Rational Sentiments and Economic Cycles *

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Abstract

We propose a rational model of endogenous cycles generated by the two-way interaction between credit market sentiments and real outcomes. Sentiments are high when most lenders optimally choose lax lending standards. This leads to low interest rates and high output growth, but also to the deterioration of future credit application quality. When the quality is sufficiently low, lenders endogenously switch to tight standards, i.e. sentiments become low. This implies high credit spreads and low output, but a gradual improvement in the quality of applications, which eventually triggers a shift back to lax lending standards and the cycle continues. The equilibrium cycle might feature a long boom, a lengthy recovery, or a double-dip recession. It is generically different from the optimal cycle as atomistic lenders ignore their effect on the composition of the pool of borrowers. Carefully chosen macro-prudential or countercyclical monetary policy often improves the decentralized equilibrium cycle.

JEL codes: D82, E32, E44, G01, G10
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1 Introduction

A growing body of empirical evidence suggests that periods of overheating in credit markets forecast recessions. Overheated, high sentiment markets are characterized by increased total quantity of credit, low interest rates, and importantly, deteriorating quality of newly issued credit. In the subsequent recessions, credit turns scarce and expensive even for ex-post high-value projects.[1]

A major conundrum for policy makers and academics alike is how economic policy should respond to this phenomena. For this, it is essential to understand the mechanism which triggers overheating and then turns credit booms into recessions, and vice-versa. That is, we need a framework where credit and real cycles arise endogenously. In this paper, we build a model where the interaction between credit market sentiments and real economic outcomes generate cycles. We also rank various policy instruments according to how efficiently they steer the economy towards higher welfare cycles.

The model captures credit market sentiments as lenders' rational choice of lending standards under imperfect information. In our model, entrepreneurs run projects and obtain credit from investors to scale up their operation. Only some entrepreneurs intend to pay back their debt. The majority of investors are not sufficiently skilled to distinguish these good entrepreneurs from the bad ones. However, they have access to a technology that can imperfectly reveal entrepreneur type.

Specifically, investors can choose to run one of two tests to decide which entrepreneurs to grant credit to. A *bold test* represents lax lending standards. This test approves the credit application of all good entrepreneurs along with some bad ones. A *cautious test* on the other hand represents tight lending standards as it only approves a fraction of good applications. It rejects all bad applications and even some good ones. Thus, tight lending standards improve the quality, but decreases the quantity of the credit issued by an investor.

When there are few bad entrepreneurs among borrowers, investors optimally choose lax lending standards. Credit market exhibits symptoms of overheating or high sentiments. A

[1]See (Greenwood and Hanson, 2013; López-Salido et al., 2017; Greenwood et al., 2020) on identifying overheated markets and their relationship with future bond excess returns and recessions. See also Morais et al. (2019) for US and international evidence on lax bank lending standards in booms, and Baron and Xiong (2017) for the negative relationship between banks' credit expansion and banks' equity returns. More generally, there is ample evidence of pro-cyclical volume and counter-cyclical value of investment in a wide range of contexts. For instance, Eisfeldt and Rampini (2006) demonstrate this for sales of property, plant and equipment, while Kaplan and Stromberg (2009) show similar evidence on venture capital deals.

mixed quality of credit is issued at a low interest rate which induces high credit growth and high output. At the same time, the flow of credit towards bad types helps them grow, leading to the deterioration of borrower quality in future periods. When the average borrower quality sufficiently deteriorates, lenders rationally switch to tight standards. Tight lending standards coincides with high credit spreads and low quantity, but high quality of issued credit. Thus the pool of credit applications improves, eventually triggering a shift back to lax lending standards. And the cycle continues. As such, there is a two-way interaction between credit sentiment and the fundamentals of the economy.

Booms are generally characterized by high output and positive output growth, and low yields in the credit market. In contrast, output is low and output growth is negative in a recession, while credit markets are fragmented.

The model gives rise to a variety of cyclical behavior depending on the underlying parameters. Often there are long booms interrupted by short recessions, akin to the usual US business cycle patterns. Alternatively, the cycle can feature a prolonged recovery period, or a double-dip recession.

The normal expansion and contractions cycles generated by the model manifests the *cleansing effect of recessions* (Schumpeter, 1939; Caballero and Hammour, 1994). This refers to the observation that in recessions, plant shutdowns are more concentrated among bad firms, and is mostly explored in the labor market. In contrast, in our model the fact that credit supply is more cyclical for bad firms delivers the cleansing effect.

We also argue that the generated cycles are not constrained efficient. It is so because investors fail to internalize that their individual choice of lending standards affect the future quality of borrowers. Nevertheless, a constrained planner often prefers a cycling economy to one with persistently lax or persistently tight lending standards. In a constrained optimal cycle, recessions induced by tight lending standards keep the fraction of bad projects at bay which makes the subsequent booms more beneficial.

We further connect the constrained optimal solution to realistic monetary and macro-prudential policies. We show that changing the risk-free rate through well designed monetary policy, and specifying capital requirements using a macro-prudential policy can be used to influence investors' lending standards. Therefore, each of these policies affects the dynamics of the state distribution, and, consequently, welfare. However, the policy maker can improve the quality of loan applications only at the expense of increasing the average cost of capital.

This trade-off determines the ranking across policies. Under our representation, we show that macro-prudential and counter-cyclical monetary policy both strongly dominate a non-state contingent monetary policy. The counter-cyclical monetary policy can improve welfare slightly more than the risk-weighted capital requirements, however, the former requires a more sophisticated regulator.

Finally, the prolonged period of low interest since the financial crisis have spurred concerns that low interest rates lead investors to "reach for yield" and take on excessive risky investment (Rampini and Viswanathan, 2019; Campbell and Sigalov, 2020). Our model provides an alternative explanation for the same phenomena: low interest rates lengthen the period of reach for yield during the business cycle by increasing the opportunity cost. This perspective also enables investigating a potential negative long run effect of government policy by dampening the cleansing effect of recessions.

Literature. To the best of our knowledge our paper is the first to provide a mechanism where economic cycles are endogenously generated by the interaction between the choice of lending standards and average borrower quality.

Our paper belongs to the growing body of literature on dynamic lending standards. In this literature, lenders' choice to acquire information on borrowers differs in booms and in recessions (Martin, 2005; Gorton and Ordonez, 2014; Hu, 2017; Asriyan et al., 2018; Fishman et al., 2019; Gorton and Ordonez, 2016). Gorton and Ordonez (2016) and the contemporaneous paper of Fishman et al. (2019) are the closest to our work. Similar to our model, the mechanism in Fishman et al. (2019) rely on the two-way interaction of lenders' information choice and borrowers' average quality. However, unlike our paper, their economy does not feature an endogenous cycle, and converges to a high or a low steady state depending on the parameters. In other words, Fishman et al. (2019) and most of the rest of the papers in this literature do not provide an endogenous mechanism repeatedly turning a boom into a recession and vice-versa. One exception is Gorton and Ordonez (2016). This paper has both an economy that converges to a good steady state, and one that cycles between multiple periods in the good state and one in the bad one. Unlike us, in this economy recessions and the corresponding tight lending standards have no welfare benefits. If possible, a planner prefers to force agents to always use lax lending standards. In our setup on the other hand, a planner often prefers a cyclical economy to a persistent boom, as tight lending standards during the downturn improves future borrowers' quality which makes the subsequent boom

more beneficial.[2]

Our paper also contributes to the literature on endogenous credit cycles (Azariadis and Smith, 1998; Matsuyama, 2007; Myerson, 2012; Gu et al., 2013). These papers present different mechanism that leads to endogenous fluctuations in granted credit quantity. However, none of them capture the interdependence of investors choice of lending standards and economic activity.

This paper is also connected to the literature on collateral based credit cycles (Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014). As in these papers, we are also interested in how a change in credit availability induces boom and busts. However, these papers focus on how exogenous shocks are amplified by the effect through the price of the collateral. In our model the price of collateral or exogenous shocks play no role.

There is a long tradition in economics starting with Keynes' metaphor of animal spirits to associate boom-bust cycles with fluctuating investors' sentiment.[3] There is a literature (Bordalo et al., 2018; Greenwood et al., 2019; Gennaioli and Shleifer, 2020) focusing on the role of extrapolative expectations. In contrast, we capture credit market sentiment as a rational choice of lending standards. Our model generates some of the leading facts of the empirical side of this literature; for instance the deterioration of credit quality in booms, or the strong correlation between high credit growth, low subsequent returns and recessions. However, as a rational model, our mechanism does not generate an exploitable anomaly under the least informed agent's information set. That is, regarding evidence that points to such anomalies, our approach can only play a complementary role to behavioral models.

Finally, from a methodological perspective, the structure of the credit market builds on Kurlat (2016) and our companion paper, Farboodi and Kondor (2018). Neither of these papers focus on endogenous economic cycles.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 solves for the equilibrium within each period. Section 4 characterizes the dynamic equilibrium. Section 5 discusses optimal policy, Section 6 introduces aggregate shocks implying stochastic cycles, and section 7 compares the implications of the model with empirical facts.

[2]The difference in welfare implications is a consequence of the different underlying mechanisms. Gorton and Ordonez (2016) argue that dynamic lending standards imply fluctuation in the *perceived* quality of the average borrowers. In our framework, they imply fluctuation in the *realized* quality of the average borrower.

[3]Angeletos and La'O (2013) provide a conceptually distinct approach to capture sentiment in a rational framework as rationally overweigh public information.

Finally, section 8 concludes.

2 Set Up. Rational Sentiments and Economic Cycles

The economy runs for an infinite number of periods. Each period is divided into two parts: morning and evening. There is one type of consumption good. It can be consumed, invested, or stored at a rate of return $1 + r_f$ between morning and evening.[4] There are two types of agents, entrepreneurs and investors. Each agent is risk-neutral and endowed with one unit of the good in the morning. There is no discounting except through possibility of death.

Entrepreneurs. There is measure one of entrepreneurs, and each one is endowed with a projects with a two-dimensional type. It is good or bad, $\tau = g, b$, and opaque or transparent, $\omega = 0, 1$. Entrepreneurs know their own type. We use entrepreneur and project type interchangeably. Let $\mu_{0,t}$ and $\mu_{1,t}$ denote the measure of opaque and transparent bad entrepreneurs at time t, respectively. We will show that at each time t, $(\mu_{0,t}, \mu_{1,t})$ is sufficient statistics for entrepreneur type distribution.

Each entrepreneur maximizes his life-time utility. At time t, entrepreneur (τ, ω) obtains credit $\ell_t(\tau, \omega)$ at interest rate $r_t(\tau, \omega)$ and invests $i_t(\tau, \omega)$ in the morning, and consumes in the evening. Each unit of investment in the morning produces $\rho > 1 + r_f$ the same evening.[5] The cost of investment has to be covered by entrepreneur's initial endowment or credit, implying the following budget constraint

$$i_t(\tau, \omega) = 1 + \ell_t(\tau, \omega). \tag{1}$$

Furthermore, each entrepreneur has to pledge his investment as collateral to obtain credit. Seizing the collateral is the only threat to enforce repayment from borrowers, thus

[4] r_f, can represent a physical return or a policy rate. In sections 3 and 4, we think of it as the rate of return on the storage technology, which can be normalized to zero. In section 5 we reintroduce r_f as the return on a risk-free asset provided by the policy maker.

[5] We have also solved the model under the alternative assumption that good (bad) investment returns $\rho_g > 1 + r_f$ ($\rho_b < 1$). The expressions are more complex without providing further intuition. Therefore, we have decided to use $\rho = \rho_b = \rho_g > 1 + r_f$. The more general solution is available in the previous circulated versions of the paper, as well as available upon request.

$(1 + r_t(\tau, \omega))\ell_t(\tau, \omega) \leq i_t(\tau, \omega)$. Using (1) this simplifies to

$$\ell_t(\tau, \omega) \leq \frac{1}{r_t(\tau, \omega)}. \tag{2}$$

The key friction of the model is that investors cannot seize the investment in bad projects, and they only have imperfect information about project type. That is, if investors could observe the type of entrepreneurs, they would only lend to good ones as repayment from bad ones cannot be enforced.

At the end of each period, some entrepreneurs exit the market ('die'). An entrepreneur exists either because he is hit by an exogenous shock with probability δ, or because he has not been able to raise credit. Thus, we assume that credit is essential for survival. When an entrepreneur dies, he is replaced with a newborn so as to keep the population fixed at 1. The type distribution of the new entrants is fixed. λ $(1-\lambda)$ of new entrants are good (bad), and $\frac{1}{2}$ $(\frac{1}{2})$ are transparent (opaque). The two dimensions of the type distribution of entrants are independent.

Investors. There are two groups of investors. A small, w_1, measure of investors are skilled, while a large, w_0, measure are unskilled. Skill is privately observable. Each investor has one unit of endowment. Let $h \in [0, w_0 + w_1]$ denote an individual investor.

Each investor lives for one period and maximizes her period utility. She makes a portfolio decision in the morning, and consumes and dies in the evening. A dead investor is replaced by the same type of investor the next day. A portfolio decision involves extending credit to entrepreneurs and/or storing part of their unit endowment until the evening.

Each investor chooses to participate in or stay out of the lending market. Skilled investors observe the type of each project. Participating unskilled investors only observe imperfect signals for the project sample that they receive instead. These signals are generated by a test of investor's choice. Each investor can opt for a *bold test* or a *cautious test*. We call the former a bold investor, and the latter a cautious investor. The cost of either test is $c \in (0, 1)$, and each unskilled investor runs exactly one test.

The tests differ in the signal they generate for opaque projects. The bold test pools all opaque projects, good or bad, with transparent good ones (a false positive error). The cautious test pools all opaque projects with transparent bad ones (a false negative error).[6]

[6]For simplicity we restrict investor's choice set to these two tests. In appendix D we enrich the model

Intuitively, one can envision the bold test to reject transparent bad projects only and pass all other ones, while the cautious test passes only transparent good projects. When an investor is indifferent between the two tests, we break the tie by assuming that she chooses the bold test.

The size of the sample that an unskilled investor tests is limited by her unit endowment. An unskilled investor can test only as many applications as she can finance if all application pass her test.

Credit Market. Credit market operates in the morning. After each unskilled investor chooses the type of her test, each participating skilled and unskilled investor advertises an interest rate, $\tilde{r}(h)$, at which she is willing to extend loans. Each entrepreneur chooses the measure of loan applications $\sigma(r;\tau,\omega) \in [0, \frac{1}{r}]$ he wishes to submit at each interest rate r. The credit market clears starting from the lowest interest rate. At each interest rate, the unskilled investors sample first.

We assume that there is no credit history for entrepreneurs. That is, investors cannot learn from the past. Furthermore, there is no saving technology available across periods. Therefore, entrepreneurs consume their wealth at the end of each period, and if survived, they start the new period with the unit endowment received in the morning. Moreover, we make the following assumption about skilled and unskilled investor wealth.

Assumption 1 *Skilled and unskilled investor capital w_1 and w_0 are such that*

(i) Skilled investor capital, w_1, is not sufficient to cover the credit demand of all opaque good entrepreneurs at any interest rate that any good entrepreneur is willing to borrow at.

(ii) Unskilled investor capital, w_0, is abundant. In particular, it covers the credit demand of all entrepreneurs that unskilled investors are willing to lend to at any equilibrium interest rate.

The formal optimization problem of investors and entrepreneurs, as well as further details on collateralization and market clearing protocol are stated in Appendix A. We next define

and allow the investors to choose among the continuum of tests lying between the bold and cautious tests. We prove that the dominant choice is always one of the extremes. Thus, this assumption is not restrictive.

the equilibrium within each period, and the full dynamic equilibrium of the economy. We will show that the type distribution of entrepreneurs summarized by (μ_0, μ_1) is a sufficient state variable for the economy. As such, we fix (μ_0, μ_1) when characterizing the stage game equilibrium.

Definition 1 (Stage Game Equilibrium) *For a fixed (μ_0, μ_1), the stage game equilibrium consists of entrepreneurs' investment schedule $i(\tau, \omega)$ and credit demand schedule $\sigma(r, \tau, \omega)$, investors' advertised interest rate schedule $\tilde{r}(h)$ and unskilled investors' choice of test, equilibrium interest rate schedule $r(\tau, \omega)$, equilibrium credit allocation schedule to entrepreneurs $\ell(\tau, \omega)$, and equilibrium allocation of applications to investors such that*

(i) each agent's choice maximizes the agent's stage game utility given the strategy profile of other agents, equilibrium interest rates and allocations,

(ii) the implied interest rate schedule $r(\tau, \omega)$, credit allocation schedule for entrepreneurs $\ell(\tau, \omega)$, and allocation of applications to investors are consistent with agents' choices and the market clearing process.

Definition 2 (Dynamic Equilibrium) *The dynamic equilibrium consists of an infinite sequence of $\{(\mu_{0,t}, \mu_{1,t})\}_{t=0}^{\infty}$, individual entrepreneurs' $i_t(\tau, \omega)$ and $\sigma_t(\tau, \omega, r)$, individual investors' $\tilde{r}_t(h)$ and unskilled investors' choice of test, equilibrium $r_t(\tau, \omega)$, $\ell_t(\tau, \omega)$ and allocation of applications to investors, all within each period, such that*

(i) there exists a finite κ and a stable invariant set $\{(m_{0,i}, m_{1,i})\}_{i=1}^{\kappa}$ such that $(\mu_{0,t}, \mu_{1,t}) = (m_{0,i}, m_{1,i})$ and

$$(\mu_{0,t+1}, \mu_{1,t+1}) = \begin{cases} (m_{0,i+1}, m_{1,i+1}) & if \quad i < \kappa \\ (m_{0,1}, m_{1,1}) & if \quad i = \kappa, \end{cases}$$

(ii) the dynamics of $(\mu_{0,t}, \mu_{0,t})$ is consistent with the birth-death process of entrepreneurs.

(iii) each agent's choice maximizes the agent's life-time utility given the strategy profile of other agents, equilibrium interest rates and allocations,

(iv) in each period t, the implied interest rate schedule $r_t(\tau, \omega)$, credit allocation schedule for entrepreneurs $\ell_t(\tau, \omega)$, and allocation of applications to investors are consistent with agents' choices and the market clearing process.

The dynamic equilibrium nests both a steady state and a cycle. If $\kappa = 1$, it is a standard steady-state equilibrium. When $\kappa > 1$, it is a *cyclical dynamic equilibrium* as it features a stable cycle of length κ.[7] In the next section, we start by describing the stage game equilibrium, and then show that each dynamic equilibrium is a sequence of stage game equilibria.

3 Stage Game Equilibrium

In order to analyze the stage game, fix the measure of opaque and transparent bad entrepreneurs, (μ_0, μ_1). We first characterize the equilibrium interest rates in the credit market, and then outline the real outcomes.

The following lemma describes entrepreneurs' credit demand.

Lemma 1 *Entrepreneurs' credit demand schedule $\sigma(r, \tau, \omega)$ is as follows.*

(i) Entrepreneur (τ, ω) chooses a reservation interest rate $r^{max}(\tau, \omega)$. He submits maximum demand, $\sigma(r; \tau, \omega) = \frac{1}{r}$ to all $r \leq r^{max}(\tau, \omega)$ and zero demand to all $r > r^{max}(\tau, \omega)$.

(ii) Good entrepreneurs never choose a reservation rate higher than $\bar{r} \equiv \rho - 1$, while bad entrepreneurs never choose a reservation rate lower than $\bar{r} \equiv \rho - 1$,

$$r^{max}(g, \omega) \leq \bar{r} \leq r^{max}(b, \omega) \qquad \forall \omega.$$

It follows from the lemma that it is sufficient to find the equilibrium reservation interest rate for entrepreneurs, instead of working out a full credit demand schedule. Furthermore, the lemma demonstrates that there exists an interest rate $\bar{r}$ above which a good entrepreneur never borrows as the repayment would be higher than the project payoff.

We next show that in each period, the unique equilibrium in the credit market is one of three distinct types, depending on the parameters. In what follows, the succeeding two definitions introduce useful objects to characterize the equilibria.

[7] As it is clear from the equilibrium definition, most of our formal analysis focuses on deterministic cycles for simpler intuition and analysis. In Section 6 we introduce an extension which leads to the replacement of determinstic cycles with more realistic stochastic ones.

Definition 3 (Interest Rates)

$$r_B(\mu_0, \mu_1, c, r_f) \equiv \frac{\mu_0}{1-\mu_1-\mu_0} + \frac{1-\mu_1}{1-\mu_1-\mu_0} r_f + \frac{1}{1-\mu_1-\mu_0} c \tag{3}$$

$$r_C(\mu_0, \mu_1, c, r_f) \equiv r_f + \frac{2}{1-\mu_1-\mu_0} c \tag{4}$$

$$r_M(\mu_0, \mu_1, c, r_f) \equiv \frac{2\mu_0}{1-\mu_1-\mu_0} + \frac{1+\mu_0-\mu_1}{1-\mu_1-\mu_0} r_f + \frac{1+\mu_1+\mu_0}{1-\mu_1-\mu_0} c. \tag{5}$$

Definition 4 (Opaque Bad Limit) *Let* $\tilde{\mu}_0(\mu_1, c, \rho, r_f) = \frac{(\bar{r}-r_f-c)(1-\mu_1)}{2+c+r_f+\bar{r}}$ *be implicitly defined by* $r_M(\tilde{\mu}_0(\mu_1), \mu_1, c, r_f) \equiv \bar{r}$. *For any measure of transparent bad entrepreneurs* μ_1, *it denotes the largest measure of opaque bad entrepreneurs for which interest rate* $r_M(.)$ *is sustainable.*

The following lemma provides a result about individual investor lending behavior.

Lemma 2 *Each unskilled investor who participates in the lending market only extends loans to the projects that pass her test.*

One of the implications of Lemma 2 is that transparent bad entrepreneurs never obtain any credit. Next, Proposition 1 states our first key result, a characterization of the three types of equilibrium in the credit market as a function of (μ_0, μ_1).

Proposition 1 *When* $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}$,

(i) $\mu_0 \in [0, \frac{c}{1+r_f}]$ *corresponds to a* **bold stage**. *In a bold stage every unskilled investor who extends credit chooses the bold test. The credit market has a pooling equilibrium where all entrepreneurs who obtain credit (all good and opaque bad), do so at the common interest rate* $r_B(\mu_0, \mu_1, c, r_f)$.

(ii) $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$ *corresponds to a* **cautious stage**. *In a cautious stage every unskilled investor who extends credit chooses the cautious test. The credit market has a separating equilibrium, where opaque good entrepreneurs obtain credit at interest rate* $\bar{r}$, *transparent good entrepreneurs obtain credit at rate* $r_C(\mu_0, \mu_1, c, r_f)$, *and bad entrepreneurs don't obtain any credit.*

(iii) $\mu_0 \in (\frac{c}{1+r_f}, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}]$ *corresponds to a* **mix stage**. *In a mix stage, among the unskilled investors who extend credit some choose the bold test while others choose the cautious test. The credit market has a semi-separating equilibrium, where opaque good and bad entrepreneurs obtain credit at interest rate* $r_M(\mu_0, \mu_1, c, r_f)$. *Transparent good entrepreneurs obtain credit at interest rate* $r_C(\mu_0, \mu_1, c, r_f)$.

Otherwise the economy is in autarky, where unskilled investors do not lend, bad entrepreneurs do not borrow, and good ones obtain credit at interest rate $\bar{r}$ *from skilled investors only.*

Since unskilled capital is abundant, unskilled investors always lend at an interest rate that makes them indifferent between paying cost c, running the test of their choice and lending to the entrepreneurs who pass the test, versus using the storage technology and earning the risk-free rate. The bold test passes many applicants, however the resulting loan portfolio involves some defaults since some bad entrepreneurs pass the test. $r_B(.)$ has to compensate the investor for these defaults. On the other hand, an investor who chooses the cautious test is always paid back since he lends to a high quality loan portfolio. However, her rejection rate is also high since even some good entrepreneurs fail the test. As running the test has a fixed cost, not lending to tested applications is costly. $r_C(.)$ has to compensate the investor for excess rejections. Lastly, $r_M(\cdot)$ is a break-even interest rate for a bold investor when not all good types are applying for loans at that rate.

In the bold stage the break-even rate for bold investors is smaller than that of cautious investors, $r_B(\cdot) \leq r_C(\cdot)$. This is the case when $\mu_0 \leq \frac{c}{1+r_f}$, in the leftmost region of Figure 1. Here there are few opaque bad entrepreneurs. Thus the rejection rate of cautious test is relatively high and cautious investors cannot compete with bold ones. As the bold test passes all the good projects, skilled investors would not receive any applications at rates higher than $r_B(.)$. Therefore, there is a single prevailing market interest rate at which all good projects and some bad ones raise funding from both skilled and unskilled investors. Skilled investors still make positive profits as they finance only good projects.

Intuitively, when there are not too many bad projects around, investors are more concerned about losing out on good projects by applying too tight lending standards. Thus lending standards are lax, and many projects including some bad ones are able to raise financing at the same relatively low rate. A bold stage realizes.

On the other hand, if there are many bad projects, investors are concerned about extending loans to bad projects that will default. Lending standards are tightened and credit

market becomes segmented. Not only bad projects are unable to raise financing, even some good ones are able to do so only at extremely high rates. In this case, $r_C(.) < r_B(.)$. This is the rightmost region in the left panel of Figure 1. As cautious investors reject opaque good entrepreneurs, skilled investors can advertise a higher interest rate and attract them. Since skilled capital is in short supply, the interest rate will be the highest rate that a good entrepreneur is willing to accept, $\bar{r}$.

Lastly, if the measure of bad projects is in some intermediate range, some investors apply lax and some tight lending standards. Thus, the mix stage arises. The third part of the proposition states that this happens if there is an intermediate range of μ_0 for which $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$, hence $r_M(\cdot)$ is a feasible interest rate. This is the middle region in the left panel of Figure 1.

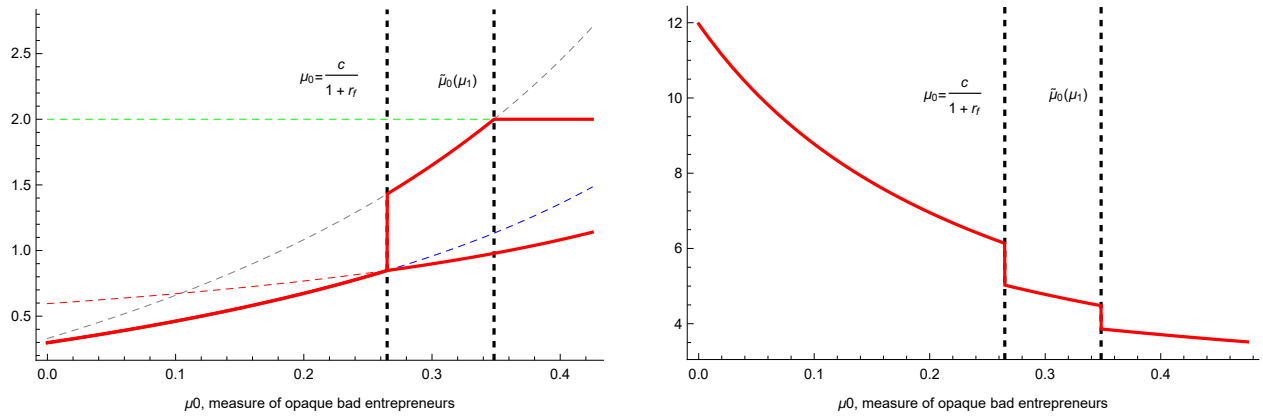
In a mix equilibrium the credit market is fragmented. Cautious unskilled investors finance only transparent good projects at low interest rate $r_C(\cdot)$ which allow them to break even. On the other hand, bold unskilled investors break even at higher interest rate $r_M(.)$ in a market where only opaque good and all bad applicants are present. In this market, some bad projects are able to raise financing from unskilled bold investors. Skilled investors lend in the same market.

A bold stage exhibits several features of an overheated, high sentiment credit market. Interest rates are uniformly low and most projects including some bad ones are financed. Thus the overall quality of initiated credit contracts is low with a significant share eventually defaulting. This is in contrast with the cautious stage which exhibits feature of a low sentiment credit market. Most importantly, this market is fragmented. Some good entrepreneurs (transparent ones) enjoy a lot of funding at low interest rates. However, not only bad projects are not funded, but also some good entrepreneurs (opaque ones) can get only limited funding at very high rates. Therefore, the total loan quantity is relatively low, but its quality is high, which leads to high subsequent realized returns.

3.1 Investment and Output

In this section, we conclude the characterization of the stage game equilibrium by deriving the implied quantity of credit, investment and output for each stage.

In the next proposition we will show that due to the informational friction some en-



(a) Interest rates (b) Output

Figure 1: Interest rates and output as a function of μ_0, for a fixed μ_1. All three types of stage equilibrium occur for some μ_0. The left panel displays the reservation interest rates r_B (dashed blue), r_C (dashed red), r_M (dashed grey), the maximum feasible rate $\bar{r}$ (dashed green, horizontal), and the equilibrium interest rates (solid curves). The right panel displays the output. In the leftmost region the stage game equilibrium is bold, in the middle range it is mix, and in the rightmost region it is cautious. The parameters are: $\rho = 3, \lambda = 0.3, \delta = 0.55, c = 0.265, r_f = 0, w_0 = 5, w_1 = 0,\ \mu_1 = 0.11$.

trepreneurs might face limited credit supply. Let $\bar{\ell}_t(\tau, \omega)$ denote the maximum credit available to entrepreneur (τ, ω). As such, the effective credit constraint is

$$\ell_t(\tau, \omega) \leq \min\left(\bar{\ell}_t(\tau, \omega), \frac{1}{r_t(\tau, \omega)}\right). \tag{6}$$

Each entrepreneur (τ, ω) faces interest rate $r_f < r(\tau, \omega) \leq \bar{r}$. Thus they always prefer to borrow up to the collateral constraint and invest all the proceeds in their projects, and the borrowing constraint (6) holds with equality. The entrepreneurs who are funded by abundant capital of the unskilled investors are unconstrained by the information friction, and borrow $\frac{1}{r(\tau,\omega)}$. This includes all good entrepreneurs in the bold stage, and transparent good entrepreneurs in the cautious stage. For the constrained entrepreneurs who are able to raise financing, $\bar{\ell}(\tau, \omega)$ is determined either by the information friction, or by the limited supply of capital at the market where they raise financing. The next proposition formalizes this result.

Proposition 2

(i) In any equilibrium transparent bad entrepreneurs are not financed by any investors,

$\ell(b,1) = 0$.

(ii) In the bold stage, all entrepreneurs face interest rate $r_B(.)$. *All good entrepreneurs borrow* $\ell(g,\omega) = \frac{1}{r_B}$. *Opaque bad entrepreneurs' are limited by unskilled investors' mistakes at interest rate* $r_B(.)$, *implying* $\ell(b,0) = \frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1}$.

(iii) In the cautious stage, all transparent good entrepreneurs face interest rate $r_C(.)$ *and borrow* $\ell(g,1) = \frac{1}{r_C}$. *Opaque good ones face* $\bar{r}$ *and are limited by the short supply of skilled capital, implying* $\ell(g,0) = \frac{2w_1}{1-\mu_0-\mu_1}$. *Opaque bad entrepreneurs are not financed,* $\ell(b,0) = 0$.

(iv) In the mix stage, all transparent good entrepreneurs face $r_C(.)$ *while opaque good ones face* $r_M(.)$. *Neither are constrained by information frictions,* $\ell(g,1) = \frac{1}{r_C}$ *and* $\ell(g,0) = \frac{1}{r_M}$. *Opaque bad entrepreneurs are limited by unskilled investors' mistakes at interest rate* $r_M(.)$, $\ell(b,0) = \frac{1}{2r_M} - \frac{w_1}{1-\mu_0-\mu_1}$.

The investment of entrepreneur (τ,ω) is given by $i(\tau,\omega) = \rho(1+\ell(\tau,\omega))$ and his output is $y(\tau,\omega) \equiv \rho i(\tau,\omega)$. Therefore, aggregate output in state (μ_0,μ_1) is given by

$$\begin{aligned} Y(\mu_0,\mu_1) &\equiv \frac{1-\mu_0-\mu_1}{2}\Big(y(g,1)+y(g,0)\Big) + \rho\Big(\mu_1 y(b,1) + \mu_0 y(b,0)\Big) \\ &= \rho\left(1 + \frac{1-\mu_0-\mu_1}{2}(\ell(g,1)+\ell(g,0)) + \mu_0\ell(b,0)\right). \end{aligned} \tag{7}$$

In a bold stage all good entrepreneurs are fully financed by bold unskilled investors at interest rate $r_B(.)$. Transparent bad entrepreneurs are excluded from the credit market. However, opaque bad ones can obtain some credit since the bold test does not distinguish them from good entrepreneurs. Yet, their credit is limited by the participating unskilled investor mistakes. Since all good entrepreneurs and even some bad ones raise a lot of credit at low rates and invest, the output is high. Thus the bold stage corresponds to a "boom".

In a cautious stage transparent good entrepreneurs are fully financed by cautious unskilled investors at interest rate $r_C(.)$. However, opaque good entrepreneurs can only obtain credit from skilled investors, who charge them the maximum interest rate $\bar{r}$. As the capital of skilled investors is in short supply, their capital limits the credit of these entrepreneurs implying low credit quantities. Furthermore, none of the bad entrepreneurs is financed. Thus investment is low in a cautious stage and it corresponds to a "downturn".

In a mix stage, some unskilled investors are cautious and some are bold. The cautious ones finance transparent good entrepreneurs at interest rate $r_C(.)$. Similar to the cautious stage, transparent good entrepreneurs use the capital supply of cautious unskilled investors and are unconstrained. On the other hand, the bold investors lend at higher rate $r_M \in (r_C, \bar{r})$. Here similar to the bold stage, opaque good entrepreneurs can use the capital supply of bold unskilled investors and are unconstrained, and opaque bad entrepreneurs obtain some credit as well and survive. Skilled investors only lend to opaque good entrepreneurs at rate $r_M(.)$.

The right panel of Figure 1 illustrates aggregate output conditional on state μ_0, in each type of stage game equilibrium. A natural observation is that for a fixed μ_1, the aggregate output is smoothly monotonically decreasing in the measure of opaque bad entrepreneurs within each class of equilibria. This is because the equilibrium interest rates are (weakly) increasing in μ_0, as depicted in left panel. A larger proportion of bad entrepreneurs increases the equilibrium interest rates due to adverse selection. This increases the cost of capital, which in turn decreases investment and total output.

4 Dynamic Endogenous Cycles

This section develops our main results about the cyclical dynamic behavior of the economy. We describe the deterministic cycles that emerge under different conditions, and explain the outcome in both the credit market and real economy in each cycle. We leave the extension resulting in stochastic cycles for Section 6.

Throughout, a *boom* or an *upturn* refers to the times when output is high and output growth is positive. These real outcomes are accompanied by low yields in the credit market. Alternatively, a *bust*, *downturn*, or *recession* happens when output is low and output growth is negative. This is accompanied by a fragmented credit market.

We first establish that within each period, the dynamic equilibrium reduces to the stage game that we established in the previous section.

Lemma 3 *In any dynamic equilibrium, the economy is in a stage game equilibrium in each period.*

The key to the dynamics of the model is the interaction between the choice of lending standards and the quality composition of the investment. This quality deteriorates in the

bold equilibrium when investors' lending standards are lax, and improves in the cautious equilibrium when their lending standards are tight. At the same time, the change in borrower quality composition induces rational shifts in investors choice of information test and implies fluctuations in sentiment. This interaction leads to endogenous economic cycles without any exogenous aggregate shock to the economy.

We first describe the law of motion for the state variables (μ_0, μ_1), and then explain the emerging cycles. To ease the notation, we omit the time-subscript whenever it does not cause any confusion.

Evolution of State Variables. Let (μ_0, μ_1) and (μ_0', μ_1') denote the state variables today and tomorrow, respectively. When at least some investors are bold, only transparent bad projects cannot raise financing. However, when all investors are cautious, opaque bad projects are not financed either. Any entrepreneur who cannot raise financing exits and is replaced by a new draw from the outside pool. The next proposition summarizes the law of motion for measure of opaque and transparent bad entrepreneurs.

Proposition 3 *Assume* $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r} = \rho - 1$ *so the economy is not in autarky.*

(i) If $\mu_0 \in \left[0, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}\right]$*, then the law of motion for* μ_0 *and* μ_1 *follows*

$$\mu_{0B}(\delta, \lambda, \mu_0, \mu_1) = (1-\delta)\mu_0 + (\delta + (1-\delta)\mu_1)\frac{\lambda}{2}, \tag{8}$$

$$\mu_{1B}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1-\delta)\mu_1)\frac{\lambda}{2}. \tag{9}$$

(ii) If $\mu_0 \in \left(\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1\right]$*, then the law of motion for* μ_0 *and* μ_1 *follows*[8]

$$\mu_{0C}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1-\delta)(\mu_0 + \mu_1))\frac{\lambda}{2}, \tag{10}$$

$$\mu_{1C}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1-\delta)(\mu_0 + \mu_1))\frac{\lambda}{2}. \tag{11}$$

The laws of motion are intuitive. For instance, consider the measure of opaque bad types μ_0. When some investors are bold, function $\mu_{0B}(\delta, \lambda, \phi, \mu_0, \mu_1)$ describes the evolution

[8] If economy is in autarky, equation (10) and (11) still govern the laws of motion.

of μ_0. It consists of survivals from the current period, plus the replacements from the outside pool. From the existing opaque bad entrepreneurs, fraction $(1-\delta)$ survive. The replacements consists of two parts itself: δ measure of all entrepreneurs are exogenously replaced. Furthermore, the remaining transparent bad types cannot raise funding and are replaced. From the replacements, a fraction $\lambda/2$ enter as opaque bad.

The other laws of motion follow a similar intuition. The opaque and transparent good entrepreneurs are subject to the same law of motion in both cases, as they always raise financing, and their measures in the outside pool is the same. As such, in the long run both measures are equal to $\frac{1-\mu_0-\mu_1}{2}$. This validates that (μ_0, μ_1) are sufficient state variables for the economy despite four types of entrepreneurs.

Notice that if the state variables were only governed by dynamics of one of μ_{iB} (equations 8-9) or μ_{iC} (equations 10-11), then (μ_0, μ_1) would converge to constants regardless of the initial conditions. This observation leads to the following Lemma, establishing conditions for a dynamic equilibrium without cycles.

Lemma 4 *Consider the pair of constants*

$$\bar{\mu}_{0B} \equiv \frac{\lambda}{2-\lambda(1-\delta)}, \bar{\mu}_{1B} \equiv \frac{\lambda\delta}{2-\lambda(1-\delta)} \quad \textit{and} \quad \bar{\mu}_{0C} \equiv \frac{\lambda\delta}{2-2\lambda(1-\delta)}, \bar{\mu}_{1C} \equiv \frac{\lambda\delta}{2-2\lambda(1-\delta)}.$$

For any λ and δ, $\bar{\mu}_{0B} > \bar{\mu}_{0C}$ and $\bar{\mu}_{1B} < \bar{\mu}_{1C}$. Furthermore,

(i) If $\bar{\mu}_{0B} \leq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1B})\}$, then $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ is a bold steady state equilibrium.

(ii) If $\bar{\mu}_{0C} \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1C})\}$, then $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ is a cautious steady state equilibrium.

$\bar{\mu}_{0B}$ and $\bar{\mu}_{0C}$ denote the measure of opaque bad entrepreneurs in the permanent steady states of the economy which correspond to a fixed information choice of investors, bold and cautious, respectively. Observe that $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ and $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ correspond to the fixed points of equations (8)-(9) and (10)-(11) described in Proposition 3, respectively. Lemma 4 implies that if the investors' optimal information choice at the fixed point coincides with the information choice that entails the fixed point, then the economy converges to a steady state. Note that the measure of opaque bad entrepreneurs in the bold steady state has to be higher that of the cautious one, $\bar{\mu}_{0,B} > \bar{\mu}_{0,C}$, as the exit rate of opaque bad entrepreneurs is lower when investors are bold.

To understand when the economy converges to a permanent steady state, it is insightful to think of $\frac{c}{1+r_f}$ as the opportunity cost of giving up on good investment for unskilled investors. When the opportunity cost is high, investors prefer to be bold than cautious since the bold test leads to taking all good investment opportunities.

Very high opportunity cost leads to a permanent overheated bold stage since investors always prefer bold to cautious test independent of fraction of bad entrepreneurs, the first part of Lemma 4. The mirror image is the second part of Lemma 4, when very low opportunity cost of giving up on good investment leads to a permanent low-sentiment cautious stage.

Throughout the rest of the paper we focus on parameters where the conditions of Lemma 4 are violated and the dynamic equilibrium is cyclical. This happens when the cost of the test is intermediate. In this situation, in the permanent bold steady state the fraction of bad entrepreneurs is high enough to make it too costly for investors to be bold, and they prefer being cautious. Alternatively, in the permanent cautious steady state, the fraction of opaque good projects is too high. It is too costly for investors to stay cautious and give up on all good investment opportunities, so they prefer to be bold. These deviations make the permanent steady states unsustainable.

Depending on the parameters, the economy admits a wide range of cyclical patterns where the two state variables cycle through a finite number of values in the long-run. We use the following two criteria to broadly classify the cycles. The first criterion is whether the cycle involves a mix stage or not. A *two-stage economy* is one with a permanent cycle which only consists of bold and cautious stages. Alternatively, a *three-stage economy* is one whose permanent cycle has all three stages, bold, mix, and cautious. The second criterion is whether the economy spends more time in the bold or cautious stage during the cycle.

Two-Stage Economy. Using Proposition 1, an economy can cycle through only bold and cautious stages if $\tilde{\mu}_0(\mu_1) \leq \frac{c}{1+r_f}$, for every realized μ_1. Furthermore, from Lemma 4, a cyclical dynamic equilibrium can arise if $\frac{c}{1+r_f} \in (\bar{\mu}_{0,C}, \bar{\mu}_{0,B})$. The next Proposition provides further details on the prevailing cycles as a function of the position of $\frac{c}{1+r_f}$ within this interval.

Proposition 4 *When $\frac{c}{1+r_f} \in (\bar{\mu}_{0C}, \bar{\mu}_{0B})$, for any λ and δ there are constants $\mu^*_{0B} < \mu^*_{0C} \in (\bar{\mu}_{0C}, \bar{\mu}_{0B})$, such that if the prevailing cyclical dynamic equilibrium is a two-stage economy then*

(i) $\frac{c}{1+r_f} \in [\mu^*_{0B}, \mu^*_{0C})$ *implies a 2-period cycle with the two-point support* (μ^*_{0B}, μ^*_{0C})*. In the long-run, the economy oscillates between a one-period bold stage and a one-period cautious stage.*

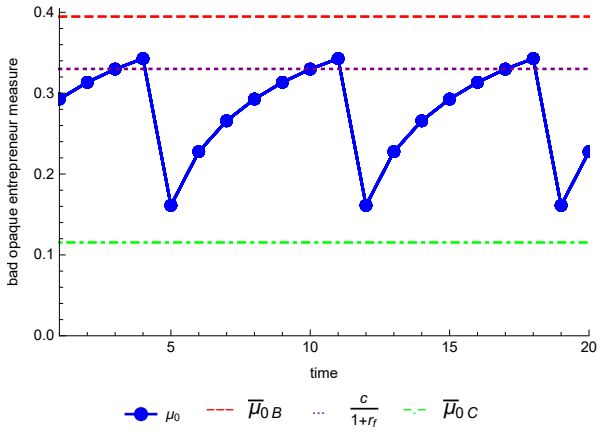
(ii) $\frac{c}{1+r_f} \in [\mu^*_{0C}, \bar{\mu}_{0B})$ *implies a* $\kappa > 2$ *period long bold-short cautious cycle. The cycle consists of* $\kappa - 1$ *consecutive periods where* μ_0 *increases, a long bold stage, followed by a one period decline in* μ_0*, a short cautious stage. A larger* $\frac{c}{1+r_f}$ *implies a longer bold cycle.*

(iii) $\frac{c}{1+r_f} \in (\bar{\mu}_{0C}, \mu^*_{0B})$ *implies a* $\kappa > 2$ *period short bold-long cautious cycle. The cycle consists of* $\kappa - 1$ *consecutive periods where* μ_0 *decreases, a long cautious stage, followed by a one period increase in* μ_0*, a short bold stage. A smaller* $\frac{c}{1+r_f}$ *implies a longer cautious stage.*

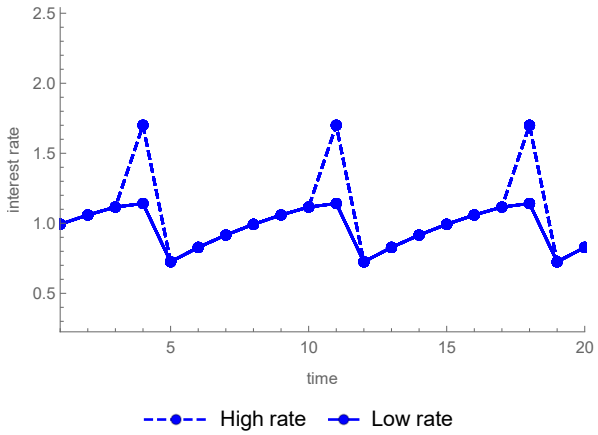
When investors have an intermediate opportunity cost $\frac{c}{1+r_f}$, the economy features deterministic endogenous cycles. The cycles are an outcome of the two-way interaction between investor sentiment and the fundamentals of the economy. When the measure of opaque bad applicants are relatively low, the opportunity cost of losing good investment is high and investors are bold. Lending standards are lax and the interest rate is low. There is a lot of credit and the economy is in a boom. However, as a result of lax lending standards the quality of the credit pool deteriorates. At some point, there are sufficiently many opaque bad applicants that investors prefer to turn cautious. Being cautious implies tight lending standards, high interest rate, large credit spread, and little credit to opaque projects. A recession hits, but this also stops opaque bad entrepreneurs from raising funding. Hence, the quality of credit improves, and the cycle continues.

Proposition 4 illustrates different types of cycles that emerge for different parameter values, and describes a close relationship between the size of $\frac{c}{1+r_f}$ and the time the economy spends in bold or cautious stages. Higher investor opportunity cost of giving up good investment implies longer booms interrupted by one period recessions. A short recession is enough to improve the quality of loan applications enough for investors to be bold again. As such, investors do not risk losing good investment at the cost of financing some bad projects.

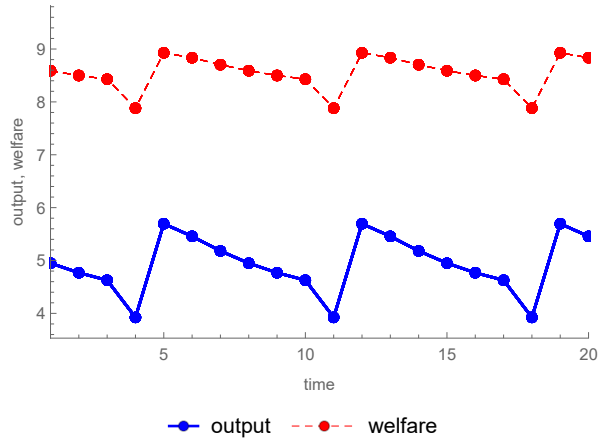
Figure 2 depicts this case, a long bold-short cautious cycle. Panel 2a shows the evolution of the state variable, the measure of opaque bad entrepreneurs. Consider starting at a low μ_0, below the threshold $\frac{c}{1+r_f}$ where the measure of opaque bad entrepreneurs is low. Investors are bold and μ_0 grows towards the higher bold steady state, $\bar{\mu}_{0,B}$. Since $\frac{c}{1+r_f} < \bar{\mu}_{0,B}$, μ_0



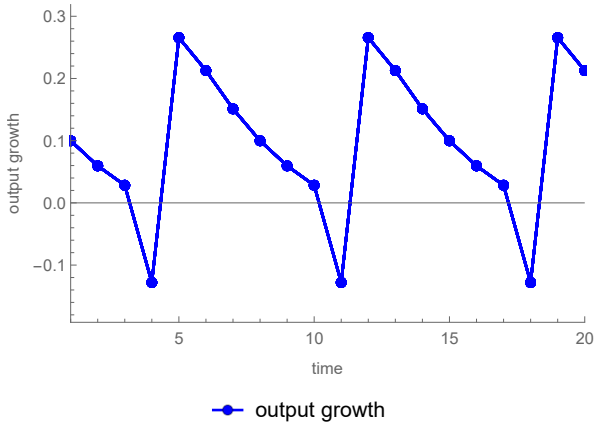
(a) State variable μ_0



(b) Interest rates



(c) Output and Welfare



(d) Output growth

Figure 2: This figure plots a two-stage economy with cycle that consists of a multi-period boom and a one period recession. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare, and Panel (d) is the output growth. The parameters are: $\rho = 2.7, \lambda = 0.6, \delta = 0.2, c = 0.33, r_f = 0, w_0 = 3.3, w_1 = 0.2$.

surpasses this threshold before reaching the bold steady state and triggers a switch to being cautious. At that point μ_0 immediately drops and moves towards the lower cautious steady state, $\bar{\mu}_{0,C}$. The length of boom and bust is determined by the number of periods the economy spends in each stage before crossing the threshold.[9]

Figure 2b plots the interest rates throughout the cycle. As shown in Proposition 1, there is no credit spread in the bold stage. However, the credit market is fragmented in the cautious stage, and the credit spread spikes.

On the other hand, lower investor opportunity cost implies longer downturns followed only by short booms. This corresponds to the economy in Figure 3. Lastly, $\frac{c}{1+r_f}$ in between these two cases implies an alteration between short booms and short downturns.

Three-Stage Economy. If $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$, the economy does not directly transition from a bold stage into a cautious stage. Instead, it passes through a number of intermediate stages in which a fraction of unskilled investors are bold and a fraction are cautious. In this *mix stage*, credit market is fragmented, and interest rates rise. As such, the output experiences a first drop. However, since there are still investors with lax lending standards, the opaque bad projects are able to get some financing. Thus the quality of credit keeps falling as the economy transitions through the mix stage. The mix stage ends when the credit quality is sufficiently low that it is not optimal for any investor to be bold anymore. All investors switch to being cautious and imposing tight credit standards. The economy enters a bust and the output experiences a second drop. However, this drop is accompanied by a dramatic improvement in quality of the credit applicants, to which the investors respond by switching to lax lending standards. The economy switches to a boom, and the cycles continues. Figure 4 depicts a three-stage economy, formally outlined in the next proposition.

Proposition 5 *For any λ and δ, if the prevailing cyclical dynamic equilibrium is a three-stage economy then the cycle has length $\kappa \geq 3$ and consists of a bold stage, followed by a mix stage, and a one period cautious stage. μ_0 increases during bold and mix stages and declines in the cautious stage.*

[9]The indifference threshold $\mu_{0t} = \frac{c}{1+r_f}$ is not a steady state equilibrium. With our tie-breaking assumption, Proposition 3 implies that the bold dynamics apply at the threshold and thus μ_{0t} increases. Any other tie breaking assumption implies a change in μ_{0t} as well. In particular, if positive measure of investors chooses to be bold, the bold dynamics applies. If all investors choose to be cautious then the cautious dynamics applies.

In the next section, we discuss the real outcomes throughout a cycle.

4.1 Dynamics Evolution of Output

This section describes the properties of the path of aggregate output along equilibrium cycles. The first lemma demonstrates that the change in total output is not smooth when the economy switches between different stages.

Lemma 5 *Consider a set of parameters for which the stage game equilibrium is not autarky. Total output, $Y(\mu_0, \mu_1)$, is discontinuous at the threshold across any two stages and jumps downward in μ_0.*

This result shows that $Y(\mu_0, \mu_1)$ is not continuous in the state variables of the economy, as clear in Figure 1b. In this sense, the economy crashes around the thresholds where agents switch *sentiments*. This crash is the consequence of a discontinuous drop in credit when some or all unskilled investors stop lending to opaque entrepreneurs. Furthermore, opaque good entrepreneurs can only borrow at a higher rate. This leads to discontinuously less investment and smaller output.

Output Growth. To illustrate upturns and downturns transparently, it is instructive to examine *output growth*. We define output growth in each period as the percentage difference between period output and initial capital of all agents,

$$g(\mu_0, \mu_1) \equiv \frac{Y(\mu_0, \mu_1)}{w_0 + w_1 + 1} - 1.$$

We believe this is the relevant measure of growth in our framework considering the OLG structure of the model, and no inter-temporal transfer of resources.

Panel 2c and 2d illustrate output level and growth along the equilibrium cycle for an economy with a long boom and a short recession. Panel 2c illustrates the cyclicality of output, and its crash when investor sentiments switches. Comparison with panel 2a shows the co-movement of output with the measure of opaque bad entrepreneurs μ_0. Unsurprisingly, a larger measure of opaque bad entrepreneurs implies smaller output. Moreover, the amplified output drop when there is a switch from an overheated to a low-sentiment credit market is

noticeable. Panel 2a shows that this switch occurs in periods 4, 11 and 18 in our example. While μ_0 increases only slightly in those periods, panel 2c shows a sizable drop in output. In these periods, the deterioration of the pool of credit applications triggers investors to become cautious. Therefore all bad projects lose financing, and opaque good projects are significantly squeezed. As panel 2b shows, the fragmentation in the credit market means the opaque good entrepreneurs face a significantly higher interest rate than before.

On the bright side though, the crash has a "purification effect" on the economy. Bad entrepreneurs exit the economy at a higher rate. This leads to a sufficient improvement in the credit application quality which triggers investors to switch to bold tests. Over the next couple of periods, the credit market becomes overheated again, and the cycle continues.

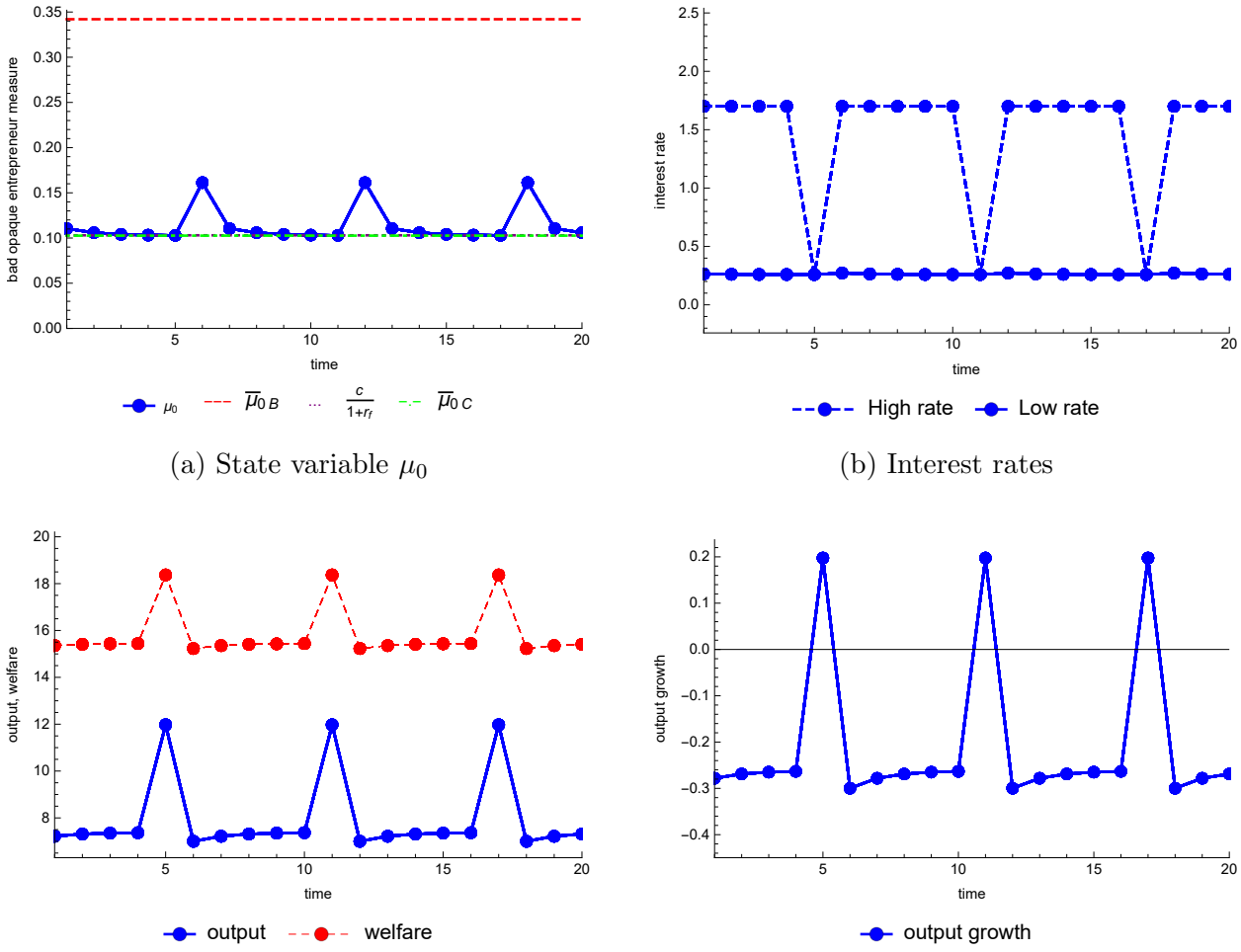
Panel 2d depicts the output growth throughout the cycle. The growth rate is positive in the boom, and negative in the downturn.

4.2 Interpreting Cycles

The richness of the cyclical behavior generated by this framework allows us to consider a few different business cycle outcomes through the lens of the model.

Normal Expansion and Contraction. This is the common post-war business cycle pattern in the US, for instance according to the NBER US Business Cycle Expansions and Contractions categorization. It consists of a long boom, followed by a short recession, followed by the same pattern. Credit market is integrated and the interest rate is low throughout the boom, while during the recession there is segmentation in credit market and interest rates increase. This is characterized in Proposition 4.ii, and depicted in Figure 2.

Prolonged Recovery. If investor opportunity cost is relatively low, the economy is trapped in a lengthy recovery period after each bust, before turning to a short boom. During the lengthy recovery period, the output and loan quality is only slowly improving, and the credit market is fragmented for a long time until credit quality improves sufficiently that investors choose to be bold and relax the lending standards. Figure 3 depicts such an economy which is characterized in Proposition 4.iii.



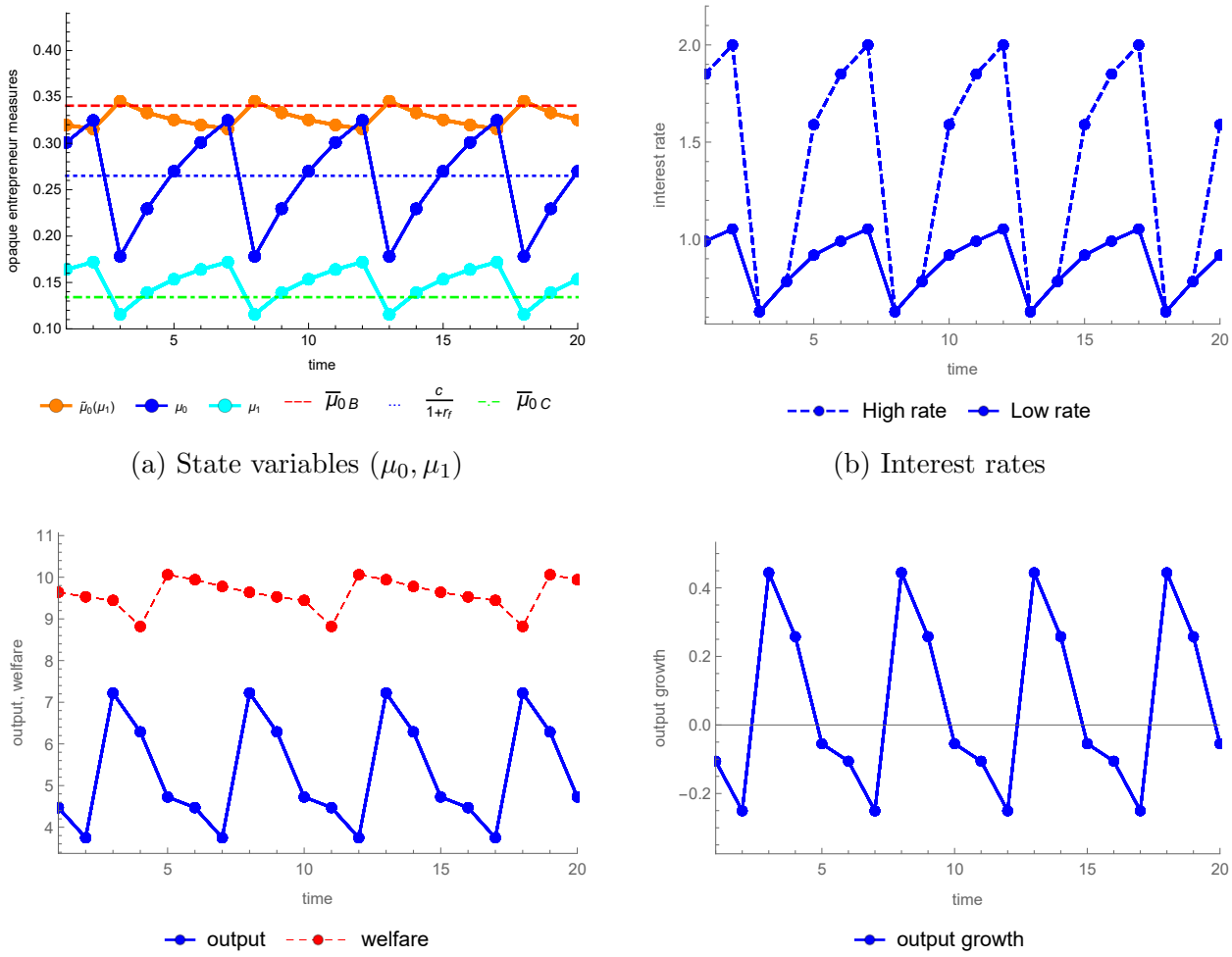
(a) State variable μ_0

(b) Interest rates

(c) Output and Welfare

(d) Output growth

Figure 3: This figure plots a two-stage economy with a cycle which has a long recovery period. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare, and Panel (d) is the output growth. The parameters are: $\rho = 2.7, \lambda = 0.54, \delta = 0.22, c = 0.103, r_f = 0, w_0 = 8.8, w_1 = 0.2$.



(a) State variables (μ_0, μ_1)

(b) Interest rates

(c) Output and Welfare

(d) Output growth

Figure 4: This figure plots a three-stage economy with a double-dip recession. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare, and Panel (d) is the output growth. The parameters are: $\rho = 3, \lambda = 0.3, \delta = 0.55, c = 0.265, r_f = 0, w_0 = 3.99, w_1 = 0.01$.

Double-Dip Recession. The recession can be exacerbated if the initial decline in credit quality is not sufficiently bad to make all investors adopt a cautious strategy and impose tight lending standards. As such, although the fragmentation of credit market leads to a drop in output, yet it does not entail an improvement in loan quality. For some time, the credit market is fragmented, but since some investors are still bold, bad loans keep getting financing and thus credit quality worsens. At some point however, the credit quality has deteriorated so much that every investor chooses to use tight lending standards. The output takes a second hit, but this time it is accompanied by an improvement in the loan quality and leads to a boom. This phenomena happens in the three-stage economy. Figure 4 illustrates an example of a double-dip recession, as explained in Proposition 5.

5 Optimal Cycles and Economic Policy

We have so far demonstrated that changes in credit market sentiments and production fundamentals feed onto each other and create endogenous cycles. As we explicitly model the mechanism which turns booms to recessions and vice-versa, our framework is well suited to explore how policy influences economic cycles.

We first provide an appropriate definition of welfare in our framework, and express some of its important properties. We use that to study the problem of a constrained planner who can choose the investor lending standards, but not the lending and investment behavior directly. This clarifies the dynamic externality which leads to the equilibrium being generically inefficient. Finally, we explore the rational for a policy maker to intervene in this economy with realistic monetary and macro-prudential policies. We explore the cost and benefits of each policy and rank their efficiency in our environment.

5.1 Welfare

The natural measure of welfare in this economy is the aggregate consumption of all entrepreneurs and investors. In equilibrium it simplifies to:

$$W(\mu_0, \mu_1) \equiv \rho(1 + \mu_0 \ell(b, 0)) + \frac{1 - \mu_0 - \mu_1}{2} \sum_{\omega=0,1} \ell(g, \omega) [\rho - (1 + r(g, \omega))]$$
$$+ w_0(1 + r_f) + w_1 \left(1 + \max_{\omega} r(g, \omega)\right). \tag{12}$$

The first term is the total production of all bad entrepreneurs which is fully consumed by them. The second term consists of the consumption of transparent and opaque good entrepreneurs, which is their production net of repayment. The third term is the consumption of unskilled investors noting that they are indifferent between lending and storage at risk-free rate. The last term is the consumption of the skilled investors.

The next proposition shows that welfare decreases in the measure of opaque bad entrepreneurs, μ_0, within any segment of the state space where the type of equilibrium does not change. Moreover, it discontinuously drops when the economy switches across two stages.

Proposition 6 *Consider a set of parameters for which the stage game equilibrium is not autarky. Welfare, $W(\mu_0, \mu_1)$, is decreasing in the measure of opaque bad entrepreneurs μ_0, and discontinuously drops in μ_0 at the threshold across any two stages.*

An increase in the measure of opaque bad entrepreneurs decreases welfare since it exacerbates the borrower adverse selection problem. The cost of capital increases, and the production falls. When some investors switch to be cautious, the problem is intensified since not only some entrepreneurs lose some (or all) financing, but also some good ones can only borrow at the high rates that skilled investors are willing to lend at.

As such, in a cycling economy, just as output, welfare is higher in the bold stage than in the cautious stage, re-enforcing our interpretation of these stages as booms and busts. Figure 2c depicts the dynamics of welfare and output under our baseline parametrization. We next provide a definition of average welfare that enables us to define the constrained optimum and compare across policies.

Definition 5 (Expected Welfare.) *For any collection of m states characterized by the*

pair of state variables $(\mu_{0,j}, \mu_{1,j})$*, the expected welfare is*

$$EW\left((\mu_{0,j}, \mu_{1,j})_{j=1}^{m}\right) \equiv \frac{1}{m}\sum_{j=1}^{m} W\left(\mu_{0,j}, \mu_{1,j}\right).$$

In what follows we are interested in the effect of policy on expected welfare of the cycle.

5.2 Optimal Cycles

In the rest of this section, we normalize the physical return of the storage technology to zero. We then model the monetary policy as the introduction of an asset by the government providing positive return r_f.

As we focus on the relationship between the choice of investors' lending standards and fundamentals, it is instructive to study the following constrained planner problem.

Definition 6 (Constrained Planner Problem) *The constrained planner maximizes the expected welfare in the ergodic state distribution by choosing a threshold* $\hat{\mu}_0^P$ *and one single test available to investors for* $\mu_0 \leq \hat{\mu}_0^P$ *and another test for* $\mu_0 > \hat{\mu}_0^P$*. He cannot choose the prevailing interest rates, lending or investment levels. Furthermore, the participation constraint of all investors and entrepreneurs has to be satisfied.*

The constrained planner has a very restricted tool to influence the economic outcomes. He can only partition the state space into two parts, and in each part choose the single test that is available to investors. As such, the planner can implement a bold (cautious) steady state by choosing a threshold $\hat{\mu}_0^P > \bar{\mu}_{0,B}$ ($\hat{\mu}_0^P < \bar{\mu}_{0,C}$). Alternatively, the planner can implement various two-stage cyclical economies by choosing different levels of $\hat{\mu}_0^P \in (\bar{\mu}_{0,C}, \bar{\mu}_{0,C})$ to partition the state space, and choose the available test to investors in each partition.[10] In the next section, we show that the policies we consider cannot outperform this very restricted constrained planner, which makes it a reasonable benchmark.

The next proposition provides a sufficient condition for the constrained planner solution to be a cyclical economy.

[10]Note that the constrained planner cannot implement a three-stage economy and cannot partition the economy into more than two segments even with only bold and cautious stages.

Proposition 7 *Let* $\lambda^{\min} \equiv \frac{2c+2r_f}{3c+3r_f+1} < \lambda^{\max} \equiv 2\frac{\rho-c-r_f-1}{2\rho-c-r_f-1}$. *For any* $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$, *there exists a* $\bar{\delta}$ *such that for* $\delta < \bar{\delta}$, *the constrained planner solution features a cycle.*

The proposition states conditions for the rational sentiment driven cycles to be the choice of a welfare maximizing constrained planner. Intuitively, the choice of the test is planner's instrument to influence the ergodic state distribution. Tight lending standards has a purifying effect: it keeps the measure of bad entrepreneurs at bay. However, if the planner forces investors to always be cautious, opaque good firms are always squeezed. Therefore, to maximize expected welfare, the planner periodically forces the investors to be cautious when the measure of entrepreneurs not paying back their loans is high.

Externality. The decentralized equilibrium features an externality because investors do not internalize that their individual choice of test influences the ergodic state distribution. Individual investors are atomistic, thus from their perspective a unilateral deviation to another test does not affect the ergodic distribution. As such, the externality would persist even if investors were long lived.

Figure 5 compares constrained planner expected welfare with policy outcomes that we will discuss in the next section. The solid green curve in Figure 5a is the planner curve. It plots the expected welfare of the corresponding cycle for different levels of planner choice of threshold $\hat{\mu}_0^P$. The blue dot represents the expected welfare in the decentralized economy, which is achieved if the planner chooses $\hat{\mu}_0^P = c$. The vertical dashed lines partition the figure into three regions. The leftmost region corresponds to a cautious steady state, the middle region to two-stage cyclical economies of various lengths and various bold/cautious compositions, and the rightmost region is a bold steady state. Welfare changes discontinuously wherever the choice of the planner changes the type of the prevailing cycle and it is flat otherwise.

Figure 5a illustrates that the constrained planner prefers to shorten the length of the boom compared to the equilibrium. Panels 5b and 5c illustrate the intuition. Panel 5b contrasts the path of the state variable μ_0 chosen optimally by the planner with the decentralized equilibrium. The planner enforces a switch to cautious test at a lower level of μ_0 which implies that the economy purifies more often. This keeps the measure of bad types in the applicant pool lower on average, which in turn makes the bold stage more productive. Panel 5c compares the welfare paths between the planner's choice and the decentralized economy. Because of the lower measure of bad types, both the booms and the recessions lead

to a higher welfare under the constrained planner solution compared to the decentralized equilibrium.

5.3 Economic Policy

We have so far established that the constrained optimal economy is often cyclical. In what follows we connect the constrained optimum to realistic monetary and macro-prudential policies. We analyze the cost and benefits of the different policies and compare their efficiency in this economy.

In order to implement monetary policy, the policy maker introduces a risk-free asset for saving within each period. This asset supply is perfectly elastic for entrepreneurs and investors alike. The monetary policy rate $r_{f,t}$ is the net return on this asset. To ensure that the budget constraint of the policy maker is satisfied in every period, we assume a lump-sum tax is imposed on investors each period which exactly covers the aggregate expenditure of providing the risk-free asset. We further assume that the policy maker must implement the same rate within each stage, but he can set a different risk-free rate in bold, cautious and mix stages, r_f^B, r_f^C and r_f^M correspondingly.

As a macro-prudential tool, we model risk-weighted capital requirements. Assume that the regulator imposes a risk weight $x \geq 1$ for each unit of risky investment. The macro-prudential policy is permanent and only depends on the risk characteristics of individual investor portfolio. As such, it is non-state-contingent.

Only bold unskilled investors lend to bad entrepreneur, so they are the only investors with a risky portfolio and subject to the macro-prudential policy. Let v_g and v_r be the bold investor's investment in the risky and risk-free asset per-unit-financing, respectively. Thus we must have $v_g x + v_r = 1$. If $x = 1$, this reduces to the budget constraint of the investor in our baseline economy. When $x > 1$ the capital requirement forces bold investors to forgo investing $v_g(x-1)$ units of their resources. We assume that the investor consumes this excess capital at the end of the period.

Let the tuple $\pi = (x, r_f^B, r_f^C, r_f^M)$ denote a policy profile. In the next proposition, we express the equilibrium associated with each policy profile.

Proposition 8 *Under the policy profile π, the equilibrium is characterized by Propositions*

1-2 with the following modified interest rate functions

$$
\begin{aligned}
r_B^{\pi}(\mu_0,\mu_1,c,\pi) &\equiv r_B(\mu_0,\mu_1,c,r_f^B) + \frac{(x-1)(c+r_f^B+1)(1-\mu_1)}{1-\mu_1-\mu_0}, \\
r_C^{\pi}(\mu_0,\mu_1,c,\pi) &\equiv r_C(\mu_0,\mu_1,c,r_f^C), \\
r_M^{\pi}(\mu_0,\mu_1,c,\pi) &\equiv x r_M(\mu_0,\mu_1,c,r_f^M) + (x-1)(1-\frac{2\mu_1}{1-\mu_0-\mu_1}c),
\end{aligned}
\tag{13}
$$

and the modified thresholds for $\frac{c}{1+r_f}$ *and* $\tilde{\mu}_0(\mu_1,c,\rho,r_f)$ *are,*

$$
\begin{aligned}
\hat{\mu}_0^{\pi}(\mu_1,c,\pi) &\equiv \frac{c}{1+r_f^C} - \frac{(1-\mu_1)}{1+r_f^C}\left((x-1)\left(1+r_f^B+c\right)+\left(r_f^B-r_f^C\right)\right), \\
\tilde{\mu}_0^{\pi}(\mu_1,c,\rho,\pi) &\equiv \frac{(1-\mu_1)(\rho-(1+r_f^M)-(x-1)(c+r_f^M+1))-(1+\mu_1)c}{\rho+x(1+c+r_f^M)},
\end{aligned}
\tag{14}
$$

respectively.

The next Corollary summarizes the effect of the policy tools on the cost of capital and the cycle in a two-stage economy.

Corollary 1 *For any state* (μ_0,μ_1)*, keeping the state constant, an increase in the risk-free rate in the bold stage* r_f^B*, in the cautious stage* r_f^C*, or in the mix stage* r_f^M *increases the cost of capital only in the corresponding stage. An increase in risk weight* x *increases the cost of capital in the bold and mix stages.*

Furthermore, in a two stage economy an increase in x*,* r_f^B*, or the common interest rate* r_f *(*$r_f = r_f^B = r_f^C$*) shortens the bold and elongates the cautious stage.*

This corollary is a direct consequence of Proposition 8, evaluating the comparative statics of interest rate and switching thresholds in equations (13) and (14) with respect to the elements of π.

Intuitively, higher risk-free rates and capital requirement lead to a tightening compared to the laissez-fair equilibrium. Increasing the bold, cautious, or mix risk-free rate implies a higher opportunity cost of lending to entrepreneurs in the corresponding stage. Alternatively, increasing the capital requirement implies a higher opportunity cost only for bold investors by directly decreasing the amount of capital that he can lend. Finally, observe that the lending rate is more sensitive to the common risk-free rate in bold versus cautious regime.

Thus, an increase in the common interest rate increases the bold lending rate more than the cautious one. As such, an increase in x, r_f^B, or r_f leads the economy to spends more time in the cautious low sentiment stage.

Policy Experiment. To gain some insight about the relative efficiency of the available policy instruments, we compare three specific policy profiles. A *simple monetary policy* specifies the same interest rate r_f regardless of the state of the economy, $\pi_{r_f} = (1, r_f, r_f, r_f)$. A *counter-cyclical monetary policy* sets a specific risk-free rate for each stage. It is straightforward to show that it is optimal for the policy maker to have a positive bold risk-free rate and zero cautious risk-free rate. We further assume that the planner sets the mix risk-free rate $r_f^M \geq \bar{r}$. This is sufficient to have $r_M \geq \bar{r}$, so that the mix stage never realizes.[11] Thus the counter-cyclical monetary policy profile is represented by $\pi_{r_f^B} = (1, r_f^B, 0, \bar{r})$. A *macro-prudential policy* consists of risk-weighted capital requirements for risky investment without providing a risk-free asset, $\pi_x = (x, 0, 0, 0)$.

In order to make the welfare effects of these policies comparable, we introduce the concept of equivalent policies bellow.

Definition 7 *Two policy profiles π and π' are equivalent to each other, or to the planner's choice $\hat{\mu}_0^P$, if they imply the same ergodic set for the state variable in the dynamic equilibrium.*

Equivalent policies are useful tools to compare effectiveness of policy tools in improving the efficiency of the equilibrium cycle. One can work out policy instruments r_f, r_f^B, and x that implement the same ergodic state distribution as a given planner threshold $\hat{\mu}_0^P \in [0, c]$. These policy instruments are generically well defined equivalent policies except for rare cases where the implies risk-free rate is so high that pushes the economy into autarky.

The critical observation is that policy equivalence does not imply the same welfare. Our main result below ranks the three policy instruments according to their relative efficiency in achieving the same constrained optimal cycle.

Proposition 9 *Consider a planner threshold $\hat{\mu}_0^P$ that implements longer or more frequent cautious stages than the equilibrium. Furthermore, consider the case where all three equiva-*

[11] Our simulations indicate that the policy maker always finds it optimal to set the mix risk-free rate sufficiently high that the realized cycle is two-stage.

lent policies $\pi_{r_f}, \pi_{r_f^B}$ and π_x lead to a two-stage economy. Within each class of policies, pick the one that corresponds to the lowest lending rate. Then the following statements hold.

(i) Expected welfare associated with all three policies is strictly lower than the constrained optimal expected welfare.

(ii) Equivalent policies $\pi_{r_f^B}$ and π_x imply the same equilibrium interest rate for every entrepreneur in every stage. However, the counter-cyclical monetary policy has a higher expected welfare.

(iii) For $\lambda \leq \frac{8}{9}$,

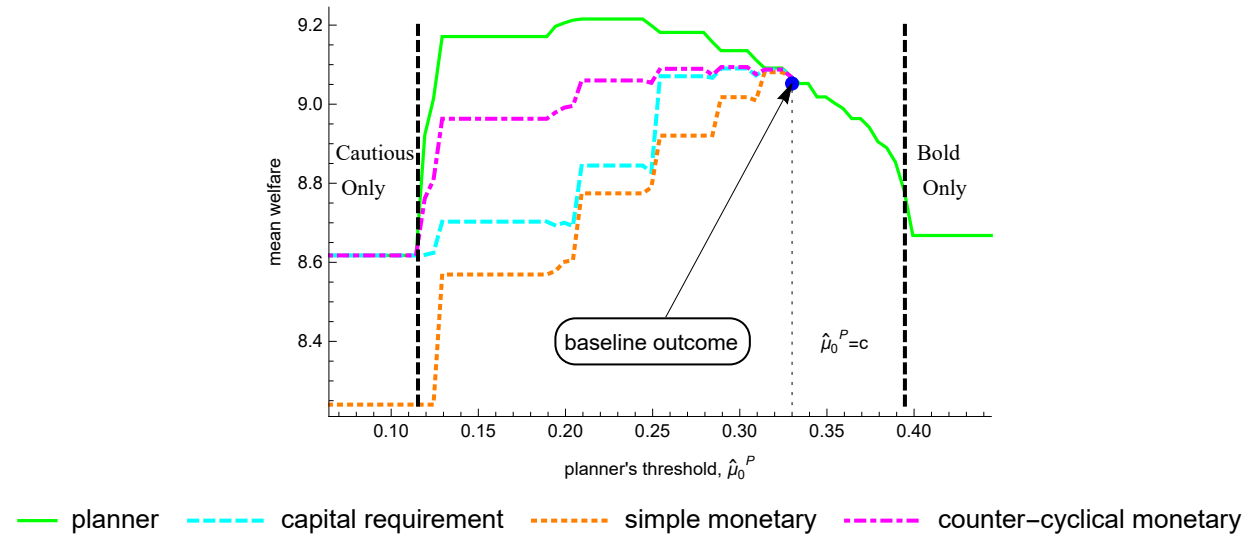
(a) $\pi_{r_f^B}$ has a higher expected welfare than π_{r_f},

(b) $\exists \bar{c}$ such that if $c \leq \bar{c}$, π_x has a higher expected welfare than π_{r_f}.

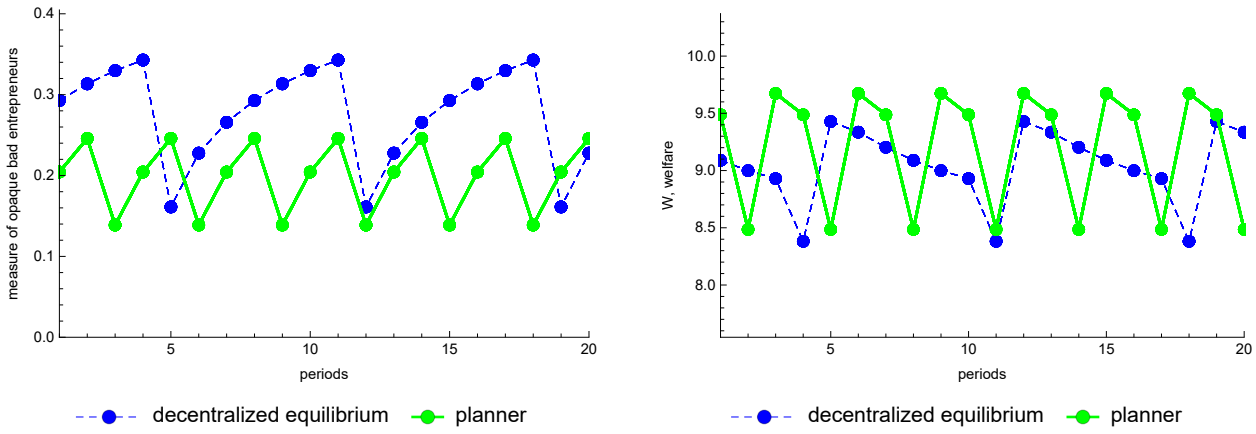
All three policies are costly compared to the constrained optimum. Unlike the planner who directly chooses the lending standards, the policy maker has to influence investors' incentives to choose among the available tests appropriately. This leads to higher lending rates under all policies, compared to the constrained optimum. Within each class of policies, we choose the one that implements the planner threshold $\hat{\mu}_0^P$ at the lowest interest rate in the lending market.

The higher cost of capital associated with each policy implies less borrowing, investment, output and consumption, which entails a welfare loss. The simple monetary policy performs the worst since it increases the cost of capital in all stages. This leads to less investment and output in every period, while the other two policies only make borrowing costlier in the bold stage.

It is interesting to note that the counter-cyclical monetary policy and its equivalent macro-prudential policy have the same effect on the cost of capital in a two-stage economy. Thus they entail the same investment and output. Yet, the counter-cyclical monetary policy has a higher expected welfare. The intuition is the following. Both policies imply the same interest rates, thus they correspond to identical credit demand. However, on the credit supply side, the macro-prudential policy implies a quantity constraint on lending. The lower per-capita credit supply requires more unskilled investors to enter the lending market to satisfy the same credit demand. As all of them have to pay the fixed cost c, the macro-prudential policy is dominated by the counter-cyclical monetary policy. Nevertheless, unless this cost is very high, it dominates the simple monetary policy.



(a) Expected welfare



(b) Measure of opaque bad projects

(c) Welfare paths

Figure 5: Expected welfare for different levels of planner choice of threshold $\hat{\mu}_0^P$, as well as the comparison between the implied paths for the measure of opaque bad entrepreneurs μ_0, and welfare, along the optimal versus the decentralized cycle. Baseline parameters are: $\rho = 2.7, \lambda = 0.6, \delta = 0.2, c = 0.33, r_f = 0, w_0 = 4.5, w_1 = 0.2$. On bottom panels planner's threshold is $\hat{\mu}_0^P = 0.21$.

Both the simple monetary policy and capital requirements can turn a two-stage economy to a three-stage economy by changing the incentives for different lending standards. In such cases a bold, high sentiment stage is replaced by a mixed sentiment one. While the implied dynamics under the two regimes is the same, welfare is typically higher if sentiments are high.

The three non-solid curves in Figure 5a illustrate the expected welfare under the three policies that implement the constrained optimal threshold $\hat{\mu}_0^P$ on the x-axis. The pink dashed-doted line represents the counter-cyclical monetary policy, the orange dotted line is the simple monetary policy, and the blue dashed line is the macro-prudential policy.

The policies perform best by implementing cycles that have shorter booms than the laissez-fair equilibrium, or slightly more recurrent low-sentiment purifying stages. Interestingly, in this region, just to the left of the baseline outcome, the counter-cyclical monetary policy and the capital requirements perform extremely close, and outperform the simple monetary policy. They both imply a two-stage economy with identical cost of capital.

To compare these two policies, it is important to note that the aggregate state is fully endogenous in this model. As such, the counter-cyclical monetary policy assumes a relatively high degree of sophistication for the policy maker. The policy maker has to solve a fixed point problem to compute the aggregate state, and predict what aggregate outcome each choice of interest rate entails. For instance, he has to foresee whether the chosen interest rate keeps the economy in a boom or moves it to a downturn. This allows the policy maker to avoid the mix stage all together when using a counter-cyclical monetary policy. In contrast, the macro-prudential policy conditions only on the individual lending choice of investors, and delivers a very similar peak performance without requiring any information about the aggregate state.

6 Persistent productivity shock, stochastic cycles

In this section, we introduce an aggregate productivity shock to our model. We argue that this extension, while keeps the analysis tractable, makes our framework more appealing. This extension implies that deterministic cycles are replaced by more realistic stochastic ones. In this extended economy, we can also study the interaction of credit cycles and productivity cycles.

Suppose that the fraction of bad entrepreneurs among the new entrants, λ_t instead of being constant at λ, follows an S state Markov process, $s = 1, ...S,$, with transition matrix $\Sigma \in [0,1]^S$ and support vector $\boldsymbol{\lambda} \in \mathbb{R}^S$. Also, we relax the assumption that the productivity of good and bad firms is equal, $\rho_g \neq \rho_b$, and let $\rho_g > \rho_b > 1 + r_f$ instead. Note that the average productivity of entrants in a given period is $(1-\lambda_t)\rho_g + \lambda_t \rho_b$. In this sense, low λ_t values correspond to an exogenous low productivity state.

Finally, in this part, we interpret investment in the storage technology as riskless, low-productivity production.

Observe that the fraction of bad enterpreneurs in the outside pool, λ_t, enters the equilibrium only through the law of motion described in Proposition 1. Therefore, the characterization of the equilibrium in Sections 3-4 remains intact except for the trivial replacement of (8)-(11) with their state-dependent, stochastic counterparts. For instance, (8) is replaced by

$$\mu_{0B}(\delta, \lambda_t, \mu_0, \mu_1) = (1-\delta)\mu_0 + (\delta + (1-\delta)\mu_1)\frac{\boldsymbol{\lambda}_{s'}}{2}, \text{ w.p. } \Sigma_{s,s'} \text{ if } \lambda_t = \boldsymbol{\lambda}_s.$$

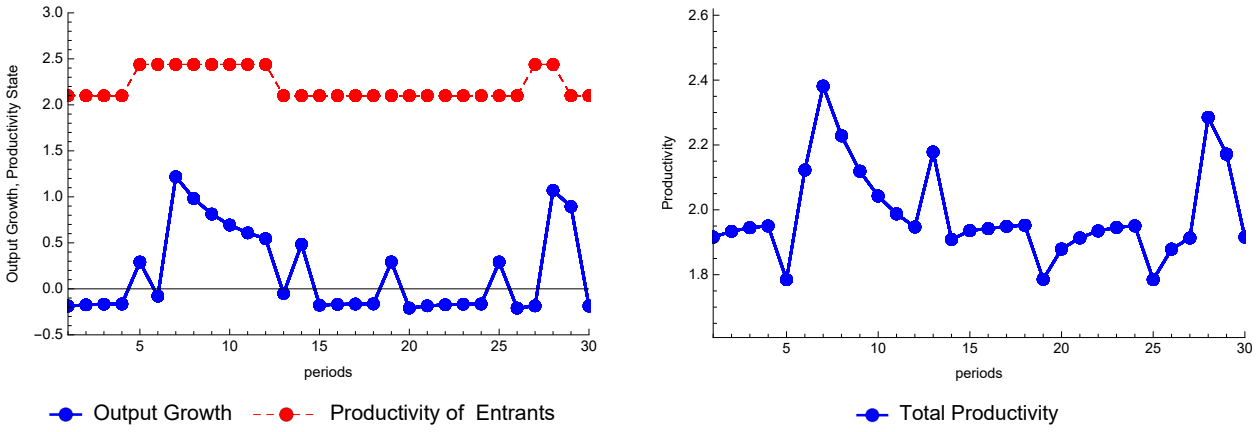
We study the properties of the extended model through a simple example.

Example 1 *Let λ_t follow a two state, persistent process. That is, $S = 2$, $\Sigma_{ss} = \alpha_s > \frac{1}{2}$ for $s = 1, 2$ and $\boldsymbol{\lambda}_1 < \boldsymbol{\lambda}_2$*

In this example, there are two persistent aggregate states. The first state corresponds to high productivity of entrants as in this case the fraction of bad entrants is low.

Figure 6 illustrates equilibrium variables under one possible path. Panel 6a shows the path of growth (solid) along with the average productivity of entrants (dashed). The latter is the exogenous productivity state. Clearly, this extension leads to stochastic cycles. Booms alternate with recessions of varying length.

Note also that the effect of productivity shocks and endogenous lending standards are intertwined. For instance, periods 14-26 correspond to an interval of low aggregate productivity state. Still, the economy switches between recessions and booms as lending standards endogenously fluctuate within this interval. This is contrast with period 27 where the high productivity state does not push the system to a boom. Because the economy is still in the cautious stage, tight lending starts limit the effect of the productivity shock, slowing down growth.



(a) Output Growth

(b) Productivity

Figure 6: The figure plots growth and productivity paths whith persistent productivity shocks to entrants. Panel (a) depicts output growth, while panel (b) depicts the productivity of new entrants (dashed) and the total productivity of all firms (solid). The parameters are: $\rho_g = 2.7, \rho_b = 1.7, \lambda_1 = 0.4, \lambda_2 = 0.6, \alpha = 0.8, \delta = 0.2, c = 0.116, r_f = 0, w_0 = 6.5, w_1 = 0.2$

It is also intresting to consider the highest growth point in period 7. This is the outcome of a the coincidence that a long cautious stage cleansed the economy from bad firms, which was reinforced by the entry of high productivity firms in period 6. Therefore, the switch to the bold stage in period 7 happened when the fraction of bad firms was unusually low explaining the high growth.

Panel 6b shows the path of the total productivity of producing firms. This is the ratio of total output of risky and risk-free production over the total investment in all the technologies. Note that total productivity depends both on the exogenous productivity of new entrants and on endogenous credit market outcomes. This is so, because, just as in the baseline model, state variables (μ_0, μ_1) and lending standards influence which firms are financed. These firms scale up increasing their weight in total productivity. This leads to an intricate relationship between total productivity and growth. In fact, while the two processes are positivly correlated as we expect, the correlation can be well below 1. In this example, it is 0.64.

7 Model and Facts

The model generates a rich set of empirical predictions despite its simple structure. When mapping the model outcomes to the data, a critical question is the empirical counterpart of the distribution of credit flow to different firm types in the model. Here we explore the empirical predictions of the model under two different approaches.

The conservative approach is to treat the heterogeneity across firms as unobservable. As such, the econometrician can only observe aggregate credit flows, without being able to identify flows to different firm types. We first describe the predictions of the model under this assumption, and then move to a less conservative assumption.

Tightness of credit, interest rates, and economic cycles Treating firm type as fundamentally unobservable, our model has the following predictions for any group of borrowers: (1) Credit standards are lax and the average quality of issued credit is deteriorating in booms. Morais et al. (2019) find both US and international evidence for lax lending standards in booms in the bank loan market. In a different context, Demyanyk and Van Hemert (2009) document that the quality of sub-prime loans deteriorated for six consecutive years before the 2007 crisis. (2) Conditions of credit supply are more favorable in booms than in recessions. Consistent with this prediction, Becker and Ivashina (2014) present various measures to argue that the cyclicality of aggregate credit is mostly due to the cyclicality in credit supply, at least for small firms in the US. (3) Within group, credit is granted at less dispersed interest rates in booms compared to recessions. We are not aware of any work focusing on the cyclicality of interest rate dispersion within a group of borrowers.

A less conservative approach is to assume that at least ex-post, it is possible to partition firms according to their transparency. Consider the following thought experiment building on the example of the commercial paper market around the European debt crisis in 2010. When global fundamentals are strong, investors choose to be bold and lend to all major banks based in developed countries. They understand that some have less healthy balance sheets than others, but they do not have the expertise to distinguish them. Instead, when global fundamentals are weak, investors choose to be cautious and only lend to major US banks as the safest strategy. If our mechanism captures main determinants of the European debt crisis, we should be able to identify a large group of investors following the former strategy before 2010, but switching to the latter after the Greek default. By observing the

difference between these two strategies, the econometrician would conclude that credit to European banks maps to opaque credit in our model.

Market fragmentation and heterogeneous portfolio rebalancing As a bold stage turns into a cautious stage, skilled and unskilled investors rebalance their portfolio in opposite directions. Unskilled investors rebalance from low-quality bonds (opaque ones) to high-quality ones (transparent ones), while skilled investors do the opposite. This implies that good entrepreneurs face heterogeneous experiences. Some good entrepreneurs (transparent good ones) enjoy abundant credit supply while others are squeezed (opaque good ones), although in the bold stage they faced the same market conditions. This market fragmentation and the implied heterogeneous effect of a downturn is a unique feature of our model.

Indeed, our suggested thought experiment is inspired by Ivashina et al. (2015) and Gallagher et al. (2018) who find a group of US money market funds that stopped lending to all European banks in 2011, but not to other banks with similar fundamentals. This pattern is implied by our mechanism considering these funds to be low-skilled investors. Moreover, Ivashina et al. (2015) find evidence that this process led to a significant disruption in the syndicated loan market, a possible channel for the real effect predicted by our model.[12]

Credit composition, the quality spread and credit market sentiment One interpretation of the credit issued to firms who are rejected by a cautious test, i.e. credit to opaque firms, is junk bond issuance. Alternatively, loans to transparent good firms map to high-grade bond issuance.

With this interpretation, our model is consistent with the well-known fact that the quality spread, the spread between AAA and BAA corporate borrowers, is counter-cyclical. As such, our paper provides an information based alternative explanation for time-varying risk-premium.

We can also interpret our predictions within the context of the growing body of evidence suggesting that periods of overheating in credit markets forecasts low excess bond returns. Importantly, Greenwood and Hanson (2013) show that the share of junk bond issuance out of total issuance inversely predicts the excess return on these bonds.[13]

[12]Farboodi and Kondor (2018) provide a substantially richer picture on market fragmentation by treating sentiment switches as exogenous.

[13]The inverse relationship between credit expansion and subsequent returns is remarkably widespread

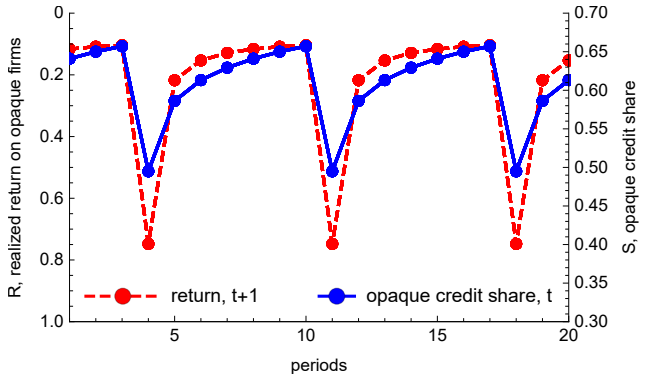


Figure 7: Model generated positive correlation between opaque credit share and its future realized excess return on the invested scale. The solid blue line plots the share of issued credit to opaque projects relative to all credit in a given period on the right scale. The dashed red line depicts the realized excess return on opaque credit, one period later, on the left scale on an inverted scale.

Figure 7 illustrates the model equivalent of this empirical pattern documented in Exhibit 1 of Stein (2013), for the two-stage economy simulated on Figure 2. As in Exhibit 1 of Stein (2013), the model predicts a positive correlation between share of junk bond issuance and its future realized excess return on the invested scale. High level of curves correspond to overheated periods with low subsequent returns. Low levels instead correspond to recessions, low sentiment credit markets with high subsequent returns.[14] Note the strong co-movement between share of opaque loans and their corresponding return on a reverse scale, both within the bold stage and across periods.

It is important to note that although our model generates a strong positive correlation between these variables, this does not amounts to an exploitable anomaly based on the

across various financial markets. For instance, Baron and Xiong (2017) document the negative relationship between bank's credit expansion and banks' equity returns, Kaplan and Stromberg (2009) find a similar inverse relationship between venture capitalists aggregate flow to new investments and their subsequent returns. A related early work is Eisfeldt and Rampini (2006), who shows that volume of transactions is pro-cyclical while return on transactions is counter-cyclical in the sales of property, plant and equipment.

[14]Formally, let $S(\mu_0, \mu_1)$ denote the share of credit to opaque firms, and $R(\mu_0, \mu_1)$ denote the net excess realized return on a portfolio of these loans. We have:

$$S(\mu_0,\mu_1) \equiv \frac{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\ell(g,0)}{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\left(\ell(g,0)+\ell(g,1)\right)} \quad R(\mu_0,\mu_1) \equiv \frac{\frac{1-\mu_0-\mu_1}{2}\ell(g,0)(1+r(g,0))}{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\ell(g,0)} - (1+r_f).$$

information set of unskilled investors.[15]

8 Conclusion

The idea that economic fluctuations can be captured by models with endogenous cycles is not new. In fact, the earliest business cycle models by John Hicks and Nicolas Kaldor followed this approach. However, as Boldrin and Woodford (1990) explain, these models fell out of favor by the late 1950's because they had been empirically rejected: actual business cycles were found not to show regular cycling behavior.[16]

In this paper, we argue that despite real world cycles being stochastic and difficult to forecast, simple models with endogenous cycles are a useful apparatus for macroeconomic theory as indispensable analytical tools for policy analysis. To asses the effect of various policies on the length and depths of booms and busts, it is essential to understand what predictably turns booms into busts and vice-versa.

We propose a model where endogenous cycles are generated by the interaction between lenders' choice of lending standards in the credit market, and the economic fundamentals. Tight credit standards screen out low quality entrepreneurs and thus the future quality of credit applications improves. Once the improvement is sufficiently significant, it triggers a switch to lax lending standards. This in turn leads to the deterioration of fundamentals, which prompts tight credit conditions again.

We show that simple policy tools allow the policy maker to control the cyclicality of the economy. By utilizing a macro-prudential policy to carefully choose capital requirements for risky investment, or through an appropriate counter-cyclical monetary policy, the policy maker can optimally use recessions to keep the stock of bad borrowers at bay. We further demonstrate that the predictions of the model match numerous stylized facts related to credit cycles.

[15] See Bordalo et al. (2018); Greenwood et al. (2019); Gennaioli and Shleifer (2020) for empirical facts pointing towards such anomalies, and bounded rational models designed to target those.

[16] See the recent work of Beaudry et al. (2020) for the argument that modern statistical techniques might refute this statement.

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A Agent Optimization Problem and Market Clearing Protocol

In this Appendix we formally define the problem of each agent, the market clearing protocol, and a robustness criterion. We also show how the agents' problem reduce to the ones set up in the main text. The structure of our credit market is a modified version of Kurlat (2016). The entrepreneur and investor problems are simplified versions of those in Farboodi and Kondor (2018).

A.1 Entrepreneur and Investor Problems in the Stage Game

Let R denote the a set of trading posts, each of which identified by an interest rate r. The problem for an entrepreneur (τ, ω) is

$$\max_{\{\sigma(r;\tau,\omega)\}_{r\in R}} \rho i(\tau,\omega) - \mathbf{1}_{\tau=g}\ell(\tau,\omega)\left(1+r(\tau,\omega)\right) \tag{A.1}$$

$$s.t.$$

$$0 \leq \sigma(r;\tau,\omega) \leq \frac{1}{r} \quad \forall r \in R$$

$$\ell(\tau,\omega) = \int_R \sigma(r;\tau,\omega) d\eta(r;\tau,\omega) \tag{A.2}$$

$$r(\tau,\omega) = \frac{\int_R r\sigma(r;\tau,\omega) d\eta(r;\tau,\omega)}{\ell(\tau,\omega)} \tag{A.3}$$

$$\ell(\tau,\omega) \leq \frac{1}{r(\tau,\omega)} \tag{A.4}$$

$$i(\tau,\omega) = \ell(\tau,\omega) + 1.$$

$\sigma(r;\tau,\omega)$ denotes the number of credit units entrepreneur (τ,ω) demands at interest rate r. $\ell(\tau,\omega)$ and $i(\tau,\omega)$ denote the total amount of credit and the investment level for entrepreneur (τ,ω), respectively.

η is the *rationing function* that assigns $\eta\left(R_0;\tau,\omega\right)$ measure of credit, per unit of application, to entrepreneur (τ,ω) who has submitted applications to the subset of trading posts $R_0 \in R$. η is an equilibrium object, determined by the choices of the agents and the market clearing protocol as explained below. The entrepreneur takes η as given.

Let $\bar{\ell}$ denote the maximum available credit for a given entrepreneur,

$$\bar{\ell}(\tau,\omega) \equiv \int_R \frac{1}{r} d\eta\left(r;\tau,\omega\right).$$

We are interested in showing that an equilibrium exists. As such, we conjecture and then verify that there exist an equilibrium in which each entrepreneur only raises credit at one single interest rate. From equations (A.3) and (A.2), $r(\tau,\omega)$ denotes the *average* interest rate that the entrepreneur raises credit at. Under the conjecture that he raises credit at a single interest rate, with some abuse of notation let $r(\tau,\omega)$ denote that unique interest rate. In particular, $r(\tau,\omega)$ does not depend on $\sigma(.)$.

Under this conjecture, the entrepreneur's problem can be rewritten as

$$\max_{\ell(\tau,\omega),r(\tau,\omega)} \rho + \ell(\tau,\omega)(\rho - \mathbf{1}_{\tau=g}(1+r(\tau,\omega))) \tag{A.5}$$
$$s.t. \qquad \ell(\tau,\omega) \leq \min\left(\bar{\ell}(\tau,\omega), \frac{1}{r(\tau,\omega)}\right).$$

This form suppresses the choice over credit applications, $\sigma(\cdot)$, and focuses on the total obtained credit $\ell(\cdot)$. For any obtained credit $\ell(\tau,\omega)$ along with equilibrium $\eta(r;\tau,\omega)$ schedule, equation (A.2) determines $\sigma(r;\tau,\omega)$.

Each investor h advertises a single rate $r(h)$. Unskilled investor h solves

$$\max_{\chi(h),\tilde{r}(h)} (1+\tilde{r}(h))\left(S_u(r;g,1) + \mathbb{1}_{\chi(h)=B} S_u(r;g,0)\right)$$
$$+(1+r_f)\Big(S_u(r;b,1) + \mathbb{1}_{\chi(h)=C}(S_u(r;b,0) + S_u(r;g,0))\Big),$$

while skilled investor h solves

$$\max_{\tilde{r}(h)} (1+\tilde{r}(h))\left(S_s(r;g,1) + S_s(r;g,0)\right).$$

$\chi(h)$ is the unskilled agent's choice of test. S_u and S_s are the sampling functions for unskilled and skilled investors.

An unskilled investors has one unit of wealth, thus she samples total one unit of applications at the interest rate she advertises. $S_u(r;\tau,\omega)$ denotes the measure of applications submitted by (τ,ω) entrepreneurs that the unskilled investor who has advertised interest rate r receives. Importantly, this measure is independent of unskilled investor's choice of test. $S_s(r;\tau,\omega)$ is the analogous object for skilled investors. The sampling functions are aggregate equilibrium objects determined by the market clearing protocol and the choices of agents, and are taken as given by investors.

We follow Kurlat (2016) to assume the following *robustness* criterion.

Assumption A.1 *Suppose that ε fraction of applications submitted at an advertised interest rate are granted unconditionally. We require that the equilibrium strategy of each entrepreneur is the limit of equilibrium strategies as ε goes to 0.*

This assumption has two implications. First, it prevents equilibrium multiplicity. Second, it implies that every type who chooses to submit loan applications at a given interest rate, submits the maximal amount. Thus $\sigma(r;\tau,\omega) > 0$ implies $\sigma(r;\tau,\omega) = \frac{1}{r}$. As such, the application pool at any given interest rate is independent of cross-sectional distribution of $i(\tau,\omega)$, and we can solve the credit market equilibrium independently of $i(\tau,\omega)$ choices. This simplifies the analysis considerably.

Market Clearing Protocol. Let r' denote the lowest interest rate which is both advertised by some investor and some entrepreneurs have submitted demand at this rate. If there is no such interest rate, then no applications are financed.

First, each entrepreneur who submits an application at that rate posts r' *down-payment* per unit of application from her endowment. Applications without a down-payment are automatically discarded. Then, each unskilled investor who has advertised rate r' obtains a measure one sample of the (non-discarded) applications submitted at that rate with the underlying distribution. As such, $S_u(r';\tau,\omega)$ is equal to the fraction of non-discarded (τ,ω) application submitted at interest rate r'. The investor runs her chosen test and grants credit to all applications that pass the test in the sample she receives.

If there are not enough applications to fill up every unskilled present investor's capacity limit, then all applications have been sampled and the sampling process stops. Otherwise, all unskilled investors sample a measure (of value) one of applications and provide financing to all applications in their sample that passes their chosen test. Their remaining endowment is invested in the risk-free asset.

If all unskilled investors reach their sampling capacity and there are remaining good projects, then they are distributed pro rata across skilled investors up to their capacity given by their one unit of endowment. As such, $S_s(r';g,\omega)$ is the ratio of remaining non-discarded (g,ω) applications at interest rate r' relative to sum of remaining non-discarded $(g,\omega)+(g,\omega')$ applications after unskilled investors make their financing decision at rate r'. Skilled investors grant credit to these projects.

Entrepreneurs who receive financing invest the credit they obtain along with the down-payment, and the invested units are posted as collateral for the loan. These invested units enter into a public registry, so they cannot serve as collateral to other loan applications. Applications that are submitted but do not receive financing are discarded, and the down-payment is returned to the entrepreneur who can only invest it in the risk-free asset.

Then, the process is repeated at the next lowest advertised interest rate at which there are applications. The process stops once there is no such rate anymore. $\eta(r;\tau,\omega)$ is computed by aggregating over all investors who grant credit to entrepreneur (τ,ω) at interest rate r.

A.2 Entrepreneur and Investor Problems in the Full Game

Since each investor lives for a single period, she solves the identical utility maximization problem in the stage game and the full game.

For entrepreneurs the only change is that they maximize the expected sum of their future utility while alive. This consists of entrepreneur's period utility, as well as his expected continuation value. That is, instead of (A.5), the value function of the entrepreneur can be written as

$$
\begin{aligned}
&V(\tau,\omega;\mu_{0,t},\mu_{1,t}) = \\
&\max_{\ell_t(\tau,\omega),r_t(\tau,\omega)} \rho + \ell_t(\tau,\omega)(\rho - \mathbf{1}_{\tau=g}(1+r_t(\tau,\omega))) + (1-\delta)\mathbf{1}_{\ell_t(\tau,\omega)>0}V(\tau,\omega;\mu_{0,t+1},\mu_{1,t+1}) \quad \text{(A.6)} \\
&s.t. \qquad \ell_t(\tau,\omega) \leq \min\left(\bar{\ell}_t(\tau,\omega), \frac{1}{r_t(\tau,\omega)}\right),
\end{aligned}
$$

where the entrepreneur takes the equilibrium dynamics of $(\mu_{0,t},\mu_{1,t})$ as given.

B Proofs

Proof of Lemma 1

The market clearing mechanism and Assumption A.1 implies that in the stage game if any agent would like to raise credit at an interest rate $r^{\max}$, she would want to submit a maximum measure of applications, $\sigma(r;\tau,\omega) = \frac{1}{r}$ at every interest rate smaller than $r^{\max}$ too. The reason is that it makes it possible that they are receiving a fraction of their credit at a lower rate (as markets clear from the lowest interest rate), and potentially even without the requirement to invest the received amount (Assumption A.1). This latter possibility is attractive for bad entrepreneurs. Because applications with no down-payment are discarded, there is no possibility of having more credit granted as intended. Agents also want to submit the maximum measure of applications at $r^{\max}$. Given the linear structure, if, at a given interest rate an agent would like borrow to invest, she also would like to borrow up to the limit $\frac{1}{r}$ and invest at that rate. This concludes the first part of the Lemma.

For the second part, observe that the objective function (A.1) implies that a good entrepreneur does not apply for credit at any interest rate $r(g,\omega) > \rho - 1$ as that would imply negative return on her investment. As we noted before, Assumption A.1 and objective (A.1) imply that bad entrepreneurs instead apply for maximum credit at any interest rate as they do not plan to pay back.

Proof of Lemma 2 and Proposition 1

The main steps of the proof are explained in the text. Here, we just have to specify the details.

As we explained, in any equilibrium unskilled investors have to lend at the break-even interest rate which makes them indifferent whether to participate. Also, they never extend credit to entrepreneurs not passing their test. This is so, because tests are informative. Therefore, if an investor extends credit even to those entrepreneurs who are not passed her test, it will increase her break-even interest rate. Therefore, if there were such a group of investors in equilibrium, non-participating investors would deviate by entering at a slightly lower interest rate, extending credit only to those who pass their test, and stealing the business of the first group. This proves Lemma 2.

Now, we derive the break-even interest rate, r_B^p, for bold investors in case all entrepreneurs submit the maximum demand at that advertised rate. The superscript refers to the fact that it is a pooled market where all entrepreneurs submit. In fact, r_B^p is defined by the indifference condition

$$(1-\mu_1-\mu_0)(1+r_B^p)+\mu_1(1+r_f)-c=1+r_f \tag{A.7}$$

Note that $((1-\mu_1-\mu_0)+\mu_0)\frac{(1-\mu_1-\mu_0)}{((1-\mu_1-\mu_0)+\mu_0)}=(1-\mu_1-\mu_0)$ is the probability of ending up financing a good project with a bold test (Pr(passed test) $\times$ Pr(good project|passed test)), while μ_1 is the probability that a entrepreneur in the sample will not pass the bold test, hence the investor invests in the risk-free asset instead. Therefore, the left hand side is the expected utility of running the bold test on a proportional sample of applications. Note that we are using the assumption that unskilled investors sample first.

Similarly, a cautious investor is indifferent to enter to a pooled market at interest rate r_C^p, which is defined as:

$$\frac{(1-\mu_1-\mu_0)}{2}(1+r_C^p)+\left(\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)\right)(1+r_f)-c=1+r_f \tag{A.8}$$

We claim that if and only if $r_B^p \leq r_C^p$ holds, r_B^p supports a bold equilibrium where the entering measure of unskilled investors is determined by the following market clearing condition. Given the fraction of bold investors' capital financing good projects, together with the capital of skilled investors (which all finance good projects) all good projects, opaque or transparent, have all their credit demand satisfied. (This market clearing condition is spelled out in the proof of Proposition 2). Then, following the intuition in the text, it is easy to check that no one has a profitable deviation: skilled or unskilled investors do not want to change their interest rate from r_B^p, and none of the entrepreneurs want to demand less than $\bar{L}$ at that rate. While, if the condition above did not hold, investors would be motivated to choose to be cautious advertising a rate $\tilde{r} \in (r_C^p, r_B^p)$.

Now consider a cautious equilibrium where all unskilled are cautious and advertise r_C^s. This implies that opaque good projects can be financed only by skilled investors. As skilled capital is scarce, they will advertise the maximum feasible rate $\bar{r}$. As unskilled capital is abundant, therefore r_C^s has to make cautious unskilled indifferent whether to enter. As all entrepreneurs demand credit at all advertised rate which is lower than their reservation rate, the pool of applicants in that low interest rate post is identical to the one in the pooled equilibrium at r_P^p. That is, r_C^s solves (A.8) and $r_C^s = r_C^p = r_C$ holds. If an unskilled investors is to deviate to a bold test, she has two options. She can advertise an interest rate $\tilde{r} \leq r_C^s$ attracting the pool of all type of entrepreneurs or it can advertise a high rate $\tilde{r} \in (r_C^s, \bar{r}]$ attracting all, but the transparent good ones. The earlier is a profitable deviation if and only if $r_B^s \leq r_C^s$ where r_B^s solves (A.7). That is, a necessary condition for a cautious equilibrium is $r_B = r_B^p > r_C$. The latter option is a profitable deviation if and only if $r_M \leq \bar{r}$ where r^I is determined by the indifference condition

$$\frac{\frac{(1-\mu_1-\mu_0)}{2}}{\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)}(1+r_M)+\frac{\mu_1}{\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)}(1+r_f)-c=(1+r_f).$$

Note that $r_M > r_B$ because it refers to an adversely selected pool of applicants. Checking that neither skilled investors nor any type of entrepreneurs want to deviate from the assigned strategies concludes the construction of the cautious equilibrium.

Finally, if $r_M < \bar{r}$ and $r_B > r_C$, then there is a mix equilibrium. In this case, skilled investors cannot offer $\bar{r}$ as they would be undercut by bold unskilled ones. Instead, skilled and bold unskilled investors advertise $r_M(.)$. This high interest rate post is cleared similarly to the one at the bold equilibrium: the fraction of entering bold unskilled investors have to be sufficient to satisfy, together with skilled investors, all the credit demand of opaque good projects. At the same time, a group of unskilled investors choose to be cautious and advertise $r_C(.)$ to serve transparent good projects. Note that the two groups of unskilled investors make the same expected profit of $1+r_f$ by the definition of $r_M(.)$ and $r_C(.)$. Again, we can check that none of the agents prefer to deviate from the assigned strategies. Given that the conditions for each type of equilibria are mutually exclusive, we have uniqueness.

Observe that the static reasoning can be applied in each period of the dynamic set up, and express the equilibrium criteria in terms of μ_0.

Proof of Proposition 2

We described in the main text how entrepreneurs' decide on investment i and borrowing ℓ taking the interest rate $r(\tau,\omega)$ and the borrowing limit $\bar{\ell}(\tau,\omega)$ as given. Then, expressions in Proposition 2 follow from the determination of $r(\tau,\omega)$ in Proposition 1 and the borrowing limits $\bar{\ell}(\tau,\omega)$ which we derive here. We also derive here $k(\mu_0,\mu_1)$, the equilibrium fraction of unskilled investors who decide to not to enter the credit market in a given state. Consider the bold stage first. The market clearing condition for credit to transparent good and opaque

entrepreneurs is

$$w_1 + (1 - k_B)\, w_0\, (1 - \mu_0 - \mu_1) = (1 - \mu_0 - \mu_1)\, \frac{1}{r_B}$$

where $k(\mu_0, \mu_1) = k_B$ in a bold stage. Then, $\bar{\ell}(b, 0)$ is determined by the endowment of unskilled investors which is allocated to bad, opaque credit by mistake:

$$\mu_0 \bar{\ell}\,(b, 0) = (1 - k_B)\, w_0 \mu_0$$

implying

$$\bar{\ell}\,(b, 0) = \frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)} \tag{A.9}$$

and

$$i\,(b, 0) = \bar{\ell}\,(b, 0)\,(1 + r_B) = \frac{(1 + r_B)}{r_B} - \frac{(1 + r_B)\, w_1}{(1 - \mu_0 - \mu_1)}.$$

Assumption 1 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{r_B}$, thus the bad entrepreneurs are constrained in a bold stage.

In the cautious stage market clearing for opaque good firms gives

$$\frac{(1 - \mu_0 - \mu_1)}{2} \bar{\ell}(g, 0) = w_1$$

implying

$$\bar{\ell}(g, 0) = \frac{2w_1}{(1 - \mu_0 - \mu_1)} \tag{A.10}$$

and investment

$$i\,(g, 0) = 1 + \frac{2w_1}{(1 - \mu_0 - \mu_1)}.$$

Assumption 1 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{2\bar{r}}$ implying that opaque good entrepreneurs are indeed constrained in this stage. The fraction of entering unskilled investors in a cautious stage, $(1 - k_C)$, is determined by the market clearing condition for the low interest rate market,

$$\frac{(1 - \mu_0 - \mu_1)}{2} \frac{1}{r_C} = (1 - k_C) w_0 \frac{(1 - \mu_0 - \mu_1)}{2}.$$

Turning to the mix stage recall from the proof of Proposition 1 that $\frac{1-\mu_0-\mu_1}{\mu_0+\mu_1+1}$ fraction of invested unskilled capital finances good, opaque projects at the high interest rate market, $2\frac{\mu_0}{\mu_0+\mu_1+1}$ finances opaque bad projects and $2\frac{\mu_1}{\mu_0+\mu_1+1}$ ends up at risk-free storage. Then

market clearing for opaque good firms then is

$$\frac{(1-\mu_1-\mu_0)}{2}\ell(g,0) = (1-k_I)\, w_0 \frac{(1-\mu_1-\mu_0)}{1+(\mu_1+\mu_0)} + w_1$$

as opaque good entrepreneurs are not constrained, this implies

$$\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1} = (1-k_I)\, w_0 \frac{1}{1+(\mu_1+\mu_0)}$$

Then market clearing for bad, opaque entrepreneurs gives

$$\mu_0 \bar{\ell}(b,0) = (1-b_I)\, w_0 2 \frac{\mu_0}{\mu_0+\mu_1+1}.$$

Substituting back $(1-b_I)$ implies

$$\bar{\ell}(b,0) = \left(\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1}\right) \tag{A.11}$$

and

$$i\,(b,0) = (1+r_M)\left(\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1}\right).$$

Assumption 1 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{2r_M}$. Also, w_0 has to be sufficiently large that $k_I, k_B, k_C \in [0,1]$. We can summarize the requirements on w_1 for later use as:

$$\frac{w_1}{(1-\mu_0-\mu_1)} < \min\left(\frac{1}{2\bar{r}}, \frac{1}{2r_M}, \frac{1}{r_B}\right) = \frac{1}{2\bar{r}}. \tag{A.12}$$

Proof of Lemma 3

Comparing (A.5) and (A.6) and using the equilibrium definitions, it is sufficient to show that maximizing life-time utility leads to the same outcome as maximizing stage game utility within each period. That is, introducing endogenous continuation does not change equilibrium strategy profiles.

First, consider a sequence of stage game equilibria consistent with the law of motion for state variables. We will show that in every period, there is no individual deviation from the optimal strategy in the stage game equilibrium which would increase the life-time utility of an entrepreneur who lives more than one period. That is, any sequence of stage game equilibria consistent with the equilibrium law of motion of the state (μ_0, μ_1) is a dynamic equilibrium. Proposition 1 implies that in any stage game equilibrium all good entrepreneurs obtain positive credit. That is, they hit the upper limit of their probability of survival, $1-\delta$. As such, they cannot increase the interest rate that they accept, compared to the stage

game $\bar{r}$, in order to improve their survival probability. On the other hand, more credit always increases bad entrepreneurs' stage game utility. Furthermore, as long as they are able to raise credit they are indifferent about the corresponding interest rate. Hence, they have no incentive to reduce their reservation interest rate below $\bar{r}$. For them there is no trade-off between stage game utility and increasing the chance of survival by obtaining more credit

Second, we show that there is no dynamic equilibrium where the economy is not in a stage game equilibrium in each period. By contradiction, assume that such dynamic equilibrium exist. This implies that there is at least one period in which some good entrepreneur obtains credit at rate $r > \bar{r}$. First note that any good entrepreneur can obtain some credit if he demands a positive amount at an interest rate which a skilled investor advertises. Furthermore, by assumption, any amount of credit is sufficient for an entrepreneur to survive, i.e. maximizes the survival probability at $1 - \delta$. Thus, a necessary condition for such an equilibrium is that all skilled investors advertise an interest rate which is larger than $\bar{r}$.

In such an equilibrium, a good entrepreneur might be willing to borrow at interest rate above $\bar{r}$, lose in the short-term but in return survive with positive probability. Let $r' \equiv \bar{r} + \Delta$ denote the lowest advertised rate by any skilled investor. Note that since continuation value of an entrepreneur is finite, Δ cannot be arbitrarily large. Furthermore, all good entrepreneurs financed at $r > \bar{r}$ would submit only a diminishingly small demand at r' because that leads to minimal current loss and guarantees maximum survival probability. They submit 0 at every higher interest rate. Moreover, assumption A.1 implies that they demand maximum credit at all rates equal or lower than $\bar{r}$, where they make positive current profit and guarantees maximum survival probability. The first consequence is that all skilled investors must advertise the same rate r' as by advertising a higher rate would not lend anything. Second, each skilled investors can only lend out a diminishingly small fraction of her endowment and thus obtains a diminishingly small return on her capital. Hence, a skilled investor can deviate to $r \leq \bar{r}$ and lend a positive measure of her endowment, which is a contradiction. Thus, such an equilibrium does not exist.

Proof of Lemma 4

See appendix C.1 for the proof.

Proof of Propositions 3

The proposition directly follows from birth-death process for entrepreneurs, the equilibrium information choice and lending choice of investors.

Proof of Proposition 4

See appendix C.4 for the proof.

Proof of Proposition 5

See appendix C.4 for the proof.

Proof of Lemma 5

Recall that $Y(\mu_0, \mu_1)$ is the population weighted sum of the outputs $\rho(1 + \ell(\tau, \omega))$ for each group of entrepreneurs (τ, ω). The statement follows from the observation that (A.9)-(A.11) and that $\ell(\tau, \omega) = \frac{1}{r(\tau,\omega)}$ in the unconstrained cases and using (A.12). $\ell(g, 0)$ discontinuously decreases in μ_0 as it crosses the threshold from below between a bold and a mix stage, or a bold and a cautious stage. Similarly, $\ell(b, 0)$ discontinuously decreases in μ_0 as it crosses the threshold from below between a bold and a mix stage, a bold and a cautious stage, or a mix and cautious stage.

Proof of Proposition 6

Proposition 6 follows from the following five Lemmas.

Lemma B.1 *Within the pooling region, welfare is decreasing in* μ_0.

Proof. Welfare in the bold stage is

$$\begin{aligned} W_B =& (1 - \mu_0 - \mu_1)(\rho - 1)(1 + \frac{1}{r_B}) + \mu_0\rho(1 + \frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)}) + \mu_1\rho \\ &+ w_0(1 + r_f) + w_1(1 + r_B) \end{aligned}$$

which we rewrite as

$$\begin{aligned} W_B =& \rho + w_0(1 + r_f) + w_1\rho \\ &+ (\rho(1 - \mu_1) - (1 + r_B)(1 - \mu_0 - \mu_1)) \left(\frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)}\right) \end{aligned}$$

Note that

$$d(\frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1})/d\mu_0 = (-\frac{1}{r_B^2}\frac{dr_B}{d\mu_0} - \frac{w_1}{(1 - \mu_0 - \mu_1)^2}) < 0$$

also

$$(1-\mu_1)(\rho-(1+r_f))-c=(1-\mu_1)(\bar{r}-r_B)+\mu_0(1+r_B)>0,$$

implying the result. ■

Lemma B.2 *Within the mix region, welfare is decreasing in* μ_0.

Proof. Welfare in the mix stage is

$$\begin{aligned}W_M=&\frac{1-\mu_0-\mu_1}{2}\left(\rho(1+\frac{1}{r_C})-\frac{1}{r_C}(1+r_C)+\rho(1+\frac{1}{r_I})-\frac{1}{r_I}(1+r_I)\right)+\\&\mu_0\rho(1+\left(\frac{1}{2}\frac{1}{r_I}-\frac{w_1}{1-\mu_0-\mu_1}\right))+\mu_1\rho\\&+w_0(1+r_f)+w_1(1+r_I)\end{aligned}$$

which we rewrite as

$$\begin{aligned}W_M=&\rho+w_1\rho+w_0(1+r_f)+\frac{1-\mu_0-\mu_1}{2}\left((\rho-1)\frac{1}{r_C}-1\right)+\\&(\rho(1-\mu_1)-(1-\mu_1-\mu_0)(1+r_I))\left(\frac{1}{2r_I}-\frac{w_1}{1-\mu_0-\mu_1}\right)\end{aligned}$$

Then, the statement follows from the observations that

$$\frac{1}{r_C},\frac{1-\mu_0-\mu_1}{2},(\rho(1-\mu_1)-(1+\mu_0-\mu_1)(1+r_f)-(1+\mu_1+\mu_0)c)$$

are decreasing in μ_0,

$$(\rho-1)\frac{1}{r_C}-1>0$$

$$(\rho(1-\mu_1)-(1-\mu_1-\mu_0)(1+r_I))=(1-\mu_1)(\rho-(1+r_I))+\mu_0(1+r_I))>0$$

as $r_C\leq\bar{r}$, and

$$\frac{1}{2r_I}>\frac{w_1}{1-\mu_0-\mu_1}$$

by (A.12), finally

$$\frac{\partial\left(\frac{1}{2r_I}-\frac{w_1}{1-\mu_0-\mu_1}\right)}{\partial\mu_0}<0$$

as

$$\frac{\partial r_I}{\partial \mu_0} < 0.$$

■

Lemma B.3 *Within the separating region, welfare is decreasing in μ_0.*

Proof. Welfare in the cautious stage is

$$\begin{aligned} W_C =& \frac{1-\mu_0-\mu_1}{2}\left(\rho(1+\frac{1}{r_C}) - \frac{1}{r_C}(1+r_C) + \rho(1+\frac{2w_1}{1-\mu_0-\mu_1}) - \frac{2w_1}{1-\mu_0-\mu_1}\rho\right) + \mu_0\rho + \mu_1\rho \\ &+ w_0(1+r_f) + w_1(1+\bar{r}) \end{aligned}$$

which we rewrite as

$$W_C = \rho + \frac{1-\mu_0-\mu_1}{2}\frac{(\rho-1-r_C)}{r_C} + w_0(1+r_f) + w_1\rho$$

Then

$$\frac{\partial\left(\frac{1-\mu_0-\mu_1}{2}\frac{(\rho-1-r_C)}{r_C}\right)}{\partial\mu_0} = \frac{1-\mu_0-\mu_1}{2}\left(-\frac{\rho-1}{r_C^2}\right)\frac{\partial r_C}{\partial\mu_0} - \frac{1}{2}\frac{(\rho-1-r_C)}{r_C} < 0$$

where we used $\frac{\partial r_C}{\partial \mu_0} > 0$. This implies the Lemma. ■

Lemma B.4 *Fix μ_1 and μ_0 at any level $\mu_0 \leq \frac{c}{1+r_f}$. Welfare is strictly larger in a pooling equilibrium than it would be in a – counterfactual – separating or mix equilibrium, $W_B(\mu_0,\mu_1) > W_C(\mu_0,\mu_1), W_M(\mu_0,\mu_1)$, as long as $\mu_0 \leq \frac{c}{1+r_f}$.*

Proof. As welfare is aggregate consumption, we can decompose $W_B(\mu_0,\mu_1) - W_C(\mu_0,\mu_1)$ as follows. The difference in transparent good entrepreneurs' consumption is

$$\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_B}+1\right) - \frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_C}+1\right)$$

which is non-negative in any point when $r_B \leq r_C$, that is, in the pooling region. The difference in opaque good plus skilled consumption is

$$\left[\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_B}+1\right) + w_1(1+r_B)\right] - \left[\frac{(1-\mu_0-\mu_1)}{2}\rho + w_1(1+\bar{r})\right] \quad \text{(A.13)}$$

note that the term in the first squared bracket is decreasing in r_B as

$$\frac{\partial\left(\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_B}+1\right)+w_1(1+r_B)\right)}{\partial r_B}=$$
$$=-\frac{1}{r_B^2}\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)+w_1\leq-\frac{1}{r_B^2}\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)+\frac{1-\mu_0-\mu_1}{r_B}=$$
$$=\frac{(1-\mu_0-\mu_1)}{r_B}(1-\frac{\rho-1}{r_B})<0$$

where we used (A.12), and equals to the term in the second left bracket when $r_B=\bar{r}$. That is, (A.13) is non-negative at any point as long as $r_B\leq\bar{r}$. Unskilled consumption is equal under the two regimes, while the difference in bad consumption is equal to

$$\mu_0\left(\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1}\right)>0.$$

The proof for $W_B(\mu_0,\mu_1)>W_M(\mu_0,\mu_1)$ is analogous, except that in the second step we show that use that opaque good plus skilled consumption has the form of

$$\left[\frac{(1-\mu_0-\mu_1)}{2}(\rho-1)\left(\frac{1}{r_x}+1\right)+w_1(1+r_x)\right] \tag{A.14}$$

with interest rates $r_x=r_B,r_M$ in the pooling and mixed cases, respectively, which term is decreasing in r_x by (A.12). That is, (A.13) is non-negative at any point as $r_B\leq r_M\leq\bar{r}=\rho-1$. Finally, the difference in bad consumption is

$$\mu_0\rho(\frac{1}{r_B}-\frac{w_1}{(1-\mu_0-\mu_1)})-\mu_0\rho\left(\frac{1}{2}\frac{1}{r_I}-\frac{w_1}{1-\mu_0-\mu_1}\right)=$$
$$=\quad\mu_0\rho\left(\frac{1}{r_B}-\frac{1}{2}\frac{1}{r_I}\right)>0.$$

■

Lemma B.5 *Welfare jumps downward in μ_0 at the mix-cautious threshold whenever that threshold exists. That is, $W_M(\mu_0,\mu_1)>W_C(\mu_0,\mu_1)$ at $\mu_0=\tilde{\mu}_0(\mu_1)$.*

Proof. Consider the definition (12) where each element corresponds to the consumption of a group of agent of a given type. Recall that at $\mu_0=\tilde{\mu}_0(\mu_1)$, $r_M=\bar{r}$ by definition. This the interest at which good investors are indifferent whether to borrow. Therefore, by Propositions 1 and 2, only the consumption of bad transparent types, $\rho(1+\ell(b,0))$ is discontinuous at $\mu_0=\tilde{\mu}_0(\mu_1)$. $\ell(b,0)$ jumps downward to 0 as opaque bad types cannot borrow when all investors turn to cautious which proves the Lemma. ■

Proof of Proposition 7

We will show that under the conditions of the proposition, there is at least one cyclical economy (the one with short-booms and short recessions) which is preferred by the planner compared to both the always bold and always cautious economies. We will argue that for this conclusion, it is sufficient to show that $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ implies

$$\max(\lim_{\delta\to 0} W_C\left(\bar{\mu}_{0C}, \bar{\mu}_{1C}\right), \lim_{\delta\to 0} W_B\left(\bar{\mu}_{0B}, \bar{\mu}_{1B}\right)) < \lim_{\delta\to 0} \frac{W_B\left(\mu^*_{0B}, \mu^*_{1B}\right) + W_C\left(\mu^*_{0C}, \mu^*_{1C}\right)}{2}.$$

Note that $\lim_{\delta\to 0} \bar{\mu}_{0B} = \frac{\lambda}{2-\lambda}$ and

$$\lim_{\delta\to 0} \bar{\mu}_{1B}, \bar{\mu}_{1C}, \mu^*_{1C}, \mu^*_{1B}, \bar{\mu}_{0C}, \mu^*_{0C}, \mu^*_{0B} = 0.$$

In an economy where investors are always bold or always cautious, welfare converges to $W_B\left(\mu^*_{0B}, \mu^*_{1B}\right)$ and $W_C\left(\bar{\mu}_{0C}, \bar{\mu}^*_{1C}\right)$ by definition. First, note that

$$\lim_{\delta\to 0} W_C\left(\bar{\mu}_{0C}, \bar{\mu}_{1C}\right) = W_C\left(0,0\right) < \lim_{\delta\to 0} \frac{W_B\left(\mu^*_{0B}, \mu^*_{1B}\right) + W_C\left(\mu^*_{0C}, \mu^*_{1C}\right)}{2} = \frac{W_B\left(0,0\right) + W_C\left(0,0\right)}{2}.$$

This is implied by Lemma B.4. Then, we show that $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ is a sufficient condition that

$$\lim_{\delta\to 0} W_C\left(\mu^*_{0C}, \mu^*_{1C}\right) > \lim_{\delta\to 0} W_B\left(\bar{\mu}_{0B}, \bar{\mu}_{1B}\right). \tag{A.15}$$

or

$$W_C\left(0,0\right) > W_B\left(\frac{\lambda}{2-\lambda}, 0\right)$$

which we can rewrite as

$$\left(\rho - 1 - \left(r_f + c\right)\right)\frac{1}{2}\frac{1}{r_f + c} > \\ > \left(\rho - 1 - \left(r_f + c\right)\right)\left(\frac{1}{r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f)} - \frac{w_1}{\left(1 - \frac{\lambda}{2-\lambda}\right)}\right).$$

This holds when $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$, because by (3) $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ is the condition for

$$\frac{1}{2}\frac{1}{r_f + c} > \frac{1}{r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f)}$$

and $r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f) < \bar{r}$ to hold simultaneously. As all inequalities are strict and all relevant

functions are continuous from the left in (μ_0, μ_1), for any $\lambda \in \left[\lambda^{\min}, \lambda^{\max}\right]$ we can pick a $\bar{\delta}(\lambda)$ that if $\delta < \bar{\delta}(\lambda)$ then our statement holds. Picking

$$\bar{\delta} = \max_{\lambda \in [\lambda_{\min}, \lambda_{\max}]} \bar{\delta}(\lambda)$$

defines the threshold for δ.

Proof of Proposition 8

Clearly, a risk weight of $x > 1$ does not influence the interest rate in a cautious stage as investors are lending to projects which they all pay back.

In a bold stage, we require

$$v_g x + v_r = 1$$

but still assume that the technology of a bold test did not change implying

$$\frac{v_g}{v_g + v_r} = (1 - \mu_1).$$

Therefore,

$$v_g = \frac{1 - \mu_1}{x(1 - \mu_1) + \mu_1}, v_r = \frac{\mu_1}{x(1 - \mu_1) + \mu_1}$$

which modifies the indifference condition determining the zero profit rate r_B^x as follows

$$\frac{1 - \mu_1}{x(1 - \mu_1) + \mu_1}(1 + r_B^x)\frac{(1 - \mu_1 - \mu_0)}{1 - \mu_1} + \frac{\mu_1}{x(1 - \mu_1) + \mu_1}(1 + r_f) - c = 1 + r_f$$

implying the expression for r_B^x in the proposition.

In the mix stage, the bold test on the high interest rate market (at which transparent good entrepreneurs do not apply for credit) implies

$$\frac{v_g}{v_g + v_r} = \frac{\frac{(1 - \mu_1 - \mu_0)}{2} + \mu_0}{\frac{(1 - \mu_1 - \mu_0)}{2} + (\mu_1 + \mu_0)}.$$

Therefore

$$v_g = \frac{\mu_0 - \mu_1 + 1}{x + 2\mu_1 + x\mu_0 - x\mu_1}, v_r = 2\frac{\mu_1}{x + 2\mu_1 + x\mu_0 - x\mu_1}$$

in the mix stage. This implies that the indifference condition determining the zero profit

rate r_M^x is modified as follows:

$$\frac{1-\mu_0-\mu_1}{x+2\mu_1+x\mu_0-x\mu_1}\left(1+r_M^x\right)+2\frac{\mu_1}{x+2\mu_1+x\mu_0-x\mu_1}\left(1+r_f\right)-c=1+r_f$$

which gives the expression of r_M^x in the proposition. Finally, by analogous arguments to the baseline case, the threshold between the bold and cautious stages is given by identity

$$r_B^x\left(\bar{\mu}_0^x\left(\mu_1,c,r_f\right),\mu_1,c,r_f,x\right)\equiv r_C\left(\bar{\mu}_0^x\left(\mu_1,c,r_f\right),\mu_1,c,r_f\right)$$

while the threshold $\tilde{\mu}^x(\cdot)$ is given by identity

$$r_M^x\left(\tilde{\mu}_0^x(\mu_1,\rho_g,c,r_f),\mu_1,c,r_f,x\right)\equiv\rho-1.$$

Proof of Proposition 9

First, we construct equivalent policies with the lowest implied cost of capital. Let $(m_{0,i},m_{1,i})_{i=1}^{\kappa}$ the invariant set corresponding to the constrained planner's choice of $\hat{\mu}_0^P$. We define i' as the index of the smallest $m_{0,i}\in(m_{0,i})_{i=1}^{\kappa}$ such that $m_{0,i}>\hat{\mu}_0^P$. Note that Proposition 1 imply that $\max(\tilde{\mu}_0^{\pi}(m_{1,i'},c,\rho,\pi),\hat{\mu}_0^{\pi}(m_{1,i'},c,\pi))=m_{0,i'}$ is sufficient to ensure that policy π is an equivalent policy to the planner's choice $\hat{\mu}_0^P$. Then, we can pick r_f, r_f^B and x for the equivalent policies $\pi_{r_f},\pi_{r_f^B},\pi_x$ as follows:

$$\begin{aligned}
r_f&=\max\left(\frac{(1-m_{1,i'})(\rho-1)-(1+m_{1,i'})c-m_{0,i'}(\rho+1+c)}{m_{0,i'}+(1-m_{1,i'})},\frac{c-m_{0,i'}}{m_{0,i'}}\right),\\
r_f^B&=\frac{c-m_{0,i'}}{1-m_{1,i'}},\\
x&=\max\left(\frac{((1-m_{1,i'})\rho-2cm_{1,i'}-\rho m_{0,i'})}{((m_{0,i'}+(1-m_{1,i'}))(c+1)))},\frac{c-m_{0,i'}}{((1+c)(1-m_{1,i'}))+1)}\right).
\end{aligned}$$

In this proposition, we focus on those economies when the implied cycle does not feature a mix stage, that is, the relevant expression for r_f and x is the second term within the max operator. Note that welfare in the bold and cautious stage is

$$\begin{aligned}
&W_B^{\pi}\left(\mu_0,\mu_1;\pi\right)=\rho+\\
&(\rho-1)\left(\left(1-\mu_0-\mu_1\right)\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)}+\mu_0\left(\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)}-\frac{w_1}{(1-\mu_0-\mu_1)}\right)\right)+\\
&+(w_0+w_1)-c\left(\left(\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)}-\frac{w_1}{(1-\mu_0-\mu_1)}\right)\right)\\
&\qquad\qquad-\left(1-\mu_1\right)(x-1)\,c\left(\frac{1}{r_B^{\pi}(\mu_0,\mu_1,c,\pi)}-\frac{w_1}{(1-\mu_1-\mu_0)}\right)\quad\text{(A.16)}
\end{aligned}$$

and

$$W_C^\pi = \rho + \frac{1}{r_C^\pi(\mu_0,\mu_1,c,\pi)}\left(\frac{1-\mu_0-\mu_1}{2}(\rho-1)-c\right) + w_0 + w_1\rho.$$

These formulas follow the calculation in the baseline case with the additional adjustment in the last line of (A.16). For that last term, the market clearing condition in a bold stage is

$$w_1 + (1-k_P)\, w_0 v_g \frac{(1-\mu_1-\mu_0)}{((1-\mu_1-\mu_0)+\mu_0)} = (1-\mu_0-\mu_1)\frac{1}{r_B}$$

where v_g is the bold investor's credit to entrepreneurs, while $\frac{(1-\mu_1-\mu_0)}{((1-\mu_1-\mu_0)+\mu_0)}$ is the fraction of good firms passing her test. Then the fraction of entering unskilled investors $(1-k_P)$ has to satisfy

$$(1-k_P)\, w_0 = \frac{x(1-\mu_1)+\mu_1}{(1-\mu_1-\mu_0)}\left((1-\mu_0-\mu_1)\frac{1}{r_B} - w_1\right).$$

This implies that the total cost paid by these entrants is

$$-c(1-k_P)\, w_0 = -\left(x(1-\mu_1)+\mu_1\right)c\left(\frac{1}{r_B} - \frac{w_1}{(1-\mu_1-\mu_0)}\right).$$

As $(x(1-\mu_1)+\mu_1) > 1$, this implies an adjustment of

$$-\left(\left(x(1-\mu_1)+\mu_1\right)-1\right)c\left(\frac{1}{r_B} - \frac{w_1}{(1-\mu_1-\mu_0)}\right)$$

which is the last term in (A.16).

For the first statement, note that monetary policy effects welfare only through the cost of capital r_B^π and r_C^π. As

$$\begin{aligned}
\frac{\partial W_B^\pi}{\partial r_B^\pi} &= -\frac{1}{(r_B^\pi)^2}\left((\rho-1)(1-\mu_1) - c(\mu_1+(1-\mu_1)x)\right) = \\
&= -\frac{1}{(r_B^\pi)^2}\left((\rho-1)(1-\mu_1) - (1-\mu_0-\mu_1)(1+r_B^\pi) + (r_f^B+1)x(1-\mu_1)\right) = \\
&= -\frac{1}{(r_B^\pi)^2}\left((\bar r - r_B^\pi)(1-\mu_1) + \left(r_f^B x + (x-1)\right)(1-\mu_1) + \mu_0(1+r_B^\pi)\right) < 0
\end{aligned}$$

and

$$\frac{\partial W_C^\pi}{\partial r_C^\pi} < 0$$

and $\frac{\partial r_B^\pi}{\partial r_f}, \frac{\partial r_C^\pi}{\partial r_f} > 0$, any of our monetary policies lead to smaller welfare than the equivalent $\hat{\mu}_0^P$. The macro-prudential policy has a similar negative effect through cost of capital as $\frac{\partial r_B^\pi}{\partial x} > 0$, along with an additional direct negative effect

$$\frac{\partial W_B^\pi}{\partial x} = -\left(1-\mu_1\right) c \left(\frac{1}{r_B^\pi(\mu_0,\mu_1,c,\pi)} - \frac{w_1}{(1-\mu_1-\mu_0)} \right) < 0$$

The term in the bracket is the loan amount of an opaque bad borrower, hence it is positive.

The additional argument for the second statement is to show that equivalent π_x and $\pi_{r_f^B}$ implies the same r_B^π and r_C^π. None of them have an effect on r_C^π and

$$\begin{aligned} & r_B^\pi(\mu_0,\mu_1,c,\pi_{r_f^B}) - r_B^\pi(\mu_0,\mu_1,c,\pi_x) = \\ & = \frac{1-\mu_1}{1-\mu_1-\mu_0} r_f^B - \frac{(x-1)(c+1)(1-\mu_1)}{1-\mu_1-\mu_0} = \\ & \qquad = \frac{1-\mu_1}{1-\mu_1-\mu_0}\left(x-1\right)\left(c+1\right) - \frac{(x-1)(c+1)(1-\mu_1)}{1-\mu_1-\mu_0} = 0, \end{aligned}$$

where, in the second step, we used expression (14) and that $\mu_0^\pi(\mu_1,c,\pi_x) = \mu_0^\pi(\mu_1,c,\pi_{r_f^B})$ under the conditions of the statement.

For the first part of the third statement, consider first the simple monetary policy $r_f = \frac{c-m_{0,i'}}{m_{0,i'}}$, and the equivalent counter-cyclical monetary policy $r_f^B = \frac{c-m_{0,i'}}{1-m_{1,i'}}$. Note that, if $r_f \geq r_f^B$ in the two equivalent policies than welfare is weakly smaller in the bold stage and strictly smaller in the cautious stage under the simple monetary policy. Hence, it is sufficient to show that

$$\frac{c-\mu_0}{1-\mu_1} < \frac{c-\mu_0}{\mu_0}$$

whenever μ_0 implies a bold stage and μ_1 is within the support of the ergodic distribution of μ_1, $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$. As $\hat{\mu}_0^\pi(\mu_1,c,\pi) < \bar{\mu}_{0B}$ in any cyclical economy, it is sufficient that

$$1-\bar{\mu}_{1C} = 1 - \frac{\delta\lambda}{2((1-\delta)(1-\lambda)+\delta)} \geq \bar{\mu}_{0B}$$

or

$$\frac{\lambda}{2-(1-\delta)\lambda} \leq 1 - \frac{\delta\lambda}{2((1-\delta)(1-\lambda)+\delta)},$$

for which $\lambda \leq \frac{8}{9}$ is a sufficient condition.

Finally, the last result and the second statement implies the final statement: if c is sufficiently small than expected welfare under π_x and $\pi_{r_f^B}$ is sufficiently close. As $\pi_{r_f^B}$ strictly dominates π_{r_f}, π_x is also more efficient than π_{r_f}.

C The Cyclical Dynamic Equilibrium: Characterization and Existence Conditions

In this appendix, we provide detailed characterization for a class of cyclical dynamic equilibria in our economy. This class is defined by the property that the finite invariant set $\{\mathbf{m}_i\}_{i=1}^{\kappa} \equiv (m_{0,i}, m_{1,i})_{i=1}^{\kappa}$, or a cyclical permutation of it, is monotonic in i. All the cases we highlight in the main text are within this class. Here, we present sufficient and necessary conditions for the existence of each member of this class. We also show uniqueness within this class, that is, at most one equilibrium within this class can exist for a given set of parameters. As we explain below, while for some parameter values cyclical equilibria exists outside of this class, they tend to have very similar properties to the ones exposed here.

C.1 Steady States; Proof of Lemma 4

Let

$$\boldsymbol{\mu}_t = \begin{bmatrix} \mu_{0t} \\ \mu_{1t} \end{bmatrix}$$
$$\mathbf{a} = \begin{bmatrix} \delta\frac{\lambda}{2} \\ \delta\frac{\lambda}{2} \end{bmatrix}$$
$$\mathbf{A}_C = \begin{bmatrix} (1-\delta)\frac{\lambda}{2} & (1-\delta)\frac{\lambda}{2} \\ (1-\delta)\frac{\lambda}{2} & (1-\delta)\frac{\lambda}{2} \end{bmatrix}$$

and

$$\mathbf{A}_B = \begin{bmatrix} (1-\delta) & (1-\delta)\frac{\lambda}{2} \\ 0 & (1-\delta)\frac{\lambda}{2} \end{bmatrix}$$

. By Proposition 3, if $\mu_0 \in \left[0, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}\right]$ then

$$\mathbf{a} + \mathbf{A}_B\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} \tag{A.17}$$

and $\bar{\boldsymbol{\mu}}_B$ solves

$$\mathbf{a} + \mathbf{A}_B\bar{\boldsymbol{\mu}}_B = \bar{\boldsymbol{\mu}}_B \tag{A.18}$$

or

$$\bar{\boldsymbol{\mu}}_B = -\left(\mathbf{A}_B - \mathbf{I}\right)^{-1}\mathbf{a} = \begin{pmatrix} \frac{\lambda}{-\lambda+\lambda\delta+2} \\ \lambda\frac{\delta}{-\lambda+\lambda\delta+2} \end{pmatrix},$$

a unique fixed point under the permanent bold regime. Clearly, the stationary steady state

$\bar{\boldsymbol{\mu}}_B$ exists if $\bar{\mu}_{0B} \leq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1B})\}$.

If $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$ then

$$\mathbf{a} + \mathbf{A}_C \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} \tag{A.19}$$

and $\bar{\boldsymbol{\mu}}_C$ solves

$$\mathbf{a} + \mathbf{A}_C \bar{\boldsymbol{\mu}}_C = \bar{\boldsymbol{\mu}}_C \tag{A.20}$$

or

$$\bar{\boldsymbol{\mu}}_C = -\left(\mathbf{A}_C - \mathbf{I}\right)^{-1} \mathbf{a} = \begin{bmatrix} \frac{1}{2}\lambda \frac{\delta}{-\lambda+\lambda\delta+1} \\ \frac{1}{2}\lambda \frac{\delta}{-\lambda+\lambda\delta+1} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{0C} \\ \bar{\mu}_{1C} \end{bmatrix},$$

a unique fixed point under the permanent bold regime. The stationary steady state $\bar{\boldsymbol{\mu}}_C$ exists if $\bar{\mu}_{0C} \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1C})\}$. Note that $\bar{\mu}_{0C} \leq \bar{\mu}_{0B}$ but $\bar{\mu}_{1C} \geq \bar{\mu}_{1B}$. Furthermore,

$$0 < \tilde{\mu}_0(\bar{\mu}_{1B}) - \tilde{\mu}_0(\bar{\mu}_{1C}) = (\bar{\mu}_{1C} - \bar{\mu}_{1B}) \frac{\left(\frac{\rho}{(r_f+1)} + \frac{c}{(r_f+1)} - 1\right)}{1 + \frac{c}{1+r_f} + \frac{\rho_f}{1+r_f}} < (\bar{\mu}_{1C} - \bar{\mu}_{1B}) < (\mu_{0B} - \mu_{0C})$$

for any $\frac{\rho}{(r_f+1)} > 1$ and $\delta, \lambda \in (0,1)$. That is, at most one of the steady states can exist. Furthermore, both systems (A.17) and (A.19) are stable as the all eigenvalues of A_B and A_C are within the unit circle. This concludes Lemma 5.2.

C.2 Monotonicity Properties

Before, we proceed, it is useful to establish some monotonicity properties when $\mu_0 \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ and $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$. We will loosely refer to this range as $[\bar{\boldsymbol{\mu}}_C, \bar{\boldsymbol{\mu}}_B]$. Observe that under each dynamics, (A.17) and (A.19) both $\mu_{0,t}$ and $\mu_{1,t}$ monotonically converge to their respective steady states, but from opposite directions. For instance, under (A.19), $\mu_{0,t} > \mu_{0,t+1} > \bar{\mu}_{0C}$ and $\mu_{1,t} < \mu_{1,t+1} < \bar{\mu}_{1C}$ This can be seen by using (A.19)

$$\begin{aligned} \mu_{0,t} - \mu_{0,t+1} &= \mu_{0,t} - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\mu_{0,t} - (1-\delta)\frac{\lambda}{2}\mu_{1,t} > \\ &= \bar{\mu}_{0C}\left(1 - (1-\delta)\frac{\lambda}{2}\right) - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\bar{\mu}_{1C} = 0 \end{aligned}$$

and

$$\mu_{1,t} - \mu_{1,t+1} = \mu_{1,t} - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\mu_{0,t} - (1-\delta)\frac{\lambda}{2}\mu_{1,t} <$$
$$= \bar{\mu}_{1C}\left(1 - (1-\delta)\frac{\lambda}{2}\right) - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\bar{\mu}_{0C} = 0.$$

Similarly, under (A.17)

$$\mu_{0,t} - \mu_{0,t+1} = \mu_{0,t} - \delta\frac{\lambda}{2} - (1-\delta)\mu_{0,t} - (1-\delta)\frac{\lambda}{2}\mu_{1,t} <$$
$$= \bar{\mu}_{0B}\left(1 - (1-\delta)\right) - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\bar{\mu}_{1B} = 0$$

and

$$\mu_{1,t} - \mu_{1,t+1} = \mu_{1,t} - \delta\frac{\lambda}{2} - (1-\delta)\frac{\lambda}{2}\mu_{1,t} >$$
$$= \bar{\mu}_{1B}\left(1 - (1-\delta)\frac{\lambda}{2}\right) - \delta\frac{\lambda}{2} = 0.$$

C.3 Monotonic Invariant Sets

Next, we construct all potential finite invariant sets, $\{\mathbf{m}_i\}_{i=1}^{\kappa} \equiv (m_{0,i}, m_{1,i})_{i=1}^{\kappa}$ for our dynamic equilibria which are monotonic in i. For each $\kappa > 2$, there exists two candidates.

(i) A $B-$cycle cycles through $\left(\mathbf{m}_i^{B,\kappa}\right)$ $i = 1, ..\kappa.$, a sequence of two-dimensional vectors with monotonically increasing first dimension and monotonically decreasing second dimension. In this cycle, the first $\kappa - 1$ steps are implied by (A.17) and then a step implied by (A.19) pushes back the economy to its starting point $\mathbf{m}_1^B$. This implies that $\mathbf{m}_1^{B,\kappa}$ has to satisfy

$$\mathbf{m}_1^{B,\kappa} = \Sigma_{n=0}^{\kappa-1}\left(\mathbf{A}_B\right)^n \mathbf{a} + \left(\mathbf{A}_B\right)^{\kappa-1}\mathbf{A}_C\mathbf{m}_1^{B,\kappa}$$

implying

$$\mathbf{m}_1^{B,\kappa} = \left(\mathbf{I} - \left(\mathbf{A}_B\right)^{\kappa-1}\mathbf{A}_C\right)^{-1}\Sigma_{n=0}^{\kappa-1}\left(\mathbf{A}_B\right)^n \mathbf{a}.$$

Clearly, there is a unique such point. Then, for any $i \in [2, \kappa]$ we have

$$\mathbf{m}_i^{B,\kappa} = \Sigma_{n=0}^{i-2}\left(\mathbf{A}_B\right)^n \mathbf{a} + \left(\mathbf{A}_B\right)_1^{i-1}\mathbf{m}_1^{B,\kappa}.$$

(ii) A $C-$cycle has the support of $\left(\mathbf{m}_i^C\right)$ $i = 1, ..\kappa$, which is monotonically decreasing in its first element, and monotonically decreasing in its second one. That is, starting from

$\mathbf{m}_{t_0+1} = \mathbf{m}_1^C$, for any non-negative integer k, if $t = t_0 + k\kappa + i$ then $\boldsymbol{\mu}_t = \mathbf{m}_i^C$. In this cycle, the first $\kappa - 1$ steps are implied by (A.19) and then a step implied by (A.17) pushes back the economy to its starting point $\mathbf{m}_1^C$. This implies that $\mathbf{m}_1^C$ has to satisfy

$$\mathbf{m}_1^{C,\kappa} = \Sigma_{n=0}^{\kappa-1} (\mathbf{A}_C)^n \mathbf{a} + (\mathbf{A}_C)^{\kappa-1} \mathbf{A}_B \mathbf{m}_1^{C,\kappa}$$

implying

$$\mathbf{m}_1^{C,\kappa} = \left(\mathbf{I} - (\mathbf{A}_C)^{\kappa-1} \mathbf{A}_B\right)^{-1} \Sigma_{n=0}^{\kappa-1} (\mathbf{A}_C)^n \mathbf{a}.$$

Clearly, there is a unique such point. Then, for any $i \in [2, \kappa]$ we have

$$\mathbf{m}_i^{C,\kappa} = \Sigma_{n=0}^{i-2} (\mathbf{A}_C)^n \mathbf{a} + (\mathbf{A}_C)^{i-1} \mathbf{m}_1^{C,\kappa}. \tag{A.21}$$

For $\kappa = 2$, two algorithms above imply the same $\{\mathbf{m}_i\}_{i=1}^2$ values

$$\mathbf{m}_1^2 = \mathbf{m}_1^{B,2} = \mathbf{m}_2^{C,2} = (\mathbf{I} - (\mathbf{A}_B)\mathbf{A}_C)^{-1} (\mathbf{a} + \mathbf{A_B a})$$
$$\mathbf{m}_2^2 = \mathbf{m}_2^{B,2} = \mathbf{m}_1^{C,2} = (\mathbf{I} - (\mathbf{A}_B)\mathbf{A}_C)^{-1} (\mathbf{a} + \mathbf{A_C a}).$$

In the main text, we denote the first element of $\mathbf{m}_1^2$ and $\mathbf{m}_2^2$ as μ_{0C}^* and μ_{0B}^* respectively.

C.4 Necessary Conditions; Proof of Propositions 4 and 5

Consider B-cycles first. For the invariant set $\left(\mathbf{m}_i^{B,\kappa}\right)_{i=1}^{\kappa}$ to be part of a cyclical dynamic equilibrium, we need that

$$\left[\mathbf{m}_\kappa^{B,\kappa}\right]_1 > \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\mathbf{m}_\kappa^{B,\kappa}\right]_2)\} \tag{A.22}$$

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2)\}, \tag{A.23}$$

where $\left[\mathbf{m}_\kappa^{B,\kappa}\right]_1$ and $\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1$ denote the largest and second largest implied μ_0 value along this invariant set. Note, that under these conditions, this is a locally stable cycle because all the eigenvalues of $(\mathbf{A}_B)^{\kappa-1} \mathbf{A}_C$ are inside the unit cycle for any κ. (The largest eigenvalue is $\frac{1}{2^{\kappa-2}} \left(\lambda^{\kappa-1} + \Sigma_{i=0}^{\kappa-2} \lambda^i 2^{\kappa-2-i}\right) \lambda (1-\delta)^\kappa < 1$).

The corresponding equilibrium is a bold-cautious two-stage economy, if

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \frac{c}{1+r_f} < \left[\mathbf{m}_\kappa^{B,\kappa}\right]_1 \tag{A.24}$$

and

$$\tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2) < \left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1, \tag{A.25}$$

and a bold-mix-cautious three stage economy[17] if

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2) < \left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 \tag{A.26}$$

and

$$\frac{c}{1+r_f} < \left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1. \tag{A.27}$$

and

$$\left[\mathbf{m}_{1}^{B,\kappa}\right]_1 < \frac{c}{1+r_f}. \tag{A.28}$$

Two important observations, which can be justified by tedious algebra, are that

(i) $\mathbf{m}_i^{B,\kappa} \in (\bar{\boldsymbol{\mu}}_C, \bar{\boldsymbol{\mu}}_B)$ and

(ii) $\left[\mathbf{m}_{\kappa}^{B,\kappa+1}\right]_1 - \left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 > 0$, that is, the relevant intervals for the thresholds to imply a B-cycle of length κ are increasing and non-overlapping.

Given Proposition 1, the characterization in Proposition 5 and case (ii) in Proposition 4 follow.

Analogously, if $\left(\mathbf{m}_i^{B,\kappa}\right)_{i=1}^{\kappa}$ is part of a cyclical dynamic equilibrium then conditions

$$\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 < \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_2)\}$$
$$\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\left[\boldsymbol{\mu}_{\kappa-1}^{C,\kappa}\right]_2)\}$$

must hold, implying a locally stable cycle because all the eigenvalues of $(\mathbf{A}_C)^{\kappa-1}\mathbf{A}_B$ are inside the unit cycle for any κ. (The largest eigenvalue has the form of $\frac{1}{2}\lambda^{\kappa-1}(1-\delta)^{\kappa}(\lambda+1)$). Also, $\mathbf{m}_i^{C,\kappa} \in (\bar{\boldsymbol{\mu}}_C, \bar{\boldsymbol{\mu}}_B)$ for all i and $\left[\mathbf{m}_{\kappa}^{C,\kappa+1}\right]_1 - \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 < 0$. That is, the relevant intervals for the thresholds to imply a C-cycle of length κ are decreasing and non-overlapping. If the corresponding cyclical dynamic equilibrium is a bold-cautious two-stage economy[18], then

$$\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 > \frac{c}{1+r_f} \geq \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 \tag{A.29}$$

[17] If (A.26)-(A.27) hold, but (A.28) is violated, we have a cautious-mix economy. This case is qualitatively similar to a bold-cautious two-stage economy, hence we do not discuss it in the main text.

[18] A mix-cautious 2-stage economy is also possible, if

$$\frac{c}{1+r_f} < \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 \leq \tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_2) < \left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1.$$

.

and

$$\tilde{\mu}_0(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_2) \leq \left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 \tag{A.30}$$

must also hold. Case (iii) in Proposition 4 is implied by these conditions. Case (i) corresponds to a cyclical dynamic equilibrium of length $\kappa = 2$. A necessary condition for this case is

$$\left[\mathbf{m}_1^2\right]_1 \leq \frac{c}{1+r_f} < \left[\mathbf{m}_2^2\right]_1 \tag{A.31}$$

and

$$\tilde{\mu}_0(\left[\mathbf{m}_2^2\right]_2) < \left[\mathbf{m}_2^2\right]_1, \tag{A.32}$$

in line with the statement.

C.5 Sufficient Conditions

There is one additional condition to make sure that a given invariant set $\{\mathbf{m}_i\}_{i=1}^{\kappa}$ is part of a cyclical dynamic equilibrium. It is that the economy is not in autarky, or

$$\min\left(\frac{r_B(\mu_0,\mu_1,c,r_f)+1}{1+r_f}, \frac{r_C(\mu_0,\mu_1,c,r_f)+1}{1+r_f}\right) < \frac{\rho}{1+r_f}$$

for any $(\mu_0,\mu_1) \in \{\mathbf{m}_i\}_{i=1}^{\kappa}$.

The following Lemma is useful to establish sufficient conditions for a cyclical dynamic equilibrium.

Lemma C.6 *Suppose that* $1-\bar{\mu}_{1C}-\bar{\mu}_{0B} > 0$, *and*

$$\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2} \frac{\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}}{\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2} - \frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}} > \bar{\mu}_{0C}. \tag{A.33}$$

Then, condition

$$\max\left(\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}} + \left(\frac{\rho}{1+r_f}-1\right)\frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}, \bar{\mu}_{0C}\right) < \frac{c}{1+r_f} < \min\left(\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2}\left(\frac{\rho}{1+r_f}-1\right), \bar{\mu}_{0B}\right). \tag{A.34}$$

defines a connected set of $\frac{\rho}{1+r_f} > 1$ *and* $\frac{c}{1+r_f}$ *values. When* $\frac{\rho}{1+r_f}$ *and* $\frac{c}{1+r_f}$ *are in this set,*

then for any $\mu_0 \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ *and* $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$ *the economy is not in autarky, that is*

$$\min\left(\frac{r_B(\mu_0,\mu_1,c,r_f)+1}{1+r_f}, \frac{r_C(\mu_0,\mu_1,c,r_f)+1}{1+r_f}\right) < \frac{\rho}{1+r_f} \tag{A.35}$$

and $\max\left(\frac{c}{1+r_f}, \tilde{\mu}(\mu_1)\right) \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$, *hence the economy is not stationary.*

Proof. For any $\mu_0 \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ and $\mu_1 \in [\bar{\mu}_{1B}, \bar{\mu}_{1C}]$, a sufficient condition for (A.35) is

$$\min\left(\left(\frac{1-\bar{\mu}_{1C}}{1-\bar{\mu}_{1C}-\bar{\mu}_{0B}} + \frac{1}{1-\bar{\mu}_{1C}-\bar{\mu}_{0B}}\frac{c}{1+r_f}\right), 1 + \frac{2}{1-\bar{\mu}_{1C}-\bar{\mu}_{0B}}\frac{c}{1+r_f}\right) < \frac{\rho}{1+r_f}$$

by the monotonicity of the functions $r_B(\cdot)$ and $r_C(\cdot)$ in μ_0 and μ_1. Note that if $1-\bar{\mu}_{1C}-\bar{\mu}_{0B} > 0$, this is equivalent to

$$\frac{c}{1+r_f} < \frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2}\left(\frac{\rho}{1+r_f}-1\right). \tag{A.36}$$

Now consider the condition $\tilde{\mu}(\mu_1) < \bar{\mu}_{0B}$. By the monotonicity of $\tilde{\mu}(\mu_1)$ in μ_1, it is sufficient that $\tilde{\mu}(\bar{\mu}_{1B}) < \bar{\mu}_{0B}$, which we rewrite as

$$\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}} + \left(\frac{\rho}{1+r_f}-1\right)\frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}} < \frac{c}{1+r_f} \tag{A.37}$$

The two conditions along with $\frac{c}{1+r_f} \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$ aggregates to (A.34). Consider the space of $\frac{c}{1+r_f}$ values on the y-axis and $\left(\frac{\rho}{1+r_f}-1\right)$ values on the x axis Then we need the set between two horizontal lines $(\bar{\mu}_{0C}, \bar{\mu}_{0B})$ and two increasing lines. The line corresponding to the left hand side of (A.37) starts at a negative value, while the one corresponding the right hand side of (A.36) starts at 0. As long as their intercept is above $\bar{\mu}_{0C}$, the set exists. The intercept is at

$$\frac{\frac{-2\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}}{\frac{(1-\bar{\mu}_{1C}-\bar{\mu}_{0B})}{2} - \frac{1-\bar{\mu}_{0B}-\bar{\mu}_{1B}}{1+\bar{\mu}_{1B}+\bar{\mu}_{0B}}} = \left(\frac{\rho}{1+r_f}-1\right)$$

therefore we need (A.33). ■

It is simple to show that $(1-\bar{\mu}_{1C}-\bar{\mu}_{0B}) > 0$ if $\lambda < \frac{8}{9}$. With tedious algebra, one can also show that (A.33) holds if $\lambda < \bar{\lambda}$ where $\bar{\lambda}$ is a specific root of a six-order polynomial and $\bar{\lambda} > \frac{3}{4}$. (The numerically solution is $\bar{\lambda} = 0.774388$). Therefore, $\lambda < \frac{3}{4}$ can be used to replace the conditions of the Lemma. One can also show that there is a real subset of $\frac{\rho}{1+r_f}$ and $\frac{c}{1+r_f}$ values satisfying (A.34) generating one of the cycles $B-$ or $C-$cycles we defined above. For this, note that sufficiently large κ, the interval $\left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1, \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1\right)$ gets arbitrarily close to $\bar{\mu}_{0C}$ from above. Hence there must be a set of $\frac{c}{1+r_f}$ values, close to $\bar{\mu}_{0C}$, which simultaneously

satisfy (A.34) and are within the interval $\left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1, \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1\right)$, implying a C-cycle of length κ.

C.6 Other Classes of Cyclical Dynamic Equilibria

Suppose, that $\tilde{\mu}_0(\bar{\mu}_{1B}) \leq \frac{c}{1+r_f}$, so we must have a 2-stage economy. As we have established, intervals of the form $\left(\left[\mathbf{m}_{\kappa-1}^{x,\kappa}\right]_1, \left[\mathbf{m}_{\kappa}^{x,\kappa}\right]_1\right)$, $x = B, C$ are non-overlapping. That is, there must be a set of parameters that

$$\frac{c}{1+r_f} \in \left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa-1}\right]_1, \left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1\right).$$

This implies that the necessary conditions established in section C.4 for a cyclical dynamic equilibrium with monotonic $(\mathbf{m}_i^{\kappa})_{i=1}^{\kappa}$ are violated. Is there a cyclical dynamic equilibrium for such set of parameters? Our simulations show that in these sets, our economy still converge to a cyclical dynamic equilibrium where $(\mathbf{m}_i^{\kappa})_{i=1}^{\kappa}$ consists of a finite number of subsequent monotonic series. For instance, when $\frac{c}{1+r_f}$ is too high for a $\kappa = 3$ B-cycle, but still too low for a $\kappa = 4$ B-cycle, then the economy converges to a cycle which is in a bold stage for 4 periods, then cautious for a single period, then bold for 3 periods and only then, after an additional cautious period, returns to its starting point . By a trivial modification of our algorithm in section C.4, it is possible to establish necessary conditions for these slightly more complex cycles. However, given that the economic properties of these cycles are very similar to the ones with monotonic $(\mathbf{m}_i^{\kappa})_{i=1}^{\kappa}$, this would not add anything to the analysis, hence, we leave it for the interested reader.

D Continuum of Tests

Assume there is a continuum of tests, indexed by $s \in [0, 1]$. Every test s passes all $\frac{1-\mu_0-\mu_1}{2}$ transparent good projects and rejects all μ_1 transparent bad projects. Furthermore, test s passes s fraction of the opaque projects, i.e. $s\frac{1-\mu_0-\mu_1}{2}$ good projects and $s\mu_0$ bad opaque projects. Thus, $s = 0$ corresponds to the cautious test, and $s = 1$ corresponds to the bold test. Tests with $s \in (0, 1)$ cover everything in between. We follow the logic as in proof of Proposition 1 to show that both the bold and the cautious equilibrium are robust to this modification. In particular, investors strictly prefer to choose the bold test when $\mu_0 < \frac{c}{1+r_f}$ and the cautious test when $\mu_0 > \frac{c}{1+r_f}$ even if the intermediate choices are also available.

Recall that the unskilled investors choose a test which allows them to advertise the lowest break even interest rate under the conjecture that at that interest rate all types will submit an application. If that were not true, unskilled investors not entering in equilibrium could choose a test and advertise an interest rate which leads to higher profit than staying outside. (We rely here on Lemma 1 (i) ensuring that if an entrepreneurs applies for a given rate in

equilibrium, he also applies for all lower rates, advertised or not.) The break even interest rate for any test characterized by s is

$$\left(\frac{1-\mu_0-\mu_1}{2}+s\frac{1-\mu_0-\mu_1}{2}\right)(1+r(s)) \\ +\left(\mu_1+(1-s)\,\mu_0+(1-s)\,\frac{1-\mu_0-\mu_1}{2}\right)(1+r_f)-c=1+r_f,$$

which in turn implies

$$\frac{(1+r_f)\left(1-\left(\mu_1+(1-s)\,\mu_0+(1-s)\,\frac{1-\mu_0-\mu_1}{2}\right)\right)+c}{\left(\frac{1-\mu_0-\mu_1}{2}+s\frac{1-\mu_0-\mu_1}{2}\right)}-1=r(s).$$

Note that

$$\frac{\partial r(s)}{\partial s}=-2\frac{c-\mu_0-\mu_0 r_f}{(s+1)^2(1-\mu_0-\mu_1)},$$

implying that whenever $\mu_0<\frac{c}{(1+r_f)}$, the smallest interest rate is implied by the test $s=1$, while in the opposite case it is $s=0$. Thus, by the same argument as in the main text, if $\mu_0<\frac{c}{(1+r_f)}$, the equilibrium advertised interest rate by unskilled investors corresponds to the test $s=1$ (bold test), and in the opposite case they choose $s=0$ (cautious test). In this sense, the continuum of intermediate tests are always dominated by either the bold or the cautious test, and restricting investor choice to these two tests is without loss of generality.