

How Competitive is the Stock Market?

Theory, Evidence from Portfolios, and Implications for the Rise of Passive Investing*

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Abstract

We develop a framework to theoretically and empirically analyze investor competition in financial markets. The classic view assumes that markets are very competitive: if a group of investors changes its behavior, other investors react such that nothing happens in equilibrium. Our framework quantifies the strength of the competitive response. We estimate a demand system of institutional investors in the US stock market accounting for two layers of equilibrium: how investors compete with each other in setting their strategies and how prices adjust to clear asset markets. We find that investors react to the behavior of others in the market: when an investor is surrounded by less aggressive traders she trades more aggressively. This reaction reduces the equilibrium consequences of changes in individual behavior by 60%. However, it also implies that the stock market is far from the competitive ideal. A consequence of this result is that the large increase in passive investing over the last 20 years has led to substantially more inelastic aggregate demand curves for individual stocks, by about 15%.

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1 Introduction

What happens to equilibrium prices when a subset of investors changes its behavior? For example, what are the implications of investors switching to passive strategies, which has occurred on a large scale over the last few decades?¹ Answering this question relies crucially on how other investors react to this change. In the standard view that “financial markets are competitive,” the answer is simple: nothing happens, because other investors pick up any slack left off by those changing their behavior.² Casually said: if you stop looking for \$20 bills on the floor, someone else will replace you. In this paper, we propose a framework to quantify investor competition in financial markets, combining information from prices and portfolio positions. We implement the framework for the U.S. stock market.

We find that investors react to the behavior of others in the market: when an investor is surrounded by less aggressive traders—that is, with a lower price elasticity of demand—she trades more aggressively. While this reaction strongly mitigates the equilibrium consequences of changes in individual behavior, the stock market is far from the competitive ideal. Our estimates suggest that competition reduces the impact of an increase in passive investing in half. Still, an increase as large as the last 20 years leads to substantially more inelastic aggregate demand curves for individual stocks, by 15%.

To get to these answers, we proceed in three steps. First, we present an equilibrium model of investor competition. Each investor’s demand elasticity responds to the elasticity of others in the market; the level of competition is the strength of this response. We demonstrate that while simple, this approach embodies the economics of investor competition across many theories such as those of Grossman and Stiglitz (1980), Kyle (1989), and Shleifer and Vishny (1997). Second, to take these ideas to the data, we distill the model into a realistic demand

¹For example, the ICI factbook (ICI, 2020) reports that the total assets of passive mutual funds in the U.S. have increased from \$11bn to \$2.8tn between 1993 and 2020.

²In the same way that efficient markets have a different meaning in the finance literature relative to the remaining of economics, this is not the standard notion of competition. Rather than asking whether investors are price-taker (the focus of Lee and Kyle (2018)), this notion captures how much substitution there is in the strategies of investors. Going back to Kreps and Scheinkman (1983), it is understood that price-taking is not the only aspect shaping competition.

system that enriches the framework of Kojien and Yogo (2019). We show how to overcome the estimation challenges created by equilibrium interactions between investors. Third, we estimate the model using detailed portfolio positions of institutional investors in the U.S. stock market. Using the model, we quantify the impact of a rise in passive investing, and decompose the sources of evolution in the demand for individual stocks.

Our model represents investor competition by its effect on demand elasticities, how much an investor changes her position in an asset as a function of its price. A more elastic demand curve implies more aggressive trading: the investor increases their position a lot when the asset is cheap. Unlike in standard price theory, demand elasticities in financial markets are not only determined by the investor's preferences for the asset but also by the behavior of other investors. Indeed, it matters why the asset is cheap, and decisions of other investors shape the reason behind price movements. For example, asset prices convey information, and depending on this information, the investor should be more or less aggressive in her response to price movements. We assume that each investor's demand elasticity combines an investor-specific component and a response to the aggregate demand elasticity prevalent in the market. The strength of this response is the level of competition. An equilibrium combines two layers. First, the elasticities of all investors must be consistent with each other: the average of all investor elasticities must be equal the aggregate elasticity. Second, the asset price is such that the sum of all demand curves evaluated at this price is equal to the supply of the asset.

This framework is simple because it reduces the problem of investor competition to an equilibrium determination of elasticities. However, this simplicity does not impede its richness: we show that our setting unifies many foundations for competition. For example, we consider a theory in which investors expend resources to obtain more precise information on the assets they invest in, like in Grossman and Stiglitz (1980) — see Veldkamp (2011) for a review. Two elements shape the demand for information: an investor's own ability to gather information and how much information other investors have acquired. If other investors have

more information, the value of acquiring information declines, and the investor chooses to obtain less of it. This force is the source of competition between investors, and how easily an investor is able to adjust her information determines how competitive the market is. Because when an investor acquires more information she trades more aggressively, the information equilibrium can be recast exactly as an elasticity equilibrium. Elasticities inherit the strategic substitutability of information choices, giving rise to competition. In contrast, in theories where investors have market power, like in Kyle (1989), it is optimal to trade more aggressively if others are more aggressive as well. In both cases, the equilibrium strategies of investors boil down to a choice of elasticity.

What happens when a group of investors becomes passive? Their investment strategy becomes irresponsive to the price of the asset, hence their demand elasticity becomes zero. This pushes the aggregate elasticity down, which prompts a response from other investors, potentially compensating the direct effect. When the competitive response is strongest, this reaction completely offsets the direct effect, the equilibrium market elasticity is unchanged. This corresponds to the ideal of “competitive financial markets”. In the other extreme, if investors do not react, the elasticity provided by the traders who became passive is just lost. We quantify the intensity of competition to obtain the pass-through of a rise in passive investing into aggregate elasticities.

We adapt our basic two-layer equilibrium setting to be able to take it to the data. We implement our framework as a demand system in the style of Koijen and Yogo (2019). The level of demand and the individual-specific component of elasticity are functions of stock characteristics. In addition, demand has a residual component unobservable to the econometrician. The portfolio choice of investors across stocks is represented by a logit system. These choices allow for a rich and realistic specification of demand, and capture heterogeneity across investors. To this set of assumptions we add a second layer: the elasticity equilibrium. The demand elasticity of an investor depends on her individual-specific component and a response to the aggregate elasticity of demand for this stock. In equilibrium, the aggregate

demand elasticities must be the average of investor-level elasticities.

To estimate the model we overcome three challenges. First, the interaction between investors through their elasticity decisions introduces a reflection problem (Manski, 1993): a market with high elasticity could be the result of either high individual elasticities or strong positive spillovers. The cross-section of stocks provides a solution to this issue because we observe the same investor in different markets. This investor faces a different mix of competing investors for each stock, therefore a different aggregate demand elasticity. This variation allows us to isolate the spillover from the individual-specific component of elasticity. We face a chicken-and-egg question: to implement this comparison we need to know the elasticity of other investors, which itself has to be estimated in the same way. We prove conditions on the graph of investor-stock connections that solve this issue.

Second, both the price and the aggregate elasticity are equilibrium quantities and therefore depend on portfolio decisions. We construct an instrument for each of these two variables using variation in investment universe across investors. Stocks that more investors can buy naturally have more money chasing them and a higher price. This instrument for the price is introduced in Koijen and Yogo (2019). Because individual components of elasticities are not known a priori, the same idea cannot be directly implemented to craft an instrument for aggregate elasticity. We construct a new model-based instrument combining the variation in investment universe with the estimated individual component of elasticities. This adds a challenge: we have to estimate the instrument and the demand system simultaneously. Still this is valid strategy because model parameters are exogenous by definition.

Third, the inclusion of rich investor heterogeneity, the need to solve for an elasticity equilibrium, and the presence of model-based instrument, all concur to a seemingly intractable estimation. However, we develop a computationally efficient algorithm that estimates the model. The key idea is to isolate low-dimensional fixed-point problems due to the elasticity equilibrium from a larger but standard linear system with high-dimensional investor-specific coefficients.

Our estimates suggest a substantial amount of competition. If the aggregate elasticity for a stock increases by 1, an individual investor decreases her own elasticity of demand by 2.2. This competitive response stabilizes the levels of aggregate elasticity. Intuitively, when a very aggressive investor trades a specific stock, other investors in this stocks adjust by becoming less aggressive. This force implies about 50% less cross-sectional variation in elasticity across stocks than estimates ignoring competitive interactions, highlighting the importance of these interactions.

We use these estimates to assess the impact of a rise in passive investing. To do so, we ask how equilibrium elasticities change when a fraction of investors exogenously switches to be passive. We obtain a simple formula for the pass-through of a change in the fraction of active investor to the aggregate elasticity. This pass-through only depends on the intensity of competition and the initial fraction of active investors. It is decreasing in both quantities. Empirically, we find this pass-through to be about 0.4. Half of a change in the fraction of active investors translates into a reduction in demand elasticity. Given an estimate that the fraction of active investors has decreased by about 30% over the last 20 years, this yields a decrease in elasticities of 13%. This is a large change: for example, in the context of the information model we studied, it would lead to less informative and more volatile prices. Again, this prediction highlights that while competitive effects are strong, the stock market is far from the competitive ideal. In competitive markets, the pass-through is 0, in which case a rise of passive investing has no impact. On the other hand, without competitive effects, the pass-through is 1, leading to a 30% decrease in elasticity.

The model also provides an account of the actual evolution of the demand for stocks over the last 20 years. The entire cross-sectional distribution of stock-level elasticity has decreased during that period, by 40%. Interestingly, the model attributes about equally this drop to two investor-specific sources of change. First, the fraction of passive investors has increased — the extensive margin. Second, the investor-specific component of the elasticity of active investors has decreased — the intensive margin. This dimension is particularly interesting

because developments in computing power and access to big data would have instead suggested that the most aggressive quantitative funds would have increased their elasticities on their own.³ However, another aspect played an important role: active investors also increased their equilibrium elasticity in response to broad decrease in aggregate elasticities. In a counterfactual exercise in which we shut down the competitive responses, we find that elasticities would have decreased twice as much, while they would have barely moved with strong competition.

Taken together, our results highlight the importance of a finer approach to competition in financial markets. No, it is not the case that “financial markets are competitive” and that all shocks are fully absorbed by other investors. But also no, it does not mean that investors do not interact together at all. This framework is a first step towards quantifying the intensity of competition and its implications. Our estimates suggest that competition played an important role in shaping the response to the rise of passive investment. Competition is likely important for many other questions about investor demand. What happens when a large set of financial institutions must change their trading because of new regulation? What happens when some sophisticated specialized investors get in financial trouble?

Contribution to the existing literature. The idea of competition among investors has a long history in finance. Grossman and Stiglitz (1980) first formalize the notion of competition for information between investors and show it does not lead to informationally efficient markets.⁴ Kyle (1989) highlights how market power also creates interaction among investors. These seminal contributions have led to a large theoretical literature pointing out rich ways in which investors react to each other and choose their trading strategy. In the context of the rise of passive investing Subrahmanyam (1991) is an early contribution highlighting liquidity concerns. More recent work includes Bond and García (2018), Malikov (2019), Lee (2020),

³Farboodi and Veldkamp (2020) develop a theory of the effect of growth in financial data technology that upends common wisdom.

⁴Coles, Heath, and Ringgenberg (2020) show that an increase in passive investing does not affect price informativeness in this baseline model.

Buss and Sundaresan (2020), and Kacperczyk, Nosal, and Sundaresan (2020). Gârleanu and Pedersen (2018) and Gârleanu and Pedersen (2021) focus on the interaction between the market for asset managers and the market for assets. Farboodi and Veldkamp (2020) focus on the choice between information about fundamentals or about demand in the context of the rise in big data. However, with the exception of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), these theories are rarely confronted to portfolio data. Our new approach, summarizing competition through choices of demand elasticity, allows us to bring the theory to the data.

We contribute to a recent literature on estimating demand systems accounting for the large heterogeneity in portfolio holdings, started by Kojien and Yogo (2019). Kojien et al. (2021), Kojien and Yogo (2020), Kojien, Richmond, and Yogo (2020), and Jiang, Richmond, and Zhang (2020) also apply this approach. Balasubramaniam, Campbell, Ramadorai, and Ranish (2021) estimate a factor model of portfolio holdings. Dou, Kogan, and Wu (2020) study how mutual funds change their portfolios in response to common fund flows. Gabaix and Kojien (2020) estimate the aggregate demand for stocks. Our key innovation on that front is to incorporate strategic interactions between investors, a long-theorized feature we find to be quantitatively important.

More broadly our paper relates to a broader literature studying the relation between portfolio quantities and asset prices. De Long et al. (1990) argue that noise trader shocks can affect prices. These ideas have found applications across multiple asset classes: stocks (Shleifer (1986), Warther (1995)), government bonds (Vayanos and Vila (2021), Greenwood and Vayanos (2014), Haddad and Sraer (2020)), options (Gârleanu, Pedersen, and Poteshman (2009)), currency markets (Gabaix and Maggiori (2015), Greenwood et al. (2019), Gourinchas, Ray, and Vayanos (2019)), or corporate bonds (Haddad, Moreira, and Muir (2021)). While our estimates concentrate on the stock market, we bring to the forefront the importance of strategic interactions between investors, which likely also matter in other markets.

Finally our results provide new insights in the debate on the consequences of the long-term

rise in passive investing. French (2008) and Stambaugh (2014) provide empirical evidence of a shift towards passive investing. Zooming in on portfolios, we uncover how passive investing is altering how all investors trade and therefore its equilibrium implications. Other work focuses on quasi-natural experiments around index or ETF inclusion such as Chang, Hong, and Liskovich (2014) or Ben-David, Franzoni, and Moussawi (2018). Sammon (2021) studies the response of stock prices around earnings announcements. Bai, Philippon, and Savov (2016), Dávila and Parlato (2018), and Farboodi et al. (2021) document long term trends in price informativeness.

2 An Equilibrium Model of Financial Markets with Investor Competition

We present our framework for investor competition. The key idea is that there are two layers to an equilibrium in financial markets. First, the price is such that the sum of investor demands equals the supply of the assets. Second, investors compete with each other: they choose how aggressively they trade as a function of how others trade. This aggressiveness is measured by their demand elasticity. First, we introduce the two layers, then we highlight the implications for the rise of passive investing. Table 1 summarizes the model.

2.1 First layer: the asset price clears the market given demand curves

For the sake of simplicity, we focus on the case of one asset in fixed supply S and a continuum of investors indexed by i . We generalize to multiple assets when moving to the data in Section 4. In an equilibrium each investor decides how much they buy as a function of the price P of the asset: a demand curve $D_i(P)$, which we can log linearize around the baseline

Table 1. The 2-layer model of investor competition.

	Individual Decision	Equilibrium Condition
Demand	$d_i = \underline{d}_i - \mathcal{E}_i \times (p - \bar{p})$	$\int D_i(p) = S$
Elasticity	$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \times \mathcal{E}_{agg}$	$\int \mathcal{E}_i D_i / S = \mathcal{E}_{agg}$

value for the price \bar{P} :

$$d_i = \underline{d}_i - \mathcal{E}_i \times (p - \bar{p}), \quad (1)$$

where lowercase letters represent log values.⁵ The elasticity of this demand curve, \mathcal{E}_i , determines how aggressive the investor is. An investor with $\mathcal{E}_i = 0$ does not react to changes in prices, while a trader with large \mathcal{E}_i increases her position a lot when the asset is cheap. Aspects beyond the price can also affect the choice of positions. For example an investor could have a preference for environmental, social, and governance (ESG) investing. We collect them all inside of the constant \underline{d}_i ; the empirical analysis will be more flexible about modelling \underline{d}_i .

Investors' elasticities play an important role in the determination of equilibrium prices. The aggregate demand curve is $D_{agg}(P) = \int D_i(P)$, and the equilibrium price solves $D_{agg}(P^*) = S$. Aggregate demand has elasticity

$$\mathcal{E}_{agg} = \frac{\int \mathcal{E}_i D_i}{\int D_i}. \quad (2)$$

How strongly aggregate demand for the asset responds to prices is the (holdings-weighted) average of individual elasticities. This aggregate elasticity shapes the behavior of the equilibrium price. If investors are very aggressive, aggregate demand is perfectly elastic, $\mathcal{E}_{agg} \rightarrow \infty$,

⁵Similarly Gabaix and Koijen (2020) consider log-linear demand curves around a reference price level.

and prices are pinned down at a fixed level. In such a situation, changes in individual investor characteristics $d_{0,i}$ or in supply S do not affect the price. This is what people sometimes describe as “efficient markets”: any deviation of the price from a fundamental value is immediately traded away by aggressive investors. On the other hand, when demand is more inelastic, small changes in the market structure can have a large effect on prices, because investors are unwilling to change their positions.

For example, if elasticities are constant, a small uniform change $\Delta \underline{d}$ to the demand of all investors results in a price change of

$$\Delta p = \mathcal{E}_{agg}^{-1} \Delta \underline{d}. \quad (3)$$

If all investors want to increase the size of their position by one percent, the price increases by the multiplier $M_{agg} = \mathcal{E}_{agg}^{-1}$ percent. Consequently, more inelastic markets experience larger price variation due to changing investor demands, and are therefore more volatile.⁶ A change in supply would have the opposite effect on the price with a multiplier $-M_{agg}$. More fleshed-out models such as the ones we present in Section 3 also relate the aggregate elasticity to other equilibrium properties such as price informativeness, liquidity, or limits to arbitrage. We confirm this relation empirically in Section XX (5.3?).

2.2 Second layer: investors set their demand elasticity in response to others

In standard price theory, the elasticity of demand reflects only an individual’s preference for a good. In particular, it does not depend on the decisions of other market participants. When choosing how many apples to put in your shopping cart, it does not matter what other shoppers are doing beyond their effect on the price level. However, in financial markets,

⁶See also Gabaix and Koijen (2020) for a discussion of the role of the elasticity of aggregate demand in financial markets.

it matters why the price is moving and consequently demand elasticities are not fixed.⁷ Investors compete for trading opportunities. If many investors trade aggressively, fewer good deals are available; therefore, there are also fewer incentives to trade with a high elasticity.

This relation adds a second layer to the equilibrium, which captures investor competition. At the individual level, the elasticity responds to the aggregate demand elasticity. But conversely, the aggregate demand elasticity is an average of individual elasticities. Formally, we represent this feedback by endogenizing individual demand elasticities as a function of the aggregate demand elasticity:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}. \quad (4)$$

The parameter χ controls the strength of the response to aggregate elasticity; it measures the extent of strategic substitution in demand elasticities.⁸ $\underline{\mathcal{E}}_i$ is a baseline level of elasticity reflecting the investor's own preferences for the asset, for example shaped by her risk aversion or her beliefs about the payoffs. Together, the individual decision equation (2) and the aggregation condition of equation (4) pin down the equilibrium of elasticities.

We call χ the competition parameter. The standard view is that financial markets are very competitive, which corresponds to large values of χ . If a group of sophisticated investors goes away, other investors pick up the slack by trading more aggressively. In the extreme case where χ goes to infinity, the competitive response is so strong that the equilibrium aggregate elasticity \mathcal{E}_{agg} is pinned down at a fixed level. Changes in individual investor behavior or the composition of investors do not affect the aggregate elasticity.

On the other hand, when $\chi = 0$, individual investors do not respond at all to the aggregate elasticity and financial markets are not competitive. We are back to standard price theory: each investor follows a strategy that is independent of the actions of other investors. Under

⁷A similar phenomenon arises in auctions with a common value component: a bidder's optimal bidding strategy respond to the strategies of other participants in the auction.

⁸We consider strategic substitutes and complements in the sense of Bulow, Geanakoplos, and Klemperer (1985) and defined in chapter 4 of Veldkamp (2011).

this view, if a group of sophisticated investors goes bankrupt, nobody else steps in to take advantage of the opportunities that are left untouched: the aggregate elasticity drops sharply.

The parameter χ offers a simple and flexible way to measure competition and its consequences, consequences we highlight in the remainder of this section. We do not take a stand on a specific microfoundation for the parameter χ . In many theories, demand elasticities are a key feature of investors' strategies and exhibit substitutability or complementarity; we devote Section 3 to discuss these theories.⁹ Rather than restricting ourselves to a specific foundation — many of these theories are operating side by side —, we measure competition directly from trading and portfolio data.

2.3 The effect of a rise in passive investing

Our framework is useful to evaluate the effect of a rise in passive investing. Consider the following thought experiment. We start from an economy with homogenous investors who, in this initial equilibrium, have elasticity $\mathcal{E}_i = \mathcal{E}_0$. The aggregate elasticity is therefore also \mathcal{E}_0 . What happens when a fraction $1 - \alpha$ of these investors becomes passive, that is keep the same holdings, but reduce their elasticity to zero? The level of competition χ determines the answer to this question.

The direct effect of this change is that now, only a fraction α of investors contribute to the aggregate elasticity. If we only consider this effect, the aggregate elasticity decreases to $\mathcal{E}_{agg} = \alpha\mathcal{E}_i$ (from the aggregation equation (2)). However the story does not end here; the active investors have a competitive response. They change their own elasticity in response to the aggregate: $\Delta\mathcal{E}_i = -\chi\Delta\mathcal{E}_{agg}$ (from equation (4)). This response compensates the direct effect when $\chi > 0$. Each active investor responds again to the response of other active

⁹Technically, other aspects of investor decisions may be the source of substitutability (e.g. information acquisition or social interactions). However, because elasticities are directly related to these other decisions, the substitutability manifests itself in the demand elasticity.

investors, until they reach a new equilibrium.¹⁰ The new aggregate elasticity is:

$$\mathcal{E}_{NEW} = \underbrace{\alpha \mathcal{E}_0}_{\text{direct effect}} + \underbrace{(1 - \alpha) \mathcal{E}_0 \frac{\alpha \chi}{1 + \alpha \chi}}_{\text{competitive response}}. \quad (5)$$

When financial markets are competitive, χ is large and $\mathcal{E}_{NEW} = \mathcal{E}_0$, the aggregate elasticity is unchanged. The drop in elasticity due to the investors that became passive is exactly compensated by a greater elasticity of the remaining active investors. On the other hand when investors are insensitive to market conditions, χ close to zero, only the direct effect operates and the elasticity declines by a factor α .

What does this imply quantitatively? In the estimation of Section 4, we find a level of competition χ of 2. Over the last 20 years, the fraction of active investors has decreased by 30%, so we set $\alpha = 70\%$. This implies that the initial elasticity is multiplied by a factor of $(2 + 1)/(2 + 1/(70\%)) = 0.875$. The rise of passive investing leads to a substantial drop in elasticity of 12.5%. This is about half of the direct effect that would have led to a decrease of 30%. However, this is still much more than the zero predicted by the competitive market paradigm.

In Section 4, we fully specify our framework to account for heterogeneity across investors and stocks, and estimate it using holdings data. This allows us to revisit the question of the rise in passive investing in the context of a realistic quantitative model in Section 5.1.

2.4 Other applications

We present two other applications for which competition plays a central role. First, we show that competition determines whether investors react more when an asset is overpriced or when it is underpriced. Second, competition determines the stability of arbitrage to price movements.

¹⁰Formally, we do not model this tâtonnement, but rather focus directly on equilibria. We present details of the calculation in Appendix Section A.1

2.4.1 Intermediary asset pricing

How do markets change when some financial institutions get distressed or when they are more tightly regulated? As these institutions trade less aggressively they provide less elasticity to the market and we expect more unstable prices. Two aspects shape this response: how large is the direct shock to the institutions, but also how other competing investors respond. Consider how the aggregate elasticity responds to a combination of shocks to individual elasticities $\{\Delta \underline{\mathcal{E}}_i\}_i$; for example, only the affected institutions receive a negative shock to their elasticity. For simplicity, we assume that the price is at its baseline, $p = \bar{p}$, and leave the general case to Appendix A.¹¹ We show the response of aggregate elasticity is

$$\Delta \mathcal{E}_{agg} = \frac{1}{1 + \chi} \mathbf{E}[\Delta \underline{\mathcal{E}}_i], \quad (6)$$

where $\mathbf{E}[\cdot]$ denote the demand-weighted population average.¹² The change in aggregate elasticity combines the average direct elasticity shock $\mathbf{E}[\Delta \underline{\mathcal{E}}_i]$ and a mitigating factor due to competition $1/(1 + \chi)$. In fully competitive markets, $\chi \rightarrow \infty$, the shock to some investors has no effect on the aggregate elasticity. This is the view of those arguing that intermediaries cannot matter for asset prices. However, for lower values of χ the direct effect is not mitigated. Theoretical models centered on intermediaries often assume $\chi = 0$ (e.g He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2013)).

As such, when analyzing how the financial health of intermediaries matters for asset pricing, one must also take into account how other investors compete with them. Consistent with this idea, Haddad and Muir (Forthcoming) show that in markets that are more sophisticated and hence with lower competition, periods of distress in the financial sector are associated with stronger movements in risk premium. Eisfeldt, Lustig, and Zhang (2017) also empha-

¹¹Unlike in the precedent calculation, we assume that there are no passive investors.

¹²Formally this corresponds to

$$\mathbf{E}[x_i] = \int x_i \frac{D_i}{S}.$$

size this role of investor competition in markets for complex assets such as mortgage-backed securities.¹³

2.4.2 Asymmetry of mispricing

In this section, we show that in very competitive markets, prices are less responsive to demand when their levels are high than when they are low. The opposite happens for low levels of competition. This distinction has important practical implications: when competition is mild, as we find in the data, we expect to observe more situations of overpricing than underpricing, and more sensitivity to demand shocks for overpriced assets. This prediction lines up with the evidence in Stambaugh, Yu, and Yuan (2012) and Stambaugh, Yu, and Yuan (2015) across a large set of anomaly strategies.¹⁴ In particular, they document that trading the short leg of the portfolio (overpriced stocks) is more profitable than the long leg (underpriced stocks) in episodes of high investment sentiment (demand shocks in our theory).¹⁵

Multiplier. First, we revisit the calculation of equation (3) in the presence of competition. We show in Appendix A that the price multiplier to an aggregate demand shock becomes:

$$M_{agg} = \frac{1}{\mathcal{E}_{agg}} \cdot \frac{1}{1 + \frac{\chi}{1+\chi} \frac{\text{Var}[\mathcal{E}_i]}{\mathcal{E}_{agg}} (p - \bar{p})}, \quad (7)$$

where $\text{Var}[\mathcal{E}_i]$ is the demand-weighted cross-sectional variance of elasticity.¹⁶ For small deviations of the price from its baseline (small values of $p - \bar{p}$), the response of prices to a change in demand is still determined by the aggregate elasticity \mathcal{E}_{agg} . For example, at first order,

¹³Other examples of large effects of intermediary health in specialized markets include Gabaix, Krishnamurthy, and Vigneron (2007) and Siriwardane (2019).

¹⁴The anomaly strategies are portfolio sorted on characteristics that predict unconditional returns, in the style of Fama and French (1993).

¹⁵As such, imperfect competition is an alternative explanation to short-sell constraints (for example, Miller (1977) or Haddad, Ho, and Loualiche (2021)) for the pervasiveness of overpricing.

¹⁶Formally this corresponds to

$$\text{Var}[\mathcal{E}_i] = \int \left(\mathcal{E}_i^2 \frac{D_i}{S} \right) - \mathcal{E}_{agg}^2.$$

the effect of a rise in passive investing on aggregate elasticity is reflected one-to-one into the multiplier. In competitive markets, the rise in passive investing does not affect the sensitivity of prices to demand, whereas it increases it when markets are not competitive. We now turn to how the multiplier changes when the price deviates from its baseline \bar{p} .

No competition. Without competition ($\chi = 0$), the aggregate multiplier is $M_{agg} = \mathcal{E}_{agg}^{-1}$. While each of the investors' elasticities is fixed at $\underline{\mathcal{E}}_i$, their contribution to aggregate elasticity depends on their relative demand. When the price is below its baseline, the more elastic investors have a stronger response than the less elastic investors: they buy relatively more of the asset. Thus more elastic investors represent a larger fraction of the market; aggregate elasticity increases. For example in response to a supply shock, the aggregate elasticity moves in the opposite direction from the price: $\Delta \mathcal{E}_{agg} = -\text{Var}[\mathcal{E}_i] \Delta p$. How much investors differ from each other, the variance of individual elasticities, controls the strength of the composition effect. The market has a higher capacity to absorb demand shocks on the downside than on the upside. Without competition, overpricing is more likely to happen than underpricing.

High level of competition. Consider now the other extreme of full competition, $\chi \rightarrow \infty$. In this case the aggregate elasticity \mathcal{E}_{agg} is constant; if more elastic investors decrease their relative share, everybody becomes more elastic and exactly compensates the initial decline. However, the multiplier changes because second term of the product in equation (7) is not constant: it is smaller than one for $p > \bar{p}$ and larger than one otherwise. What happens then? If the price is above its baseline, the increase in individual elasticities due to the competitive response implies a decrease in demand. Because all investors become more aggressive, they demand less of the asset, which prevents the price from increasing much, leading to a smaller multiplier. The demand effect goes in the opposite direction when the price is below its baseline. Overall this leads to an opposite behavior of the multiplier M_{agg} relative to no competition. The multiplier is larger with underpricing than overpricing: underpricing is

more likely to happen than overpricing.

2.4.3 Limits to arbitrage

An important insight is that engaging in arbitrage trades (or more broadly exploiting mispricing) is a risky activity and this risk limits the effectiveness of arbitrageurs (De Long et al. (1990), Shleifer and Vishny (1997), Shleifer and Summers (1990), Brunnermeier and Pedersen (2008)).

If an asset is underpriced ($p < \bar{p}$), we expect arbitrageurs (high elasticity investors) to take on large positions. When the mispricing worsens, the arbitrageurs suffer large losses due to their large exposure. If they are unable to raise additional capital, they have to liquidate some of their positions which pushes the price down even further. This feedback creates a natural instability of arbitrage activity: shocks that worsen the mispricing hurt the arbitrageurs and deepen the mispricing. Mitchell, Pulvino, and Stafford (2002) and Mitchell and Pulvino (2012) document this instability in action.

Competition plays an important role in this process. Without competition, the arbitrage capacity destroyed by arbitrageurs' losses is gone altogether. With competition, other investors become more elastic in response to the decline of the arbitrageurs. They purchase more of the asset, thereby partially compensating the lower positions of the arbitrageurs.

We can illustrate this mechanism in our model by considering how the price responds to a demand shock that affects disproportionately investors with high elasticity. We assume that each investor's baseline demand changes by an amount $\{\Delta \underline{d}_i\}_i$ and compute the equilibrium price response:

$$\Delta p = M_{agg} \times \left[\mathbf{E}(\Delta \underline{d}_i) + \frac{\chi}{1 + \chi} (p - \bar{p}) \text{Cov}(\mathcal{E}_i, \Delta \underline{d}_i) \right], \quad (8)$$

where expectation and covariance represent demand-weighted moments. Without competition, $\chi = 0$, the price response is simply the product of the aggregate multiplier M_{agg} with the average demand shock $\mathbf{E}(\Delta \underline{d}_i)$. A shock that hurts disproportionately investors with

large positions (for example the asset is underpriced and these investors have a high demand elasticity) pushes the price down. The second term captures the role of competition. When the asset is underpriced ($p < \bar{p}$), and demand decreases more for high elasticity investors ($\text{Cov}(\mathcal{E}_i, \Delta \underline{d}_i) < 0$), we obtain a positive response which compensates the direct effect. A symmetric compensation occurs when the asset is overpriced (see Appendix A). Interestingly, the stabilizing role of competition is stronger the further away prices are from their baseline.¹⁷

3 Why are financial markets not fully competitive?

The standard view is that financial markets are very competitive. Investors are constantly on the lookout for good opportunities, ready to come in if another market participant steps down. This corresponds to $\chi = +\infty$ in our framework. In practice, many forces limit competition. We discuss the most prominent ones in this section: costly information acquisition, bounded rationality, liquidity, peer effects, and investment mandates. Our 2-layer equilibrium model captures the main insight of these theories in a parsimonious way.

3.1 Costly information acquisition

The basic idea of investor competition is that if some active investors exit the market, there are more investment opportunities to take advantage of, and other investors go after them by trading more aggressively. In practice, knowing that there are more investment opportunities is not enough, investors have to evaluate them. The costs of this process of learning (information gathering, hiring analysts, etc.) naturally limit the ability to compete.

We formalize this intuition in a model in the style of Grossman and Stiglitz (1980) with information acquisition as in Veldkamp (2011), and show it maps tightly to our two-layer equilibrium.¹⁸ We focus here on the main results and leave details of the setting and deriva-

¹⁷Duffie (2010) and Duffie and Strulovici (2012) study how the competitive response unfolds over time.

¹⁸Bond and García (2018) and Malikov (2019) provide theoretical analyses of the rise of passive investing in this family of theories.

tions to Appendix B.

There is one period and one asset, and a continuum of agents indexed by i . Each agent has CARA preferences with risk aversion ρ_i . The gross risk-free rate is 1, and the (random) asset payoff is f . The asset is in noisy supply $\bar{x} + x$ with \bar{x} an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$. Initially, each agent is endowed with an independent signal μ_i of the fundamental f , distributed $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$.¹⁹ Each agent can acquire a private signal $\eta_i \sim \mathcal{N}(f, \sigma_{i,\eta}^2)$ at monetary cost $c_i(\sigma_{i,\eta}^{-2} + \sigma_i^{-2})$, with $c_i(\cdot)$ a non-decreasing positive function.²⁰ That is, obtaining more precise signals is more costly. The signal being private implies in particular that signal realizations are uncorrelated across agents conditional on the fundamental f .

Optimal asset demand is linear in the price: $d_i = \underline{d}_i - \mathcal{E}_i p$.²¹ The slope of the demand curve characterizes how aggressively an investor changes her portfolio when the price moves. We find:

$$\mathcal{E}_i = \frac{1}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \quad (9)$$

Two elements shape the investor's demand elasticity: her risk aversion and her private information. An investor with more precise information about the asset is more confident in her forecast of the asset returns, and therefore trades more aggressively. Looking ahead, we can already see that constraints to the ability to change information acquisition will limit the ability of the investor to change her elasticity and compete with others.

Before that, we show that the aggregate elasticity, $\mathcal{E}_{agg} = \int_i \mathcal{E}_i di$, is the appropriate notion

¹⁹Following Veldkamp (2011), we assume agents start with a flat prior on f , hence their initial belief is $f \sim \mathcal{N}(\mu_i, \sigma_i^2)$.

²⁰This parametrization is without loss of generality relative to a cost function that would only depend on the acquired signal $\sigma_{\eta,i}$.

²¹For all of this subsection, we do a small abuse of notation: lowercase letters represent levels rather than logarithms and \mathcal{E}_i denotes the slope of the demand curve, rather than the elasticity *stricto sensu*. This approach lends itself to the linearity of the CARA-Normal framework, but is less appealing for empirical applications.

for how the collective actions of all investors shape the price. In equilibrium, the price follows

$$p = A + f - \mathcal{E}_{agg}^{-1}x, \quad (10)$$

where A is a constant. The price responds one-to-one to the fundamental f , but is also affected by noise trading x . The aggregate elasticity controls the impact of noise: if everybody trades aggressively against abnormal price movements, noise traders cannot push the price far away from fundamentals. In line with this intuition, a market with higher aggregate elasticity also has less volatile returns ($\text{Var}(f - p) = \mathcal{E}_{agg}^{-2}\sigma_x^2$) and more informative prices ($\text{Var}(f|p)^{-1} = \mathcal{E}_{agg}^2\sigma_x^{-2}$).

Investor competition occurs through information choices. The aggregate elasticity impacts price dynamics, which in turns affect the incentives to acquire information and trade in an elastic way. When choosing how much information to acquire, investors trade off the cost of a more precise signal with the benefit of a more informed trading strategy. The utility gain from precise information is proportional to knowledge of the fundamental, which combines private information (corresponding to \mathcal{E}_i) and information learned from prices (corresponding to \mathcal{E}_{agg}). Focusing on elasticities, this leads to the following optimization problem:

$$\max_{\mathcal{E}_i} \frac{1}{2} \log (\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2}) - \rho_i c_i(\rho_i \mathcal{E}_i) \quad (11)$$

This problem is the counterpart to equation (4): the choice of individual elasticity \mathcal{E}_i responds to the aggregate elasticity \mathcal{E}_{agg} . To a first-order approximation, the competition parameter is the response of the optimal individual elasticity to the aggregate elasticity: $\chi = -\partial \mathcal{E}_i / \partial \mathcal{E}_{agg}$.²² In this model, competition χ is always positive. If others acquire less information and become less aggressive, there are incentives to look for information and step in to replace them. However, competitive forces only partially offset the initial change, $\chi < \infty$. In particular,

²²The relation between \mathcal{E}_i and \mathcal{E}_{agg} is not linear in general. In Appendix B.3 we find a two-parameter family of simple cost functions under which this relation is exactly linear as in equation (4). Each of the two parameters maps to in closed-form to the value of competition χ and individual elasticity $\mathcal{E}_{0,i}$.

costs to adjust information limit the ability to react and result in lower χ . Formally, we show in Appendix B.5 that χ is decreasing in a form of curvature of the information cost function, $c_i''/c_i'^2$.²³

3.2 Other costs of adjusting elasticity

In this first approach, we saw that the costs of adjusting information strategies limit the ability of investors to compete with each other. Other practical reasons hinder flexibility in setting a trading strategy, and bring χ down towards zero.

One such aspect is risk. Following an aggressive high-elasticity trading strategy entails taking more extreme positions, and hence more risk. Risk itself is endogenous to the aggressiveness of other traders: in more efficient markets, prices are tightly related to fundamentals, while without any active traders, prices are more sensitive to the whims of noise traders.²⁴ Thus, it is unappealing to follow aggressive strategies exactly when they are most needed, which limits the process of competition. In Appendix Section C.1, we present a setting where investors do not make information choices but learn from prices. We show that endogenous risk shapes competition; for example when all risk is endogenous investors do not compete, $\chi = 0$, while otherwise there is positive competition.²⁵

Another aspect is institutional. Many financial institutions face strong mandates from their ultimate investors in terms of what strategies they are allowed to follow. While these restrictions can be viewed as an optimal contract solving information asymmetry between final investors and the asset manager, they are costly in terms of competition. Investment mandates limit the ability of institutions to react to changes in the behavior of other investors, pushing χ down relative to an unconstrained setting. Beber et al. (2018) show how explicit mandates and constraints in active mutual funds prospectuses strongly limit their investment

²³Coles, Heath, and Ringgenberg (2020) show that full competition arises in the baseline setup of Grossman and Stiglitz (1980) without adjustment costs.

²⁴De Long et al. (1990) first highlighted the importance of endogenous risk for dynamic arbitrage.

²⁵In this case, the model coincide with that of the previous section when the information cost is infinitely steep, $c_i''/c_i'^2 \rightarrow \infty$.

opportunity set. Banks and insurance companies investment strategies are also restricted, this time by their regulatory framework (for example, Basel III capital regulation).

3.3 Bounded rationality

Competition between investors relies on their understanding of the market structure. For example, in the rational expectations equilibrium of Section 3.1, each investor knows the strategies followed by everyone else. Practically, how would investors figure out other people's strategies? In our model and in the microfounded models, investors only need to know the aggregate elasticity \mathcal{E}_{agg} and not the actions of each of the other investors. In the real world, institutions can track changes in investment styles directly (e.g. industries, factors, arrival of activist investors) or through their impact on prices (e.g. price impact, volatility, price informativeness). While this information is useful, it is still a leap to assume investors can follow exactly the optimal policies in frictionless models.

First, the information available to investors about the aggregate elasticity might be imperfect.²⁶ In such a setting the response to other investors is dampened which reduces the intensity of competition. For example, assume an investor wants to react to aggregate elasticity with a coefficient χ_0 , but she only observes a noisy signal about \mathcal{E}_{agg} . Then, her elasticity choice is

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi_0 \theta \cdot \mathcal{E}_{agg} + \epsilon. \quad (12)$$

Because the investor cannot separate the noise from the information about \mathcal{E}_{agg} , she responds to her signal with a Bayesian shrinkage factor $0 < \theta < 1$. The residual ϵ is due to the noise in the signal. Appendix C.3 provides derivations and explicit expressions for these quantities. The effective level of competition is $\chi_0 \theta$ and incorporates the baseline competition parameter χ_0 with the dampening factor θ .

²⁶Imperfect information about other investors' strategies is different from imperfect information about fundamentals or noise traders.

Second, investors have to be sophisticated enough to understand the equilibrium mechanisms leading them to compete. A recent strand of research considers equilibria in which investors miss the actions of others (Eyster and Rabin (2005), Greenwood and Hanson (2014), Eyster, Rabin, and Vayanos (2019), Bastianello and Fontanier (2021)). Neglecting equilibrium forces can either amplify or mitigate competition. On the one hand, investors could simply ignore how the elasticity choice of others affect their investment opportunities. In this case, we will not observe any competition. On the other hand, investors might understand the direct effect of changes in elasticity but fail to realize that others react to those as well, a form of partial equilibrium thinking as in Bastianello and Fontanier (2021). For example, all investors understand there is a rise in passive investing but fail to realize that others will react by trading more aggressively. We include partial equilibrium thinking into the calculation from Section 2.3 on the effect of a rise in passive investing. We show in Appendix C.4 that the new aggregate elasticity becomes:

$$\mathcal{E}_{NEW}^{PET} = \alpha\mathcal{E}_0 + (1 - \alpha)\chi\alpha\mathcal{E}_0. \quad (13)$$

Because investors do not account for the response of others, they overreact to the initial change in elasticity. With partial equilibrium thinking, the competitive response is stronger by a factor $1 + \alpha\chi$ than in the baseline (see equation (5)). This leads to a higher final level of aggregate elasticity and is therefore akin to more competition.

3.4 Strategic complementarities

Finally, some forces generate strategic complementarity rather than substitutability, which yields negative values of competition χ . In these situations, when some investors become less aggressive, other investors also pull out of markets instead of replacing them.

One such case arises when investors worry about the price impact of their trades. In Appendix Section C.2, we show that a model of market power in the style of Kyle (1989)

yields a negative value of χ .²⁷ Specifically, the standard CARA elasticity becomes

$$\mathcal{E}_i = \frac{1}{\rho_i \sigma^2 + \underbrace{(\mathcal{E}_{agg} - \mathcal{E}_i)^{-1}}_{\lambda_{-i}}}. \quad (14)$$

The investor responds to the price based on her risk aversion and the risk of the asset, $\rho_i \sigma^2$, and the slope of the residual demand curve for the asset, what Kyle (1989) calls λ_{-i} . When other investors are more price elastic, it enhances liquidity in the market. In turn, this facilitates my ability to trade and I can be more responsive to prices. This type of complementarity holds in a broader family of theories of liquidity such as Vayanos and Wang (2007).

Strategic complementarities can also arise through social interactions. When investors follow their peers, as in Hong, Kubik, and Stein (2004), changes in some investors are amplified by similar decisions from other investors. If I see others around me trade a stock more aggressively, I also want to trade that stock more aggressively. This herding leads to negative values of χ .

4 Estimating investor competition

In this section, we estimate the level of investor competition χ and demand elasticities in the context of the U.S. stock market. First, we enrich our model to account for the heterogeneity of stocks and investors. Then, we design and implement a new identification strategy for demand estimation when there are strategic interactions.

4.1 Quantitative model

Individual decisions. In practice, agents invest in many assets. Therefore, an empirical model must make sure that portfolio positions add up to total assets for each investor. In

²⁷We also show that the measure of price impact Kyle's λ is closely related to the inverse of aggregate elasticity.

addition, it should also account for the portfolio aspect of financial decisions, that is some form of substitution across assets. Kojien and Yogo (2019) show that a logit framework satisfies both of these requirements. We denote each security by the index k , the total assets of an investor by A_i , and the portfolio share of investor i in security k by w_{ik} . Therefore $d_{ik} = \log(w_{ik}A_i) - p_k$. The framework of Kojien and Yogo (2019) corresponds to specifying a log-linear model for relative portfolio shares w_{ik}/w_{i0} instead of the individual demand directly, with index 0 being the outside asset. We follow this approach. For each investor, we take as given total assets under management, A_i , and investment universe, \mathcal{K}_i , that is the set of assets they can invest in.

Second, we need to specify the baseline levels of demand and elasticity \underline{d}_i and $\underline{\mathcal{E}}_i$. We assume that each of those combines potentially distinct sets of asset characteristics using investor specific coefficients. Going back to the setting of Section 3, an interpretation of this assumption is that investors form priors on different assets based on their characteristics; for example, characteristics could capture factor loadings. This corresponds to expressing the baseline demand as $\underline{d}_{ik} = \underline{d}_{0i} + \underline{d}'_{1i}X_k^{(d)} + \epsilon_{ik}$ and the baseline elasticity as $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i}X_k^{(e)}$, where the two vectors of characteristics are $X_k^{(d)}$ and $X_k^{(e)}$. We also account for asset-specific changes in demand by including a shock ϵ_{ik} in \underline{d}_{ik} . For example, ϵ_{ik} captures the private signal η and noise trading x of the model of Section 3.

Finally, we estimate the model using only the cross-section in each time-period. Thus, we allow all quantities and parameters of the model to depend on time. For ease of notation we drop the subscript t . Putting it all together, our model of portfolio demand is²⁸

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i}X_k^{(d)} - \mathcal{E}_{ik} p_k + \epsilon_{ik}, \quad (15)$$

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i}X_k^{(e)} - \chi \mathcal{E}_{agg,k}. \quad (16)$$

²⁸To match equation (15) with equation (1), recall that: $d_{ik} = \log \frac{A_i w_{ik}}{P_k}$. Then $\underline{d}_{ik} = \underline{d}_{0i} + \underline{d}'_{1i}X_k + \log(A_i) + \log(w_{0i}) + \epsilon_{ik}$.

Starting from the relative shares $\omega_{ik} = w_{ik}/w_{i0t}$, the actual shares can be obtained by

$$w_{ik} = \frac{\omega_{ik}}{1 + \sum_{k \in \mathcal{K}_i} \omega_{ik}}, \quad (17)$$

$$w_{i0} = \frac{1}{1 + \sum_{k \in \mathcal{K}_i} \omega_{ik}}. \quad (18)$$

Interestingly, the demand system of Koiien and Yogo (2019) is a special case of this framework. In their framework, demand elasticities are fixed structural parameters.²⁹ This corresponds to setting $\underline{\mathcal{E}}_{1i} = 0$ and $\chi = 0$. Therefore, their model implicitly assumes no competition at all. For example, when some investors are removed from the markets, the other ones do not step in with larger elasticities. This is the extreme opposite from the standard hypothesis of perfect competition, which corresponds to $\chi \rightarrow \infty$. Our framework lets us quantify how close or far reality is from these extremes.

Passive investors. We also include passive investors. By passive, we mean that these are investors whose demand does not respond to prices. Index funds are a specific example of such investors. Our notion is larger though, because it accommodates arbitrary fixed portfolios. To represent such behavior, we simply replace equation (16) by $\mathcal{E}_{ik} = 0$. Separating out these investors is important, not only because of their low level of elasticity, but also because they do not respond to aggregate trading conditions. We denote the set of active investors for asset k by $Active_k$ and the total assets under management of this group of investors as $|Active_k|$.

Equilibrium prices and elasticities. Going from individual decisions to an equilibrium relies on market clearing. As in the model of Section 2, two equilibrium objects play a role in individual decisions: prices, p_k , and aggregate elasticities, $\mathcal{E}_{agg,k}$. The corresponding

²⁹Technically in the logit model the demand elasticity is $1 - (1 - w_{ik})(1 - \mathcal{E}_{ik})$. For values of w_{ik} that are small relative to one, as in the data, this expression is close to \mathcal{E}_{ik} . Hence we refer to \mathcal{E}_{ik} as the demand elasticity throughout the paper.

equilibrium conditions are

$$\sum_i w_{ik} A_i = P_k, \quad \forall k, \quad (19)$$

$$\sum_i \frac{w_{ik} A_i}{P_k} \mathcal{E}_{ik} = \mathcal{E}_{agg,k}, \quad \forall k. \quad (20)$$

We normalize the number of shares available to 1 to obtain the market-clearing condition for assets, equation (19). Said otherwise, p_k denotes the log market capitalization. In the appendix, we present the details of an algorithm to solve for the equilibrium efficiently; we rely on a quasi-Newton method.

4.2 Data

We estimate the model for the U.S. stock market. We use stock level data from CRSP: price, dividend, shares outstanding. We merge the CRSP file with COMPUSTAT for balance sheet information and compute stock level characteristics: book equity, profitability, and investment.

We obtain portfolio holdings data from the 13F filings to the SEC from 2001 to 2020. We build the dataset from the SEC EDGAR website following the method of Backus, Conlon, and Sinkinson (2019, 2020). The SEC requires that every institution with more than \$100m of assets under management file a quarterly report of their stock positions. We find that collectively the holdings reported in the 13F account for 80% of the total stock market capitalization.

We follow Kojen and Yogo (2019) to construct the final panel dataset.

4.3 Identification

To estimate the model described above we have to overcome three difficulties: (i) first, the classic problem of endogeneity in demand estimation; (ii) a reflection problem induced by the interactions between investors; and (iii) how to implement the estimation given that one

of the “regressors”, the aggregate elasticity, is unknown.

4.3.1 Identifying demand

Combining equation (15) and (16), the model is similar to a regression equation:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik}. \quad (21)$$

The parameters are \underline{d}_{0i} , \underline{d}_{1i} , $\underline{\mathcal{E}}_{0i}$, $\underline{\mathcal{E}}_{1i}$, and χ . There are two challenges to identify these parameters: residual demand ϵ_{ik} is unobservable and aggregate elasticities $\mathcal{E}_{agg,k}$ is itself a function of $\underline{\mathcal{E}}_i$ and w_{ik} as expressed in the equilibrium condition (20). To solve these issues we make identification assumptions.

The simplest possible assumption takes residual demand as exogenous to all other variables to get the moment condition

$$\mathbf{E} \left[\epsilon_{ik} | X_k^{(d)}, X_k^{(e)}, p_k, \mathcal{E}_{agg,k} \right] = 0. \quad (22)$$

Then, we could estimate (21) using ordinary least squares. The independence of ϵ_{ik} from X_k is naturally motivated by taking the supply of assets as exogenous, as in endowment economies (Lucas, 1978). Furthermore, the independence from p_k and $\mathcal{E}_{agg,k}$ relies on the logic that residual demands do not matter for equilibrium outcomes because they “cancel out” in the aggregate. This rules out both the presence of non-atomistic investors and correlated demand shocks — see the equilibrium conditions in equations (19) and (20). Both of these last assumptions are not likely to hold for institutional investors. Therefore we relax these assumptions and propose an alternative identification strategy.

We assume that the variation in total assets and the investment universe is exogenous to the residual demand, an assumption shared with Kojen and Yogo (2019). They argue that the investment universe is often determined by mandates, which are predetermined rules on which assets can be held. Similarly assets under management (AUM) are also predetermined.

Building on this we construct instruments for equilibrium outcomes p_k and $\mathcal{E}_{agg,k}$. The instrument for the price of asset k is

$$\hat{p}_{k,i} = \log \left(\sum_{j \neq i} A_j \frac{\mathbf{1}_{k \in \mathcal{K}_j}}{|\mathcal{K}_j|} \right), \quad (23)$$

where $\mathbf{1}_{k \in \mathcal{K}_j}$ is an indicator variable of when stock k is in investor j investment universe. This instrument corresponds to how much money would flow to stock k if all investors other than i had an equal-weighted portfolio.³⁰ Variation in the instrument comes from variation across investors' investment universe. For example, a stock with large investors has more money flowing towards it. Given our assumption of downward-sloping demand for stocks, a larger exogenous demand generates higher prices that are uncorrelated with residual demand.

Our setting includes the additional equilibrium variable $\mathcal{E}_{agg,k}$, for which we develop a new instrument:

$$\hat{\mathcal{E}}_{agg,k} = \frac{1}{1 + \chi |Active_k|} \frac{\sum_{j \in Active_k} A_j / |\mathcal{K}_j| \cdot \mathbf{1}_{k \in \mathcal{K}_j} \cdot \underline{\mathcal{E}}_{jk}}{\sum_{j \in Active_k} A_j / |\mathcal{K}_j| \cdot \mathbf{1}_{k \in \mathcal{K}_j}} \quad (24)$$

This instrument is the solution to the elasticity equilibrium defined by equations (16) and (20), where we have replaced the endogenous weights w_{ik} with counterfactual weights under the assumption that each investor holds an equal-weighted portfolio.³¹ The variation in this instrument also comes from variation across investors' investment universe. However, the asset flows are weighted by individual elasticity: a stock with more intrinsically inelastic investors (for example, passive mutual funds) will tend to have a lower aggregate elasticity.

³⁰We consider an alternative with portfolio weights proportional to book equity.

³¹Our instrument for aggregate elasticity is the solution to the following problem:

$$\hat{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{ik} - \chi \hat{\mathcal{E}}_{agg,k}; \quad \sum_j \hat{w}_{jk} A_j / \exp(\hat{p}_k) \hat{\mathcal{E}}_{jk} = \hat{\mathcal{E}}_{agg,k},$$

where the counterfactual weights \hat{w}_{jk} are defined as:

$$\hat{w}_{jk} = \frac{\mathbf{1}_{k \in \mathcal{K}_j}}{|\mathcal{K}_j|}.$$

We are interested in the response to the interaction of aggregate elasticity with the price (see equation (21)). To isolate this interaction from linear effects, we also include a linear control for aggregate elasticity, similarly to $X_k^{(d)}$.³²

The two instruments allow us to weaken the moment condition (22) to

$$\mathbf{E} \left[\epsilon_{ik} | X_k^{(d)}, X_k^{(e)}, \hat{p}_{i,k}, \hat{\mathcal{E}}_{agg,k} \right] = 0. \quad (25)$$

The instrument for the aggregate elasticity depends on the model parameters ($\underline{\mathcal{E}}_{0i}$ and $\underline{\mathcal{E}}_{1i}$). This is not an issue for identification as parameters are by definition not endogenous. However, this precludes us from using standard methods such as two-stage least squares to estimate the model. Appendix Section E.1 lists the unconditional moments derived from condition (25) that we use for estimation. In Section 4.3.3, we detail our numerical procedure for estimating the model.

Relevance condition. To evaluate the strength of our instruments, we run what would be a first-stage regression in a standard two-stage least square estimation. First, we regress the price onto the instrument and the other characteristics for each manager. For each date, we compute the first and the fifth percentile of the (Kleibergen and Paap, 2006) F-statistics across managers. Figure 1 reports the histogram of these percentiles across all dates. At least 95% of the F-statistics in any given date are above 18 (panel A). We also confirm the relevance of the elasticity instrument. In the panel, we regress the product of the price interacted with the aggregate elasticity onto their instrumented version and the other characteristics. We represent the histogram of the F-statistic of this regression for each date in panel C; the F-statistic is always above 10.³³

³²To maintain tractability in estimation we assume that the coefficient on this linear control, like χ , is constant across investors.

³³Appendix Figure IA.6 reports the results for the book-equity weighted instrument.

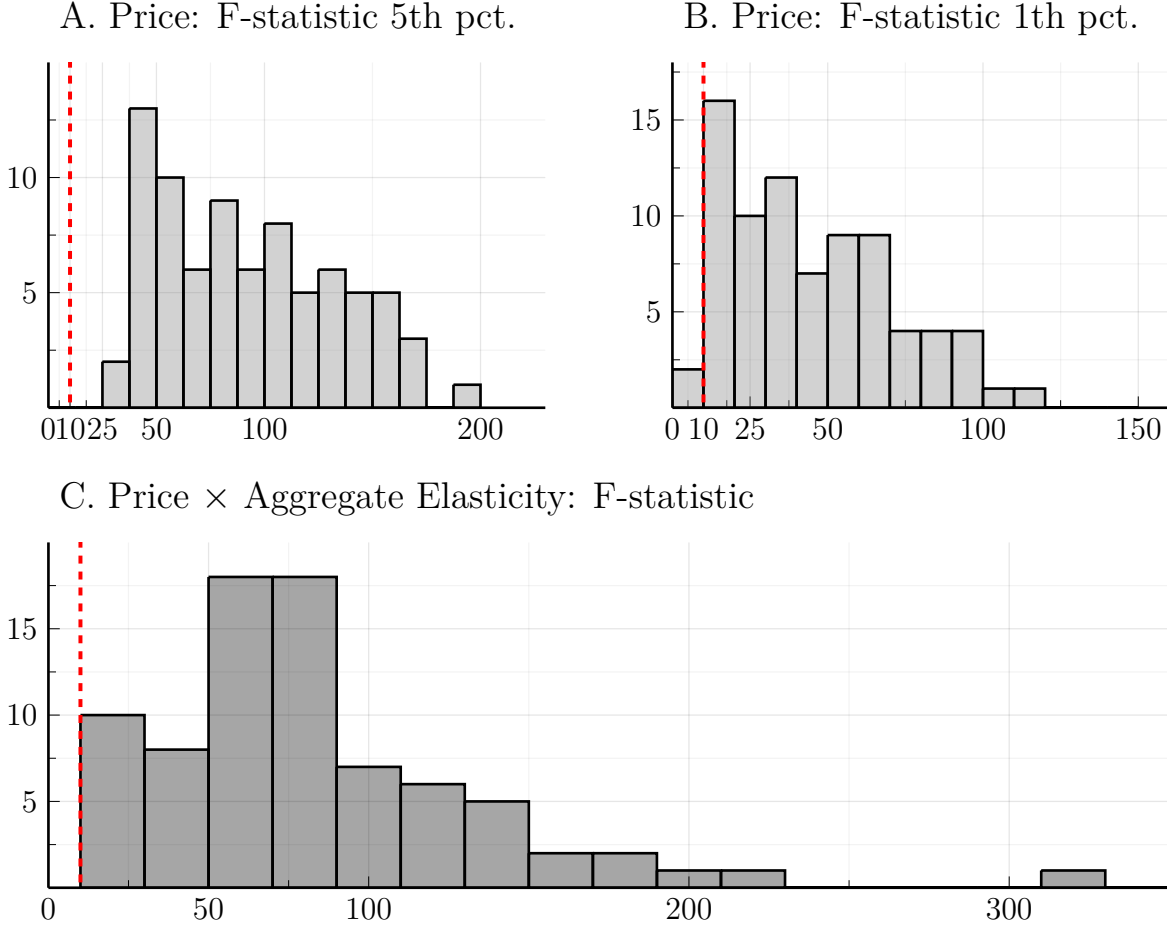


Figure 1. Relevance condition for the price and elasticity instruments. Figure 1 shows the F-statistic of the first-stage for the price and aggregate elasticity variables. For the price, we estimate the F-statistic (Kleibergen-Paap) at the manager level for each year. We summarize these statistics at every date with the 5th percentile (Panel A) and 1st percentile (Panel B). Panel A shows that 95% of all F-statistics are above 10 across all dates, and Panel B shows that 99% of all F-statistics are above 10 across most dates. In Panel C we regress the elasticity interacted with the price onto their instrumented version and report the F-statistic for each date. The sample period is 2001–2020.

4.3.2 The reflection problem.

While our instruments provide us with as many moment conditions as parameters, we discuss how the estimation can disentangle the individual component of elasticity from competitive effects. Individual investor elasticities \mathcal{E}_i depend on an investor specific term, $\underline{\mathcal{E}}_i$ and on the

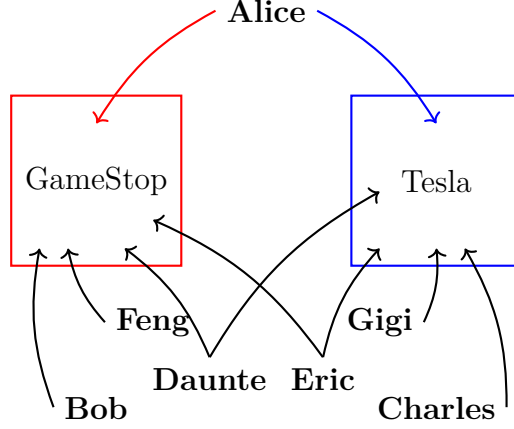


Figure 2. Illustration of Identification Strategy.

aggregate elasticity \mathcal{E}_{agg} as

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_{ik} - \chi \mathcal{E}_{agg,k}. \quad (26)$$

We need to disentangle whether investors are elastic because of their own characteristics or in response to other investors on the market. For example, if we see in a market that all investors behave in a very elastic manner, it could be that each of them is fundamentally very elastic, high $\underline{\mathcal{E}}_i$. But it could also be the consequence of a strong positive feedback where $\chi < 0$. This identification problem is the reflection problem (Manski, 1993).

Two features of our model let us solve the reflection problem. First, there is variation in investor composition across stocks, \mathcal{K}_i . Second, we assume that the investor specific component of elasticity depends on observable asset characteristics, $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k$. To measure the effect of competition, the ideal experiment would be to compare the behavior of the same investor for the same stock with variation in the characteristics of the other investors.

With the second assumption, two stocks with the same characteristics X_k , will elicit the same baseline elasticity $\underline{\mathcal{E}}_i$ for the same investor. In addition because the coefficients on stock characteristics is investor specific, we focus on variation within the same investor

across different stocks. Finally to estimate χ , we need variation in $\mathcal{E}_{agg,k}$ across stocks. The different investment universe for different investors guarantees such a source of variation. Figure 2 illustrates this idea: we need to compare how Alice trades differently when facing different groups of other investors, such as for GameStop and Tesla. Last, we need to ensure that the system of equations for all investors and stocks given by (26) and the equilibrium condition for aggregate elasticity (20) has a unique solution. In our example, to estimate Alice’s behavior, we also simultaneously need to figure out the elasticity for Bob, Charles, Daunte, etc. The following theorem formalizes the intuition behind the needed identifying variation and proves uniqueness.

Theorem 1. *A decomposition of demand elasticities $\{\mathcal{E}_{ik}\}_{i,k}$ into individual elasticities $\{\underline{\mathcal{E}}_i\}_i$ and the competition parameter χ is unique if:*

- (a) *The graph \mathcal{G} of investor-stock connections is connected.*
- (b) *Position-weighted averages of demand elasticities are not constant across stocks: there exists k and k' such that $\sum_{i \in I_k} w_{ik}/P_k A_i \underline{\mathcal{E}}_i \neq \sum_{i \in I_{k'}} w_{ik'}/P_{k'} A_i \underline{\mathcal{E}}_i$.*

We derive and discuss this theorem in Appendix E.2. In particular, we explain that the two conditions for the result to apply are satisfied in our setting.

4.3.3 Implementation

Last, we need to implement the estimation free of the identification issues discussed above. We cannot estimate (21) using off-the-shelf methods. This is because the competition parameter χ and the aggregate elasticities $\mathcal{E}_{agg,k}$ must not only satisfy moment conditions but also respect the two-layer equilibrium relations. A naïve approach to solve all these conditions simultaneously is computationally untractable due to the large dimension of the parameter space.

However, we develop an algorithm that leads to rapid computation: estimating the model for a given quarter takes about two minutes on a personal computer. The basic idea of our

method is to focus on two nested equilibrium questions. On the one hand, if one knows the effect of aggregate elasticity, solving the values of aggregate elasticities can be done using an iteration process: run standard instrumental regressions at the investor level to estimate their demand then update aggregate elasticities using equation (20); repeat until obtaining convergence. On the other hand, if one knows all the equilibrium quantities, finding the effect of aggregate elasticity is a low-dimensional fixed point problem involving a single large panel regression; we solve it using the standard Newton method. In the interest of space, Appendix Section E.3 details this estimation procedure.

4.4 Estimates

We estimate the model for each quarter from 2001Q1 to 2020Q4. Recall that our identification comes from the cross-section, such that for each time period the model is estimated independently.

4.4.1 Competition χ

The average value of the competition parameter is $\chi = 2.2$. We show a summary of the estimates of the competition parameter χ across quarters in row 1 of Table 2; Appendix Figure IA.3 represents the whole distribution. The estimates show little variation around their median. There are no detectable trends in the time series of the estimates as seen in Appendix Figure IA.4.

A competition parameter of 2.2 implies a substantial competitive response at the individual level. If all other investors become more aggressive and increase their elasticity by 1, an atomistic investor would respond by decreasing her elasticity by 2.2. However, the estimate of competition points to an equilibrium behavior far from perfect competition. Recall that the perfect competition benchmark corresponds to $\chi \rightarrow +\infty$ and no-competition benchmark to $\chi = 0$. For example our simple calculation in equation (5) shows that we need large values of χ for strong equilibrium effects. Making 50% of investors passive, a value of χ of

at least 18 is necessary to compensate 90% of the drop in aggregate elasticity. This is an order of magnitude larger than our estimate of 2.2, and actually than all of our estimates. We investigate the implications of the competition parameter χ for the impact of the rise of passive investing in Section 5.

Robustness. We assess the robustness of the estimates along several dimensions. Table 2 reports the results of the estimation for these alternative specifications in rows 2 to 6. Overall the estimates of competition χ do not vary substantially across specifications — the median value of χ is always between 1.91 and 2.46.

First, in row 2, we consider an alternative construction of the instrument where the counterfactual portfolio positions are weighted by book equity instead of being equally weighted. While these weights are potentially more realistic and can strengthen the relevance condition, their ad-hoc nature might weaken the plausibility of the exogeneity condition. This leads to a median χ of 1.91, close to our baseline. Row 3 includes additional controls for stock characteristics to the regression (X_k): profitability, investment, and dividend yield. The estimates do not change much. However, we find that including many additional parameters sometimes hinders the convergence of the estimation algorithm (Appendix Table IA.1 restricts to a sample where all the methods converge, 2003Q3 to 2020Q4, and finds very similar results). Rows 4 and 5 consider an alternative weighting scheme. In our baseline all investors contribute equally to the estimate of χ , while row 4 weighs them by their assets under management. If our model was misspecified and the competitive response varied by investor size, this change would lead to different estimates. This is not the case here, with extremely close estimates. Row 5 weighs by book assets under management to avoid contamination by prices, also leading to virtually identical estimates. Row 6 addresses the details of how we deal with investors with few positions. In the baseline, active investors with fewer than 1,000 stock holdings are grouped together based on their assets under management such that each group on average contains 2,000 stock holdings. A finer way to construct these groups is to make them based on investor types, but data coverage is incomplete. The estimates in row

Table 2.
Estimates of competition χ under alternative specifications

	Estimates for χ		
	Median	25th pct.	75th pct.
(1) Baseline Specification	2.15	1.81	2.76
(2) BE-weighted Instrument for \mathcal{E}_{agg}	1.91	1.52	2.31
(3) Additional Controls	2.51	2.09	3.5
(4) AUM-weighted Regression	2.3	1.81	2.8
(5) Book-weighted Regression	2.27	1.76	2.78
(6) Investor-Type Grouping	2.42	1.93	2.94
(7) Constant χ	1.95		
(8) No Instrument for \mathcal{E}_{agg}	1.21	0.77	1.56
(9) No Instruments	0.96	0.67	1.38

Table 2 presents statistics of estimates of χ across dates (2001Q1 - 2020Q4) under various specifications. Our baseline specification (1) estimates χ given aggregate elasticities $\mathcal{E}_{agg,k}$ each period via the regression:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} + \xi \mathcal{E}_{agg,k} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik},$$

where $X_k^{(d)}$ contains log book equity and log book equity squared. $X_k^{(e)}$ is log book equity. Active investors with fewer than 1,000 stock holdings are grouped together based on their assets under management such that each group on average contains 2,000 stock holdings. The regression is weighted such that each group's weights sum to the same constant. Specification (2) shows estimates of χ based on the book-equity weighted instrument. Specification (3) adds additional characteristics to $X_k^{(d)}$; profitability, investment and dividends relative to book equity. Specification (4) value-weights the regression by weighting investors by their AUM. Specification (5) similarly value-weights the regression by weighting investors by their book assets. An investor's book assets are determined as follows: A stock's book equity is first allocated to investors proportional to the number of shares they hold, and subsequently summed up within investor across stocks to form an investor's books assets. Specification (6) follows Kojen and Yogo (2019) in grouping investors both by investor type and AUM. Institutional investors whose type we cannot determine are bundled together in a separate group. Specification (7) imposes for χ to be constant across time in the estimation, with each year receiving equal weight. Specification (8) reports results without instrumenting for the aggregate elasticity \mathcal{E}_{agg} . Specification (9) additionally removes the instrument for prices.

6 updates our estimation based on these data, which results in little change. Finally, row 7 constrains the competition parameter χ to be constant over time. This estimation is more computationally demanding, and yield a value of χ of 2.07 close to the baseline.

We also estimate the model without using instruments. Row 8 removes the instrument for aggregate elasticity. In this case we find an average value of χ of 1.2. This estimate far below that from any other specification suggests that it is important to account for the endogeneity

of elasticities — because they depend on actual portfolio weights which themselves depend on residual demand. Also removing the instruments for prices — row 9 — leads to even lower estimates, suggesting a deeper bias.

4.4.2 Stock-level Elasticities

The model also delivers estimates of aggregate elasticity, $\mathcal{E}_{agg,k}$, for each stock. Figure 3 represents these elasticities for 2011Q3 as a function of stock market capitalization. Each green dot corresponds to the elasticity estimate of one stock in our model for that date. We compare our estimates to a model where individual level elasticities are fixed, where $\underline{\mathcal{E}}_{1,i} = 0$ and $\chi = 0$. These estimates are represented with red dots in Figure 3.

There is substantial cross-sectional variation in elasticities, lending credence to our ability to identify the competition parameter χ . In both sets of estimates, the demand curve for individual stock is inelastic with average values around 0.45. This magnitude is far from the typical asset-pricing benchmark of perfectly horizontal demand curves with infinite elasticity. However, it is consistent with other empirical estimates, in particular based on portfolio data; see for example the discussion in Chang, Hong, and Liskovich (2014) and Koijen and Yogo (2019).

Figure 3 demonstrates a few ways in which accounting for the endogeneity of demand elasticities is important. First, the full model estimates exhibit less variation than the model with constant elasticities. With constant individual elasticities, variations in investor composition directly translates in variation in aggregate elasticities. However, with positive levels of competition χ , investors react to each other and soften those variations. For example, if an active investor with high-elasticity takes positions in a stock, other investors respond by trading less aggressively. Thus, stocks become more similar to each other.

Second, the full model exhibits a stronger negative relation between the size of a stock and its elasticity. Koijen and Yogo (2019) point out that large stocks tend to have more inelastic investors overall. Once we allow individual elasticities to respond to stock characteristics

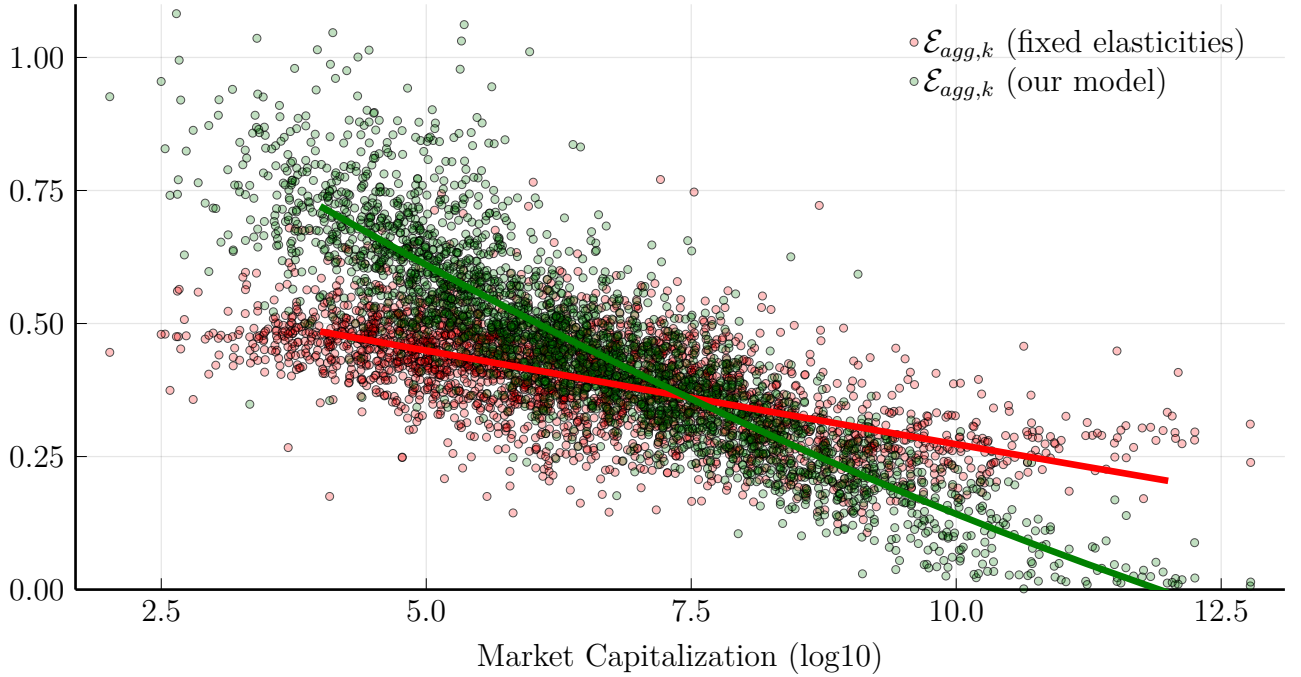


Figure 3. Aggregate elasticity at the stock level: $\mathcal{E}_{agg,k}$. Figure 3 represents estimates of the aggregate elasticity $\mathcal{E}_{agg,k}$, where each stock is represented by a circle, as a function of their market capitalization (in logarithm) for the date 2011Q3. Green circles are our estimates, while red circles correspond to a model where elasticities are fixed.

and the aggregate elasticity, the data reveals an additional source for this relation: the same investor behaves more inelastically for large stocks than small stocks. This additional source of variation within investor rather than across investors leads to a steeper relation between size and elasticity. We estimate a linear relation between size and elasticity at the investor-level for computational tractability; this linearity leads to extremely low values of elasticity for the very largest stocks.

Table 3 shows that these conclusions hold not only for this specific date, but across our sample. We report the distribution across dates of various statistics of the cross-section of \mathcal{E}_{agg} . In particular, we confirm that our estimates have less variation in elasticity across stocks (Panel B), by about 50%, and a steeper relation between elasticity and stock size (Panel C), by about 25%.

Table 3.
Properties of Aggregate Elasticity \mathcal{E}_{agg}

Panel A: Statistics of average elasticity across stocks				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.489	0.411	0.46	0.54
Fixed elasticity	0.389	0.357	0.389	0.442
Panel B: Regression coefficient (by dates) of elasticity on size				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	-0.109	-0.117	-0.1	-0.0855
Fixed elasticity	-0.0286	-0.0309	-0.0272	-0.0249
Panel C: Residual cross-sectional standard deviation of elasticity				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.0498	0.0389	0.0441	0.0521
Fixed elasticity	0.0842	0.0739	0.0826	0.0917

Table 3 presents statistics of the aggregate elasticity $\mathcal{E}_{agg}(k, t)$. We estimate the elasticity in our model accounting for competition χ and without competition as in Kojen and Yogo (2019) (denoted by fixed elasticity). Panel A has summary statistics of the average elasticity by date. Panel B shows summary statistics of the coefficient β_t from the regression $\mathcal{E}_{agg}(k, t) = \alpha_t + \beta_t size_{k,t} + \varepsilon_{k,t}$ by date. Panel C reports summary statistics of the cross-sectional standard deviation of the residual from the regression described in Panel B. The sample period is 2001 to 2020.

The negative relation between size and elasticity might appear surprising given the evidence on prices suggesting that large stocks are more informationally efficient.³⁴ However there are good reasons to think that institutions are more reluctant to change their positions for large stocks than for small stocks. Mechanically, the largest stocks occupy a larger share of portfolios. As of July 2021, the five largest corporations in the U.S. stock market account for about 18% of total market capitalization.³⁵ As a consequence, a large change in portfolio weight would have a large effect on an institution's portfolio return. Many institutions are either benchmarked to the index or have hard dollar limits on how much they can trade a given stock, and hence they would be unwilling to take on such large changes. As an

³⁴See Lo and MacKinlay (1990), Jegadeesh and Titman (1993), Lakonishopk, Shleifer, and Vishny (1994), and Hong, Lim, and Stein (2000).

³⁵The total market capitalization of Apple, Microsoft, Amazon, Alphabet (Google), and Facebook amount to \$8.8tn for total U.S. market capitalization of \$49tn.

illustration, Figure 4 decomposes trading activity—the sum of squared relative change in portfolio position—across percentiles of portfolio weights; Appendix Section F.1 details this calculation. There is much less trading activity for the larger portfolio positions: the top 50% of portfolio positions only account for 9% of trading activity. As such, the interpretation of our results is not so much that large stocks experience more mispricing but rather that high investor elasticity cannot be the explanation for the evidence on their returns.³⁶

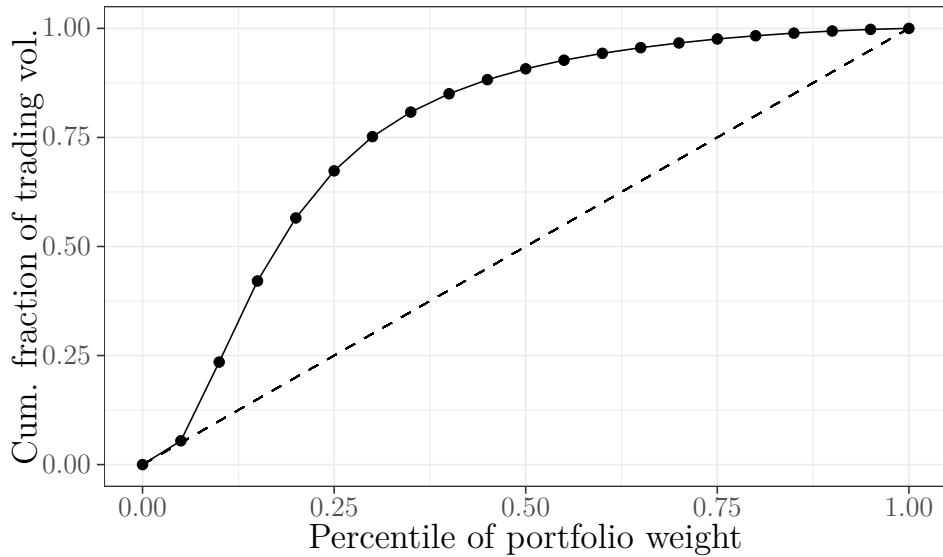


Figure 4. Trading activity across portfolio positions. Figure 4 presents the cumulative share of trading activity (defined in equation (IA.139)) by quantiles of investor portfolio weights.

5 Implications

5.1 The rise of passive investing

The last 20 years have seen a large increase in passive investing, a fact documented in French (2008). More recently Stambaugh (2014) shows that both the fraction of mutual funds that

³⁶In the model of Section 3, both elasticity and the quantity of noise trading determine price informativeness. Farboodi et al. (2021) use a richer structural model to decompose informativeness into data, growth, and volatility.

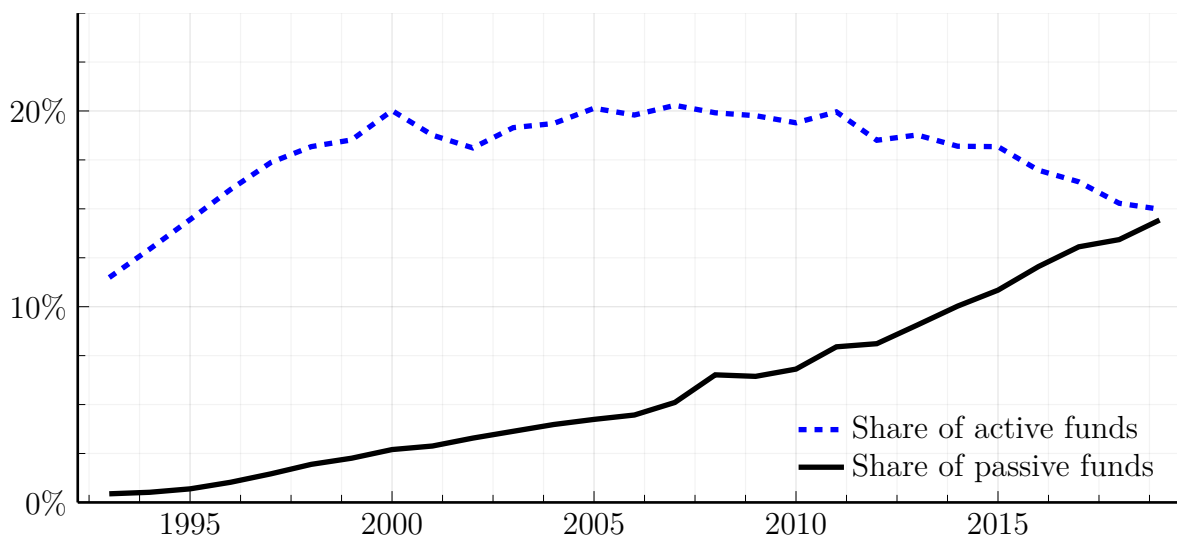


Figure 5. Share of Passive and Active Funds. Figure 5 shows the share of domestic mutual funds and ETFs as a fraction of the US stock market capitalization for passive funds (black solid line) and active funds (blue dashed line); Source ICI (2020).

are actively managed (at the extensive margin) and the active share of the portfolio of active equity mutual funds (at the intensive margin) have declined. We update and confirm these trends in Figure 5. The share of passive funds of the U.S. stock market has grown from nearly zero at the beginning of the 1990s to more than 15% in 2019. Concurrently, the share of active funds topped out at the end of the 1990s and has declined from 20% to 15% from 2000 to 2019.³⁷ Our model takes a more comprehensive view of who are the passive investors, not restricting ourselves to mutual funds. With this approach we find that the share of passive strategies has grown by 22 percentage points over the last 20 years (see Appendix Figure IA.8).

Has the shift to passive portfolios impacted the behavior of prices? Understanding how investors react to changes in the behavior of other investors is crucial to answer this question. In the standard view of competitive markets, when some investors stop looking for profitable trading opportunities, some other investors step in to replace them; prices do not change. In

³⁷We report the dollar numbers in Figure IA.7. Net assets of passive funds has grown from virtually zero to \$5.4tn in 2019, whereas the net assets of active funds only increased from \$600bn in 1993 to \$5.5tn in 2019.

contrast, if investors do not respond to others, the demand for stock becomes more inelastic which strongly affects the behavior of prices. For example in the theory of Section 3.1, more inelastic demand leads to prices that are more volatile and less informative. Our model, and in particular the parameter χ , accounts for the strength of this reaction. We use the estimated parameters to quantify the impact of the rise in passive investing on aggregate demand elasticities.

Starting with the demand system from Section 4, we impose an exogenous change in the fraction of active investors and compute the new equilibrium elasticities. Of course the rise of passive investing is not a purely exogenous phenomenon. However, most plausible explanations of this phenomenon are independent from the rest of the demand system. For example, the development of financial technology made it cheaper to pursue passive strategies: fees on passive funds have dropped dramatically and ETFs became available. Or, one subset of investors, maybe after listening to finance professors, realized they were making mistakes when pursuing active strategies. Such shocks are equivalent to an exogenous change in the fraction of passive investors as long as they do not directly affect the demand for the remaining investors.

Computing the effect of the rise of passive investing corresponds to the calculation of equation (5), accounting for heterogeneous investors. Combining the individual demand elasticity \mathcal{E}_{ik} in equation (16) with the equilibrium condition of (20), we have

$$\mathcal{E}_{agg,k} = \sum_{i \in Active_k} \frac{w_{ik} A_i}{\sum_{j \in Active_k} w_{jk} A_j} \cdot \underline{\mathcal{E}}_{ik} \times |Active_k| \times \frac{1}{1 + \chi |Active_k|}. \quad (27)$$

The aggregate elasticity is the product of three terms: (i) the average baseline elasticity among active investors, weighted by their respective positions; (ii) the fraction of the asset held by active investors, $|Active_k|$; and (iii) an adjustment for the competitive response of active investors to the aggregate elasticity, which depends on χ .

From this expression we obtain the effect of a change in the fraction of active investing.

Changing $|Active_k|$ while holding everything else constant corresponds to the assumption that the set of active investors that become passive is a representative sample of the active population. This leads to a simple formula:

$$\frac{d \log \mathcal{E}_{agg,k}}{d \log |Active_k|} = \frac{1}{1 + \chi |Active_k|}. \quad (28)$$

The pass-through from a rise in active investment to aggregate elasticity is determined by two numbers: the competition parameter χ and the fraction of active investors. In competitive financial markets, when χ is large, aggregate elasticity does not respond to shift in passive investing, the pass-through is zero. At the opposite end, when $\chi = 0$ such that investors do not respond to market conditions, the pass-through is 100%; an increase in the fraction of passive investors translates into a one-to-one decrease in aggregate demand elasticity. Furthermore, because only active investors change their elasticities in response to others (passive investors always have an elasticity of zero), starting with a larger fraction of active investors leads to a smaller pass-through.

We can readily compute the pass-through: it solely depends on two observable quantities, χ and $|Active_k|$. In Section 4, we estimated the competition parameter and found that $\chi = 2.15$. Recall we measure the total quantity of passive investors as investors with an elasticity of zero in the Koijen-Yogo demand system. Not surprisingly, we find a trend down from 81% in 2001 to 59% in 2020. Taking the average across dates for the share of active investors, 68%, and for the competition parameter, $\chi = 2.15$, we find a value of the pass-through of³⁸

$$\frac{1}{1 + \chi |Active_k|} = \frac{1}{1 + 2.15 \times 0.68} = 40.6\%. \quad (29)$$

This implies that competition is strong enough to compensate about 60% of the direct effect of a rise in passive investing. It is still far from the full cancellation of the competitive market

³⁸When the share of active investors is at 81% as in 2001 the pass-through is 36.5%, while when it is towards its lowest value of 59% at the end of the sample it is 44%.

view.

We multiply this pass-through by the rise in the proportion of passive investing to obtain the total effect on elasticity. We consider different takes for the size of the exogenous change. First we use our comprehensive measure of passive investing. The decline from 81% to 59% corresponds to a 32% drop, leading to elasticities lowered by $40.6\% \times 32\% = 13\%$. Translating the elasticities into price multipliers, this implies that the price impact of buying \$1 of a stock went up roughly from \$2.5 to \$2.9. Second, we look at a narrower measure of the rise in passive investing centered around the assets under management of passive mutual funds and ETFs. Their fraction of total market capitalization has increased by 15 percentage points in the last 30 years. Starting from a baseline of 81% of active investors, this represents a 19% drop in the total fraction of active investors. With our pass-through of 0.4, this reduces elasticities by 8%.

5.2 Decomposing the evolution of the demand for stocks

In the previous exercise, we isolated the causal effect of a change in passive investing on equilibrium demand elasticities. Next, we propose a positive account of the data: we decompose the actual changes in elasticity over the last twenty years in light of our model.

5.2.1 The downward trend in aggregate elasticity

Figure 6 presents the time series of the distribution of equilibrium elasticities across stocks. For each date, we compute quantiles of the cross-section of aggregate elasticities, $\mathcal{E}_{agg,k}$. We find a downward trend in equilibrium elasticities across the whole distribution of stocks. The average elasticity (bold solid line) goes from 0.46 to 0.31, a 40% drop. The one exception to the trend is the early part of the sample with an increase in elasticities between 2000 and 2004. The tails of the distribution also decrease. The 90th percentile (upper dashed line) drops from 0.64 to 0.53 (a 19% decrease). The 10th percentile (lower dashed line) also drops from 0.25 to 0.12. The downward trend in equilibrium elasticities affects the whole

distribution of stocks. We further our understanding of what is behind the decline in the next section through a simple decomposition.

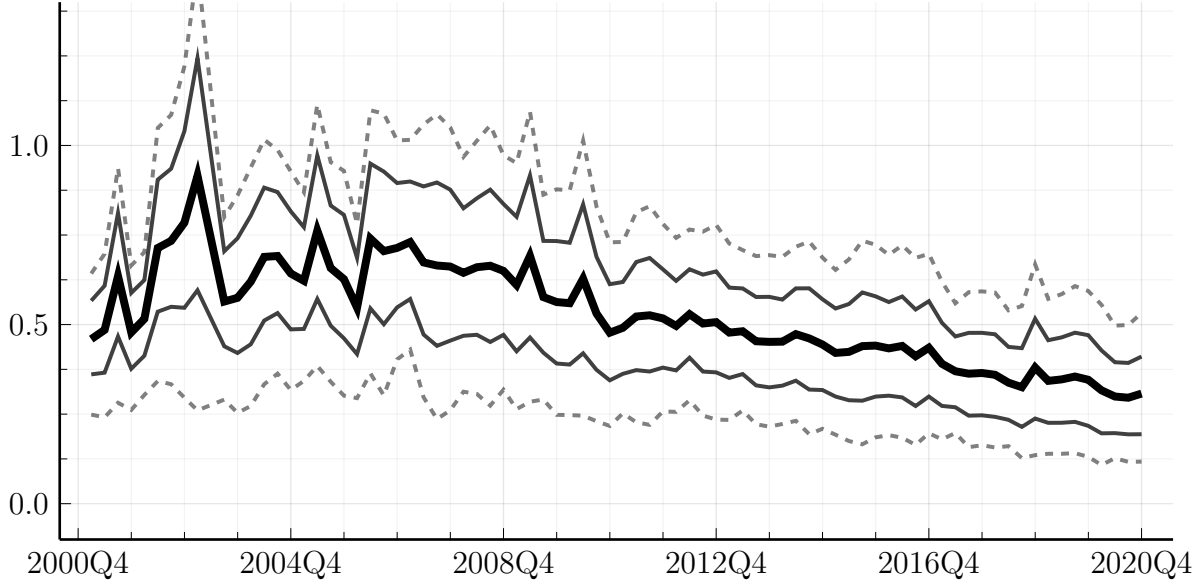


Figure 6. Distribution of aggregate elasticity across stocks.

Figure 6 traces out the distribution of aggregate elasticity $\mathcal{E}_{agg,k}$ over time. The bold line represents the average elasticity across stocks for each year. The solid lines represents the 25th and 75th percentile and the dashed lines the 10th and 90th percentile.

5.2.2 Sources of change in elasticity

In Section 4, we estimated the demand elasticities for each investor-stock in each quarter from 2000 to 2020. While our identification strategy is purely cross-sectional, we can use the time-series dimension of our estimates as a description of the evolution of the demand for stocks over time. To make parameters such as the investor-specific demand elasticity \mathcal{E}_i comparable across periods, we use the model estimates under the assumption that the competition parameter is constant over time (row 6 of Table 2).

We decompose changes in elasticity from year to year into three components using equation (27). We denote by $\langle \mathcal{E}_{ik} \rangle$ the position-weighted average of the individual-specific component of the elasticity of active investors, \mathcal{E}_{ik} ; this corresponds to the first term in equation (27).

We derive the effect of a change in investor composition,

$$\begin{aligned}
\underbrace{d\mathcal{E}_{agg,k}}_{\text{Change in aggregate elasticity}} &= \underbrace{|Active_k| \cdot d\langle \mathcal{E}_{ik} \rangle}_{\text{Individual elasticity of active investors}} + \underbrace{\langle \mathcal{E}_{ik} \rangle \cdot d|Active_k|}_{\text{Share of active investors}} + \\
&+ \underbrace{\left(\underbrace{\frac{-\chi|Active_k|}{1 + \chi|Active_k|} \cdot d(\langle \mathcal{E}_{ik} \rangle |Active_k|)}_{\text{Response to direct changes}} + \underbrace{\frac{-|Active_k|}{(1 + \chi|Active_k|)^2} \langle \mathcal{E}_{ik} \rangle \cdot d|Active_k|}_{\text{Fraction responding}} \right)}_{\text{Competitive response}}.
\end{aligned} \tag{30}$$

The first component corresponds to the average individual-level component elasticity of active investors; how their own characteristics contribute to the elasticity. The second component accounts for changes in the share of active investors over time and their ultimate effect on the elasticities. These forces correspond respectively to the intensive and extensive margin of individual elasticities. The last component corresponds to the equilibrium forces and are largely shaped by the parameter χ . It has two parts. First, the equilibrium response itself which dampens the total effect of both the extensive and intensive margins ($d(\langle \mathcal{E}_{ik} \rangle |Active_k|)$). When competition is low and $\chi = 0$, this term is zero and there is no equilibrium response; when competition is high and $\chi \rightarrow +\infty$, the equilibrium coefficient goes to -1 and there is full compensation of the direct effect on aggregate elasticity. The second part represents the change of the equilibrium effect itself when the number of active investors changes; only active investors' demands respond to others and the more there are, the larger the equilibrium response will be.

We present the three terms of this decomposition over time in Figure 7 and we summarize the total effects in Table 4.³⁹ We smooth the series to make the secular trends easier to identify. Over time we estimate that the aggregate stock-level elasticity has decreased by 40% on average. Consistent with the importance of the rise in passive investing discussed in

³⁹Because we cannot continuously integrate equation 30, we use the natural discrete approximation of the first two terms and compute the third one as a residual.

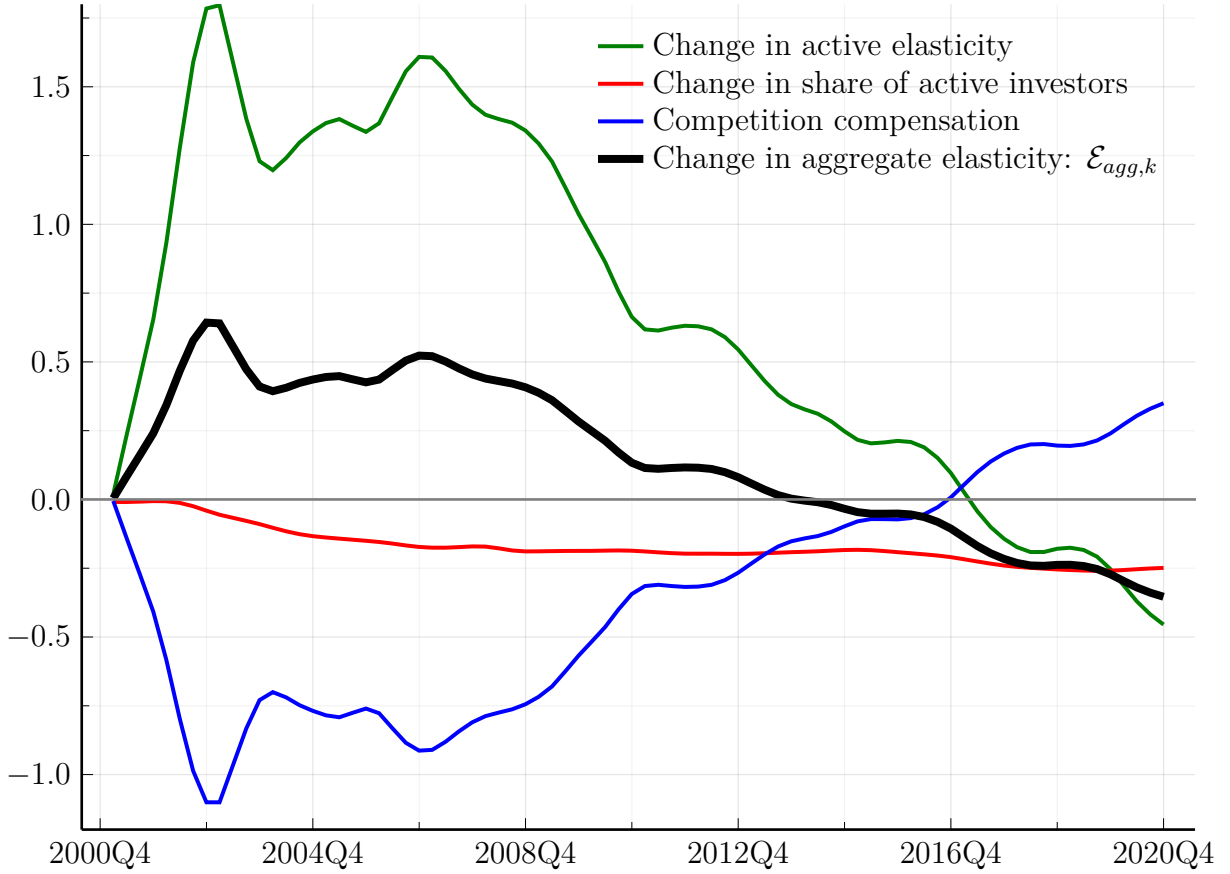


Figure 7. Decomposition of the change in aggregate elasticity.

Figure 7 shows the decomposition derived in equation (30) over time. We compute each term of the decomposition for each date and accumulate the changes over time.

Section 5.1, we find that the direct effect of the decrease in the fraction of active investors contributes 227% of the total drop in elasticity. Interestingly, investors also change their own elasticities at the intensive margin. If individual elasticities increase until 2006, they experience a sharp drop after and decline by 38% overall. Appendix Figure IA.9 confirms this pattern holds in the entire cross-section of investors. This second direct force adds 95% to the drop in aggregate elasticities. However, the competitive response strongly mitigates these individual changes in equilibrium. Competition reverses around two thirds of the decline, leading to the total change in aggregate elasticity of -40% .

Table 4. Decomposition of the change in aggregate elasticity \mathcal{E}_{agg}

Aggregate elasticity	Decomposition		
	Active share	Active elasticity	Competition
Total change (2001-2020)			
−33%	77%	120%	−96%

Table 4 reports the total change in aggregate elasticity and its decomposition, as derived in equation (30). We compute each term of the decomposition for each date and accumulate the changes over time. We report each term as a fraction of the total change in elasticity.

5.2.3 Evolution under counterfactual levels of competition

Finally we can ask how the changes in the individual components of investor demand would have affected the aggregate elasticities under different competition regimes. We start from the equilibrium levels of demand elasticity at the beginning of our sample (2001Q1). We feed into the model the two direct components highlighted above: how individual elasticities, $\underline{\mathcal{E}}_{ik}$, change over time and who becomes passive. However we make different assumptions on how investors react to changes in the behavior others. We show the time series of the results in Figure 8. The black line represents the actual evolution of the average aggregate elasticity across stocks; the colored lines show the counterfactual results.

We first consider the case of competitive investors, corresponding to $\chi \rightarrow +\infty$. In this situation any change in individual behavior is completely counteracted by other investors. The aggregate elasticities for each stock are pinned down at their initial level. The only source of variation in the average elasticity over time are changes in the composition of the universe of stocks. This is the red line in Figure 8, which experiences very little change over our sample. This result also confirms that the trends that we have documented of a decline in aggregate elasticities are not the consequence of changes in which stocks are traded.

The other extreme is the situation where investors do not react to others at all and $\chi = 0$. Then, all the changes in individual investor behavior directly feed into aggregate elasticities. This leads to a more dramatic drop in elasticities overtime than our baseline estimates. This is the green line in Figure 8. We observe a strong decrease, about twice as large as the

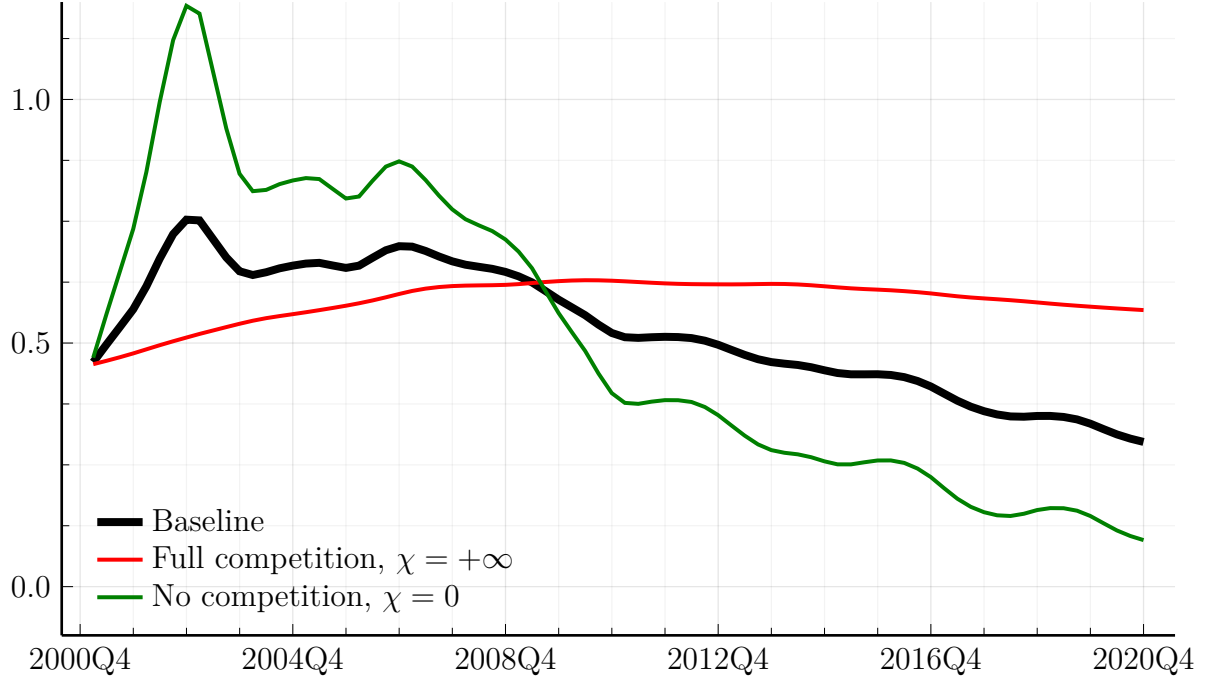


Figure 8. The evolution of aggregate elasticity under alternative competition regimes.

Figure 8 shows the evolution of aggregate elasticity $\mathcal{E}_{agg,k}$ under alternative competition regimes. The bold black line presents our baseline estimate. The top red line shows the elasticity with fully competitive investors ($\chi \rightarrow \infty$). The bottom green line shows the elasticity with no competition ($\chi = 0$).

baseline.

Overall these results confirm that changes in the behavior of investors have profoundly changed the aggregate demand curves for individual stocks. Competition among investors played an important role in mitigating the total impact of those changes. However, competition was not strong enough to fully negate the course of a downward trend in aggregate elasticities.

5.3 Implications in the cross-section of stocks

5.3.1 The competition mechanism in the cross-section

In our model, the response to a change in the share of passive investors occurs through the competitive response: the other active investors change their elasticity. However, other types

of adjustments could happen, for example, the composition of active investors could change. Also, the identity of who becomes passive beyond their demand elasticity might shape the response, as is the case in some more sophisticated theories.

While these possibilities are not explicitly part of our empirical model, they would manifest themselves through the changes in aggregate elasticity in response to changes in passive investing. We investigate their presence by zooming in on different sources of variations in passive investing than our baseline estimates (the ones caused by our instrument).

We regress annual log changes in stock-level elasticity on changes in the fraction of active investors

$$\log(\mathcal{E}_{agg,k,t}) - \log(\mathcal{E}_{agg,k,t-1}) = \beta (\log(|Active_{k,t}|) - \log(|Active_{k,t-1}|)) + \alpha_k + \gamma_t + e_{k,t}. \quad (31)$$

The inclusion of time and stock fixed effects allow to focus on variation independent of the average variation. A benchmark value for the coefficient β is the pass-through from equation (28), about 40%. However, if changes in individual-level elasticities or other types of changes in investor composition, are correlated with the active share, this would push β away from the theoretical pass-through. So effectively, we are assessing whether changes in investor behavior beyond the competition effect are correlated with changes in passive investing .

Table 5 presents the result, using the unconstrained cross-sectional model estimates. Column 1 is a univariate regression; column 2 and 3 add date then stock fixed effects. Throughout we find a coefficient of about 0.4, close to the theoretical pass-through.⁴⁰ This result supports the interpretation that our measured competition is the main driver of the response of aggregate elasticity to changes in passive investing. Furthermore, because our model estimates are only based on cross-sectional evidence, this result from including the time series dimension provides additional support for our theory. Going in this direction, in Appendix Table IA.3,

⁴⁰Statistical significance is not completely meaningful in this setting, because the left-hand-side of the regression is model-generated.

we confirm that the regression results are mostly unchanged when using the estimates that impose a constant value of χ through time.

We also consider what happens around index inclusions. For these events the source of the variation in passive investing is known because index funds are forced to change their portfolio after reclassification. Following Chang, Hong, and Liskovich (2014), Ben-David, Franzoni, and Moussawi (2018), and Chinco and Sammon (2022), we exploit the mechanical rule that allocates stocks between the Russell 1000 and 2000 indexes. We use the index-switching event as an instrument for the share of passive investors; column 5 of Table 5 reports the result. The first stage is significant with reclassification changing active ownership by about 5% (see Appendix Table IA.4). The coefficient is 0.44, again very close to the theoretical pass-through.

5.3.2 Behavior of asset prices

Our empirical model focuses on the estimation of demand elasticities for two reasons. First, elasticities are the quantity through which investor competition manifests itself across many theories. Second, these elasticities are a key determinant of the behavior of asset prices. For example, an asset with highly elastic investors will tend to be more liquid, because these investors are willing to provide liquidity. Similarly aggressive investors limit the influence of excess fluctuations in prices, which often results in less volatility or more price informativeness. In this section, we document the relation between aggregate elasticity and some of these aspects of asset prices in the cross-section.

In the spirit of our structural model, we run the following regressions:

$$Y_{k,t} = \beta \mathcal{E}_{agg,k,t} + \alpha_t + \gamma'_t X_{k,t} + e_{k,t}, \quad (32)$$

where $Y_{k,t}$ is a stock-level outcome, α_t are time fixed effects, and $X_{k,t}$ controls for stock characteristics. This OLS specification is likely biased because $\mathcal{E}_{agg,k,t}$ correlates with unobserved aspects of the stocks. Therefore, our preferred specification is 2SLS in which we instrument

Table 5. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share

	Log Change in Elasticity				
	(1)	(2)	(3)	(4)	(5)
Change in Active share	0.446*** (0.044)	0.475*** (0.036)	0.457*** (0.036)	0.426*** (0.034)	0.389*** (0.066)
Date Fixed Effects		Yes	Yes	Yes	Yes
Stock Fixed Effects			Yes		
Controls				Yes	Yes
Estimator	OLS	OLS	OLS	OLS	IV
N	50,292	50,292	49,661	50,292	10,619
R^2	0.076	0.461	0.497	0.569	0.748
First-stage F statistic					9.444
First-stage p value					0.000

Table 5 reports a panel regression of annual log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the annual log change in the active share $|Active_k|$. Column 2 adds date fixed effects. Column 3 adds stock fixed effects. Column 4 uses date fixed effects and controls for lagged book equity and annual log changes of log book equity. Column 5 instruments the log change in the active share $|Active_k|$ between Q1 and Q2 in any given year by two indicator variables corresponding to stocks switching between Russell 1000 and 2000 in either direction. It filters to stocks with CRSP market capitalization ranked between 500 to 1500 as of the end of Q1 to maintain comparability between stocks. The sample period is 2001-2020 for columns 1-4, and 2007-2020 for column 5. Standard errors are 2-way clustered by date and stock for columns 1-4, and clustered by date for column 5.

for $\mathcal{E}_{agg,k,t}$ using $\hat{\mathcal{E}}_{agg,k,t}$. We have already shown in Section 4.3.1 that the first stage of this estimation is strongly significant.

Table 6 reports the results. In columns 1 to 4, we measure the effect of aggregate elasticity on daily stock volatility. The first two columns use total volatility and the latter two use idiosyncratic volatility (from the three-factor model of Fama and French (1993)) While the relation is weak without instrumenting, the IV specifications reveal a strongly negative relation. Consistent with most theories, stocks with more elastic investors have less volatile returns. This result also ties together our mechanism with the results of Ben-David, Franzoni, and Moussawi (2018) on index inclusions. When a stock has more passive investors following an index switch, its aggregate elasticity declines due to a lack of competition (Table 5), which results in more volatility, as documented in their paper.

Columns 5 and 6 consider the measure of price informativeness of Dávila and Parlatore (2018). We find no significant relationship. However, the large standard errors reveal that the relation is difficult to estimate precisely rather than a tight zero. Columns 7 and 8 use the illiquidity measure of Amihud (2002). The IV specification is consistent with the theory: illiquidity is lower for stocks with more elastic investors. An interesting aspect of this connection is that our elasticity estimates focus on low-frequency aspects of portfolios while the Amihud (2002) measure highlights high-frequency properties of returns.

Overall, these results support the view that estimating the demand for stocks is useful to get to a better understanding of the behavior of financial markets. Specifically, demand elasticities appear to shape many aspects of this behavior.

Table 6. Determinants of the behavior of asset prices and stock-level elasticities $\mathcal{E}_{agg,k}$

	Total Volatility		Idiosyncratic Volatility		Price informativeness		Illiquidity	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Elasticity	0.081 (0.068)	−0.757*** (0.164)	0.008 (0.049)	−0.740*** (0.134)	−0.078 (0.306)	−0.270 (0.731)	1.250*** (0.066)	−0.475** (0.228)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Estimator	OLS	IV	OLS	IV	OLS	IV	OLS	IV
N	219,531	219,531	206,140	206,140	66,677	66,677	216,983	216,983
R^2	0.218	0.178	0.247	0.209	0.020	0.019	0.744	0.610

Table 6 reports panel regressions of measures of volatility, price informativeness and illiquidity on stock level elasticity $\mathcal{E}_{agg,k}$. All variables are standardized for each date (demeaned and standardized). Uneven columns show results from OLS regressions. Even columns show results from instrumental variables regressions that use our instrument for stock elasticity defined in equation (24). For columns (1) and (2), we compute the total daily volatility of stocks. For columns (3) and (4) we compute daily idiosyncratic volatility with respect to the Fama and French (1993) three-factor model based on daily CRSP data within a quarter. Columns (5) and (6) take the measure of price informativeness provided by Dávila and Parlato (2018). Columns (7) and (8) use the Amihud (2002) measure for illiquidity as the dependent variable, calculated again based on daily CRSP data within a quarter. All specifications are weighted by lagged market equity. We follow our main specification for the estimation of elasticity and control non-linearly for book equity to maintain comparability between stocks. The sample period starts in 2001 for all columns, and ends in 2020 for specifications 3-4 and 7-8, 2019 for specifications 1-2, and 2017 for specifications 5-6, based on respective data availability. Standard errors are 2-way clustered by date and stock.

6 Conclusion

The idea that investors compete with each other is fundamental in financial markets. In theories of financial decisions, how others trade has a large impact on how you trade. We put forward a framework to measure competition and analyze its impact on equilibrium outcomes. Our framework is simple yet encompasses many theories of investor competition. In the US stock market we find evidence that investors do react to each other but also that this response is much weaker than anticipated by classic views. The effects of changes in the composition of investors on the demand for stock is reduced by 60%. This implies for example that the rise in passive investing leads to substantially more inelastic markets.

The presence of measurable competition bears on many other important issues in finance. To assess the impact of financial regulation on some market participants, for example the Basel III leverage constraint on banks, one cannot ignore how other institutions will respond. Likewise to understand how the distress of some financial institutions creates fire-sale spillovers, one must realize that other investors will step up. Our framework measures how many actually will. Recent work in international finance emphasizes the importance of cross-border flows and global imbalances. What happens if a large sovereign institution stops investing in one market, like China with US treasuries? Again the competition among investors will be a crucial input in determining the final impact of such momentous shift. Last, the rise and availability of big data promises to change the face of institutional investing.

References

2020. *Investment Company Fact Book – A Review of Trends and Activities in the Investment Company Industry*. Investment Company Institute, 60 ed.
- Amihud, Yakov. 2002. “Illiquidity and Stock Returns: Cross-Section and Time-Series Effects.” *Journal of Financial Markets* 5 (1):31–56.
- Backus, Matthew, Christopher Conlon, and Michael Sinkinson. 2019. “Common ownership in america: 1980-2017.” Tech. rep., National Bureau of Economic Research.
- Backus, Matthew, Christopher T Conlon, and Michael Sinkinson. 2020. “Common Ownership Data: Scraped SEC form 13F filings for 1999-2017.”
- Bai, Jennie, Thomas Philippon, and Alexi Savov. 2016. “Have financial markets become more informative?” *Journal of Financial Economics* 122 (3):625–654.
- Balasubramaniam, Vimal, John Y Campbell, Tarun Ramadorai, and Benjamin Ranish. 2021. “Who Owns What? A Factor Model for Direct Stockholding.” Mimeo, Harvard University.
- Bastianello, Francesca and Paul Fontanier. 2021. “Partial Equilibrium Thinking in General Equilibrium.” Tech. rep., Harvard.
- Beber, Alessandro, Michael W. Brandt, Jason Cen, and Kenneth A. Kavajecz. 2018. “Mutual Fund Performance: Using Bespoke Benchmarks to Disentangle Mandates, Constraints and Skill.” Tech. rep., Duke University.
- Ben-David, Itzhak, Francesco Franzoni, and Rabih Moussawi. 2018. “Do ETFs Increase Volatility?” *The Journal of Finance* 73 (6):2471–2535.
- Bond, Philip and Diego García. 2018. “The equilibrium consequences of indexing.” Mimeo, University of Washington.
- Brunnermeier, Markus K. and Lasse Heje Pedersen. 2008. “Market Liquidity and Funding Liquidity.” *The Review of Financial Studies* 22 (6):2201–2238.
- Brunnermeier, Markus K and Yuliy Sannikov. 2013. “A Macroeconomic Model with a Financial Sector.” *American Economic Review* Forthcomin.
- Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer. 1985. “Multimarket Oligopoly: Strategic Substitutes and Complements.” *Journal of Political Economy* 93 (3):488–511.
- Buss, Adrian and Savitar Sundaresan. 2020. “More risk, more information: How passive ownership can improve informational efficiency.” Mimeo.
- Chang, Yen-Cheng, Harrison Hong, and Inessa Liskovich. 2014. “Regression Discontinuity and the Price Effects of Stock Market Indexing.” *The Review of Financial Studies* 28 (1):212–246.

- Chinco, Alex and Marco Sammon. 2022. “Excess Reconstitution-Day Volume.” Mimeo, CUNY, Baruch College.
- Coles, Jeffrey L., Davidson Heath, and Matthew C. Ringgenberg. 2020. “On Index Investing.” Tech. rep., Utah.
- Dávila, Eduardo and Cecilia Parlatore. 2018. “Identifying price informativeness.” Mimeo, National Bureau of Economic Research.
- De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. 1990. “Noise Trader Risk in Financial Markets.” *Journal of Political Economy* 98 (4):703–738.
- Dou, Winston, Leonid Kogan, and Wei Wu. 2020. “Common fund flows: Flow hedging and factor pricing.” *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper* .
- Duffie, Darrell. 2010. “Presidential Address: Asset Price Dynamics with Slow-Moving Capital.” *The Journal of Finance* 65 (4):1237–1267.
- Duffie, Darrell and Bruno Strulovici. 2012. “Capital Mobility and Asset Pricing.” *Econometrica* 80 (6):2469–2509. URL <http://dx.doi.org/10.3982/ECTA8822>.
- Eisfeldt, Andrea L, Hanno Lustig, and Lei Zhang. 2017. “Complex asset markets.” Tech. rep., National Bureau of Economic Research.
- Eyster, Erik and Matthew Rabin. 2005. “Cursed Equilibrium.” *Econometrica* 73 (5):1623–1672.
- Eyster, Erik, Matthew Rabin, and Dimitri Vayanos. 2019. “Financial Markets Where Traders Neglect the Informational Content of Prices.” *The Journal of Finance* 74 (1):371–399.
- Fama, Eugene F. and Kenneth R. French. 1993. “Common risk factors in the returns on stocks and bonds.” *Journal of Financial Economics* 33 (1):3–56. URL <http://www.sciencedirect.com/science/article/pii/0304405X93900235>.
- Farboodi, Maryam, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran. 2021. “Where has all the data gone?” Tech. rep., RFS Forthcoming.
- Farboodi, Maryam and Laura Veldkamp. 2020. “Long-Run Growth of Financial Data Technology.” *American Economic Review* 110 (8):2485–2523.
- French, Kenneth R. 2008. “Presidential Address: The Cost of Active Investing.” *The Journal of Finance* 63 (4):1537–1573.
- Gabaix, Xavier and Ralph SJ Koijen. 2020. “In search of the origins of financial fluctuations: The inelastic markets hypothesis.” Mimeo, Harvard University.

- Gabaix, Xavier, Arvind Krishnamurthy, and Olivier Vigneron. 2007. “Limits of Arbitrage: Theory and Evidence from the Mortgage-Backed Securities Market.” *The Journal of Finance* 62 (2):557–595.
- Gabaix, Xavier and Matteo Maggiori. 2015. “International Liquidity and Exchange Rate Dynamics.” *The Quarterly Journal of Economics* 130 (3):1369–1420.
- Gârleanu, Nicolae and Lasse Heje Pedersen. 2018. “Efficiently Inefficient Markets for Assets and Asset Management.” *The Journal of Finance* 73 (4):1663–1712.
- . 2021. “Active and Passive Investing: Understanding Samuelson’s Dictum.” *The Review of Asset Pricing Studies* Raab020.
- Gârleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Potesman. 2009. “Demand-Based Option Pricing.” *The Review of Financial Studies* 22 (10):4259–4299.
- Gourinchas, Pierre-Olivier, Walker Ray, and Dimitri Vayanos. 2019. “A preferred-habitat model of term premia and currency risk.” Mimeo, Berkeley.
- Greenwood, Robin and Samuel G. Hanson. 2014. “Waves in Ship Prices and Investment.” *The Quarterly Journal of Economics* 130 (1):55–109.
- Greenwood, Robin, Samuel G. Hanson, Jeremy C. Stein, and Adi Sunderam. 2019. “A Quantity-Driven Theory of Term Premiums and Exchange Rates.” Mimeo, Harvard University.
- Greenwood, Robin and Dimitri Vayanos. 2014. “Bond Supply and Excess Bond Returns.” *The Review of Financial Studies* 27 (3):663–713.
- Grossman, Sanford J. and Joseph E. Stiglitz. 1980. “On the Impossibility of Informationally Efficient Markets.” *American Economic Review* 70 (3):393–408.
- Haddad, Valentin, Paul Ho, and Erik Loualiche. 2021. “Bubbles and the Value of Innovation.”
- Haddad, Valentin, Alan Moreira, and Tyler Muir. 2021. “When Selling Becomes Viral: Disruptions in Debt Markets in the COVID-19 Crisis and the Fed’s Response.” *The Review of Financial Studies* Hhaa145.
- Haddad, Valentin and Tyler Muir. Forthcoming. “Do intermediaries matter for aggregate asset prices?” *Journal of Finance*.
- Haddad, Valentin and David Sraer. 2020. “The Banking View of Bond Risk Premia.” *The Journal of Finance* 75 (5):2465–2502.
- He, Zhiguo and Arvind Krishnamurthy. 2013. “Intermediary Asset Pricing.” *American Economic Review* 103 (2):732–770.
- Hong, Harrison, Jeffrey D. Kubik, and Jeremy C. Stein. 2004. “Social Interaction and Stock-Market Participation.” *The Journal of Finance* 59 (1):137–163.

- Hong, Harrison, Terence Lim, and Jeremy C. Stein. 2000. “Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies.” *The Journal of Finance* 55 (1):265–295.
- Jegadeesh, Narasimhan and Sheridan Titman. 1993. “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency.” *The Journal of Finance* 48 (1):65–91.
- Jiang, Zhengyang, Robert Richmond, and Tony Zhang. 2020. “A portfolio approach to global imbalances.” Mimeo.
- Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp. 2016. “A Rational Theory of Mutual Funds’ Attention Allocation.” *Econometrica* 84 (2):571–626.
- Kacperczyk, Marcin T, Jaromir B Nosal, and Savitar Sundaresan. 2020. “Market power and price informativeness.” Mimeo, Imperial College.
- Kleibergen, Frank and Richard Paap. 2006. “Generalized reduced rank tests using the singular value decomposition.” *Journal of Econometrics* 133 (1):97–126.
- Koijen, Ralph S. J. and Motohiro Yogo. 2019. “A Demand System Approach to Asset Pricing.” *Journal of Political Economy* 127 (4):1475–1515.
- Koijen, Ralph S.J., François Koulischer, Benoît Nguyen, and Motohiro Yogo. 2021. “Inspecting the mechanism of quantitative easing in the euro area.” *Journal of Financial Economics* 140 (1):1–20.
- Koijen, Ralph SJ, Robert J Richmond, and Motohiro Yogo. 2020. “Which Investors Matter for Equity Valuations and Expected Returns?” Mimeo, National Bureau of Economic Research.
- Koijen, Ralph SJ and Motohiro Yogo. 2020. “Exchange rates and asset prices in a global demand system.” Mimeo, National Bureau of Economic Research.
- Kreps, David M. and Jose A. Scheinkman. 1983. “Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes.” *The Bell Journal of Economics* 14 (2):326–337.
- Kyle, Albert S. 1989. “Informed Speculation with Imperfect Competition.” *The Review of Economic Studies* 56 (3):317–355.
- Lakonishopk, Josef, Andrei Shleifer, and Robert W. Vishny. 1994. “Contrarian Investment, Extrapolation, and Risk.” *The Journal of Finance* 49 (5):1541–1578.
- Lee, Jeongmin. 2020. “Passive Investing and Price Efficiency.” Mimeo.
- Lee, Jeongmin and Albert S. Kyle. 2018. “When are financial markets perfectly competitive?” Tech. rep., Maryland.
- Lo, Andrew W. and A. Craig MacKinlay. 1990. “When Are Contrarian Profits Due to Stock Market Overreaction?” *The Review of Financial Studies* 3 (2):175–205.

- Lucas, Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6):pp. 1429–1445.
- Malikov, George. 2019. "Information, participation, and passive investing." Mimeo. Working Paper.
- Manski, Charles F. 1993. "Identification of Endogenous Social Effects: The Reflection Problem." *The Review of Economic Studies* 60 (3):531–542.
- Miller, Edward M. 1977. "Risk, Uncertainty, and Divergence of Opinion." *Journal of Finance* 32 (4):1151–1168.
- Mitchell, Mark and Todd Pulvino. 2012. "Arbitrage crashes and the speed of capital." *Journal of Financial Economics* 104 (3):469–490. Market Institutions, Financial Market Risks and Financial Crisis.
- Mitchell, Mark, Todd Pulvino, and Erik Stafford. 2002. "Limited Arbitrage in Equity Markets." *The Journal of Finance* 57 (2):551–584.
- Sammon, Marco. 2021. "Passive Ownership and Price Informativeness." Mimeo.
- Shleifer, Andrei. 1986. "Do Demand Curves for Stocks Slope Down?" *The Journal of Finance* 41 (3):579–590.
- Shleifer, Andrei and Lawrence H. Summers. 1990. "The Noise Trader Approach to Finance." *Journal of Economic Perspectives* 4 (2):19–33.
- Shleifer, Andrei and Robert W. Vishny. 1997. "The Limits of Arbitrage." *The Journal of Finance* 52 (1):35–55.
- Siriwardane, Emil N. 2019. "Limited Investment Capital and Credit Spreads." *The Journal of Finance* 74 (5):2303–2347.
- Stambaugh, Robert F. 2014. "Presidential Address: Investment Noise and Trends." *The Journal of Finance* 69 (4):1415–1453.
- Stambaugh, Robert F., Jianfeng Yu, and Yu Yuan. 2012. "The short of it: Investor sentiment and anomalies." *Journal of Financial Economics* 104 (2):288–302. Special Issue on Investor Sentiment.
- . 2015. "Arbitrage Asymmetry and the Idiosyncratic Volatility Puzzle." *The Journal of Finance* 70 (5):1903–1948.
- Subrahmanyam, Avanidhar. 1991. "A Theory of Trading in Stock Index Futures." *The Review of Financial Studies* 4 (1):17–51.
- Vayanos, Dimitri and Jean-Luc Vila. 2021. "A Preferred-Habitat Model of the Term Structure of Interest Rates." *Econometrica* 89 (1):77–112.

- Vayanos, Dimitri and Tan Wang. 2007. “Search and endogenous concentration of liquidity in asset markets.” *Journal of Economic Theory* 136 (1):66 – 104.
- Veldkamp, Laura L. 2011. *Information Choice in Macroeconomics and Finance*. Princeton University Press.
- Warther, Vincent A. 1995. “Aggregate mutual fund flows and security returns.” *Journal of Financial Economics* 39 (2):209–235.

A Equilibrium Model of Financial Markets with Investor Competition

We derive the aggregate multiplier under different counterfactuals. First we consider an aggregate shock to the supply of a stock, then a shift in demand across investors, before concluding by examining a change in investors' elasticities.

A.1 The effect of a rise in passive investing

In Section 2.3, we ask how does the elasticity change when a fraction of investors become passive. We start with an economy with homogenous investors and assume in initial equilibrium $p = \bar{p}$. Because all investors are identical, we have $\mathcal{E}_i = \mathcal{E}_{agg}$. Combined with elasticity decision equation (4), this gives $\underline{\mathcal{E}}_i = (1 + \chi)\mathcal{E}_i$. We denote the initial equilibrium elasticity of investors by $\mathcal{E}_i = \mathcal{E}_{agg} = \mathcal{E}_0$.

Assume that a fraction $1 - \alpha$ of investors becomes passive such that their elasticity becomes zero and their level of demand is unchanged. We have to determine the new elasticity of active investors \mathcal{E}_i . The individual decisions and equilibrium conditions are:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi\mathcal{E}_{agg}, \quad (\text{IA.1})$$

$$\mathcal{E}_{agg} = \alpha\mathcal{E}_i + (1 - \alpha) \cdot 0. \quad (\text{IA.2})$$

Solving this system gives aggregate elasticity:

$$\mathcal{E}_{agg} = \frac{\alpha}{1 + \alpha\chi}\mathcal{E}_i. \quad (\text{IA.3})$$

$$\mathcal{E}_{agg} = \frac{\alpha}{1 + \alpha\chi}(1 + \chi)\mathcal{E}_0 = \alpha\mathcal{E}_0 + \frac{\alpha\chi}{1 + \alpha\chi}(1 - \alpha)\mathcal{E}_0. \quad (\text{IA.4})$$

The first part of the equation corresponds to the direct effect of the rise in passive investing and the second part represents the compensation from the equilibrium response.

A.2 Aggregate multiplier and the asymmetry of the demand elasticity

How do prices decrease in the two-layer equilibrium in response to an exogenous shift in the supply of a stock ΔS . We start by defining aggregation operators which weight investors' outcomes by the fraction of the stock they own:

$$\check{\mathbf{E}}[x_i] = \int x_i \frac{D_i}{S}, \quad (\text{IA.5})$$

$$\check{\text{Var}}[x_i] = \int x_i^2 \frac{D_i}{S} - \left(\int x_i \frac{D_i}{S} \right)^2 \quad (\text{IA.6})$$

$$\check{\text{Cov}}[x_i, y_i] = \int (x_i - \check{\mathbf{E}}[x_i])(y_i - \check{\mathbf{E}}[y_i]) \frac{D_i}{S}. \quad (\text{IA.7})$$

To see the impact on the equilibrium of the change in supply we start by deriving the change in individual demand:

$$\Delta D_i = \Delta e^{d_i - \mathcal{E}_i(p - \bar{p})} = -\mathcal{E}_i D_i \Delta p - (p - \bar{p}) D_i \Delta \mathcal{E}_i, \quad (\text{IA.8})$$

such that change in aggregate demand which corresponds to the change in aggregate supply reads:

$$\Delta S = \Delta D = -\Delta p S \int \mathcal{E}_i D_i / S - (p - \bar{p}) S \int \Delta \mathcal{E}_i D_i / S \quad (\text{IA.9})$$

$$= -\Delta p S \mathcal{E}_{agg} + \chi(p - \bar{p}) S \Delta \mathcal{E}_{agg}, \quad (\text{IA.10})$$

where we have used in the last equation the individual elasticity equation $\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}$ leading to the uniform change in individual investors' elasticity

$$\Delta \mathcal{E}_i = -\chi \mathcal{E}_{agg}. \quad (\text{IA.11})$$

To estimate the change in aggregate elasticity we write:

$$\Delta \mathcal{E}_{agg} = \int \Delta (\mathcal{E}_i D_i / S) = \int \Delta \mathcal{E}_i D_i / S - \frac{\Delta S}{S} \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta D_i / S \quad (\text{IA.12})$$

$$= -\chi \Delta \mathcal{E}_{agg} - \frac{\Delta S}{S} \mathcal{E}_{agg} - \Delta p \check{\mathbf{E}} [\mathcal{E}_i^2] + \chi(p - \bar{p}) \mathcal{E}_{agg} \Delta \mathcal{E}_{agg}. \quad (\text{IA.13})$$

$$\Delta \mathcal{E}_{agg} = -\frac{1}{1 + \chi - \chi(p - \bar{p}) \mathcal{E}_{agg}} \left(\mathcal{E}_{agg} \frac{\Delta S}{S} + \check{\mathbf{E}} [\mathcal{E}_i^2] \Delta p \right), \quad (\text{IA.14})$$

Then we can plug the change in aggregate elasticity into the main equation expressing the change in aggregate demand and solve for Δp as a function of the change in supply ΔS .

$$\frac{\Delta S}{S} = -\mathcal{E}_{agg} \Delta p - \frac{\chi(p - \bar{p})}{1 + \chi - \chi(p - \bar{p}) \mathcal{E}_{agg}} \left(\mathcal{E}_{agg} \frac{\Delta S}{S} + \check{\mathbf{E}} [\mathcal{E}_i^2] \Delta p \right) \quad (\text{IA.15})$$

After some algebra we find the change in prices in response to an exogenous change in supply:

$$\Delta p = \frac{1}{\mathcal{E}_{agg}} \cdot \frac{1}{1 + \frac{\chi}{1+\chi} \frac{p-\bar{p}}{\mathcal{E}_{agg}} \check{\mathbf{Var}}[\mathcal{E}_i]} \frac{\Delta S}{S} = M_{agg} \cdot \frac{\Delta S}{S}, \quad (\text{IA.16})$$

where we define the aggregate multiplier M_{agg} as:

$$M_{agg} = \frac{1}{\mathcal{E}_{agg}} \cdot \frac{1}{1 + \frac{\chi}{1+\chi} \frac{p-\bar{p}}{\mathcal{E}_{agg}} \check{\mathbf{Var}}[\mathcal{E}_i]} \frac{\Delta S}{S}. \quad (\text{IA.17})$$

A.3 Limits to arbitrage

A more realistic experiment considers a shift in demand across investors. We consider an experiment where demand across investors changes by $\{\Delta d_i\}_i$. Solving for the equilibrium response of the price and elasticity is similar to the simple case of a change in aggregate

supply. First, we consider the effect of a change in investors' demands on aggregate demand:

$$\Delta D = \Delta \int e^{\underline{d}_i - \mathcal{E}_i(p - \bar{p})} = \int \Delta \underline{d}_i D_i - \int \mathcal{E}_i \Delta p D_i - \int (p - \bar{p}) \Delta \mathcal{E}_i D_i \quad (\text{IA.18})$$

$$\frac{\Delta D}{S} = \check{\mathbf{E}}[\Delta \underline{d}_i] - \mathcal{E}_{agg} \Delta p + \chi(p - \bar{p}) \Delta \mathcal{E}_{agg}, \quad (\text{IA.19})$$

where we have used $\check{\mathbf{E}}[\mathcal{E}_i] = \mathcal{E}_{agg}$ and $\Delta \mathcal{E}_i = -\chi \Delta \mathcal{E}_{agg}$. To solve for the change in price as a function of the change in investors' demands we use the second market clearing condition of aggregate elasticities to find $\Delta \mathcal{E}_{agg}$:

$$\Delta \mathcal{E}_{agg} = \int \Delta \mathcal{E}_i D_i / S + \int \mathcal{E}_i \Delta D_i / S \quad (\text{IA.20})$$

$$= -\chi \Delta \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta \underline{d}_i D_i / S - \Delta p \int \mathcal{E}_i^2 D_i / S - (p - \bar{p}) \int \mathcal{E}_i \Delta \mathcal{E}_i D_i / S \quad (\text{IA.21})$$

$$= (-\chi + \chi(p - \bar{p}) \mathcal{E}_{agg}) \Delta \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta \underline{d}_i D_i / S - \Delta p \int \mathcal{E}_i^2 D_i / S \quad (\text{IA.22})$$

$$= \frac{1}{1 + \chi - \chi(p - \bar{p}) \mathcal{E}_{agg}} \cdot (\check{\mathbf{E}}[\mathcal{E}_i \Delta \underline{d}_i] - \check{\mathbf{E}}[\mathcal{E}_i^2] \Delta p). \quad (\text{IA.23})$$

Now we plug this expression back into the expression for the change in aggregate demand above:

$$\frac{\Delta D}{S} = \check{\mathbf{E}}[\Delta \underline{d}_i] - \mathcal{E}_{agg} \Delta p + \frac{\chi(p - \bar{p})}{1 + \chi - \chi p \mathcal{E}_{agg}} \cdot (\check{\mathbf{E}}[\mathcal{E}_i \Delta \underline{d}_i] - \check{\mathbf{E}}[\mathcal{E}_i^2] \Delta p) \quad (\text{IA.24})$$

The supply is fixed such that $\Delta D / S = 0$, which after rearranging terms gives the final expression for the change in price:

$$\Delta p = \frac{1}{\mathcal{E}_{agg}} \frac{1}{1 + \frac{\chi}{1 + \chi} \frac{p - \bar{p}}{\mathcal{E}_{agg}} \check{\text{Var}}[\mathcal{E}_i]} \left(\check{\mathbf{E}}[\Delta \underline{d}_i] + \frac{\chi}{1 + \chi} (p - \bar{p}) \check{\text{Cov}}(\mathcal{E}_i, \Delta \underline{d}_i) \right) \quad (\text{IA.25})$$

We recognize the first term as the standard aggregate multiplier (obtained when we derived the response to change in supply) and write the price response as:

$$\Delta p = M_{agg} \cdot \left(\underbrace{\check{\mathbf{E}}[\Delta \underline{d}_i]}_{\text{average demand shock}} + \frac{\chi}{1 + \chi} (p - \bar{p}) \underbrace{\check{\text{Cov}}(\mathcal{E}_i, \Delta \underline{d}_i)}_{\text{average elasticity composition}} \right). \quad (\text{IA.26})$$

A.4 Exogenous change in the baseline elasticity

Last, we consider a change in investors' baseline elasticity $\Delta \underline{\mathcal{E}}_i$. We start with the relative change in aggregate demand:

$$\frac{\Delta D}{S} = -(p - \bar{p}) \int \Delta \mathcal{E}_i D_i / S - \mathcal{E}_{agg} \Delta p \quad (\text{IA.27})$$

$$= -(p - \bar{p}) \check{\mathbf{E}} [\underline{\mathcal{E}}_i] + \chi(p - \bar{p}) \Delta \mathcal{E}_{agg} - \mathcal{E}_{agg} \Delta p \quad (\text{IA.28})$$

We use market clearing to get $\Delta D/S = 0$:

$$\Delta p = -\frac{1}{\mathcal{E}_{agg}} \left((p - \bar{p}) \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] - \chi \Delta \mathcal{E}_{agg} \right). \quad (\text{IA.29})$$

The change in aggregate elasticity in the case of changes in baseline elasticities is different than in the previous cases:

$$\Delta \mathcal{E}_{agg} = \int \Delta (\mathcal{E}_i D_i / S) = \int \Delta \underline{\mathcal{E}}_i D_i / S - \chi \Delta \mathcal{E}_{agg} + \int \mathcal{E}_i \Delta D_i / S \quad (\text{IA.30})$$

$$= \int \Delta \underline{\mathcal{E}}_i D_i / S - \chi \Delta \mathcal{E}_{agg} - (p - \bar{p}) \int \mathcal{E}_i \Delta \mathcal{E}_i D_i / S - \Delta p \int \mathcal{E}_i^2 D_i / S \quad (\text{IA.31})$$

$$= \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] - \chi \Delta \mathcal{E}_{agg} - (p - \bar{p}) \check{\mathbf{E}} [\mathcal{E}_i \Delta \underline{\mathcal{E}}_i] + \chi(p - \bar{p}) \mathcal{E}_{agg} \Delta \mathcal{E}_{agg} - \Delta p \check{\mathbf{E}} [\mathcal{E}_i^2] \quad (\text{IA.32})$$

After rearranging the terms and plugging in the expression for the change in the price, we find:

$$\left(1 + \chi + \chi(p - \bar{p}) \frac{\check{\text{Var}}[\mathcal{E}_i]}{\mathcal{E}_{agg}} \right) \Delta \mathcal{E}_{agg} = \left(1 + (p - \bar{p}) \frac{\check{\text{Var}}[\mathcal{E}_i]}{\mathcal{E}_{agg}} \right) \check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] - (p - \bar{p}) \check{\text{Cov}} (\mathcal{E}_i, \Delta \underline{\mathcal{E}}_i) \quad (\text{IA.33})$$

Plugging back into the change in the price we have:

$$\Delta p = -M_{agg} \frac{p - \bar{p}}{1 + \chi} \left(\check{\mathbf{E}} [\Delta \underline{\mathcal{E}}_i] + (p - \bar{p}) \check{\text{Cov}} (\mathcal{E}_i, \Delta \underline{\mathcal{E}}_i) \right) \quad (\text{IA.34})$$

B Model of Information Acquisition

B.1 Setup

There is one period and one asset, and a continuum of agents indexed by $i \in [0, 1]$. Each agent has CARA preferences with risk aversion ρ_i :

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \quad (\text{IA.35})$$

and initial wealth W_i . The gross risk-free rate is 1, and the (random) asset payoff is f . The asset is in noisy supply $\bar{x} + x$ with \bar{x} an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

Each agent has a prior that $f \sim \mathcal{N}(\mu_i, \sigma_i^2)$. Following Veldkamp (2011), agents start with a flat prior on f and receive signal μ_i such that the signal is distributed $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$. Each agent can acquire a private signal $\eta_i \sim \mathcal{N}(f, \sigma_{i,\eta}^2)$ at cost $c_i(\sigma_i^{-2} + \sigma_{i,\eta}^{-2})$, with $c_i(\cdot)$ a non-decreasing positive function. That is, obtaining more precise signals is more costly. The signal being private implies in particular that signal realizations are uncorrelated across agents conditional on the fundamental f .

We focus on rational expectations equilibria, and among those linear equilibria specifically. These are equilibria in which the price takes the form:

$$p = A + Bf + Cx. \quad (\text{IA.36})$$

An equilibrium is a set of coefficient (A, B, C) , information choices $\sigma_{i,\eta}^2$, demand curves $D_i(p|\eta_i)$ such that:

- (a) Each demand function and information choice maximizes expected utility, taking as given the price function.
- (b) The market for the asset clears: $\bar{x} + x = \int D_i(p|\eta_i) di$.

To solve the model, we process in three steps: first we solve for the price given information decisions; then we will maximize utility within the model with respect to the information choice, keeping separation between information decision and portfolio allocation.

B.2 Solving Prices given Information

We are going to solve for the price function $p = A + Bf + Cx$. First we solve for allocations given the information choice and finally we use market clearing to pin down the price.

Agents form posterior on the fundamental f based on their prior μ_i , signal η_i , and from prices. The signal agents can extract from prices about f is:

$$s(p) = \frac{p - A}{B} = f + \frac{C}{B}x. \quad (\text{IA.37})$$

Given the three signals, we are able to derive the posterior belief about f which will be

distributed as $\mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$ as follows:

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \sigma_{i\eta}^{-2} + \frac{B^2}{C^2} \sigma_x^{-2} \quad (\text{IA.38})$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + \frac{B^2}{C^2} \sigma_x^{-2} s(p) \right) \quad (\text{IA.39})$$

Asset Demand. Abstracting from the cost of acquiring information, the utility function is for a given asset holding q_i :

$$U_i(q_i) = -\mathbf{E} [\exp (-\rho_i (f q_i - p q_i))] \quad (\text{IA.40})$$

$$= -\exp \left(-\rho_i q_i (\mathbf{E}[f] - p) + \frac{\rho_i^2}{2} q_i^2 \text{Var}[f] \right). \quad (\text{IA.41})$$

The first order condition with respect to q_i gives us immediately:

$$\begin{aligned} & -\rho_i (\mathbf{E}[f] - p) + \rho_i^2 q_i \text{Var}[f] = 0 \\ \Leftrightarrow & \quad q_i = \frac{1}{\rho_i \text{Var}[f]} (\mathbf{E}[f] - p) \\ \Leftrightarrow & \quad q_i = \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\mu}_i - p) \end{aligned} \quad (\text{IA.42})$$

Market Clearing. The market clearing condition reads:

$$\int q_i di = \bar{x} + x. \quad (\text{IA.43})$$

Given asset demand this translates into:

$$\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\mu}_i - p) di = \bar{x} + x \quad (\text{IA.44})$$

The goal now is to find (A, B, C) , which we identify directly from the market clearing condition. First we replace the expressions for the price function and the posteriors mean and variances in the market clearing equation:

$$\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} \left[f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x \right] di - \int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} [A + Bf + Cx] di = \bar{x} + x \quad (\text{IA.45})$$

First, we identify all the terms that are linear in f and find that $B = 1$. Next, we group the terms that are linear in x we have:

$$\begin{aligned}
& \int \frac{1}{\rho_i} \frac{1}{C} \sigma_x^{-2} di - \int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} C di = 1 \\
\iff & \int \frac{1}{\rho_i} \left[\frac{1}{C} \sigma_x^{-2} - C \hat{\sigma}_i^{-2} - C \sigma_{i\eta}^{-2} - \frac{1}{C} \sigma_x^{-2} \right] di = 1 \\
\iff & C = - \left[\int \frac{1}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2}) di \right]^{-1}. \tag{IA.46}
\end{aligned}$$

Where we used the expression of the posterior found above to substitute into the second equation. Last we gather the constant terms to find A and

$$A = -\bar{x} \left[\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} di \right]^{-1} \tag{IA.47}$$

B.3 Optimal Information

Computing Expected Utility Conditional on the signal and the price, expected utility is:

$$U_i(q_i) = -\mathbf{E} [\exp(-\rho_i (f q_i - p q_i)) | p, \eta] \tag{IA.48}$$

$$= -\exp \left(-\rho_i q_i (\mathbf{E}[f|p, \eta] - p) + \frac{\rho^2}{2} q_i^2 \text{Var}[f|p, \eta] \right) \tag{IA.49}$$

$$= -\exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p, \eta] - p)^2}{\text{Var}[f|p, \eta]} \right), \tag{IA.50}$$

where the last line is derived using standard properties of quadratic functions.⁴¹

We can write:

$$\mathbf{E}[f|p, \eta] - p = \underbrace{(\mathbf{E}[f|p, \eta] - \mathbf{E}[f|p])}_z + (\mathbf{E}[f|p] - p) \tag{IA.51}$$

Conditional on p , z has mean 0 and its variance σ_z^2 can be obtained from:

$$\begin{aligned}
\underbrace{f - \mathbf{E}[f|p]}_{\text{variance: } (\sigma_i^{-2} + \sigma_x^{-2}/C^2)^{-1}} &= \underbrace{(f - \mathbf{E}[f|p, \eta])}_z + z \tag{IA.52} \\
&\quad \text{variance: } \hat{\sigma}_i^2
\end{aligned}$$

⁴¹For a function $f(x) = ax^2 + bx$, the maximum is reached for $x^* = -b/(2a)$ and its value is $f(x^*) = -b^2/(4a)$.

Using equation (7.32) in Veldkamp (2011), this maps to:⁴²

$$F = -\frac{1}{2} \frac{1}{\hat{\sigma}_i^2} \quad (\text{IA.53})$$

$$G = -(\mathbf{E}[f|p] - p) \frac{1}{\hat{\sigma}_i^2} \quad (\text{IA.54})$$

$$H = -\frac{1}{2} (\mathbf{E}[f|p] - p)^2 \frac{1}{\hat{\sigma}_i^2} \quad (\text{IA.55})$$

So expected utility conditional on the price is:

$$\begin{aligned} U_0|_p &= -(1 - 2\sigma_z^2 F)^{-1/2} \exp \left(\frac{1}{2} G^2 (1 - 2\sigma_z^2 F)^{-1} \sigma_z^2 + H \right) \\ &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2} \left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1} \frac{1}{\hat{\sigma}_i^2} \sigma_z^2 - 1 \right] \right) \\ &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2} \left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1} (\frac{1}{\hat{\sigma}_i^2} \sigma_z^2 - 1 - \frac{\sigma_z^2}{\hat{\sigma}_i^2}) \right] \right) \\ &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2} \left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1} \right] \right) \\ U_0|_p &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2 + \sigma_z^2} \right). \end{aligned} \quad (\text{IA.56})$$

Expected utility is:

$$\begin{aligned} \mathbf{E}[U_0|_p] &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \mathbf{E} \exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma}_i^2 + \sigma_z^2} \right) \\ &= - \left[\frac{\sigma_i^{-2} + \sigma_x^{-2}/C^2}{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \sigma_x^{-2}/C^2} \right]^{1/2} \cdot \mathbf{E} \left[\exp \left(-\frac{1}{2} \frac{(\mathbf{E}[f|p] - p)^2}{(\sigma_i^{-2} + \sigma_x^{-2}/C^2)^{-1}} \right) \right] \end{aligned} \quad (\text{IA.57})$$

where we use (IA.52) in the second equality.

Optimal information. To derive the optimal information choice, we trade off utility with the cost of acquiring information which translates in utility terms:

$$U_0^{(c)} = U_0 \cdot \exp(\rho_i c_i (\sigma_i^{-2} + \sigma_{i\eta}^{-2})) \quad (\text{IA.58})$$

⁴²There is a general formula for the mean of the exponential of the quadratic of a normal variable. If we take the multivariate normal $z \sim \mathcal{N}(0, \Sigma)$:

$$\mathbf{E}[\exp(z' F z + G' z + H)] = |I - 2\Sigma F|^{-1/2} \exp \left(\frac{1}{2} G'(I - 2\Sigma F)^{-1} \Sigma G + H \right)$$

$c_i(\cdot)$ is a function that is increasing in the signal precision and can depend on the i . The first order condition that determines the information choice is (maximizing $-\log(-U)$):

$$\begin{aligned}
& \max_{\sigma_{i\eta}^{-2}} -\log(-U_0) - \rho_i c_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2}) \\
\iff & \max_{\sigma_{i\eta}^{-2}} -\log(-U_0) - \rho_i c_i(\rho_i \mathcal{E}_i) \\
\iff & \frac{1}{2} \frac{1}{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \sigma_x^{-2}/C^2} = \rho_i c'_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \\
\iff & \frac{1}{2} \frac{1}{\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2}} = \rho_i c'_i(\rho_i \mathcal{E}_i).
\end{aligned} \tag{IA.59}$$

Example 1: linear cost function. We consider the case of a constant marginal cost for any information acquired past the initial endowment: $c_i(x) = c_{1,i} \max(x - \sigma_i^{-2}, 0)$. Note that in this case not all agents acquire information since $\sigma_{i\eta}^{-2} > 0$, so the actual precision is:

$$\sigma_{i\eta}^{-2} = \max\left(\frac{1}{2\rho_i c_{1,i}} - \sigma_i^{-2} - C^{-2}\sigma_x^{-2}, 0\right) \tag{IA.60}$$

This demand for information and the formula for the coefficient C characterizes the equilibrium. We recall that:

$$C^{-2} = \left[\int \frac{1}{\rho_j} (\sigma_j^{-2} + \sigma_{j\eta}^{-2}) dj \right]^2 \tag{IA.61}$$

Example 2: linear response to aggregate elasticity. To relate to the model of Section 2, we ask if there is a reasonable family of cost functions that give rise exactly to equation (4). We are looking for a cost function such that $\mathcal{E}_i = \alpha - \beta \mathcal{E}_{agg}$. Equivalently, this corresponds to $\mathcal{E}_{agg} = \frac{1}{\beta}(\alpha - \mathcal{E}_i)$. Plugging in the first order condition, this gives:

$$2\rho_i^2 c'_i(\rho_i \mathcal{E}_i) = \frac{1}{\mathcal{E}_i + \frac{\sigma_x^{-2}}{\rho_i \beta^2} (\alpha - \mathcal{E}_i)^2} \tag{IA.62}$$

$$=_{def} \tilde{c}'_i(\mathcal{E}_i) = \frac{1}{\frac{\sigma_x^{-2}}{\rho_i \beta^2} \mathcal{E}_i^2 + \left(1 - 2\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right) \mathcal{E}_i + \frac{\alpha^2 \sigma_x^{-2}}{\rho_i \beta^2}} \tag{IA.63}$$

The denominator of the right-hand-side is a second degree polynomial, we solve for its roots. The discriminant is:

$$\Delta = \left(1 - 2\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right)^2 - 4\frac{\sigma_x^{-2}}{\rho_i \beta^2} \frac{\alpha^2 \sigma_x^{-2}}{\rho_i \beta^2} \tag{IA.64}$$

$$= 1 - 4\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} \tag{IA.65}$$

Let us assume $\Delta < 0$. This is equivalent to $\rho_i \beta^2 < 4\alpha \sigma_x^{-2}$. In this case, we have, using

standard results on the primitive of the inverse of a polynomial:

$$\tilde{c}_i(\mathcal{E}_i) = \frac{2 \arctan \left(\frac{2 \frac{\sigma_x^{-2}}{\rho_i \beta^2} \mathcal{E}_i + \left(1 - 2 \frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right)}{\sqrt{4 \frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} - 1}} \right)}{\sqrt{4 \frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} - 1}} + K \quad (\text{IA.66})$$

The cost function is convex as long as the argument of the arctangent is negative, so:

$$\mathcal{E}_i \leq \alpha - \frac{\rho_i \beta^2}{2 \sigma_x^{-2}}. \quad (\text{IA.67})$$

We can see that if the right-hand-side of this condition is positive, the condition of $\Delta < 0$ is automatically satisfied.

After rescaling, $2\rho_i c_i(\rho_i \mathcal{E}_i) = \tilde{c}_i(\mathcal{E}_i)$, or equivalently $c_i(x) = \frac{1}{2\rho_i} \tilde{c}_i(x/\rho_i)$ we have:

$$c_i(x) = \frac{1}{\rho_i} \frac{1}{\sqrt{2\alpha\tilde{\beta} - 1}} \arctan \left(\frac{\tilde{\beta} \frac{x}{\rho_i} + (1 - \alpha\tilde{\beta})}{\sqrt{2\alpha\tilde{\beta} - 1}} \right) + \tilde{K} \quad (\text{IA.68})$$

with $\tilde{\beta} = 2\sigma_x^{-2}/(\rho_i \beta^2)$, and the condition $\alpha - \rho_i \beta^2/(2\sigma_x^{-2}) \geq 0$ becomes $\alpha\tilde{\beta} \geq 1$. We can collect these results in a proposition.

Proposition 2. *For any $a > 0$ and $b > 0$ so that $ab > 1$, assume the information cost follows the function:*

$$\begin{aligned} c_i(x) &= 0, \text{ if } x < 0, \\ c_i(x) &= \frac{1}{\rho_i} \frac{1}{\sqrt{2ab - 1}} \arctan \left(\frac{b \frac{x}{\rho_i} + (1 - ab)}{\sqrt{2ab - 1}} \right) + K, \text{ if } 0 \leq x/\rho_i \leq a - b^{-1} \\ c_i(x) &= +\infty, \text{ if } x/\rho_i \geq a - b^{-1}, \end{aligned} \quad (\text{IA.69})$$

where K is such that $c_i(0) = 0$. This cost function is increasing and convex. Then the optimal elasticity is:

$$\mathcal{E}_i = \mathcal{E}_{0,i} - \chi \mathcal{E}_{agg}, \quad (\text{IA.70})$$

with $\mathcal{E}_{0,i} = a$ and $\chi = \sqrt{(2\sigma_x^{-2})/(\rho_i b)}$.

B.4 Demand Elasticity

We recall the demand schedule for agent i :

$$\begin{aligned}
q_i &= \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\mu}_i - p) \\
&= \frac{1}{\rho_i} \hat{\sigma}_i^{-2} (\hat{\sigma}_i^2 [\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + C^{-2} \sigma_x^{-2} s(p)] - p) \\
&= \frac{1}{\rho_i} (\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + C^{-2} \sigma_x^{-2} (p - A) - \hat{\sigma}_i^{-2} p) \\
&= \frac{1}{\rho_i} (\sigma_i^{-2} \mu_i + \sigma_{i\eta}^{-2} \eta_i + (C^{-2} \sigma_x^{-2} - \hat{\sigma}_i^{-2}) p - C^{-2} \sigma_x^{-2} A). \tag{IA.71}
\end{aligned}$$

We can read the demand elasticity as:

$$\mathcal{E}_i = -\frac{dq_i}{dp} = -\frac{1}{\rho_i} (C^{-2} \sigma_x^{-2} - \hat{\sigma}_i^{-2}) = \frac{1}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \tag{IA.72}$$

In the model the regression of q_i on p would not give us the proper elasticity. There is a bias in the regression because p is correlated with μ_i and η_i . It is still possible to recover the elasticity using an instrument; for example the supply shock x covaries with p but is uncorrelated with μ_i and η_i .

We can express the equilibrium in terms of demand elasticities. We define the aggregate demand elasticity as:

$$\mathcal{E} = \int \mathcal{E}_j dj = -C^{-1}. \tag{IA.73}$$

We can rewrite the choice of information as a choice elasticity:

$$\mathcal{E}_k = \underbrace{\frac{1}{2\rho_i^2 c_{1,i}}}_{\text{Investor characteristics}} - \frac{\sigma_x^{-2}}{\rho_i} \underbrace{\mathcal{E}^2}_{\text{market elasticity}} \tag{IA.74}$$

B.5 Flexibility in Information Acquisition

We turn to the study of flexibility in the acquisition of information. First we look at how $\sigma_{i\eta}$ changes as we vary the aggregate elasticity C^{-2} . We take (IA.59) and using the implicit function theorem:

$$\begin{aligned}
\frac{2}{\rho_i} (\sigma_i^{-2} + \sigma_{i\eta}^{-2} + C^{-2} \sigma_x^{-2}) - \frac{1}{c'_i(\sigma_{i\eta}^{-2})} &= 0 \\
\chi = -\frac{d\sigma_{i\eta}^{-2}}{dC^{-2}} &= \frac{2/\rho_i \sigma_x^{-2}}{2/\rho_i + c''_i/c'_i} = \frac{\sigma_x^{-2}}{1 + \frac{\rho_i}{2} \frac{c''_i}{c'^2_i}} \tag{IA.75}
\end{aligned}$$

The response depends on the curvature of the information acquisition cost function. If the curvature is zero (as is the case in our linear cost example), then the response is highest.

Whereas a large curvature will elicit a weaker response.

B.6 Price Informativeness

We define price informativeness for investor i as the ratio of the precision of their belief about the fundamental when they condition on their private information and on the price and the precision of their belief using private information only:

$$\begin{aligned}\mathcal{I}_i &= \frac{\text{Var}(f|\mu_i, \eta_i, p)^{-1}}{\text{Var}(f|\mu_i, \eta_i)^{-1}} = \frac{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2}}{\sigma_i^{-2} + \sigma_{i\eta}^{-2}} \\ &= 1 + \mathcal{E}_{agg} \frac{\mathcal{E}_{agg}}{\rho_i \mathcal{E}_i} \sigma_x^{-2}\end{aligned}\tag{IA.76}$$

We also define the absolute price informativeness of the price as

$$\mathcal{I}_{abs} = \text{Var}(f|p)^{-1} = \mathcal{E}_{agg}^2 \sigma_x^{-2}.\tag{IA.77}$$

C Other foundations for competition χ

C.1 Learning from Prices

We consider a model in which agents can learn from prices which highlights a distinct mechanism from that of the previous section. Two main assumptions differ: agents cannot acquire information, and there is residual uncertainty about the asset payoff that cannot be learned. This setting leads to a new determinant of demand elasticity, beyond risk aversion and prior information. When many traders are aggressive, prices are more informative. How should one react? On the one hand, the extra information implies that price variation are less indicative of future return, and that pushes the investor to trade less aggressively. On the other hand, the extra information implies that returns appear less risky, and that pushes the investor to trade more aggressively. Increased price informativeness reveals relatively more about the fundamental than the payoff risk, exactly because of the presence of residual uncertainty.⁴³ Therefore the first effect dominates: the investor responds by being less aggressive, $\chi > 0$. This response is stronger when residual uncertainty is higher.

C.1.1 Setup

The asset trades at endogenous price p and pays off $f + \epsilon$, with $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. There is a continuum of mass 1 of agents indexed by i . Each agent has CARA preferences with risk aversion ρ_i . Each agent has a flat prior on f and receives an independent signal μ_i such that $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$. The asset is in noisy supply $\bar{x} + x$ with \bar{x} a constant and $x \sim \mathcal{N}(0, \sigma_x^2)$.

We look for a rational expectations equilibrium, with:

$$p = A + Bf + Cx \quad (\text{IA.78})$$

C.1.2 Equilibrium

Learning from the price. After observing the price, agent i 's posterior belief about the fundamental f is $\mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$, with:

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \frac{B^2}{C^2} \sigma_x^{-2} \quad (\text{IA.79})$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2} \mu_i + \frac{B^2}{C^2} \sigma_x^{-2} s(p) \right) \quad (\text{IA.80})$$

where the signal from the price is:

$$s(p) = \frac{p - A}{B} = f + \frac{C}{B} x \quad (\text{IA.81})$$

Taking the average over agents of type i (we will use a law of large numbers in the

⁴³In the model of the previous section, the two effects exactly cancelled out. The response was coming from changes in information acquired, which is shut down here.

population), we have

$$E_i [\hat{\mu}_i] = \hat{\sigma}_i^2 \left(\sigma_i^{-2} f + \frac{B^2}{C^2} \sigma_x^{-2} \left[f + \frac{C}{B} x \right] \right) \quad (\text{IA.82})$$

$$= f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x \quad (\text{IA.83})$$

Asset Demand. Asset demand q_i is given by the standard optimum portfolio choice:

$$q_i = \frac{1}{\rho_i} \frac{E[f + \epsilon | \mu_i, p] - p}{\text{var}[f + \epsilon | \mu_i, p]} \quad (\text{IA.84})$$

$$= \frac{1}{\rho_i} \frac{\hat{\mu}_i - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \quad (\text{IA.85})$$

Market Clearing. The total demand for the asset must equal its supply:

$$\int q_i di = \bar{x} + x, \quad (\text{IA.86})$$

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x - A - Bf - Cx \right] = \bar{x} + x. \quad (\text{IA.87})$$

This gives:

$$B = 1, \quad (\text{terms in } f) \quad (\text{IA.88})$$

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} - C \right] di = 1. \quad (\text{terms in } x) \quad (\text{IA.89})$$

Plugging in the definition of $\hat{\sigma}_i^2$, we obtain

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{1}{C^2} \frac{\hat{\sigma}_i^2}{\sigma_x^2} - 1 \right] di = C^{-1}, \quad (\text{IA.90})$$

$$\int \frac{1}{\rho_i} \frac{1}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{1}{C^2} \sigma_x^{-2} \frac{1}{\sigma_i^{-2} + \frac{1}{C^2} \sigma_x^{-2}} - 1 \right] di = C^{-1}, \quad (\text{IA.91})$$

$$\int \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left[\frac{1}{C^2} \sigma_x^{-2} - \sigma_i^{-2} - \frac{1}{C^2} \sigma_x^{-2} \right] di = C^{-1}. \quad (\text{IA.92})$$

Therefore we have:

$$C^{-1} = - \int \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \frac{1}{\sigma_i^2} di, \quad (\text{IA.93})$$

$$-C^{-1} = \int \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \frac{1}{C^2} \sigma_x^{-2} \right)} \frac{1}{\sigma_i^2} di. \quad (\text{IA.94})$$

Define $\tilde{C} = -C$, which is positive. We can rewrite:

$$\tilde{C}^{-1} = \int \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \frac{1}{\tilde{C}^2} \sigma_x^{-2} \right)} \frac{1}{\sigma_i^2} di. \quad (\text{IA.95})$$

The left-hand-side of this equation is decreasing in \tilde{C} . The right-hand-side is increasing in \tilde{C} . If $\tilde{C} \rightarrow 0$, the left-hand-side goes to ∞ and the right-hand-side goes to 0. If $\tilde{C} \rightarrow \infty$, the left-hand-side goes to 0 and the right-hand-side has a finite positive limit. Therefore, there is a unique solution to the equation, and a unique linear equilibrium.

C.1.3 Equilibrium Elasticities

We now derive demand elasticities. We show how individual demand elasticities respond to the aggregate elasticity. Demand is given by:

$$q_i = \frac{1}{\rho_i} \frac{\hat{\mu}_i - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \quad (\text{IA.96})$$

$$= \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2 \left(\sigma_i^{-2} \mu_i + \frac{B^2}{\tilde{C}^2} \sigma_x^{-2} s(p) \right) - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2}. \quad (\text{IA.97})$$

Therefore the slope of the demand curve is:

$$\mathcal{E}_i = -\frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left(\frac{1}{\tilde{C}^2} \sigma_x^{-2} - \hat{\sigma}_i^{-2} \right) \quad (\text{IA.98})$$

$$= -\frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left(\frac{1}{\tilde{C}^2} \sigma_x^{-2} - \sigma_i^{-2} - \frac{1}{\tilde{C}^2} \sigma_x^{-2} \right) \quad (\text{IA.99})$$

$$= \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \frac{1}{\sigma_i^2}. \quad (\text{IA.100})$$

Here, we observe clearly the intuition for the role of price informativeness. When prices are more informative, low $\hat{\sigma}_i^2$, expected returns respond less to the price, the numerator of the first fraction. However, the perceived risk of the asset also decreases, the denominator of the first fraction. Because of residual uncertainty σ_ϵ^2 , the effect on the asset risk is weaker than the effect on expected return: the ratio decreases and the trader becomes less aggressive.

More aggregate elasticity leads to more informative prices, so this mechanism will lead to a negative response of individual elasticity to aggregate elasticity. Formally, note that $\int_i \mathcal{E}_i = \mathcal{E}_{agg} = \tilde{C}^{-1}$. Plugging in, we obtain:

$$\mathcal{E}_i = \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2} \right)} \frac{1}{\sigma_i^2} \quad (\text{IA.101})$$

$$= \frac{1}{\rho_i} \frac{1}{\sigma_i^2 + \sigma_\epsilon^2 + \sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}^2} \quad (\text{IA.102})$$

Clearly, the individual elasticity \mathcal{E}_i is decreasing in the aggregate elasticity \mathcal{E}_{agg} . Lin-

earizing this expression, we obtain the counterpart of competition $\chi > 0$:

$$\chi = -\frac{d\mathcal{E}_i}{d\mathcal{E}_{agg}} \quad (\text{IA.103})$$

$$= \frac{1}{\rho_i} \frac{2\sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}}{(\sigma_i^2 + \sigma_\epsilon^2 + \sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}^2)}^2. \quad (\text{IA.104})$$

C.2 Price Impact

We now consider a model in which investors have non-negligible price impact and take it into account when making trading decisions in the style of Kyle (1989). This leads to a foundation for a negative competition parameter χ . Intuitively, when other traders are aggressive, I face a very elastic residual supply curve when sending orders to the market. This implies that my trades will not have a large price impact, hence I can also trade aggressively.

C.2.1 Setup

There are I investors indexed by i . Each agent has CARA preferences with risk aversion ρ_i :

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \quad (\text{IA.105})$$

and initial wealth W_i . The gross risk-free rate is 1, and the random asset payoff f is distributed $\mathcal{N}(\mu, \sigma^2)$. The asset is in noisy supply $\bar{x} + x$ with \bar{x} an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

As in Kyle (1989) we are interested in rational expectation equilibria with imperfect competition. We look for a linear pricing rule $p = A + Cx$. We solve for the individual demand strategies and look for linear strategies of the form:

$$d_i = \underline{d}_i - \mathcal{E}_i p \quad (\text{IA.106})$$

C.2.2 Solving for optimal demand strategies

Investor i maximizes their profit taking as given the residual demand from other investors' demand schedule. We use market clearing to find the residual supply curve:

$$\begin{aligned} \sum_i d_i &= \bar{x} + x \\ d_i &= \bar{x} + x - \sum_{k \neq i} \underline{d}_k + \left(\sum_{k \neq i} \mathcal{E}_k \right) p \\ p(d_i) &= \underbrace{\left(\sum_{k \neq i} \mathcal{E}_k \right)^{-1}}_{\lambda_{-i}} d_i + \underbrace{\left(\sum_{k \neq i} \mathcal{E}_k \right)^{-1} \cdot \left(\sum_{k \neq i} \underline{d}_k - \bar{x} - x \right)}_{p_{-i}}. \end{aligned} \quad (\text{IA.107})$$

To find the optimal demand of investor i for the asset, we write their program⁴⁴

$$\begin{aligned} \max_d \mathbf{E}\{f - p(d)|p_{-i}\}d - \frac{\rho_i}{2} \text{Var}\{f - p(d)|p_{-i}\}d^2 \\ \max_d (\mu - p_{-i})d - \lambda_{-i}d^2 - \frac{\rho_i}{2}d^2\sigma^2. \end{aligned} \quad (\text{IA.108})$$

The first order condition gives us:

$$d_i = \frac{\mu - p}{\rho_i\sigma^2 + \lambda_{-i}}. \quad (\text{IA.109})$$

We can already see that stronger λ_{-i} leads to less aggressive trading because of a larger price impact. Remember that λ_{-i} is the aggregate of demand elasticities of other investors, a quantity closely related to aggregate elasticity. We now close the equilibrium to show this relation more clearly.

C.2.3 Solving for aggregate demand elasticity

Given our original demand $d_i = \underline{d}_i - \mathcal{E}_i p$, we are able to identify the linear terms as:

$$\underline{d}_i = \frac{\mu}{\rho_i\sigma^2 + \lambda_{-i}}; \quad \mathcal{E}_i = \frac{1}{\rho_i\sigma^2 + \lambda_{-i}} = \frac{1}{\rho_i\sigma^2 + (\mathcal{E}_{agg} - \mathcal{E}_i)^{-1}}, \quad (\text{IA.110})$$

where we define the aggregate elasticity:

$$\mathcal{E}_{agg} = \sum_i \mathcal{E}_i. \quad (\text{IA.111})$$

Next we show that there is a unique solution for the aggregate elasticity. From the expression in equation (IA.110), we remark that \mathcal{E}_i solves a quadratic. We rule out the larger of the two roots and the solution is⁴⁵

$$\mathcal{E}_i = \frac{1}{2} \left(\frac{2}{\rho_i\sigma^2} + \mathcal{E}_{agg} - \sqrt{\left(\frac{2}{\rho_i\sigma^2}\right)^2 + \mathcal{E}_{agg}^2} \right) \quad (\text{IA.112})$$

To show that there is a unique stable equilibrium we consider the fixed point problem $F(x) = x$, with F defined by:

$$f_i(x) = \frac{1}{2} \left(\frac{2}{\rho_i\sigma^2} + x - \sqrt{\left(\frac{2}{\rho_i\sigma^2}\right)^2 + x^2} \right), \quad (\text{IA.113})$$

$$F(x) = \sum_i f_i(x). \quad (\text{IA.114})$$

⁴⁴Note that expectation and variances are conditional on the residual demand curve p_{-i} which is equivalent to conditioning on p

⁴⁵The larger root is such that $\mathcal{E}_i > \mathcal{E}_{agg}$ which violates $\sum_i \mathcal{E}_i = \mathcal{E}_{agg}$.

The function F is positive, increasing, and concave. Moreover $F(0) = 0$, $F'(0) = I/2$, and $\lim_{x \rightarrow +\infty} F'(x) = 0$, we conclude that there is a unique non-zero solution for \mathcal{E}_{agg} as long as $I \geq 3$.

The relation derived in (IA.112) between \mathcal{E}_i and \mathcal{E}_{agg} is not linear. We can approximate this equation linearly by $\mathcal{E}_i = \bar{\mathcal{E}}_i - \chi \mathcal{E}_{agg}$ with

$$\chi = -\frac{1}{2} \left(1 - \frac{\mathcal{E}_{agg}}{\sqrt{\mathcal{E}_{agg}^2 + \left(\frac{2}{\gamma_i \sigma^2}\right)^2}} \right) < 0 \quad (\text{IA.115})$$

This expression gives bounds on the value of χ : $-1/2 \leq \chi < 0$.

C.3 Imperfect Information

Assume your optimal elasticity is $\mathcal{E}_i = \bar{\mathcal{E}}_i - \chi E[\mathcal{E}_{agg} | \mathcal{F}_i]$. With perfect knowledge, you obtain: $\mathcal{E}_i = \bar{\mathcal{E}}_i - \chi \mathcal{E}_{agg}$. Assume the agent observes a signal $\hat{\mathcal{E}}_{agg} = \mathcal{E}_{agg} + \epsilon$ (with σ_ϵ^2) and has a prior $\mathcal{E}_{agg} \sim \mathcal{N}(\bar{\mathcal{E}}, \sigma^2)$. Then we have:

$$\mathbf{E}[\mathcal{E}_{agg} | \hat{\mathcal{E}}_{agg}] = \frac{1}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma^2}} \left(\frac{1}{\sigma_\epsilon^2} \hat{\mathcal{E}}_{agg} + \frac{1}{\sigma^2} \bar{\mathcal{E}} \right) \quad (\text{IA.116})$$

Therefore:

$$\mathcal{E}_i = \bar{\mathcal{E}}_i - \chi \frac{1}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma^2}} \frac{1}{\sigma^2} \bar{\mathcal{E}} - \chi \frac{\frac{1}{\sigma_\epsilon^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma^2}} \hat{\mathcal{E}}_{agg} \quad (\text{IA.117})$$

C.4 Partial Equilibrium Thinking

We repeat the calculation on the rise in passive investing in Section 2.3 in a situation with partial equilibrium thinking.

We assume that all investors are homogenous and their initial elasticity is $\mathcal{E}_i = \mathcal{E}_0$. What happens to the economy if a fraction $1 - \alpha$ of these investors become passive, their elasticity reduces to zero. To model partial equilibrium thinking, we assume that active investors only react to the effect of the switch to passive on aggregate elasticity and do not take into account the collective response.

Because investors are infinitesimal, this corresponds to forecasting a change in elasticity of $\Delta \mathcal{E}_{agg}^{\text{forecast}} = -(1 - \alpha) \mathcal{E}_0$. This implies that each active investor changes her elasticity by

$$\Delta \mathcal{E}_i = -\chi \Delta \mathcal{E}_{agg}^{\text{forecast}} = \chi(1 - \alpha) \mathcal{E}_0. \quad (\text{IA.118})$$

Aggregating across all investors, the new aggregate elasticity is:

$$\mathcal{E}_{NEW}^{PET} = \alpha \mathcal{E}_0 (1 + \chi(1 - \alpha)) = \alpha \mathcal{E}_0 + (1 - \alpha) \chi \alpha \mathcal{E}_0. \quad (\text{IA.119})$$

The new elasticity with partial equilibrium thinking is in contrast to our baseline model in Section 2.3, \mathcal{E}_{NEW} :

$$\mathcal{E}_{NEW}^{PET} - \mathcal{E}_{NEW} = (1 - \alpha)\mathcal{E}_0\alpha\chi\frac{\alpha\chi}{1 + \alpha\chi}. \quad (\text{IA.120})$$

The difference is positive when $\chi > 0$. Because investors do not account for the response of others, they overreact to the initial change in elasticity. This leads to a higher final level of aggregate elasticity and is therefore akin to more competition.

D Data Appendix

D.1 Institutional Holdings Data

TBD.

D.2 CRSP and Compustat

TBD

D.3 Identifying Passive Investing

Passive investors are insensitive to prices. Consequently, at each date we identify passive investors as investors with elasticity close to zero in a Kojien and Yogo (2019) type demand system:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} - \mathcal{E}_i^{fixed} p_k + \epsilon_{ik}, \quad (\text{IA.121})$$

where $X^{(d)}$ contains log book equity. An investor is defined as passive if their fixed elasticity is close to zero, i.e. $\mathcal{E}_i^{fixed} < \kappa$, for small κ . We choose $\kappa = 0.06$ to calibrate the level of passive investing, and define a stock's passive share as the ownership-weighted average of an indicator that is 1 if the investor is passive, and zero otherwise. That is,

$$|Active_k| = 1 - \sum_i \underbrace{\frac{w_{ik} A_i}{\exp(p_k)}}_{\text{ownership share}} 1_{\{\mathcal{E}_i^{fixed} < 0.06\}}. \quad (\text{IA.122})$$

Appendix Figure IA.8 shows the the cross-sectional median of $|Active_k|$ over time. We validate our measure in the setting of Russell index switching. Specification (4) of Appendix Table IA.4 shows that our measure of passive investing in the cross-section strongly responds to stocks switching between the Russell 1000 and 2000 indices, by about 4% of total ownership.

D.4 Additional Data Definitions

TBD

E Identification Strategy

E.1 Moment Conditions

We estimate the model using the method of moments. All of the moment conditions derive from the identifying assumption of equation (25). We list these moments here:

$$\mathbf{E} [\epsilon_{jk} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.123})$$

$$\mathbf{E} [\epsilon_{jk} X_k^{(d)} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.124})$$

$$\mathbf{E} [\epsilon_{jk} \hat{p}_{k,i} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.125})$$

$$\mathbf{E} [\epsilon_{jk} X_k^{(e)} \hat{p}_{k,j} \mathbf{1}_{\{j=i\}}] = 0, \forall i \quad (\text{IA.126})$$

$$\mathbf{E} [\epsilon_{jk} \hat{\mathcal{E}}_{agg,k}] = 0 \quad (\text{IA.127})$$

$$\mathbf{E} [\epsilon_{jk} \hat{\mathcal{E}}_{agg,k} \hat{p}_{k,j}] = 0 \quad (\text{IA.128})$$

There are exactly as many moment conditions as model parameters.

E.2 Solving the Reflection Problem

One challenge for identification is the reflection problem. How can we separate the individual component of demand elasticity from the competitive response to other investors. We show that the presence of variation in investor population across stocks allows to solve this problem. To isolate this argument from other identification concerns, we assume that we observe individual elasticities, \mathcal{E}_{ik} . For exposition purposes, we focus on a simplified version of the model in which $\underline{\mathcal{E}}_i$ does not depend on asset characteristics.

We provide sufficient conditions for the uniqueness of a decomposition of the individual elasticities into investor specific elasticities $\underline{\mathcal{E}}_i$ and the competitive response controlled by χ . After proving this result, we come back to the economic content and the empirical relevance of these conditions.

Before stating the theorem, we introduce a few notations. We define the undirected graph \mathcal{G} of investor-stock connections. The vertices (the nodes) are the investors i and the stocks k . There is an edge between i and k if and only if $i \in I_k$. There are no edges between two investors or two stocks.

Theorem 3. *A decomposition of demand elasticities $\{\mathcal{E}_{ik}\}_{i,k}$ into individual elasticities $\{\underline{\mathcal{E}}_i\}_i$ and the competition parameter χ is unique if:*

- (a) *The graph \mathcal{G} of investor-stock connections is connected.*
- (b) *Position-weighted averages of demand elasticities are not constant across stocks: there exists k and k' such that $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i \neq \sum_{i \in I_{k'}} w_{ik'}/p_{k'} A_i \underline{\mathcal{E}}_i$.*

Proof. Let us assume that there exist two distinct decompositions $(\{\underline{\mathcal{E}}_i^{(1)}\}_i, \chi^{(1)}) \neq (\{\underline{\mathcal{E}}_i^{(2)}\}_i, \chi^{(2)})$ and the two conditions (a) and (b) hold. Each decomposition for $l \in \{1, 2\}$

satisfies the two conditions of the elasticity layer

$$\mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{ik}, \quad \text{for all } k \in \mathcal{K} \quad (\text{IA.129})$$

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_i^{(l)} - \chi^{(l)} \mathcal{E}_{agg,k}, \quad \text{for all } k \in \mathcal{K} \text{ and } i \in I_k. \quad (\text{IA.130})$$

We subtract the decomposition of \mathcal{E}_{ik} for $l = 1$ from the decomposition for $l = 2$ and obtain:

$$(\chi^{(2)} - \chi^{(1)}) \mathcal{E}_{agg,k} = \underline{\mathcal{E}}_i^{(2)} - \underline{\mathcal{E}}_i^{(1)}, \quad \text{for all } k \in \mathcal{K} \text{ and } i \in I_k. \quad (\text{IA.131})$$

Here we see immediately that if $\chi^{(1)} = \chi^{(2)}$, then for all i , $\underline{\mathcal{E}}_i^{(1)} = \underline{\mathcal{E}}_i^{(2)}$, thus violating the initial assumption of distinct decompositions. Hence, we focus on the case of $\chi^{(1)} \neq \chi^{(2)}$.

We define the function:

$$f(x) = \begin{cases} (\chi^{(2)} - \chi^{(1)}) \mathcal{E}_{agg,x} & \text{for } x \in \mathcal{K} \\ \underline{\mathcal{E}}_x^{(2)} - \underline{\mathcal{E}}_x^{(1)} & \text{for } x \in I. \end{cases} \quad (\text{IA.132})$$

We restate the equality of equation (IA.131) as:

$$f(x) = f(x'), \quad \text{if and only if there is an edge between } x \text{ and } x' \text{ on } \mathcal{G}. \quad (\text{IA.133})$$

Therefore, since the graph \mathcal{G} is connected: $\forall x, x', f(x) = f(x')$, and f is a constant. We write the constant $f = a$, and plug in the constant in the aggregation of individual elasticities:

$$\mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{ik} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} - \chi^{(1)} \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{agg,k} \quad (\text{IA.134})$$

$$\iff (1 + \chi^{(1)}) \mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} \quad (\text{IA.135})$$

$$\iff (1 + \chi^{(1)}) \frac{a}{\chi^{(2)} - \chi^{(1)}} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} \quad \text{for all } k, \quad (\text{IA.136})$$

where we use $\mathcal{E}_{agg,k} = a / (\chi^{(2)} - \chi^{(1)})$. Equation (IA.136) violates assumption (b), which concludes the proof. ■

The intuition behind theorem 3 is that identification relies on comparing the behavior of one investor for two different stocks with different populations of investors. If this investor trades less aggressively when surrounded by more aggressive investors, we conclude that competition χ is positive. A challenge to implement this comparison is that we already need to know the elasticity of these other investors. This is a chicken-and-egg question. The ability to find a unique solution to this problem relies on being able to cycle through investors with enough variation in composition: this is the essence of conditions (a) and (b).

To better understand why these conditions are important, we show examples of how the model is not identified when either (a) or (b) is violated. Starting with (a), let us consider the case where each stock has its own non-overlapping population of investors. In this case, there is no identification. Because a given investor only invests in one stock, it

is not possible to tell if this investor is aggressive because of her own characteristics or in response to the other investors. As an example that violates condition (b), consider the case in which all investors have the same size and relative portfolio positions such that: $\forall k, k', w_{ik}A_i/p_k = w_{ik'}A_i/p_{k'}$. Investor composition is the same for all stocks and therefore there is no information in comparing different stocks. Relatedly, we could also consider a violation of (b) where all individual elasticities are identical across investors: $\underline{\mathcal{E}}_i = \underline{\mathcal{E}}$. Then, for all k we have $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i = \underline{\mathcal{E}}$: the aggregate elasticity for all stocks is identical. Intuitively, even though there is variation in investor composition across stocks, all investors behave the same way in terms of elasticity. This is equivalent to having a single investor, and we cannot separate individual elasticities from the response to other investors.

How can we assess these conditions empirically? The graph \mathcal{G} of investors-stocks connections can be observed directly in our data and we can assess immediately that condition (a) is satisfied using known algorithms such as depth-first-search. Condition (b) is potentially more challenging because it relies on parameter estimates $\underline{\mathcal{E}}_i$. However, the inspecting the condition shows it holds generically. Condition (b) stipulates the equality of K linear forms applied to the vector $(\underline{\mathcal{E}}_i)_i$. It is violated if and only if $(\underline{\mathcal{E}}_i)_i \in \bigcap_{k \geq 1} (w_k - w_1)^\perp$, a set of measure 0 for almost all combinations of w_k . In addition, there is still the possibility of verifying whether the condition is satisfied empirically, once the econometrician has found a set of parameter estimates.

E.3 Numerical Procedure

The main numerical challenge is that we need to solve for the equilibrium elasticity $\mathcal{E}_{agg,k}$ at the same time as we are estimating the model, and in particular the competition parameter χ and the parameter ξ . We describe a tractable approach to do so. We solve a series of nested problems.

Step 1. Given a guess for (χ, ξ) and $\{\mathcal{E}_{agg,k}\}_k$, we can estimate all remaining model parameters by two-stage least squares regression investor by investor. This corresponds to the run for each investor i the regression:

$$\log \frac{w_{ik}}{w_{i0}} - p_k - \chi \mathcal{E}_{agg,k} p_k - \xi \mathcal{E}_{agg,k} = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} \right) p_k + \epsilon_{ik}, \quad (\text{IA.137})$$

where p_k and $X_k^{(e)} p_k$ are instrumented by $\hat{p}_{k,i}$ and $X_k^{(e)} \hat{p}_{k,i}$. Running these regressions is equivalent to solving moment conditions (IA.123) to (IA.126).

Step 2. Given a guess for (χ, ξ) , we look for equilibrium values of $\{\mathcal{E}_{agg,k}\}_k$. We start from the aggregate elasticities implied by the model of (Kojen and Yogo, 2019). We run step 1 above. With the newly estimated $\underline{\mathcal{E}}_i$ and the parameter χ , we solve explicitly for the equilibrium elasticity they imply by solving the linear system of equations (16) and (20). We update our guessed aggregate elasticity by taking a weighted average of the previous iteration and these new implied values with weights of 75% and 25%. We repeat this updating process until the values of $\{\mathcal{E}_{agg,k}\}_k$ converge. This step ensures that our estimated model satisfies the 2-layer equilibrium.

Step 3. We estimate χ and ξ . We start from a guess for (χ, ξ) and run step 2 to find the aggregate elasticities it implies. With these values, we estimate the pooled regression of equation (21):

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} + \xi \mathcal{E}_{agg,k} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik}, \quad (\text{IA.138})$$

using two-stage least squares with all the instruments of the investor-level regression, $\hat{\mathcal{E}}_{agg,k}$ and $\hat{\mathcal{E}}_{agg,k} \hat{p}_{k,i}$. This is a very large scale regression with many fixed effects and investor-specific coefficients. We speed up the estimation of this large-scale regression tremendously by taking advantage of the Frisch-Waugh-Lovell theorem. We absorb all individual-level variables using investor-specific regressions, and are left with only the coefficients χ and ξ to estimate in the pooled data.

The pooled regression gives us a new guess for (χ, ξ) . We use a bivariate quasi-Newton method to find a fixed point for (χ, ξ) . With such a fixed point, we are sure that our estimates satisfy simultaneously all the moment conditions of Appendix Section E.1 and the 2-layer equilibrium.

F Additional Empirical Results

F.1 Trading big and small stocks

We investigate whether firms trade big and small stocks differently. Our estimates of elasticities by stocks suggest that the demand for large stocks is more inelastic (see Figure 3). To explain this result, one hypothesis is that large stocks mechanically tend to receive a high portfolio weight and that investors are unwilling to adjust their largest positions. For example, a 10% relative increase in portfolio weight would create much larger tracking error to the index for large positions than for small positions. Also, the granular nature of large stocks imply that they have fewer substitutes.

To complement our structural results and investigate this hypothesis, we compare the trading activity of investors across the distribution of their portfolio. For a given investor-quarter, we compute for each stock the squared relative change in the number of shares:

$$\text{Trading Activity}_{i,k,t} = \left[\left(\frac{A_{i,t}w_{ik,t}}{p_{k,t}} - \frac{A_{i,t-1}w_{ik,t-1}}{p_{k,t-1}} \right) / \frac{A_{i,t}w_{ik,t}}{p_{k,t}} \right]^2 \quad (\text{IA.139})$$

We sort positions by portfolio weights, and compute the ratio of the cumulative sum of trading activity to the total sum. This gives us a relation between the percentile of portfolio weight and the cumulative share of total trading activity. We average this relation within size groups of investors and present our results in Figure IA.1 for various dates.

If trading activity for all portfolio weights, this curve should coincide with the 45 degree line. Instead, we see the curve is always above the 45 degree line and particularly flat along the largest investor positions. This implies there is relatively less trading activity for the largest stocks. In addition, we observe that this pattern is more pronounced for the largest investors (panel D) than for small investors (panel A). Because larger investors are more important for the biggest stocks, this will amplify the lack of trading activity for the biggest stocks.

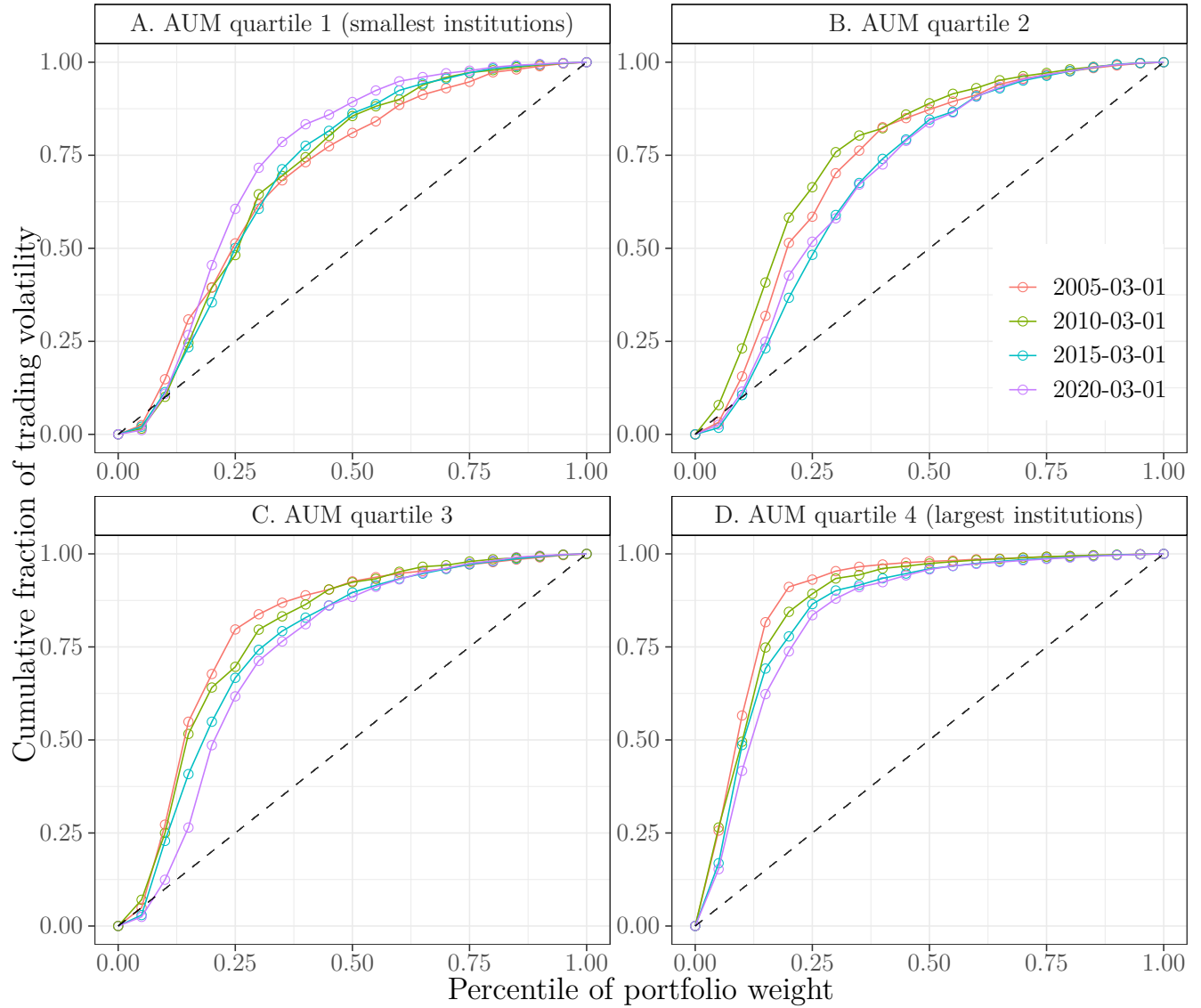


Figure IA.1. Trading activity across portfolio positions. Figure IA.1 presents the cumulative share of trading activity (defined in equation (IA.139)) by quantiles of investor portfolio weights. We aggregate the statistics by date and quartiles of assets under management.

G Appendix Tables

Table IA.1.

Estimates of competition χ under alternative specifications starting 2003Q3

	Estimates for χ		
	Median	25th pct.	75th pct.
(1) Baseline Specification	2.21	1.85	2.75
(2) BE-weighted Instrument for \mathcal{E}_{agg}	1.91	1.54	2.29
(3) Additional Controls	2.49	2.09	3.28
(4) AUM-weighted Regression	2.3	1.86	2.77
(5) Book-weighted Regression	2.24	1.77	2.68
(6) Investor-Type Grouping	2.46	1.96	2.93
(7) Constant χ	2.07		
(8) No Instrument for \mathcal{E}_{agg}	1.27	0.88	1.59
(9) No Instruments	1.01	0.73	1.41

Table IA.1 presents statistics of estimates of χ across dates (2003Q3 - 2020Q4) under various specifications. Our baseline specification (1) estimates χ given aggregate elasticities $\mathcal{E}_{agg,k}$ each period via the regression:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} + \xi \mathcal{E}_{agg,k} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \mathcal{E}_{agg,k} \right) p_k + \epsilon_{ik},$$

where $X_k^{(d)}$ contains log book equity and log book equity squared. $X_k^{(e)}$ is log book equity. Active investors with fewer than 1,000 stock holdings are grouped together based on their assets under management such that each group on average contains 2,000 stock holdings. The regression is weighted such that each group's weights sum to the same constant. Specification (2) shows estimates of χ based on the book-equity weighted instrument. Specification (3) adds additional characteristics to $X_k^{(d)}$; profitability, investment and dividends relative to book equity. Specification (4) value-weights the regression by weighting investors by their AUM. Specification (5) similarly value-weights the regression by weighting investors by their book assets. An investor's book assets are determined as follows: A stock's book equity is first allocated to investors proportional to the number of shares they hold, and subsequently summed up within investor across stocks to form an investor's books assets. Specification (6) follows Koijen and Yogo (2019) in grouping investors both by investor type and AUM. Institutional investors whose type we cannot determine are bundled together in a separate group. Specification (7) imposes for χ to be constant across time in the estimation, with each year receiving equal weight. Specification (8) reports results without instrumenting for the aggregate elasticity \mathcal{E}_{agg} . Specification (9) additionally removes the instrument for prices.

Table IA.2.

Summary Statistics of Aggregate Elasticity \mathcal{E}_{agg} with book-equity weighted instrument

Panel A: Statistics of average elasticity across stocks				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.504	0.428	0.482	0.569
Fixed elasticity	0.389	0.357	0.389	0.442
Panel B: Regression coefficient (by dates) of elasticity on size				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	-0.111	-0.12	-0.104	-0.091
Fixed elasticity	-0.0286	-0.0309	-0.0272	-0.0249
Panel C: Residual cross-sectional standard deviation of elasticity				
	Average	25th pct.	Median	75th pct.
Elasticity \mathcal{E}_{agg}	0.0548	0.0438	0.05	0.0591
Fixed elasticity	0.0842	0.0739	0.0826	0.0917

Table IA.2 presents statistics of the aggregate elasticity $\mathcal{E}_{agg}(k, t)$. We estimate the elasticity in our model accounting for competition χ and without competition as in Kojen and Yogo (2019) (denoted by fixed elasticity). Panel A has summary statistics of the average elasticity by date. Panel B shows summary statistics of the coefficient β_t from the regression $\mathcal{E}_{agg}(k, t) = \alpha_t + \beta_t size_{k,t} + \varepsilon_{k,t}$ by date. Panel C reports summary statistics of the cross-sectional standard deviation of the residual from the regression described in Panel B. The sample period is 2001 to 2020.

Table IA.3. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share using estimates from the model with a constant χ

	Log Change in Elasticity				
	(1)	(2)	(3)	(4)	(5)
Change in Active share	0.321*** (0.058)	0.462*** (0.028)	0.441*** (0.026)	0.412*** (0.025)	0.428*** (0.049)
Date Fixed Effects		Yes	Yes	Yes	Yes
Stock Fixed Effects			Yes		
Controls				Yes	Yes
Estimator	OLS	OLS	OLS	OLS	IV
N	50,292	50,292	49,661	50,292	10,619
R^2	0.017	0.269	0.282	0.318	0.627
First-stage F statistic					9.444
First-stage p value					0.000

Table IA.3 reports a panel regression of annual log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the annual log change in the active share $|Active_k|$. We use the estimates from the model with a constant value of χ over time. Column 2 adds date fixed effects. Column 3 adds stock fixed effects. Column 4 uses date fixed effects and controls for lagged book equity and annual log changes of log book equity. Column 5 instruments the log change in the active share $|Active_k|$ between Q1 and Q2 in any given year by two indicator variables corresponding to stocks switching between Russell 1000 and 2000 in either direction. It filters to stocks with CRSP market capitalization ranked between 500 to 1500 as of the end of Q1 to maintain comparability between stocks. The sample period is 2001-2020 for columns 1-4, and 2007-2020 for column 5. Standard errors are 2-way clustered by date and stock for columns 1-4, and clustered by date for column 5.

Table IA.4. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share around Russell index reconstitution

	Log Change in Elasticity			Log Change in Active share
	(1)	(2)	(3)	(4)
Log Change in Active share	0.389*** (0.066)	0.431*** (0.021)		
Log Change in Book Equity	-1.158*** (0.049)	-1.159*** (0.049)	-1.151*** (0.050)	0.018 (0.018)
Lagged Book Equity	0.005 (0.007)	0.005 (0.007)	0.007 (0.007)	0.003 (0.002)
Switch from Russell 2000 to 1000			0.017*** (0.004)	0.035*** (0.008)
Switch from Russell 1000 to 2000			-0.011 (0.006)	-0.043*** (0.012)
Date Fixed Effects	Yes	Yes	Yes	Yes
Estimator	IV	OLS	OLS	OLS
N	10,619	10,619	10,619	10,619
R^2	0.748	0.748	0.670	0.090
First-stage F statistic	9.444			
First-stage p value	0.000			

Table IA.4 reports a panel regression of log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the log change in the active share $|Active_k|$ around Russell index reconstitution, in particular between end of Q2 and Q1, from 2007 to 2020. The panel filters to stocks with CRSP market capitalization ranked between 500 to 1500 as of the end of Q1 to maintain comparability between stocks. The active share $|Active_k|$ is instrumented by two indicator variables corresponding to stocks switching between Russell 1000 and 2000 in either direction. We use the estimates for $\mathcal{E}_{agg,k}$ from the model with a variable value of χ over time. Column 2 and 3 show the corresponding OLS and reduced-form regressions, respectively. Column 4 is the first-stage regression. Standard errors are clustered by date.

H Appendix Figures

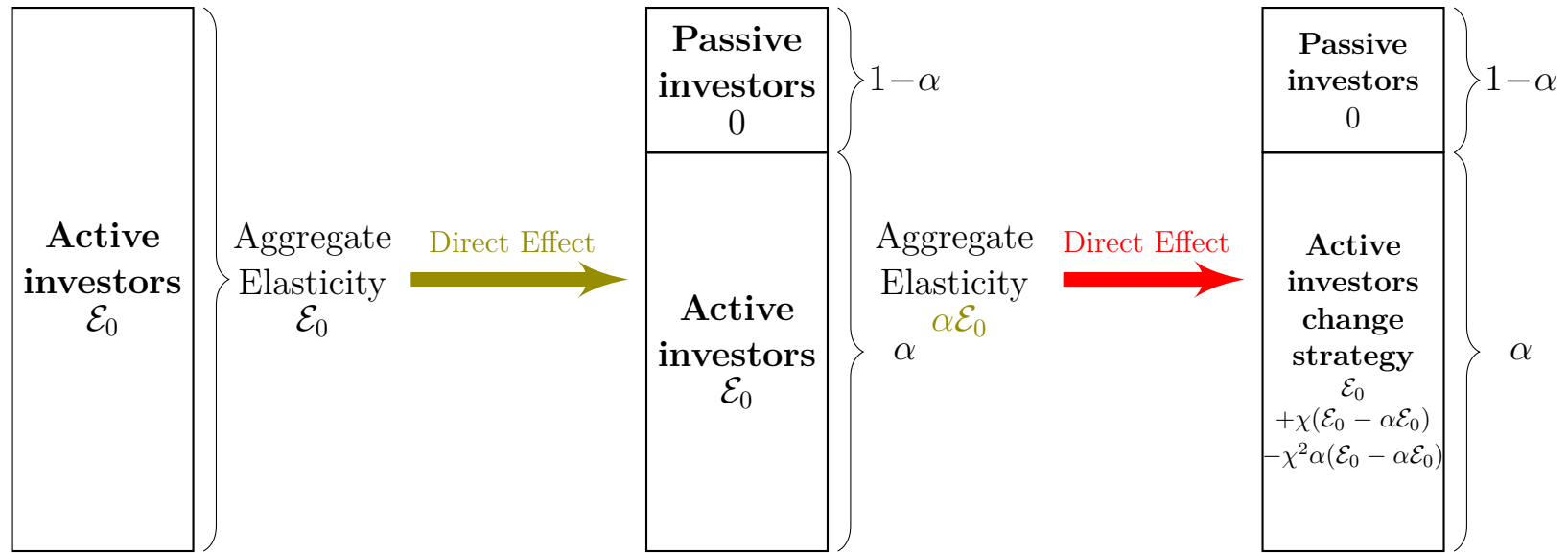


Figure IA.2. Effect of a Passive Change.

Figure IA.2 shows the time-series of the estimates for the competition parameter χ .

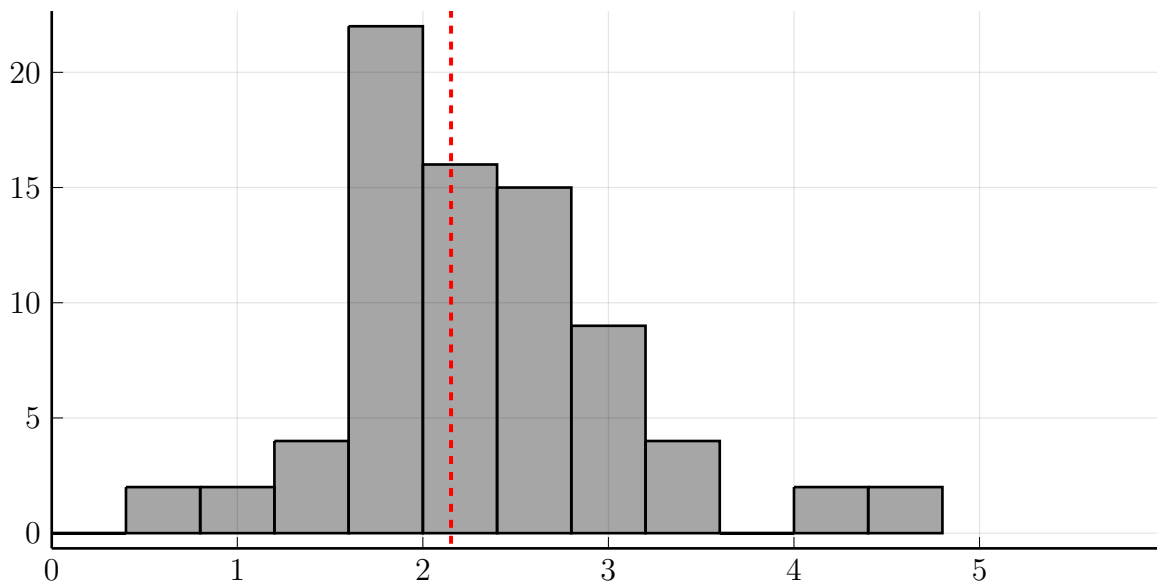


Figure IA.3. Distribution of the estimates of competition χ across dates. Figure IA.3 presents an histogram of our estimates of the competition parameter χ for each date between 2001 and 2020. The median estimate over the time-period is $\chi = 2.15$ (dashed redline).

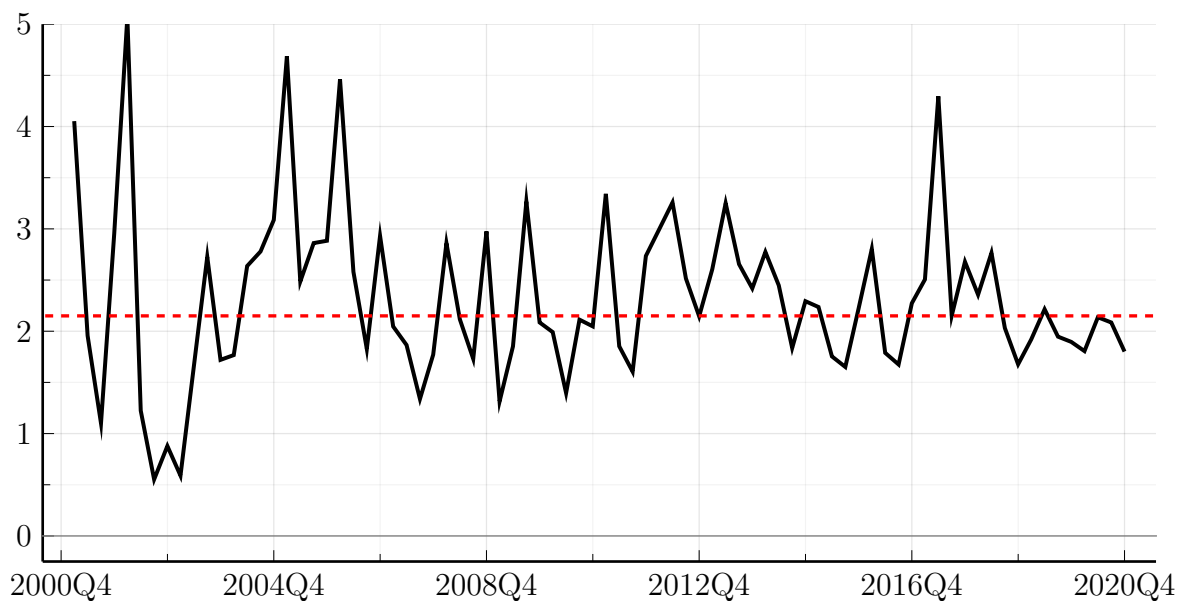


Figure IA.4. Time-series of competition χ . Figure IA.4 shows the time-series of the estimates for the competition parameter χ .

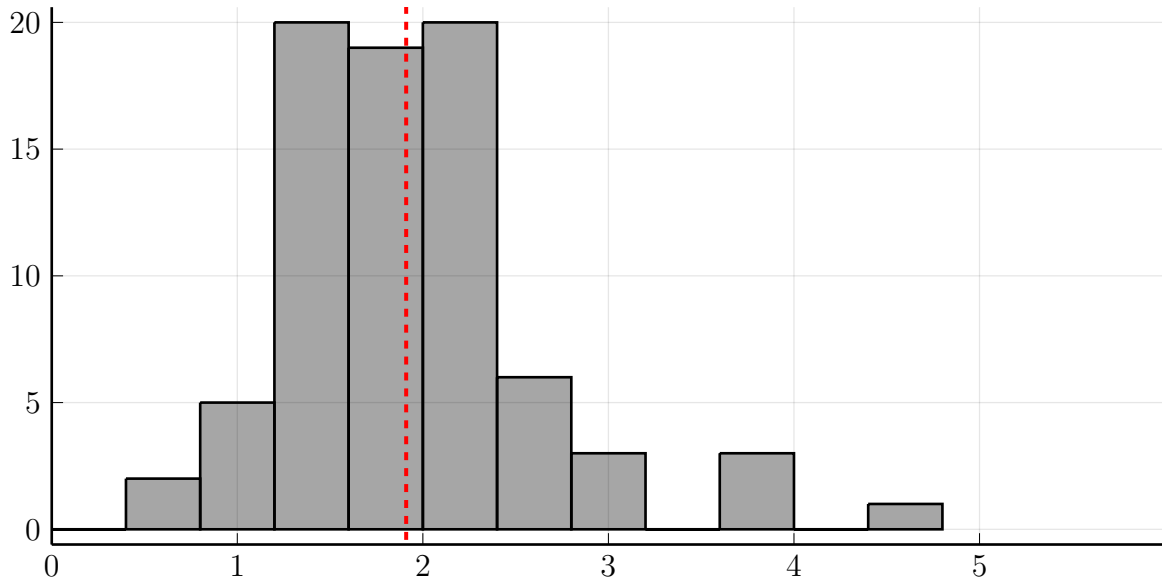


Figure IA.5. Estimates of χ using the book-equity weighted instrument.

Figure IA.5 presents an histogram of our estimates of the competition parameter χ where the instrument for aggregate elasticity weights portfolios by book equity, for each date between 2001 and 2020. The median estimate over the time-period is $\chi = 1.91$ (dashed redline).

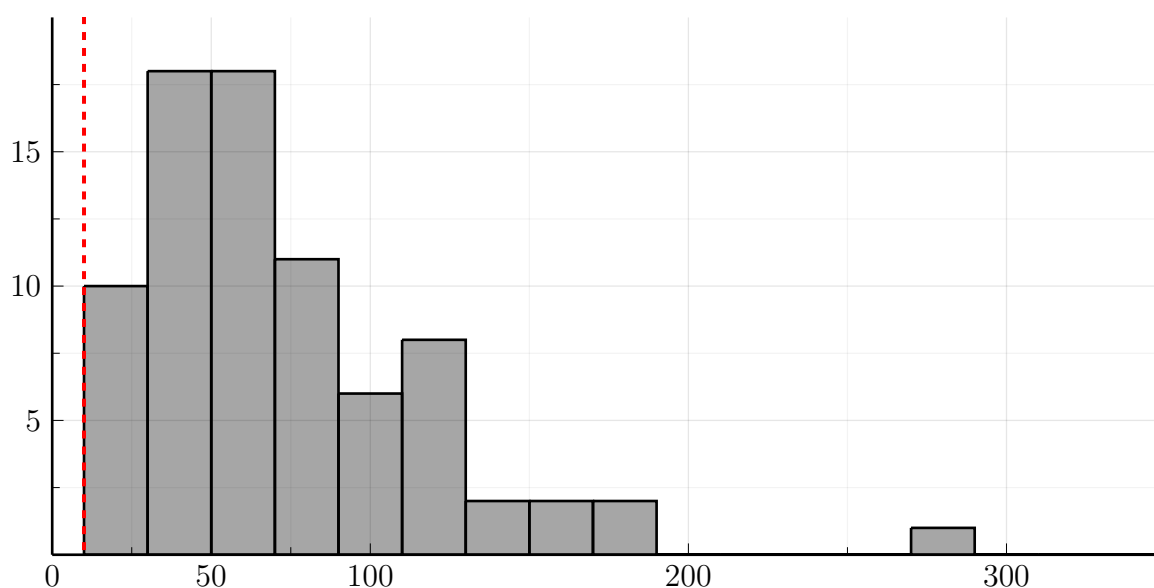


Figure IA.6. Relevance condition for the the book-equity weighted elasticity instrument.

Figure IA.6 shows the first-stage F-statistics (Kleibergen-Paap) for the instrument of price times the aggregate elasticity when the instrument of the aggregate elasticity is weighted by book equity.

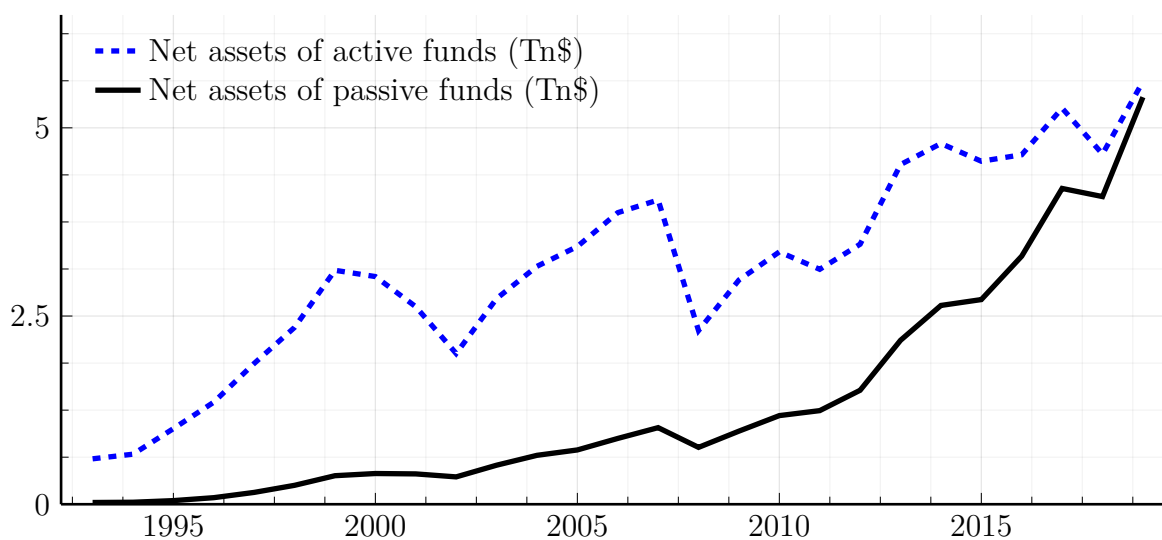


Figure IA.7. Net Assets of Passive and Active Funds.

Figure IA.7 shows the net assets of domestic mutual funds and ETFs in trillions of dollars (year-end) for passive funds (black solid line) and active funds (blue dashed line); Source ICI (2020).

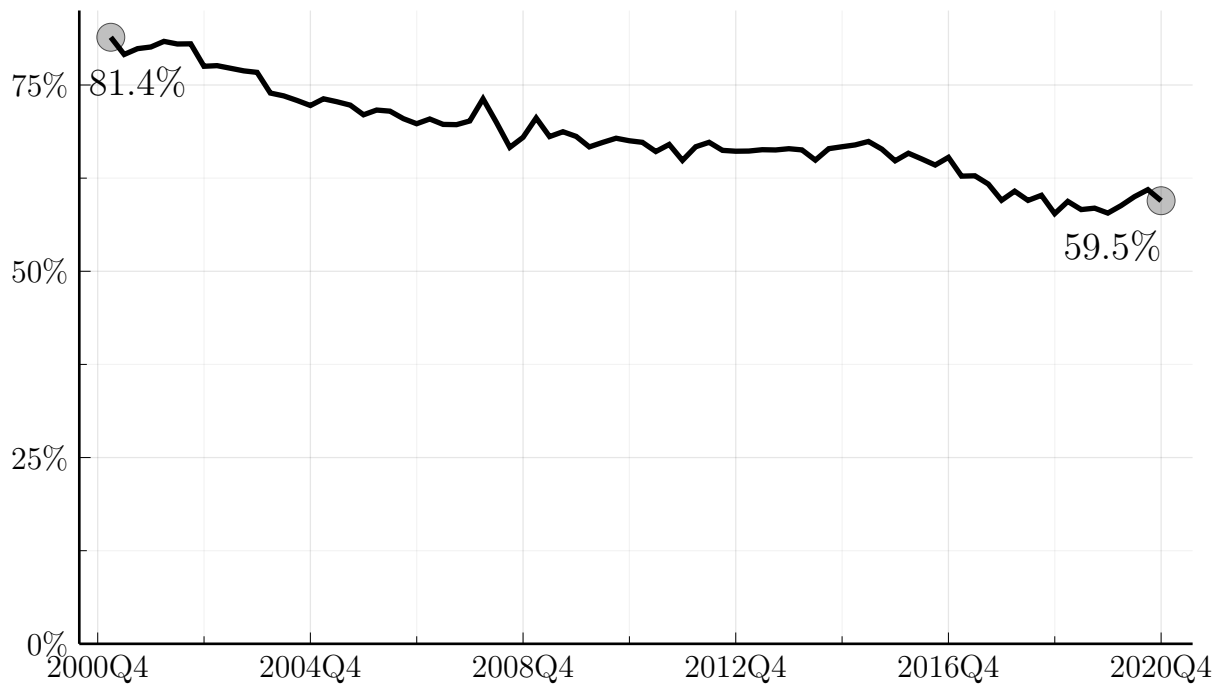


Figure IA.8. Fraction of Active Investors.

Figure IA.8 reports the fraction of active investors according to our model. For each stock, we compute the ratio of total position of active investors and the market capitalization. We report the average across stocks.

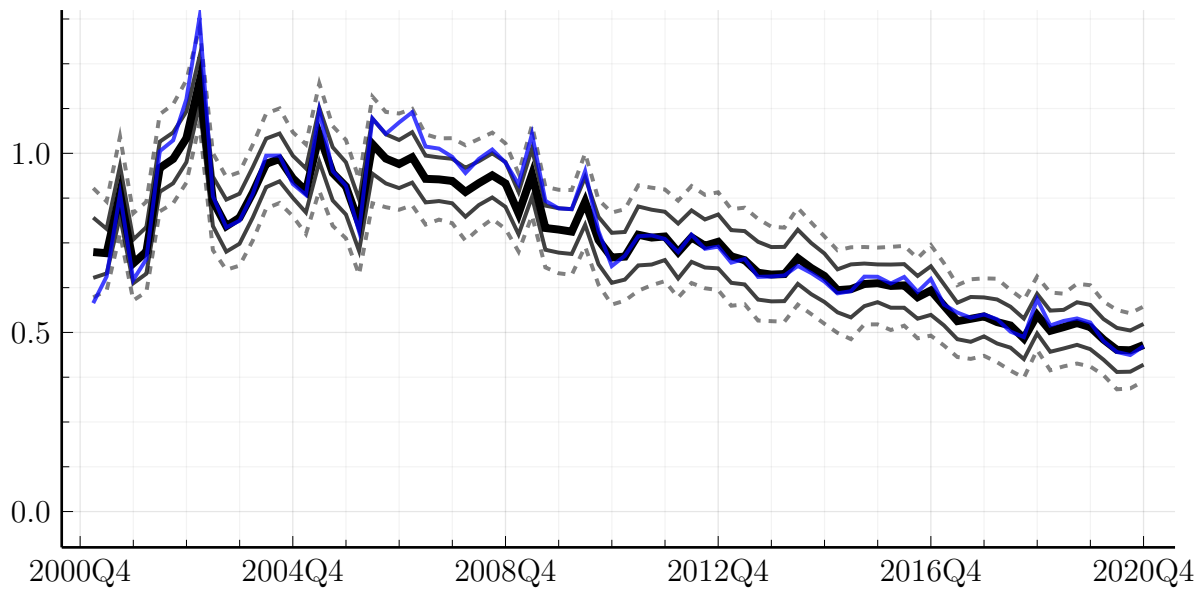


Figure IA.9. Distribution of individual-specific elasticities $\underline{\mathcal{E}}_{ik}$.

Figure IA.9 shows the quantiles of the distribution of individual elasticities $\underline{\mathcal{E}}_{ik}$ across investors for each stock and each date. We average the quantiles for each date to plot their time series. The black bold line is the average across investors. The two thin grey lines represent the 25th and 75th percentiles. The two dashed grey lines represent the 10th and 90th percentiles. And the solid blue line represents the average individual elasticities of the household investor.

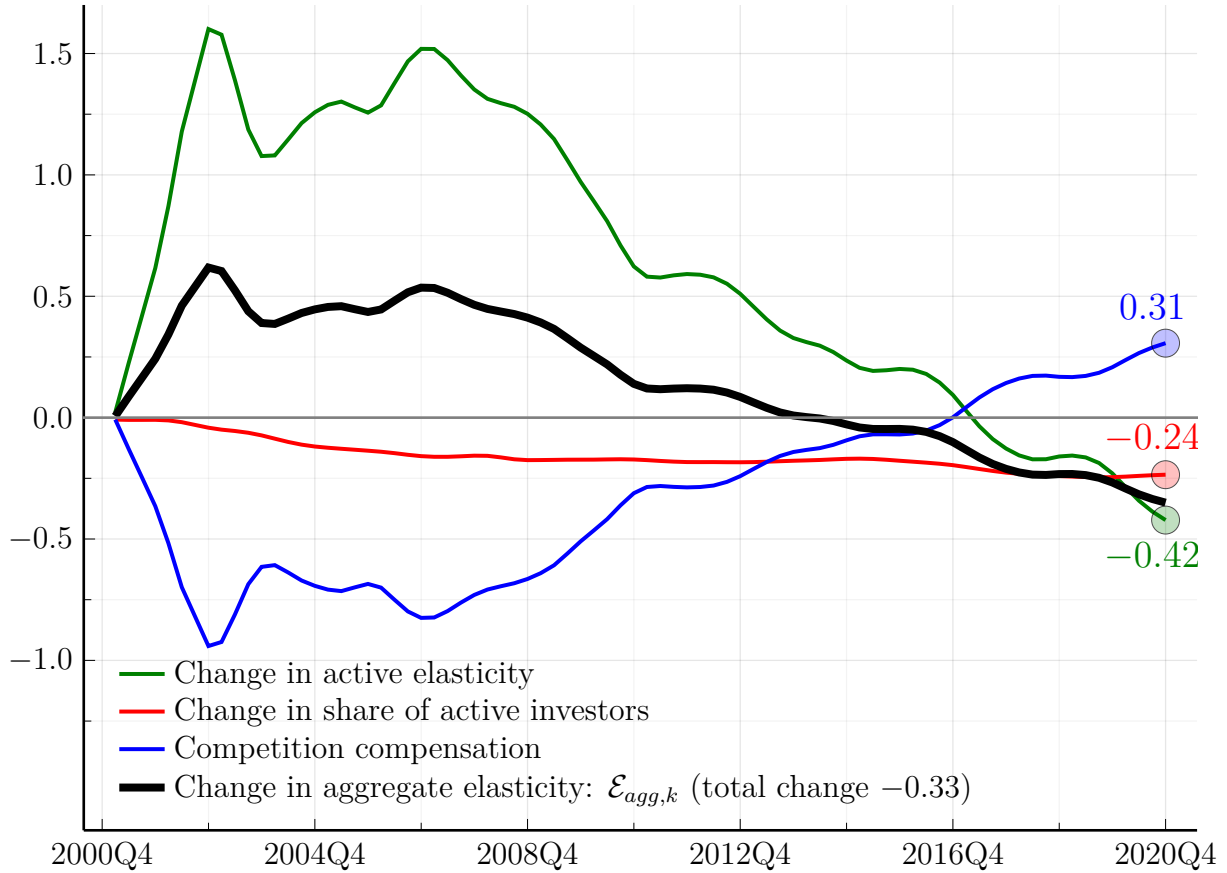


Figure IA.10. Decomposition of the change in aggregate elasticity for the the book-equity weighted elasticity instrument.

Figure IA.10 shows the decomposition derived in equation (30) over time, based on elasticities estimated from the book-equity weighted instrument for $\mathcal{E}_{agg,k}$. We compute each term of the decomposition for each date and accumulate the changes over time.