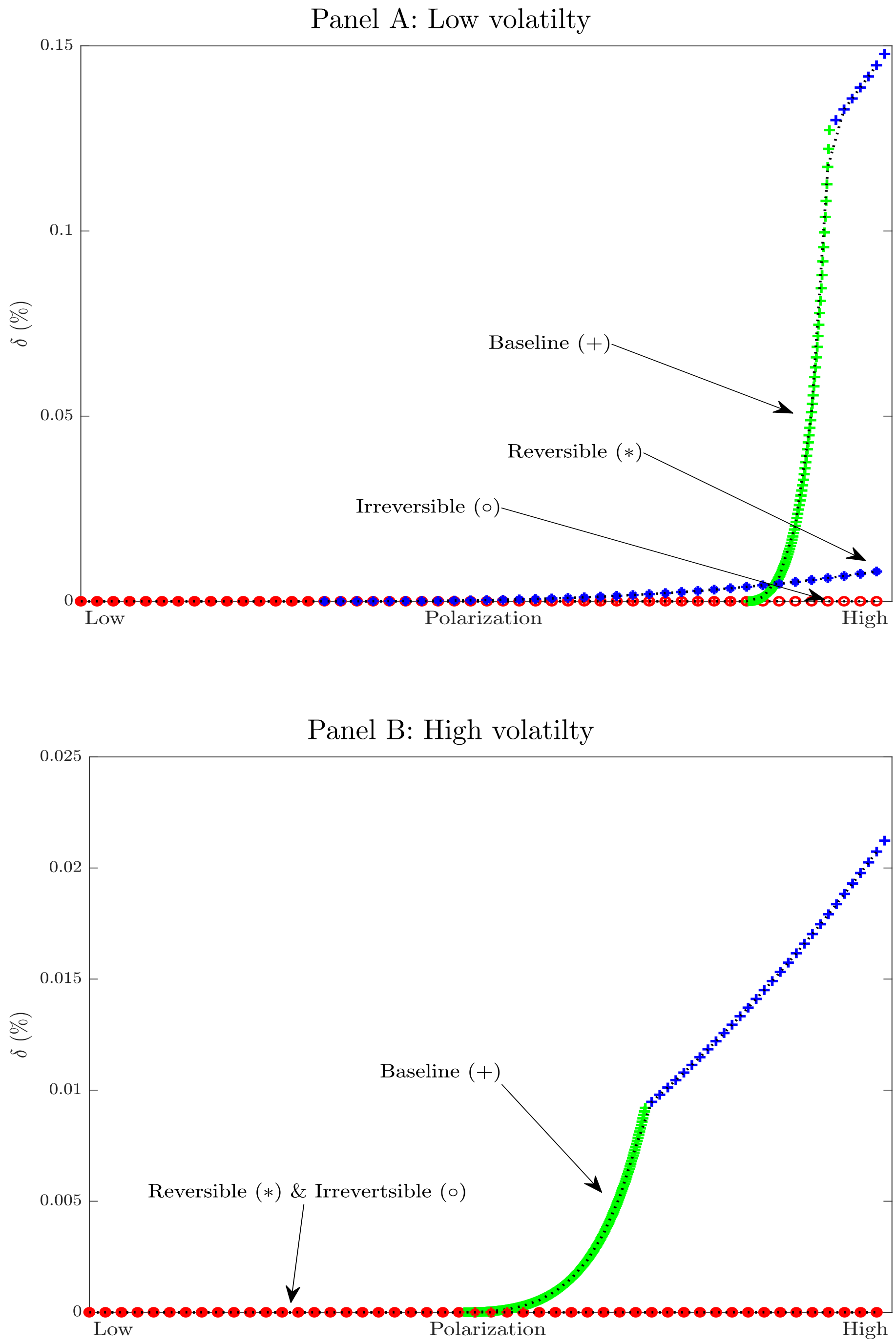


Figure 4: Coordination premium.

The figure shows the coordination premium δ associated with underinvestment (equation (29)) in a three-member group (P, M, O), as a function of polarization. We set $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$ and define polarization as the spread $\varepsilon > 0$. In Panel A cash flow volatility is set to $\sigma = 0.3$ and, in Panel B, $\sigma = 0.5$. In each panel the lines marked with '+' refer to the investment thresholds for the baseline case of $A/I = 0.5$, the lines marked with '$*$' refer to the case of reversible ($A/I = 0.9$), while the lines marked with $\circ$ refer to the case of irreversible ($A/I = 0.05$) projects. The color of the marker in each line indicates the pivotal member: P (blue), M (red), or O (green). We set $I = 1$ and $\mu_M = r/2$ with $r = 0.05$, and choose $\varepsilon \in (0, r/2)$.

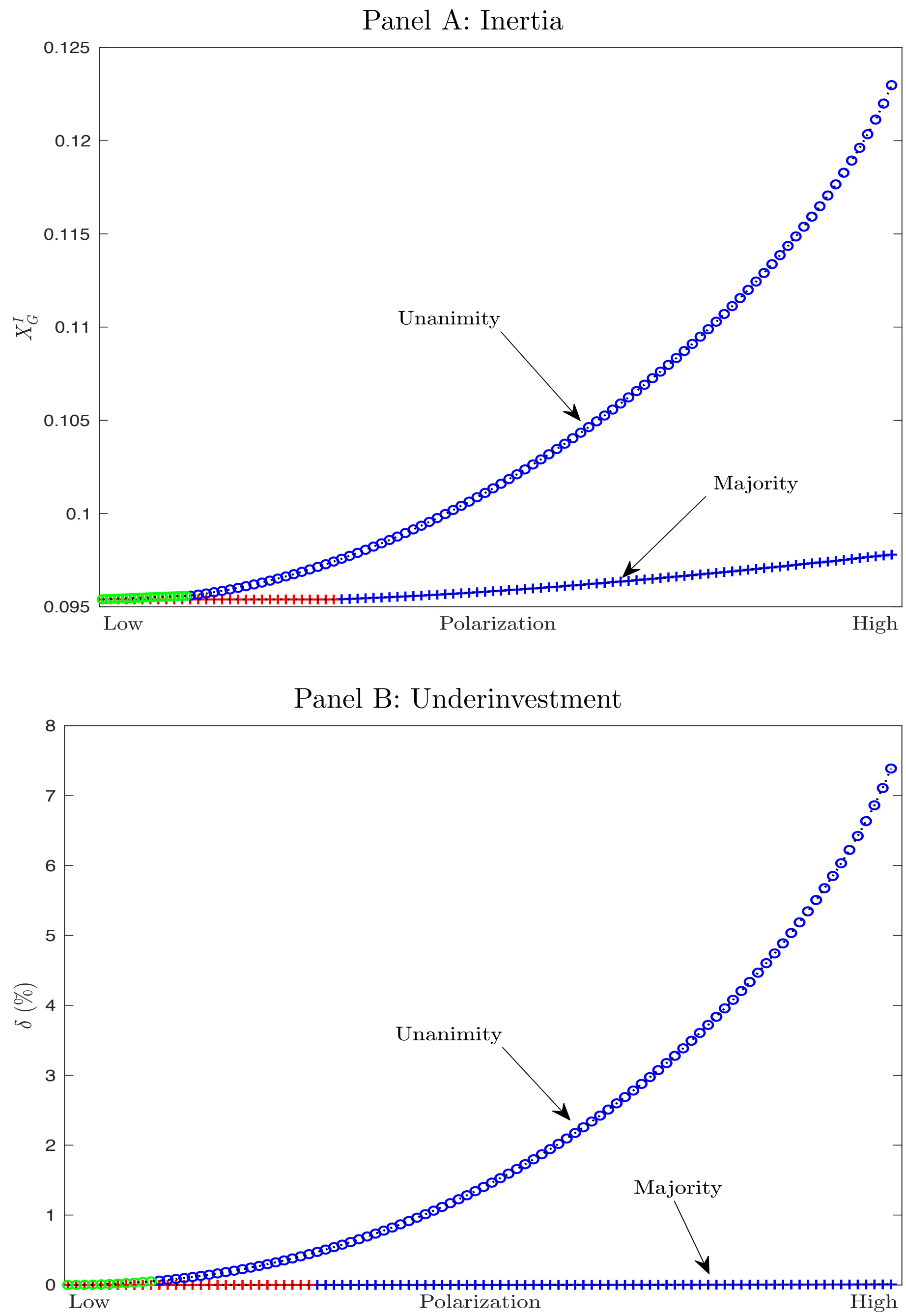


Figure 5: Inertia and underinvestment: Majority vs. unanimity.

The figure shows the group investment threshold X_{G}^I (top panel) and the coordination premium δ associated with underinvestment defined in equation (29) (bottom panel), as a function of polarization in a three-member group (P, M, O). We set $\mu_P = \mu_M - \varepsilon$ and $\mu_O = \mu_M + \varepsilon$ and define polarization as the spread $\varepsilon > 0$. The lines marked with '+' refer to the case of a group governed by majority while the lines marked with '$\circ$' refer to the case of a group governed by unanimity. The color of the marker in each line indicates the pivotal member n_I: P (blue), M (red), or O (green). We set $I = 1$, $A = 0.5$, $\sigma = 0.3$, and $\mu_M = r/2$ with $r = 0.05$, and choose $\varepsilon \in (0, r/2)$.

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