Can the cure kill the patient? Corporate credit interventions and debt overhang*

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Abstract

Interventions in corporate credit markets were a major innovation in the policy response to the 2020 recession. This paper develops and estimates a model to quantify their impact on borrowing and investment. Even during downturns, credit interventions can be a bad policy idea, because they exacerbate debt overhang and depress investment in the long run. However, if the downturn is accompanied by financial market disruptions, they initially help forestall inefficient liquidations. These short term benefits quantitatively dominate the long run overhang costs. Additionally, constraining shareholder distributions, and targeting high-leverage firms substantially increases the "bang for the buck" of credit interventions.

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1 Introduction

In early March 2020, as the impact of the pandemic came into focus, the Federal Reserve moved to provide aggressive support to the US economy, lowering the upper range of the Fed Funds rate to 0.25% and redeploying unconventional asset purchase programs from the Great Recession. But it didn't stop there. Later in March, as credit markets showed continued signs of strain, the Fed announced interventions of a new kind: direct purchases of debt of non-financial firms. By the end of March, after the passage of the CARES act, Treasury followed with its own program to provide funding to businesses.

The goal of this paper is to provide a quantitative framework to evaluate the potential effects of this new kind of intervention —– which we refer to as "business funding programs", or BFPs, for short —– on corporate borrowing, investment, and default decisions. To this end, we develop a structural model with heterogeneous firms that we estimate using data on US non-financial businesses. We then use the model to (i) study the effects of a crisis featuring a transitory, aggregate decline in cash flows and increase in risk prices; (ii) evaluate, in this context, the medium and long-run effects of BFPs similar to those used in 2020; (iii) compare these effects across alternative BFP designs.

The framework we propose emphasizes one main downside of BFPs: they can increase corporate leverage. Higher leverage in turn creates debt overhang, depressing investment and eventually slowing down the recovery, as firms find themselves operating with large amounts of legacy debt. The model allows us to weigh this downside against the main benefit of BFPs: they help reduce deadweight losses from excess liquidations caused by times of stress in financial markets.

The key take-aways from our analysis depend on the extent of financial markets stress during the downturn. If financial markets continue to function normally, BFPs have ambiguous effects on aggregate activity. First, BFPs that provide funding of any kind at the *same price* as private markets have no effects —– a type of "policy irrelevance" result. Second, BFPs that provide funding through *subsidized debt* purchases, without altering funding conditions in equity markets, always depress aggregate investment. The subsidy creates a motive for debt issuance beyond the tax shield, leading to excessive debt accumulation and, through the debt overhang channel, persistently low investment. Third, BFPs that trigger a reduction in *equity* risk prices stimulate aggregate investment, via a standard Tobin's q channel. Notably, this channel is operative even when actual BFP debt purchases are small.

Instead, when the downturn is accompanied by a "sudden stop" in financial markets, BFPs implemented via *subsidized debt* purchases improve aggregate outcomes, both in the short and in the long-run. While it is true that this type of BFP tends to depress investment during the recovery because of debt overhang, these negative long-run effects are *quanti-*

tatively small, compared with the short-run benefits of providing credit support. The main reason for the small size of the long-run debt overhang effects is that in our estimated model, the "typical" firm operates in a portion of the state-space where the sensitivity of investment to leverage is low, limiting the effects of the incremental debt associated with the BFPs.

We also use the model to compare, quantitatively, the effects of alternative program designs. Targeting debt purchases towards high-leverage firms improves their aggregate effects per unit of fiscal cost. By contrast, using grants rather than loans has a lower return per unit of fiscal cost, even when the grants come with "strings attached" —– such as dividend restrictions. This latter result also stems from the quantitatively low debt overhang effects of loan-based BFPs, whose fiscal costs are also substantially lower than grants.

While BFPs could create other distortions than debt overhang, isolating and quantifying debt overhang distortions is useful for at least two reasons.[1] First, a large literature in corporate finance and macroeconomics has argued that debt overhang can distort investment and destroy firm value.[2] This echoes one of the main criticisms of BFPs voiced in the financial press and by policymakers: they could create cohorts of "zombie firms", characterized by high debt burdens and low investment.[3] Second, in the particular context of 2020, corporate leverage coming into the crisis was elevated. The top panel of Figure 1 shows that the revenue share of relatively high-leverage firms (those debt-to-ebitda ratios above 3) was at a 15-year high when the recession started. This high initial leverage makes incremental BFP debt all the more likely to depress investment.

BFPs implemented in the US in 2020 are new and unconventional instruments in the fiscal and monetary policy toolkit. Section 2, which provides a brief overview of those BFPs, is organized around four of their main design features: type of funding; price subsidies; targeting; and constraints on participating firms. These features, which we study in our model, also help summarize BFPs implemented in other countries, either in 2020 or before that, making our results relevant for a broad set of government interventions.

Section 3 then describes our framework. It consists of an industry populated by firms that invest, borrow, issue equity, pay dividends to shareholders and make default decisions. An individual firm's problem has two components. First, on the real side, investment decisions follow a standard Q-theory model. The marginal product of capital is assumed to be exogenous and constant, but firms are subject to temporary "capital quality shocks", which are i.i.d. over time. Second, on the financing side, firms have access to both debt and equity markets. Debt is long-term, as in Leland et al. (1994), but it can be readjusted continuously

[1] Moral hazard or adverse selection may also lead to distorted firm investment incentives, and magnify the fiscal costs of the interventions. We do not consider these additional mechanisms in our framework, though we recognize their potential importance in assessing the costs and benefits of these programs.

[2] We discuss these papers, and how our work relates to them, in the literature review section below.

[3] See, among many others, Financial Times (2020) and Group of 30 (2020).

and at no cost, as in DeMarzo and He (2021). Firms choose to issue debt because it is tax-advantaged. However, debt is also defaultable, and default entails dead-weight losses. The optimal debt issuance policy trades-off these two forces. Firms in the model exhibit debt overhang, in the sense that the optimal investment policy function has a negative slope with respect to leverage. Our assumptions conveniently lead to leverage being a sufficient statistic summarizing a firm's state. Aggregate moments of the economy then depend on the cross-sectional distribution of leverage and its dynamic evolution. In the absence of aggregate shocks, the economy is on a balanced-growth path, in which capital, investment and output all grow at the same endogenously-determined rate.

In Section 4, we then use data on US public firms to carefully estimate the model.[4] We fit three particular moments that are crucial to our analysis: the aggregate investment rate; leverage; and the sensitivity of investment to leverage. In the data, this sensitivity is approximately -1: a one-unit increase in debt-to-ebitda is, on average, associated with a decline in investment of 1 percentage point (p.p.). Our model is able to replicate these moments well and closely matches a number of non-targeted financial moments. In the estimated model, the effect of debt overhang in steady-state is large: aggregate growth is approximately 0.6% per annum (p.a.), compared to 2.1% p.a. in a no-debt economy.

We then introduce an aggregate, temporary shock in the estimated model, featuring (i) a 25% decline in the marginal product of capital, and (ii) a 85 p.p. increase in Sharpe ratios. As Section 5 explains, the features of this crisis are chosen in order to replicate aggregate changes in ebitda and stock prices of US firms observed in the early stages of the 2020 recession. We then analyze BFPs in the context of this crisis, under two alternative scenarios: one where financial markets function normally; and a "sudden stop" scenario, where equity and debt markets shut down for the duration of the crisis.

In the first scenario, the aggregate effects of BFPs are ambiguous. Generally, these effects are driven by how BFPs change the *effective* cost of debt and equity capital for firms. We highlight these results in Section 6. BFPs that do not change firms' effective cost of capital have no effects. Firms in the model are able to dynamically adjust their financing policies, so they substitute BFP funding for private funding, while their investment and default choices are unchanged. So long as the funding is provided at the same price as private markets, and so long as market participants' risk free rates and risk-prices are not affected by the BFP, growth and investment are identical to laissez-faire.

On the other hand, a program of *subsidized debt* purchases depresses aggregate investment, so long as it leaves equity markets' financial conditions unchanged. With the subsidy, the marginal benefit of an extra unit of debt now includes an extra term: the wedge between

[4]Our data are from 2019, since part of our focus is on understanding the medium and long-run effects of the US 2020 BFPs. Our focus on publicly traded firms is primarily driven by data considerations: in the US, publicly-available, representative balance sheet data for private businesses is limited.

the cost of debt as perceived by shareholders, and the effective cost of debt induced by the BFP. When this wedge is positive, firms take on more debt. Since this higher leverage is persistent, it leads to a prolonged period of depressed investment in the recovery. Quantitatively, however, the effect is small: each additional 100 bps loan rate subsidy reduces expected aggregate capital 5 years after the beginning of the crisis by only 0.3%.

Finally, if financial markets continue to function normally during the downturn, a government intervention whose announcement triggers a decrease in Sharpe ratios prevalent in equity markets immediately stimulates the economy. In that case, the decrease in firms' effective cost of equity capital leads to an increase in marginal q, prompting firms to increase investment. The effects are large: each additional 10 p.p. decline in Sharpe ratios is associated with a 0.5% increase in expected aggregate capital 5-years after the beginning of the crisis. Crucially, these expansionary effects can be achieved even if debt purchases are small, so that the efficacy of BFPs should not be assessed purely by the quantity of funding extended to firms.

In the second scenario —– when the downturn is accompanied by a sudden stop in financial markets —– the aggregate response of default and investment in laissez-faire is much larger, as we show in Section 7. On impact, there is a wave of liquidations, driven by firms that would normally rely on financial markets to roll over debt or raise equity capital. Additionally, during the crisis, investment remains depressed, as firms can no longer finance it via debt or equity issuances, and instead have to rely on internally generated funds. In the recovery, investment resumes at an accelerated pace (relative to the no-crisis balanced-growth path) since surviving firms generally have lower leverage.

In this context, a BFP consisting of *subsidized debt* purchases involves a trade-off between reducing liquidations and stimulating corporate investment (during the crisis) vs. increasing leverage and reducing investment (during the recovery). Our main quantitative result is that the short term benefits of such intervention outweigh its long term costs. The BFP reduces by more than 90% the destruction of capital due to short-run defaults. On the other hand, while the additional leverage leads to investment rates that are indeed lower during the recovery, the difference relative to laissez-faire is small. On balance, 5 years after the shock, expected aggregate ebitda with the BFP is 6% below the no-crisis balanced growth path, compared to approximately 20% in the laissez-faire.

This quantitative finding begs the question of why, despite the fact that debt overhang has substantial effects on overall growth in the steady-state of our estimated model, BFPs do not make the problem substantially worse. There are two reasons for this: (i) the bulk of firms in the model operate in a region where the slope of the investment policy function with respect to leverage is small; (ii) the BFP has limited effects on overall leverage. The following back of the envelope calculation can clarifies this point. The BFP increases a firm's debt-to-ebitda by 1 (the average duration of the shock, one year) multiplied by 25% (the size of the decline in cash-flows, and hence of the earnings replacement provided in the main loan program we consider), or approximately 0.25. This is relative to a mean of approximately 2.1. Given the small slope of investment with respect to leverage in the region where most firms operate, this increase does not have large effects on future investment.

Two degrees of freedom in the design of subsidized-debt BFPs are particularly interesting: the size of loans provided to firms, and the degree of targeting. Both affect the efficacy of the BFP, measured as the improvement in expected aggregate outcomes relative to laissez-faire per unit of fiscal cost. The *marginal* gains associated with the subsidized-debt BFP decrease with the size of the BFP loans, and therefore with the fiscal cost of the intervention. Moreover, these gains can be substantially improved if the intervention is more targeted. Specifically, if the government provides loans at a fixed dollar price (potentially below par), firms whose debt value is above such level end up not participating in the program, while firms below such level maximize the amount of public funding they can raise. This creates a negative selection effect, which focuses the support on firms with the highest leverage, helping them avoid bankruptcy and the related deadweight losses. The cost is higher leverage during the recovery. But, as discussed above, this has modest aggregate effects, so that this more narrow targeting raises the returns of the intervention per unit of fiscal cost.

We also consider a wholly different intervention design: grants, instead of loans. This program is the analog, in our model, to the US Treasury's PPP. The model suggests that this design has very low efficacy, as measured by the modest improvement in expected future capital stock per unit of fiscal cost. Indeed, firms receiving grants end up mostly re-distributing them as dividends, rather than using them to increase corporate investment. With grants, restrictions on dividend distributions end up doubling the "bang for the buck" of taxpayer dollars, as firms that would otherwise be paying shareholders end up instead investing more. However, even with "strings attached", grants still have a substantially lower efficacy than loans: their fiscal cost is much larger, but their main benefit relative to loans — avoiding debt overhang — is quantitatively small.

Overall, our results generally caution against using BFPs as a tool for economic stabilization, except in times of extreme stress in financial markets. On the other hand, in such times, concerns regarding debt overhang and corporate "zombification" appear to be quantitatively dominated by the short-run benefits of BFPs, and, at any rate, can be mitigated by targeting support to firms that are closer to default.

Literature review and contribution Our paper relates to four strands of literature.

First, it builds on a theoretical literature that studies how debt overhang affects investment. Following the seminal insight of Myers (1977), this literature has developed dynamic models in which debt in place can affect firms' decisions to undertake new investment.[5] Our model more specifically extends the continuous-time framework of DeMarzo and He (2021). Due to a lack of commitment, firms' managers make strategic default decisions, as in Leland et al. (1994); but different from this article, our firms continuously re-adjust leverage, which is crucial to understanding the incentive effects of BFPs.[6] We combine this framework with a standard investment problem, analogous to Hayashi (1982). The resulting investment decisions are distorted downward by the presence of debt, leading to a debt overhang problem. Two papers adopting closely related models for debt overhang are Hennessy (2004) and Perla, Pflueger and Szkup (2020), though neither studies aggregation and the transmission of shocks that affect the distribution of corporate leverage.[7] Relatedly, our results also speak to the phenomenon of "zombie firms" — firms that are kept alive by their lenders because these lenders have limited incentives to recognize loan losses (Caballero, Hoshi and Kashyap, 2008). While we do not consider distorted lender incentives explicitly, some firms in our model behave as "zombies", in that they actively delay default when they are highly levered, in particular by reducing investment (a behavior made worse by the BFPs).

Second, our work speaks to an empirical literature that studies the effects of leverage on investment decisions.[8] In Section 4, we compare the steady-sate implications of our model to the findings of this literature. However, our focus is not on steady-state costs of debt overhang, but on its dynamic effects following a crisis and the use of BFPs. One of our key findings is that while *steady-state* debt overhang costs can be large, quantitatively, they matter less for the *response* of firms to BFPs.

Third, our work relates to a theoretical literature that studies the effects of corporate or sovereign debt overhang on macroeconomic activity (Krugman et al., 1988; Lamont, 1995; Philippon, 2010), and how policy can best address this overhang (Philippon and Schnabl, 2013). Relative to that literature, our goal in this paper is to provide a framework that can be used to quantify more precisely corporate debt overhang and its relation with monetary or fiscal policy interventions. Importantly, we allow for cross-sectional heterogeneity in leverage, which is generated by idiosyncratic capital quality shocks, as in Khorrami and

[5]See, among others, Mello and Parsons (1992); Mauer and Ott (2000); Moyen (2007); Manso (2008), and Diamond and He (2014).

[6]As in DeMarzo and He (2021), firms in our model take so much leverage that the tax benefits of debt end up fully dissipated by bankruptcy costs. By contrast, much of the existing work on debt overhang instead focuses on models in which long-term debt is fixed (e.g. Moyen 2007), or the investment decision is limited to the exercise of a growth option (e.g. Childs, Mauer and Ott 2005).

[7]Additionally, though that paper does not focus on debt overhang, our model shares common features with Hennessy and Whited (2007). The main differences are (a) our model has constant returns to scale in production, allowing for easier aggregation; (b) we assume frictionless equity markets and a simpler corporate tax structure; (c) we allow for long-term debt, potentially magnifying the effects of debt overhang; and (d) we study a continuous-time framework in which equilibrium computations are considerably simpler.

[8]See, among many others, Whited (1992), Opler and Titman (1994), Lang, Ofek and Stulz (1996), Matsa (2011), or Wittry (2020).

Tourre (2021). This heterogeneity is crucial to understanding why the interventions we consider have limited negative aggregate effects. Relatedly, recent empirical work by Jordà et al. (2020) questions whether corporate debt overhang substantially lowers investment during recoveries.

Finally, our paper adds to recent work on the credit market interventions by the Federal Reserve and the US Treasury. Brunnermeier and Krishnamurthy (2020) emphasize qualitatively the trade-off between deadweight losses of bankruptcy and debt overhang in a one-period model of the firm. Hanson et al. (2020) also discuss the potential effects of two of the Fed programs we discuss in this paper. They do not focus on debt overhang, but rather highlight the benefits of BFPs when there are externalities (aggregate demand or otherwise) associated with firm default. Our approach in this paper is both more quantitative, and more specifically focused on debt overhang, which we think is an important dimension of BFPs, particularly for larger, publicly-traded firms. Some of our conclusions also differ: for instance, we highlight the potential for lending programs to distort investment downward if the cost of credit is subsidized.[9]

2 An overview of business funding programs

In this section, we provide a brief overview of Business Funding Programs (BFPs) implemented in the US in 2020. We define BFPs as fiscal or monetary programs consisting of either outright purchases of securities issued by non-financial corporations, or grants. We organize the discussion around four key design dimensions: the form of the government participation; eligibility conditions; pricing; and constraints on participating firms. These key features, summarized in Table 1, will be part of our focus when analyzing BFPs in the model. Additionally, we distinguish between the effects of the announcement of these programs, from their actual implementation, a distinction we will also explore in our model.

While this section focuses on the US, BFPs were also implemented in several other advanced economies, both before and during 2020. Appendix A.1.2 discusses these other instances of BFPs in more detail.

Timing On March 23rd, 2020, the Fed announced its intention to directly intervene in corporate credit markets. The announcement marked an unprecedented step in its commitment

[9]Our work is also related to English and Liang (2020), who study the structure of the Main Street Lending Program and who argue for a better targeting of such program, longer loan terms, smaller minimum loan sizes, and stronger incentives for banks to participate in it. Our model speaks directly to the argument that the government credit programs should be aimed at high-leverage firms who are in need of funds in order to avoid liquidation during the crisis. Greenwood, Iverson and Thesmar (2020) also discuss the potential hurdles to efficient restructuring during the COVID crisis, though they do not focus on BFPs specifically.

to support the flow of credit to firms.[10] Two Fed credit programs were created, the Corporate Credit Facilities (CCF), and the Main Street Lending Program (MSLP). Treasury soon followed with its own program, the Paycheck Protection Program (PPP). The CCF and MSLP stopped new purchases on December 31st, 2020. The PPP stopped accepting its first wave of applications on August 8th, 2020; a second wave resumed on January 11th, 2021, following passage of the Consolidated Appropriations Act of 2021.

Design We highlight four key dimensions of the 2020 US BFPs.[11]

1. **Form of government participation** Both Fed programs (the CCF and the MSLP) involved the central bank buying and holding *debt* securities of non-financial firms, either directly, or indirectly, through ETFs. The Primary Market Corporate Credit Facility (PMCCF) focused on purchases of corporate bonds and syndications on the primary market, while the Secondary Market Corporate Credit Facility (SMCCF) focused on buying single-name corporate bonds or bond ETFs in secondary markets. The MSLP involved the Fed buying and holding loans originated by participating private banks. By contrast, while loans under the PPP were made by private banks and guaranteed by the SBA, they were, from the beginning, designed to be partially or totally forgiven, provided borrowers met certain criteria.[12] Thus the two Fed programs involved loans, while the PPP effectively involved grants.

2. **Eligibility conditions** Only investment-grade firms, or firms downgraded after March 22nd, 2020 were eligible for direct purchases under the CCF.[13] Besides this rating criterion, the CCF did not feature any other eligibility restrictions. The MSLP was subject to tighter restrictions, most importantly a size limit (participating firms were to have fewer than 15,000 employees, or less than $5bn revenue, in 2019), and a leverage constraint (firms could borrow up to no more than 4 to 6 times 2019 EBTIDA, depending on the MSLP facility, effectively excluding firms that were highly levered coming into the crisis). Loans purchased by the MSLP also had to be contractually senior to other forms of debt. Similarly to the MSLP, the PPP was subject to firm size limits (less than

[10] To our knowledge, the Federal Reserve had only used its 13(3) powers to directly lend to non-financial firms once before, in a small-scale program implemented by the New York Fed during the Great Depression (Rosa, 1947; Sastry, 2018). Some of the Great Recession programs that the Fed resumed in February and March of 2020 were meant to support indirectly credit supply, including the Commercial Paper Funding Facility, the Term Asset-Backed Securities Loan Facility, as well as bank supervisory actions meant to facilitate lending. In this paper, we focus on programs involving direct interventions in corporate credit markets.

[11] Appendix A.1.1 contains a detailed description of the structure of each individual program. On the CCF, see also Boyarchenko et al. (2020); on the MSLP, see also Crouzet and Gourio (2020); and on the PPP, see also Granja et al. (2020) and Hubbard, Strain et al. (2020).

[12] A criterion for forgiveness was the retention of employees. Information on the program is available here.

[13] High-yield firms were indirectly eligible, to the extent that bond ETFs purchased under the SMCCF had exposure to high-yield firms.

500 employees, a tangible net worth below $25m as of March 2020, and average net income below $5m in the two years prior to application).

3. **Pricing** The three programs also differ in their pricing of the government participations. All CCF purchases were to be done at market prices. By contrast, loans purchased under the MSLP and the PPP were effectively subsidized. Interest rates on MSLP loans were set at LIBOR plus 3%, while any interest rate on PPP loans prior to loan forgiveness were capped at 1%.

4. **Constraints on participating firms** Inclusion in CCF purchases did not require the issuer of the security to meet particular constraints. There was also no cap on CCF purchases from individual issuers.[14] By contrast, the MSLP put much tighter restrictions on participating borrowers. Borrowers were prevented from using proceeds from MSLP loans to refinance existing debt, and also had to follow limits on distributions to shareholders outlined in the CARES act. On top of the overall leverage constraint, loans were capped at $30m to $300m, depending on the facility. The PPP also involves constraints: borrowed funds can only be used to cover interest, payroll, rent, and utilities, and the loans are capped to 2.5 times average monthly payroll costs, up to a maximum amount of $10m.

Thus, the main similarities between the CCF and MSLP is that they both involved purchases of debt securities subject to eligibility conditions (though the extent of these conditions differed across programs). On the other hand, (a) MSLP pricing was subsidized, whereas CCF rates was not; (b) MSLP participation put constraints on borrowers, whereas inclusion in CCF purchases did not. Like the CCF and the MSLP, PPP eligibility is conditional, and like the MSLP, participating borrowers are subject to constraints on the use of funds. However, the PPP effectively provides grants to participating firms, with no obligation of repayment. Our analysis of the model will shed light on whether these design variations matter for the effects and effectiveness of these programs, and why.

Announcement and implementation effects Measured in terms of take-up, the PPP was a success: as of March 28th, 2021, a total $734.1bn of loans had been extended. Of this amount, $521.2bn of loans were issued during 2020; $194.5bn had been forgiven as of March 21st, 2021, with the remainder either under review or awaiting application for forgiveness. By contrast, while the CCF and the MSLP were set up to similar scales (the CCF could purchase up to $750bn of securities, and MSLP up to $600bn of loans), effective purchases

[14] The CCF facilities could not hold more than 25% of an individual issuance (for the PMCCF) or 10% of total issuances (for the SMCCF) of a particular firm. Additionally, the PMCCF allowed new issuances to be used to refinance existing debt.

were much smaller. The bottom panel of Figure 1 reports cumulative purchases under the CCF (only the SMCCF was effectively used by the Fed). The SMCCF purchases through December 31st, 2020 reached approximately \$14bn, two-thirds of it in the form of ETFs.[15] Likewise, total purchases under the MSLP fell far short of the program's capacity. As of January 31st, 2021, the program had bought participations in 1830 loans, for a total principal amount of \$17.4bn.

The fact that take-up rates in the Fed programs were small does not necessarily indicate that they had a limited impact. The March 23rd *announcement* of the Fed's decision to intervene in corporate credit markets —– as opposed to the actual purchases —– had an appreciable effect on the cost of debt of corporations. The bottom panel of Figure 1 shows that on the two announcement dates of March 23rd (the initial announcement of the facilities) and April 9th (their expansion), corporate credit spreads for BBB rated firms fell substantially, despite the fact that purchases under the CCF did not begin until May 12th (for ETFs) and June 16th (for single-name bonds).[16] Gilchrist et al. (2020) analyze in detail the effect of the Fed's announcements on bond markets. Comparing changes in spreads across eligible and ineligible bond issues, they conclude that the announcements reduced corporate bond spreads by 70bps on average.[17] By contrast, they estimate the effect of actual purchases on credit spreads to have been approximately 5bps. Thus, the CCF may have had an impact on credit conditions even without a large take-up rate.

We will propose an interpretation of these "announcement" effects (as opposed to "implementation" or take-up effects) through the lense of our model, by analyzing the effects of a change in risk prices in credit markets on leverage and investment rates.[18]

3 Model

In this section, we develop a partial equilibrium model of firm's investment, financing and default. We discuss its key economic mechanisms and some of its important limitations. Since we are interested in the behavior of certain aggregates, we also discuss the aggregation of firm-level decisions and the resulting balanced growth path.

[15] The bottom panel of Figure 1 reports cumulative gross purchases. As of March 11th, 2020, the SMCCF portfolio consisted of ETFs shares with a total market value of at \$8.59bn, and single-name bonds with \$5.13bn total par value.

[16] The March 23rd announcement is available here, and the April 9th announcement is available here.

[17] In a sample of eligible and ineligible bonds from the same issuers, they find smaller effects (20bps).

[18] Because data on the cost of credit for smaller, private firms is generally unavailable in the US, it is unclear whether the PPP and MSLP had announcement effects separate from their implementation. Granja et al. (2020) and Hubbard, Strain et al. (2020) discuss the effects of the first wave of the PPP in detail, and find mixed effects. In particular, Granja et al. (2020) emphasizes the relatively poor targeting of the PPP, and argues that many firms used the funds to build up cash buffers.

3.1 Model description

We first describe an individual firm's problem, and then turn to the aggregation of firm decisions. Our model is cast in partial equilibrium: for tractability, we specify the stochastic discount factor ("SDF") of investors in financial markets exogenously.

3.1.1 Firm problem

The problem of an individual firm extends the framework studied in DeMarzo and He (2021) by allowing for a continuous choice of investment rates subject to convex adjustment costs and by modifying the consequences of default. Firms' borrowing decisions are motivated by the tax deductibility of the debt interest expense. Their investment decisions are distorted downwards by the presence of long term debt. Creditors providing financing to the firm are competitive, and price the debt rationally, anticipating the firm's optimal policies. For now, we assume that shareholders and creditors are risk-neutral. In Section 3.3.1 we extend our model to the case where creditors and shareholders price their claims with non-trivial and potentially different SDFs, and where firm's cash-flows are exposed not only to idiosyncratic but also to priced aggregate shocks.

Technology The production technology of firm j yields revenue $y_t^{(j)} = ak_t^{(j)}$ per unit of time. $k_t^{(j)}$ represents efficiency units of capital of firm j, while a represents the marginal product of capital. While $k_t^{(j)}$ is specific to a firm, a is identical across firms. A firm has at its disposal an investment technology with adjustment costs, such that $\Phi(g_t^{(j)})k_t^{(j)}dt$ spent allows the firm to grow its capital stock by $g_t^{(j)}k_t^{(j)}dt$, where Φ is increasing and convex. $g_t^{(j)}$ represents the rate of growth of firm j's capital stock (or equivalently, the net investment rate), while $\Phi(g_t^{(j)})$ is the investment-to-capital ratio. Firm j's capital $k_t^{(j)}$ satisfies

$$dk_t^{(j)} = k_t^{(j)} \left(g_t^{(j)}dt + \sigma dZ_t^{(j)} \right).$$

$Z_t^{(j)}$ is a Brownian motion, representing idiosyncratic shocks hitting the production technology of the firm; the shocks are identically and independently distributed across firms. In our numerical calculations, we will assume that

$$\Phi\left(g\right) := \delta + g + \frac{\gamma}{2}g^2.$$

δ is the depreciation rate, while γ governs the magnitude of capital adjustment costs.

Capital structure The firm has access to debt and equity markets. In our baseline model, we assume that both markets are frictionless; in Section 3.3.1, we extend our model to the case where there are frictions. We note $b_t^{(j)}$ the principal amount of the firm j's (exponentially amortizing) long term debt. Its tax liability between t and $t+dt$ is equal to

$$\Theta\left(ak_t^{(j)} - \kappa b_t^{(j)}\right) dt.$$

κ is the debt coupon rate, while Θ is the corporate tax rate. The motive for the firm to take on debt stems from the tax deductibility of the debt interest expense.

Shareholders cannot commit to always repaying the debt issued by the firm, which is thus credit-risky. Upon default of firm j at time τ, bankruptcy costs cause capital to jump downwards by a factor α_k, so that $k_\tau^{(j)} = \alpha_k k_{\tau-}^{(j)}$. At the same time, shareholders and creditors renegotiate the firm's debt, resulting in the firm emerging from bankruptcy with a lower debt burden $b_\tau^{(j)} = \alpha_b b_{\tau-}^{(j)}$. We impose the condition $0 < \alpha_b < \alpha_k < 1$, so that bankruptcy costs are strictly positive and so that the restructured firm's debt-to-ebitda ratio is strictly lower than its pre-bankruptcy value. We denote $N_t^{(j)}$ the related default counting process.

When firm j issues \$1 face value of bonds, it raises proceeds equal to $d_t^{(j)}$, which represents the (endogenous) debt price of the firm (per unit of face value). The dividends paid to shareholders of firm j at any time are equal to:

$$\pi_t^{(j)} k_t^{(j)} dt := \left[ak_t^{(j)} - \Phi\left(g_t^{(j)}\right) k_t^{(j)} - (\kappa + m)\, b_t^{(j)} + \iota_t^{(j)} k_t^{(j)} d_t^{(j)} - \Theta\left(ak_t^{(j)} - \kappa b_t^{(j)}\right)\right] dt.$$

In the above, $\iota_t^{(j)} k_t^{(j)} dt$ is the notional amount of bonds issued between t and $t+dt$ by firm j (and sold at a price $d_t^{(j)}$ per unit of face value). $1/m$ represents the debt weighted average life. Negative dividends should be interpreted as equity issuances.

In this setting, the firm cannot commit to a particular debt issuance policy, so that the evolution of the debt balance $b_t^{(j)}$ is given by:

$$db_t^{(j)} = \left(\iota_t^{(j)} k_t^{(j)} - m b_t^{(j)}\right) dt.$$

Firm problem From now on, we omit the firm's superscript j for notational simplicity. Shareholders take the debt price process d_t as given. Their equity value E is defined via:

$$E(k,b) := \sup_{g,\iota,\boldsymbol{\tau}} \mathbb{E}^{k,b}\left[\int_0^{+\infty} e^{-rt} \pi_t k_t dt\right], \tag{1}$$

where $\boldsymbol{\tau}$ represents a sequence of default times chosen by the firm's management, and the superscript notation $\mathbb{E}^{k,b}$ denotes the expectation operator conditional on the initial state $(k_0, b_0) = (k, b)$. Creditors take the firm's default strategy $\boldsymbol{\tau}$ (and its related counting process

N_t) as given. They price one unit of debt rationally, via:

$$D(k,b) := \mathbb{E}^{k,b}\left[\int_0^{+\infty} \alpha_b^{N_t} e^{-(r+m)t}(\kappa+m)dt\right] \tag{2}$$

We focus on a Markov perfect equilibrium in which the equity value E is homogeneous of degree one in (k,b), the debt price function D is homogeneous of degree zero in (k,b), the firm policies ι and g are functions homogeneous of degree zero in (k,b), and the continuation region is a cone in the (k,b) plane. For $x := b/k$, we can thus write $D(k,b) = d(x) := D(1,x)$, $E(k,b) = ke(x) := kE(1,x)$, while the financing and growth policies can be expressed as $\iota(x)$ and $g(x)$. In this Markov perfect equilibrium, the default policy will be cutoff in x_t:

$$\tau = \inf\{t \geq 0 : x_t \geq \bar{x}\}$$

Highly levered firms will choose to raise equity capital from shareholders. However, at the boundary $\bar{x}$, shareholders will find it too costly to continue injecting equity capital and will instead let the firm default. We show in Appendix A.2.2 that $e(x)$ satisfies

$$e(x) = \sup_{\tau,g,\iota} \tilde{\mathbb{E}}^x\left[\int_0^{+\infty} e^{-\int_0^t (r-g_u)du} \alpha_k^{N_t} \pi_t dt\right], \tag{3}$$

$$dx_t = \left[\iota_t - (g_t+m)\, x_t\right] dt - \sigma x_t d\tilde{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1\right) x_{t-} dN_t, \tag{4}$$

with $\tilde{Z}_t := Z_t - \sigma t$ a Brownian motion under the measure $\tilde{\mathbb{P}}$ (see Appendix A.2.2). Creditors price the debt rationally, anticipating the Markov financing policy $\iota(x)$, investment policy $g(x)$, and default cutoff $\bar{x}$ used by shareholders. The debt price satisfies

$$d(x) = \mathbb{E}^x\left[\int_0^{+\infty} \alpha_b^{N_t} e^{-(r+m)t}(\kappa+m)dt\right] \tag{5}$$

$$dx_t = \left[\iota(x_t) - \left(g(x_t) + m - \sigma^2\right) x_t\right] dt - \sigma x_t dZ_t + \left(\frac{\alpha_b}{\alpha_k} - 1\right) \bar{x} dN_t$$

Equity and debt valuation equations The equity price e (per efficiency unit of capital) satisfies a Hamilton-Jacobi-Bellman (HJB) variational inequality that takes the following form:

$$0 = \max\left[\alpha_k e\left(\frac{\alpha_b}{\alpha_k}x\right) - e(x), \max_{\iota,g}\left[-(r-g)e(x) + a - \Phi(g) - (\kappa+m)\,x - \Theta(a-\kappa x) + \iota d(x) + \left[\iota - (g+m)\,x\right] e'(x) + \frac{\sigma^2}{2}x^2 e''(x)\right]\right] \tag{6}$$

The first term inside the first maximum operator encodes the shareholder default option, while the second term encodes the HJB in the continuation region, in which shareholders' discounted gains' process must be a martingale. The debt price satisfies

$$(r+m)d(x) = \kappa + m + \left[\iota(x) - \left(g(x) + m - \sigma^2\right)x\right]d'(x) + \frac{\sigma^2}{2}x^2 d''(x) \tag{7}$$

$$d(\bar{x}) = \alpha_b d\left(\frac{\alpha_b}{\alpha_k}\bar{x}\right). \tag{8}$$

The differential equation for d takes into account the policies $g(x)$ and $\iota(x)$ resulting from the shareholder optimization problem (6), while the boundary condition (8) uses the cutoff policy $\bar{x}$ used by the firm's management. Equations (6) and (7) are coupled functional equations for e and d, and it is a-priori not obvious that a solution to this system of equations exists. Shareholders' inability to commit to a particular capital structure policy will however simplify our analysis, and leads to a decoupling of this system of functional equations.

Optimality conditions The first order condition of the shareholders' problem with respect to debt issuances leads to a relationship between debt and equity prices:

$$d(x) + e'(x) = 0. \tag{9}$$

As shown in Appendix A.2.3, condition (9) implies that the debt issuance policy is:

$$\iota(x) = \frac{\Theta\kappa}{-d'(x)}. \tag{10}$$

Debt issuances increase with the tax shield, and decrease with the slope of the bond price function. The optimal capital growth rate $g(x)$ follows a q-theory optimal rule:

$$\Phi'\left(g(x)\right) = e(x) - xe'(x) := q(x), \tag{11}$$

where $q = \partial_k E$ is Tobin's q. Default optimality leads to the smooth-pasting condition:

$$e'(\bar{x}) = \alpha_b e'\left(\frac{\alpha_b}{\alpha_k}\bar{x}\right). \tag{12}$$

3.1.2 Aggregation and balanced growth

Let $K_t := \int_j k_t^{(j)} dj$ be the aggregate capital stock. Denote by $\omega_t^{(j)} := k_t^{(j)}/K_t$ the share of aggregate capital owned by a particular firm. Let $f_t(x, \omega)$ be the time-t joint density over leverage and capital shares. Aggregate capital satisfies:

$$dK_t = K_t \left[\underbrace{\int_j \omega_t^{(j)} g\left(x_t^{(j)}\right) dj dt}_{=\hat{g}_t dt} + \underbrace{\int_j \sigma \omega_t^{(j)} dZ_t^{(j)} dj}_{=0} - (1-\alpha_k) \underbrace{\int_j \omega_t^{(j)} dN_t^{(j)} dj}_{=\hat{\lambda}_t^d dt} \right] \tag{13}$$

In equation (13), the law of large numbers allows us to simplify the capital growth equation since (a) the aggregation of idiosyncratic shocks does not contribute to aggregate growth, while (b) capital destructions through default contribute a locally deterministic term $-(1-\alpha_k)\hat{\lambda}_t^d dt$, representing the capital-share weighted credit loss rate in our economy. The aggregate capital stock thus grows at a locally deterministic rate $\mu_{K,t} = \hat{g}_t - (1-\alpha_k)\hat{\lambda}_t^d$.

Appendix A.2.7 shows that the capital-share-weighted default rate $\hat{\lambda}_t^d$ and growth rate $\hat{g}_t$ can be computed using moments of the density $\hat{f}_t(x) := \int_\omega \omega f_t(x,\omega) d\omega$, which represents the percentage of the total capital stock at firms with leverage x. This density satisfies a modified Kolmogorov forward equation, which encodes the dynamic properties of such density as a function of policy decisions made by firms:

$$\partial_t \hat{f}_t(x) = \mathcal{L}^* \hat{f}_t(x) - \mu_{K,t} \hat{f}_t(x), \tag{14}$$

where $\mathcal{L}^*$ is a linear differential operator discussed in Appendix A.2.7. The capital-share-weighted default rate $\hat{\lambda}_t^d$ and growth rate $\hat{g}_t$ can then be computed via:

$$\hat{\lambda}_t^d = -\frac{1}{2}\sigma^2 \bar{x}_t^2 \partial_x \hat{f}_t(\bar{x}_t), \qquad \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx.$$

In a balanced-growth path, aggregate capital K_t and debt $B_t := \int_j b_t^{(j)} dj$ grow at a constant rate μ_K, while the default rate $\hat{\lambda}_t^d$ and the growth rate $\hat{g}_t$ are constant and equal to $\hat{\lambda}^d$ and $\hat{g}$, satisfying $\mu_K = \hat{g} - (1-\alpha_k)\hat{\lambda}$. The stationary density $\hat{f}$ is then the solution to the time-independent integro-differential equation (14), where the time-derivatives have been set to zero. The density $\hat{f}$ will be used as the starting point for the experiments we build in order to analyze the impact of BFPs on economic aggregates.

3.2 Discussion

Investment and debt overhang Corporate debt depresses investment in our model. Shareholders are less inclined to invest when the firm is highly levered, since some of the value stemming from the related decrease in leverage is captured by creditors via higher debt prices. We show in Appendix A.2.5 that e is a convex function. Using equation (11), this yields

$$g'(x) = \frac{-e''(x)}{\Phi''(g(x))} < 0$$

Investment therefore decreases with leverage. Appendix A.2.5 establishes a second, related result. If we denote g^* the optimal growth policy of a firm that is born with no debt and subsequently never borrows, then $g(x) \leq g^*$ for all leverage x. That is, levered firms invest less than a firm that cannot take any leverage. Thus, debt overhang reduces investment at the margin (as leverage increases), and overall (relative to a no-debt firm).

Figure 2a illustrates these properties graphically. It reports the optimal gross investment rate $\Phi(g(x))$ as a function of the debt to ebitda ratio $z(x) := x/a$, as well as the stationary cross-sectional density $\hat{f}$, using the estimated parameters from Section 4.[19] Investment declines with leverage, and is lowest at the default boundary. This debt overhang channel will be amplified by government interventions involving loans to businesses.

"Zombie" firms and creative destruction The investment behavior of our firms can be related to that of "zombie" firms. Appendix A.2.6 shows that the default boundary $\bar{x}$ of our firm is higher than the default boundary $\bar{x}^*$ of a firm that commits to using the no-leverage optimal investment rule g^* (but is otherwise free to choose its capital structure). Thus, highly levered firms sacrifice investment and instead pay dividends (or reduce share issuances) and postpone default. This decision is privately optimal, but it depresses investment, as some firms remain active at higher leverage than they otherwise would, had they followed the no-leverage investment rule g^*. Firms with leverage $x > \bar{x}^*$ can be thought of as zombie firms; a short sequence of bad productivity shocks will push them to default, following which they emerge with lower debt burdens and higher investment rates. This creative destruction will be altered whenever a credit market intervention maintains those firms alive.

Financial policies Figure 2b and Figure 2c report equity values and the dividend policy as a function of debt to ebitda, whereas Figure 2d and Figure 2e show debt prices and issuance rates. Equation (10) helps understand the issuance behavior of firms: since the debt price is decreasing in leverage, $d'(x) < 0$ and the firm never buys back its own debt. Numerically, given that d is concave, the bond issuance rate is a declining function of debt to ebitda.

Dividends are thus declining with leverage: at low debt levels, the firm pays large dividends, mostly financed by proceeds from bond issuances. Instead, at higher debt levels, firm cash-flows are depressed by (a) higher debt servicing costs, (b) lower levels of debt issuances

[19] We report policies as a function of the debt to ebitda ratio $z := b/(ka)$ instead of leverage $x = b/k$ because debt to ebitda is the leverage metric we use in the estimation of the model, as it allows us to sidestep the question of how to measure productive capital k in the data.

and (c) lower prices obtained for each dollar face amount of debt issued. Even if shareholders cut investment, this latter force is not sufficient to offset the former effects, leading to dividends being decreasing in leverage. At sufficiently high leverage, dividends are negative — in other words, the firm issues new shares.

Lastly, Figure 2d shows that the debt price is always strictly less than its risk-free value (equal to 1 since we set $r = \kappa$), and that credit spreads $cs(x)$ (defined in Appendix A.2.4) are increasing in leverage and bounded away from zero, even when the firm has no leverage $(x = 0)$. Indeed, creditors of a no-debt firm take into account the fact that the firm will be issuing large amounts of debt, thus increasing future leverage and future default risk.

The role of limited commitment Firms in the model take on leverage aggressively in order to monetize the deductibility of debt interest expenses. Given their inability to commit to a future financing strategy, firms lever up to the point where expected future default costs exactly wipe out the tax benefits of debt. This result is the focus of DeMarzo and He (2021). As shown in Appendix A.2.3, condition (9) can be used to show that, for *any* leverage ratio x, the equity value is the same as that of a firm whose shareholders never issue any additional debt in the future. Shareholders' time-consistency problem is "self-defeating", as it undermines their ability to benefit from the tax shield.[20] The result is particularly useful for our purpose, as it ensures that firms in the model actively take on debt, thus giving debt overhang the best possible chance to matter for investment decisions.[21]

Default resolution Our model assumes that upon default, a firm does not exit but is instead restructured. The restructuring involves dead-weight losses in productive capital (i.e. $\alpha_k < 1$). A large literature documents and measures those losses associated with defaults and liquidations; in Section 4, we use this literature to calibrate the value of α_k. Avoiding these deadweight losses is also one of the key reasons why credit interventions may be beneficial following an aggregate shock. Restructuring involves creditors accepting a haircut ($\alpha_b < 1$), though shareholders are not completely wiped out. Strictly speaking, this violates the absolute priority rule (APR). This model feature is primarily motivated by a theoretical consideration: zero recovery rates for equityholders, but positive recovery rates for creditors would not be consistent with a smooth debt issuance equilibrium.[22] However, we note that this assumption has some empirical support. Bris, Welch and Zhu (2006) for example

[20] This result holds in continuous-time, but holds only "approximately" in the discrete-time model analog.

[21] By contrast, a model with commitment, such as Leland et al. (1994), tends to under-predicts leverage relative to the data.

[22] As discussed in DeMarzo, He and Tourre (2021), it is important for the smooth MPE of this model to exist that the condition $e'(x) + d(x) = 0$ holds both in the continuation region, as well as in the default region. Concretely, when shareholders are wiped out in bankruptcy, the smooth pasting condition $v'(\bar{x}) = 0$ must hold, which is consistent with a smooth MPE only if $d(\bar{x}) = 0$.

show that the APR is violated in 12.2% of the Chapter 11 reorganizations they consider. Weiss (1990) also documents violations of APR in the majority of the bankruptcy cases in his sample. Our bankruptcy resolution protocol is thus consistent with these APR violations.

Cash holdings Our model does not feature cash holdings: cash inflows received by a firm are either distributed to creditors or used for investment, with the balance distributed as dividends (rather than potentially stored as cash reserves). Thus, a sudden decline in the marginal product of capital a, accompanied by a sudden stop in capital markets, cannot be mitigated by cash reserves held by a firm, potentially exacerbating the effects of such shock. Abstracting from cash holdings allows us to keep the model tractable, with a unique state variable (leverage) driving the ex-post firm-level heterogeneity.[23]

3.3 Extensions

In this section, we discuss a set of extensions that will be useful when analyzing BFPs and their potential effects on economic aggregates.

3.3.1 Priced aggregate shocks

In our baseline model, investors are risk-neutral and firms' capital is only exposed to idiosyncratic shocks $Z_t^{(j)}$. This assumption is convenient, as it allows us to obtain a balanced-growth path in which economic aggregates grow at a constant rate μ_K. In order to analyze BFPs that not only have an impact on lending, but also on asset prices, we extend our baseline model so that firms can also be exposed to priced aggregate shocks. In this extension, the efficiency units of capital satisfy

$$dk_t^{(j)} = k_t^{(j)} \left(g_t^{(j)} dt + \sigma \left(\rho dZ_t + \sqrt{1-\rho^2} dZ_t^{(j)} \right) \right),$$

where Z_t is an aggregate shock process, while $Z_t^{(j)}$ is an idiosyncratic shock process, both hitting the production technology of the firm; the idiosyncratic shocks are identically and independently distributed across firms, and independent of the aggregate shock Z_t.

Denote r_e (resp. r_d) the risk-free rate, and ν_e (resp. ν_d) the market price of risk faced by equity (resp. debt) investors. The SDF $\xi_{n,t}$ for investor type $n \in \{e, d\}$ satisfies

$$\frac{d\xi_{n,t}}{\xi_{n,t}} = -r_n dt - \nu_n dZ_t$$

[23]See Anderson and Carverhill (2012) for a model with liquidity in addition to debt as a firm state variable.

When these two SDFs do not coincide, credit and equity markets must be segmented, in order to rule out arbitrage opportunities.

Appendix A.2.8 derives the counterparts to equations (6)-(7) when firms are exposed to priced aggregate shocks. We show in particular that investment follows a q-theory optimal rule identical to (11), and that it is monotone decreasing in the risk free rate r_e and the market price of risk ν_e faced by equity market investors. Moreover, the equity value e (and thus debt prices, since $d(x) = -e'(x)$) is independent of creditors' SDF, as in DeMarzo, He and Tourre (2021). These observations will be essential when studying BFPs whose announcement have an effect on the pricing of aggregate risk.

We also show that debt issuance decisions are not only driven by tax motives, but also by the segmentation between credit and equity markets, and in particular shareholder's perceived mispricing of bonds. In such environment, the financing policy satisfies

$$\iota(x) = \frac{\Theta\kappa}{-d'(x)} + \frac{\left[\tilde{R}_d(x) - R_d(x)\right] d(x)}{-d'(x)}, \tag{15}$$

with $R_d(x)$ the expected debt return demanded by credit market investors, while $\tilde{R}_d(x)$ instead represents the required debt return *as perceived by shareholders*:

$$R_d(x) := r_d - \rho\nu_d \frac{\sigma x d'(x)}{d(x)} \qquad \tilde{R}_d(x) := r_e - \rho\nu_e \frac{\sigma x d'(x)}{d(x)}$$

Formula (15) is a generalization of equation (10) for the case with priced aggregate risks, when credit and equity markets are potentially segmented. In addition to the tax motive, the debt issuance rate increases with the wedge $\tilde{R}_d(x) - R_d(x)$ between equity and credit markets' perceived market cost of debt capital. The conditional equity volatility $\sigma_e(x)$, equity risk premium $p_e(x)$, and the market cost of equity capital $R_e(x)$ then satisfy

$$\sigma_e(x) := \sigma\left(1 - \frac{xe'(x)}{e(x)}\right) \qquad p_e(x) := \rho\nu_e\sigma_e(x) \qquad R_e(x) := r_e + p_e(x)$$

With aggregate shocks, our economy no longer admits a balanced-growth path, since the density $\hat{f}_t$ is exposed to the Brownian motion Z_t, as discussed in Appendix A.2.8.

3.3.2 Frictions and constraints

When analyzing the effects of BFPs on firm-level and economy-wide outcomes during a crisis, we will also be interested in studying environments with frictions – either (i) due to imperfections in credit markets, or (ii) due to constraints imposed onto firms following a government intervention. In either of these environments, the key indifference relation

$e'(x) + d(x) = 0$ will no longer hold; debt pricing will have a feedback effect onto equity valuations, and the investment margin will be distorted anytime the relevant friction "binds".

Sudden stop In such case, firms loose access to all financial markets during the crisis. This curtails their ability to service debt and finance investments; firms' optimal policies satisfy

$$\max_{g,\iota} \qquad ge(x) + \iota d(x) - \Phi(g) + (\iota - xg)\, e'(x) \qquad \text{s.t.} \qquad \pi(x) \geq 0, \iota = 0 \tag{16}$$

The dividend constraint caps the net investment rate to $\bar{g}(x)$, decreasing in x:

$$\bar{g}(x) := \Phi^{-1}\left(a - (\kappa + m)x - \Theta\left(a - \kappa x\right)\right)$$

As $\bar{g}(x)$ reaches the lowest feasible net investment rate $-1/\gamma$, the firm can no longer disinvest at a rate high enough to avoid raising capital in financial markets. With the sudden stop, firms at such level of indebtedness have to default; this occurs at the leverage cutoff $\bar{x}_{ss}$:

$$\bar{x}_{ss} = \frac{(1-\theta)a - \Phi(-1/\gamma)}{(1-\Theta)\kappa + m} \tag{17}$$

Shareholder payout restrictions If firms are prevented from making payouts to shareholders (which must be the case only for a temporary period of time to ensure positive equity values), firms' optimal policies satisfy a problem identical to (16), except for (i) the issuance constraint $\iota = 0$, which disappears, and (ii) the dividend constraint, whose sign is flipped: $\pi(x) \leq 0$. Denote $\lambda_-(x) \geq 0$ the multiplier on the dividend constraint, then

$$\lambda_-(x) = 1 + \frac{e'(x)}{d(x)} \geq 0 \text{ (strict inequality when } \pi(x) = 0) \qquad \text{and} \qquad \Phi'\left(g(x)\right) = \frac{q(x)}{1 - \lambda_-(x)}$$

The marginal cost of investing is equal to Tobin's q, except when the dividend payment restriction binds, in which case investment is distorted *upwards*.

4 Estimation

We now estimate the steady-state of the model using US data on non-financial firms.

4.1 Data

We start by describing the data we use in our estimation. Since part of our goal is to assess the interventions that started in late 2020Q1, we primarily use data from 2019.

Sources and variable definitions Our main data source is the Compustat fundamentals annual file. We focus on firm-year observations from fiscal years 2018 and 2019. Aside from Compustat, we also use the BEA's Fixed Assets Tables in order to construct measures of the growth rate in prices of capital goods, Π_K, and the aggregate, real growth rate of the capital stock of non-financial corporations, as well NIPA data to construct a value added deflator Π for the output of non-financial firms. Finally, we use default rates for rated firms from S&P (2019), and estimates of debt recovery rates from Ou, Chiu and Metz (2011), though we do not target either of these moments in our estimation.

Our estimation uses empirical proxies for five model variables: $\Phi(x)$ (the investment rate), $z(x)$ (the debt-to-ebitda ratio), $\kappa z(x)$ (the interest coverage ratio), $\pi(x)/e(x)$ (the equity payout rate), $\iota(x)/x$ (the gross debt issuance rate).[24] The definition of these empirical proxies in terms of Compustat variables is reported in Appendix A.3.1.[25] Our treatment of the Compustat data is standard and follows other work (Whited, 1992; Frank and Goyal, 2003; Hennessy, 2004; Hennessy and Whited, 2007), with two main exceptions. First, we use net rather than gross debt since firms in our model do not accumulate cash. We report results of the model estimated using gross debt ratios in our robustness checks. Second, we use gross rather than net stocks of PP&E when measuring investment since the latter produces investment rates that are both very high and very volatile, likely driven by the difference between accounting and economic measures of depreciation. We consider a perpetual inventory estimate of the PP&E stock in our robustness checks.

Sample selection, summary statistics, and moments used in the estimation Sample selection criteria are described in Appendix A.3.1. They are standard, except for two. First, either short-term (`dlc`) or long-term (`dltt`) debt must be strictly positive in both 2018 and 2019 for an observation to be included in the sample, since firms in our model all have strictly positive debt. Second, we restrict the sample to firms with strictly positive ebitda, since, as we explain below, the moments we target are weighted by ebitda. In 2019, negative ebitda firms account for 2.1% of total assets and 2.0% of total revenue of non-financial Compustat firms, and their ebitda was 2.4% of aggregate ebitda (in absolute value).

Our selected sample has 1589 observations. We winsorize the variables $\Phi(x)$, $z(x)$, $\kappa z(x)$, $\pi(x)/e(x)$, $\iota(x)/x$, and x at their 1st and 99th percentile within the selected sample. Finally,

[24] To facilitate comparison with existing work, we also report values for book debt to book assets, denoted by x in all the tables where we report results. We do not use this moment in our estimation.

[25] The variable y_t, in our model, maps to ebitda in the data. However, we assume that the firm operates an AK technology with capital as the only input. This technology could be microfounded using a constant-returns to scale gross-output production function with fully flexible labor and intermediate inputs. With this microfoundation, output would be proportional to ebitda, and the marginal product of capital a would reflect the wage rate, the cost of intermediate inputs, and total factor productivity.

by analogy with the model, we define the weight associated with an observation i as:

$$w_i \equiv \frac{N \times \text{ebitda}_i}{\sum_j \text{ebitda}_j},$$

where the sum is over the selected sample, and N is the size of that sample.[26]

The first column of Table 2, Panel C, reports the data moments that we target in our baseline estimation. With one exception, these moments are weighted averages of our variables of interest:

$$\hat{y} \equiv \frac{1}{N} \sum_i w_i y_i,$$

where y_i is observation of variable y for firm i. The exception is the moment $\hat{\Gamma}$, the slope of investment with respect to the ratio of debt to ebitda, which is computed as:

$$\hat{\Gamma} \equiv \frac{100 \cdot cov(\Phi(x), z(x))}{var(z(x))},$$

$$cov(\Phi(x), z(x)) \equiv \frac{1}{N} \sum_i w_i \left(\Phi_i z_i - \hat{\Phi}\hat{z}\right), \quad var(z(x)) \equiv \frac{1}{N} \sum_i w_i \left(z_i - \hat{z}\right)^2.$$

The values report for $\hat{\Phi}$, $\hat{z}$ and $\hat{\Gamma}$ in Table 2, Panel C, are those targeted in the estimation. The rest of the moments in the table are untargeted but will serve to evaluate model fit. The other columns of the table report alternative values of the moments we consider when different variable definitions or sample selection criteria are used. We come back to this in our robustness section below. Additionally, below (in the paragraph titled "Implications for the strength of debt overhang"), we compare the values we obtain for our targeted moments with existing evidence from other papers.

4.2 Results

Methodology and identification We estimate a version of the model in which there are no aggregate shocks, and risk prices are zero ($\nu_d = \nu_e = 0$). We will use this estimated model as our starting point in our quantitative analysis. This version of the model has ten parameters, $\{r, \kappa, m, \delta, \Theta, \alpha_k, \alpha_b, a, \sigma, \gamma\}$. We normalize $\kappa = r$, so that the price of risk-free debt is equal to 1. We then calibrate six parameters, $\{r, m, \delta, \Theta, \alpha_k, \alpha_b\}$, to values drawn from prior evidence; these values are reported in Table 2 and discussed in more detail in Appendix A.3.1.

In order to estimate the remaining three parameters, a (the marginal product of capital), σ (the volatility of capital quality shocks), and γ (the convexity of capital adjustment costs), we use a two-step efficient Generalized Method of Moments estimator (see, e.g., Erickson and

[26]Unweighted summary statistics for the resulting sample are reported in Appendix Table A-1.

Whited 2000), with appropriately weighted observations, to reflect the fact that we target moments under the distorted distribution $\hat{f}$. The method is standard and is described in Appendix A.3.2. We note that a key advantage of our model is that, because the stationary distribution of the model is simple to obtain, all the moments used in the estimation can be computed quickly and accurately.

We use an exactly identified approach. We match three data moments: the average gross investment rate $100 \cdot \hat{\Phi}$; the average ratio of debt to ebitda $\hat{z}$; and the cross-sectional sensitivity of investment to the ratio of debt to ebitda, $\hat{\Gamma}$. The main intuition for identification is the following. First, the average investment rate primarily identifies the marginal product of capital a, as higher returns to capital increase the average value of marginal q. Second, the ratio of debt to ebitda primarily identifies the volatility of capital quality shocks, because the debt price function $d(x)$ becomes steeper as volatility increases. Third, the slope of investment with respect to leverage helps identify the adjustment cost parameter γ: all else equal, in response to a capital quality shock that increases their leverage, firms cut back investment when adjustment costs are higher, making the investment policy function steeper with respect to leverage. Figure 3 reports comparative statics of the targeted moments with respect to the three estimated parameters, clarifying these monotonic relationships.[27]

Baseline results Estimation results are reported in Table 2, Panel B. The three parameters (α, σ, γ) are precisely estimated, and the targeted moments are well matched.

Our point estimates compare to existing estimates of similar parameters as follows. The point estimate for capital adjustment costs is $\gamma = 7.16$. There is a fair amount of variation in values of this parameter in the existing literature, even within those that use a Q-theoretic approach to estimating γ. Recently, Falato et al. (2020) report estimates of γ ranging from approximately 2 to 20, with higher values obtained in versions of their model that do not allow for intangible capital, as is the case in our baseline model. Belo et al. (2019) also report a similar range of estimates.[28] Our point estimate is in the middle of this range. However, we note that different from these papers, which use the slope of investment with respect to q in order to estimate γ, our paper uses the slope of investment with respect to leverage (the underlying state variable, in the model). So, the moment underlying our estimate of γ is entirely different from these models.

The point estimate for a implies that the average before-tax returns to (physical) capital

[27] We discuss the intuition for identification in more detail in Appendix A.3.4.

[28] Other estimates include: Hayashi (1982), who finds a value of approximately $\gamma = 20$ using aggregate data on the US corporate sector; Gilchrist and Himmelberg (1995), who estimate a value of approximately $\gamma = 3$ in their sample of rated firms; and Hall (2001), who considers values ranging from $\gamma = 2$ to $\gamma = 8$. Cooper and Haltiwanger (2006) estimates a much lower value of γ, but in a model with concavity in the revenue function, and without using estimates of the investment-Q relationship. Their estimated model implies an investment-Q slope of approximately 0.2, consistent with $\gamma = 5$.

in the model are 24% per year. Using Flow of Funds data, Crouzet and Eberly (2020) find an average return to capital (defined in the same way as this paper) of 22.1% for the 2001-2017 period. Other recent work on the rise in corporate profits have also founds returns to capital of the same magnitude (see, e.g. Barkai 2020). Finally, the volatility of capital quality shocks, σ, can be compared to the values used in Chen (2010). The main values used in his calibrated model, which are chosen to match both the aggregate volatility of corporate profits in NIPA data and historical estimates of default rates, imply an overall volatility of firm cash flows of $\sqrt{0.14^2 + 0.24^2} = 0.28$, close to our point estimate of 0.31 per annum.

Non-targeted moments Table 2, Panel C compares model and data values for a set of key financial and real moments which our estimation does not target, including inverse interest coverage ratios, the equity payout rate, the debt issuance rate, the debt recovery rate, equity volatility, credit spreads, the default rate, and the growth rate of the aggregate stock of capital. Appendix A.3.5 discusses in more detail the data sources for these non-targeted moments. The model is generally successful at matching these moments, with two main exceptions. First, debt recovery rates are lower in our model than in the data, driven by the high debt haircut α_b. Second, our model produces average credit spreads (398bps p.a.) that are higher than estimates of average yield spreads on corporate bonds of industrial firms.[29] We note that our sample includes non-rated firms, and that calibrations of the model with lower tax rates or lower coupon rates would reduce the level of spreads in the model. However, we acknowledge that, because of the extreme nature of the limited commitment problem in this model, spreads will generally tend to be elevated relative to the data.

We also note that the estimated model generally underestimates the amount of cross-sectional dispersion in the data.[30] It downplays the importance of two groups of firms: those with close to (or exactly) zero debt to ebitda; and those with very high debt to ebitda, driven by temporarily low ebitda. Note that this need not systematically bias the *average* effects of debt overhang on investment in one particular direction, since both very high- *and* very low-leverage firms are under-represented relative to the data.

Implications for the strength of debt overhang Figure 2a, along with the investment policy function, reports the steady-state distribution of debt-to-ebitda in the estimated model. By construction, the policy function and the distribution are such that the model matches the average marginal effect of leverage on investment — the moment $\hat{\Gamma}$ in our notation above. The value of this moment indicates that a one unit increase in the ratio of debt-to-ebitda low-

[29]For instance, Feldhütter and Schaefer (2018) report average bond yield spreads of 87bps for investment-grade firms, and 417bps for high-yield firms; this is our source for the data in Table 2.

[30]This is also highlighted in Appendix Figure A-1. There, we report the empirical and model-based cumulative distribution functions for the distorted density of net debt to ebitda.

ers investment, on average, by 1.04 percentage point; alternatively, a one standard-deviation (approximately 0.64 units of debt-to-ebitda) increase in debt-to-ebitda lowers the investment rate by 0.67 percentage points.

Analogs of this number have been estimated in empirical work on debt overhang, though the pseudo-elasticity measured is generally defined differently. For instance, Lang, Ofek and Stulz (1996) find that investment falls by 0.105 p.p. for each p.p. increase in *book leverage* (as opposed to the ratio of debt to ebitda) (see their Table 3, column 1). Using the ratio of ebitda to book assets in our data, this number can be translated into an increase in net investment of 2.62 p.p. for each unit increase in debt-to-ebitda.[31] Similarly, the estimates of Ahn, Denis and Denis (2006) imply a sensitivity of net investment ranging from 0.95 p.p to 3.35 p.p. per unit increase in ebitda; the estimates of Cai and Zhang (2011) imply a sensitivity of 0.93; and the estimates of Wittry (2020) imply a sensitivity of 0.95.[32] Thus, the sensitivity of investment to leverage in our sample is consistent with existing work, though it is at the lower range of existing estimates.[33]

While these marginal effects may seem small, the total costs of debt overhang in this model are relatively large. One can compare the ergodic enterprise value per unit of capital (Tobin's average Q), $\bar{v} := \int v(x)\hat{f}(x)dx$, to the un-levered enterprise value e^*. The former is 1.111, while the latter is 1.166, suggesting that steady-state costs of debt overhang represent approximately 4.7% of total firm value. This measure of debt overhang cost is, in magnitude, higher than those estimated by Moyen (2007), who finds costs of 0.5% (when the benchmark is the un-levered enterprise value) or 4.7%-5.1% (when the benchmark is the firm value under the assumption that managers make investment decisions maximizing enterprise value).[34] Likewise, in the calibrated model, debt substantially depresses aggregate investment. At the estimated parameters, the growth rate of the economy would be 2.1% per annum (p.a.) in a version of the model in which firms never borrow. Compared to an aggregate growth rate of 0.5% p.a. in the model with debt, reported in Table 2, this indicates that debt overhang depresses aggregate growth by 1.6% p.a. in the model.

[31] In our model, a 1 p.p. change in $x = b/k$ (say, from 0.50 to 0.51) translates into a $1/(100 \times a)$ unit change in debt to ebitda $b/(ak)$, where a is the ratio of ebitda to book assets. In our model, this ratio is approximately $a = 0.25$. Thus, a one unit change in debt to ebitda is associated with a $100 \times a = 25$ p.p. change in $x = b/k$.

[32] Ahn, Denis and Denis (2006) find estimates of the pseudo-elasticity of investment to book leverage ranging from 0.038 to 0.135 (see their Table 4). Cai and Zhang (2011) find an estimate of 0.0375 (see their Table 5). Wittry (2020) finds an estimate of 0.038 (his Table 10).

[33] Another point of comparison with existing evidence is the slope of equity valuations with respect to leverage. In our model, equity value is 21.93% lower, and enterprise value is 4.37% lower, when evaluated at a leverage that is one standard deviation above the ergodic mean. By contrast, Wittry (2020) finds that a one-standard deviation increase in debt is associated with firms either foregoing or delaying projects worth 6.34% of total equity value.

[34] It is however worthwhile pointing out that those latter estimates use a model where shareholders have some degree of commitment over financing decisions.

Robustness We finally explore how robust our findings are to alternative variable definitions and sample selection criteria. The columns marked (1) to (10) in Appendix Table A-2 report key moments for the ten robustness checks we consider, and Appendix Table A-3 repeats our baseline estimation exercise using these alternative moments. Among others, we consider the results obtained when using gross instead of net debt measures, when measuring physical capital using a perpetual inventory method, when defining productive assets more broadly than PP&E, and when restricting the sample to rated firms. The strongest effects of debt overhang correspond to the model estimated using an adjustment to physical capital that includes intangibles. In this case, estimated capital adjustment costs are low, so that investment is very sensitive to a marginal increase in leverage. By contrast, the weakest effects of debt overhang correspond to the case where assets (rather than ebitda) are used to weight moments; the (weighted) slope of investment with respect to leverage is only -0.40, instead of -1.04 in our baseline case, indicating that asset-weighting tends to over-emphasize large and low-leverage firms. Appendix A.3.6 contains a more detailed discussion of our different robustness checks.

5 The effects of a crisis without government intervention

We now introduce into our model a crisis meant to capture a real and financial shock temporarily affecting all firms. We describe the aggregate dynamics of the economy after it has entered the crisis state, distinguishing between two cases: (i) a crisis during which financial markets continue to function normally, and (ii) a crisis during which financial markets undergo a "sudden stop". Throughout this section, we assume that there are no government interventions for the duration of the crisis. In the following sections, we will use these results as the backdrop against which to assess the effects of government interventions.

5.1 A transitory crisis state

We introduce a transitory crisis in the model of Section 3 as follows. Starting on the balanced-growth path (with an leverage distribution $\hat{f}_0$ equal to its ergodic value $\hat{f}$), we assume that the economy enters *unexpectedly* into a "crisis state", in which certain structural parameters change. It remains in this state for a random, exponentially distributed time period (parameter χ), about which firms form correct expectations. We indicate changes in structural parameters during the crisis state by underlining them. Specifically, the marginal product of capital for all firms drops to $\underline{a} < a$, while the price of risk increases from zero to $\underline{\nu} > 0$.[35]

[35] In our base case crisis scenario, we assume that credit and equity markets are integrated, in other words we assume that $r_e = r_d = r$, and that $\nu_e = \nu_d = \nu$.

Additionally, the crisis may involve a sudden stop in capital markets; we describe this in more detail below.

Appendix A.2.9 details the pair of equations that the equity price (conditional on the crisis state) $\underline{e}$ and the debt price $\underline{d}$ satisfy. Both equations are the counterparts to equations (6)-(7), adapted to account for the lower level of a, the higher level of risk prices and the random transition time back to the "normal" regime.

To quantify the economic impact of the crisis and of policy interventions, we compute the expected path of aggregates quantities in the model. We outline briefly our method for doing so (see Appendix A.2.9.2 for details). Consider for example the expected future aggregate capital stock K_t, conditional on an initial cross-sectional distribution of (capital-weighted) leverage $\hat{f}_0$ and an initial aggregate amount of capital K_0. It satisfies:

$$\mathbb{E}_0\left[K_t\right] = \int_j \mathbb{E}_0\left[k_t^{(j)}\right] dj = K_0 \int \underline{h}_k\left(x,t\right)\hat{f}_0\left(x\right) dx,$$

where $\underline{h}_k(x,t)$ represents the expected future capital stock at horizon t for a firm that starts at time zero with a unit amount of capital and leverage x. Appendix A.2.9.2 discusses the PDE that the function $\underline{h}_k$ is solution of. In that section, we introduce similarly defined functions that allow us to compute the expected path of various economic aggregates of interest.

5.2 Crisis dynamics when financial markets function normally

Calibration We use a calibration that replicates some of the key features of the initial response of US non-financial public firms to the onset of the 2020 recession. First, we set $1/\chi = 1$, so that firms expect the crisis state to persist for one year.[36] Second, we set the marginal product of capital a during the crisis to $\underline{a} = 0.75a$, so as to match the measured decrease in aggregate ebitda for the publicly traded companies in our sample between 2019q4 and 2020q2, which was approximately 25%.

A crisis featuring only a cash-flow shock leads to an on-impact drop in equity prices (resp. increase in average credit spreads) of 5.3% (resp. 12bps) (first line of Table 3, Panel B). These price responses are substantially smaller than those observed in the data during the early days of the crisis and before any government intervention, as shown in Table 3, Panel A.[37] Thus, in order to also match the empirical price response, we assume that in the crisis state, Sharpe ratios increase from zero (our steady state assumption) to $\underline{\nu} = 85\%$. This value is chosen so as to replicate the change in US stock prices between February 19, 2021 (their

[36]Since it is difficult to gauge how long firms initially expected the crisis to last, in Section 7, we will study the robustness of our conclusions to this parameter choice.

[37]The table also provides more detail on the data sources used to measure these price responses.

pre-crisis peak) and the date of the first major policy intervention, March 23, 2020.[38] Table 3 shows that such a risk price shock helps the model produce credit spreads increases in the crisis state that are closer, in magnitude, to those observed during the months of February and March of 2020, though we do not target such spread response in our estimation.

Aggregate dynamics Starting from the steady-state of the model, the effects of the shock on expected future aggregates are reported in Figure 4 (solid blue lines). On impact, 0.4% of firms (in terms of aggregate capital) default (top left panel), and the aggregate gross investment rate falls from 11.3% p.a. to 7.3% p.a. (top right panel). Stock prices drop by 34% (middle left panel), while credit spreads increase by 323bps (middle right panel), as surviving firms' leverage ratios are immediately closer to the optimal default boundary $\underline{\bar{x}} < \bar{x}$. Average debt to capital (bottom left panel) initially falls before slowly recovering towards its long run value. The decline is driven by a selection effect (firms that initially exit are the most highly-levered ones) and by the fact that the intensity of debt issuance declines during the crisis.[39] As the economy recovers, investment progressively increases back toward its long-run average. Throughout the crisis, surviving firms finance some of their investment via share issuances (bottom right panel). The shock leads to a permanent response of aggregate capital, which ends up 4.8% below what it would have been absent the crisis (top right panel). Despite the large magnitude of the shock, firms' access to financial markets and the expectation that the crisis will be relatively short-lived allows them to partly smooth the shock and to limit its impact on capital growth, even absent government interventions.

5.3 Crisis dynamics with a sudden stop

Calibration As in the crisis without sudden stop, we assume that the marginal product of capital a during the crisis falls to $\underline{a} = 0.75a$, and that the Sharpe ratio rises from its steady-state value of zero to $\underline{\nu} = 85\%$. Different from the previous case, financial markets remain shut so long as the economy is in the crisis state. As discussed in Section 3.3.2, the sudden stop curtails investment and imposes an exogenous default boundary $\bar{x}_{ss}$ to firms. Table 3, Panel B., shows that, relative to the previous case, the sudden stop exacerbates both the drop

[38] While in the data, the effects of the crisis on ebitda and Sharpe ratios is measured over this one-month time period, for simplicity we assume in the model that the effects occur instantaneously. To get a sense for the magnitude of this increase, consider for example the asset pricing model of Lettau and Wachter (2007), who assume that Sharpe ratios follow AR(1) dynamics, with a half-life $\ln(2)/0.13 = 5.3$ years, ergodic average of 40%, and a conditional volatility of 12% p.a. This calibration implies an ergodic standard deviation of the risk-price distribution $\sigma_{erg} = 23.5\%$; thus, our modeled shock shifts risk prices from 1.7 σ_{erg} below the ergodic mean (before the shock) to 1.9 σ_{erg} above the ergodic mean (after the shock).

[39] Debt issuance intensity declines since the slope of the debt price function gets steeper (in absolute value). Debt to ebitda instead jumps on impact, from its steady state value of 2.1, to an immediate peak of 2.9, driven almost entirely by the exogenous decrease in the marginal product of capital a at the start of the crisis; debt to ebitda then gradually decreases towards its long run value.

in equity prices (they now fall by 40%, instead of 34%) and the increase in credit spreads (they now increase by 873 bps p.a., instead of 323 bps p.a.).

Aggregate dynamics Figure 4 reports the aggregate dynamics of the economy following the combination of the previous shock with a sudden stop (orange solid lines). Since firms can no longer finance investment and debt service payments by accessing debt and equity markets, the crisis-time default boundary is located at a much lower debt-to-capital ratio and in a region of the state space where the initial density of firms $\hat{f}_0$ is relatively high. This not only leads to a wave of immediate defaults — corresponding to 9.4% of the total capital stock (top left panel) as the crisis starts — but also to much higher expected future credit losses and credit spreads, for a prolonged period of time. This effect is exacerbated by the inability of firms to invest more than what is permitted by internally generated resources: as discussed in Section 3.3.2, investment is capped at a maximum $\bar{g}(x)$ that is decreasing in leverage. Constrained investment further exacerbates firms' drift towards higher leverage and eventual default. The crisis with a sudden stop leads to a 19.8% permanent decline in aggregate capital, vs. 4.8% when markets function normally. Overall, the sudden stop exacerbates the quantitative effects of the crisis.

Summarizing, without a sudden stop, our calibrated crisis leads to a long-run capital decline that is dampened by the fact that firms can use financial markets to smooth the effects of the shock and avoid a large cut-back in investment. Instead, with a sudden stop, investment declines significantly more for the duration of the crisis; moreover, the financial market shutdown triggers a wave of defaults early on, causing large deadweight losses. In the rest of the paper, we study how business funding programs would alter these outcomes.

6 Interventions with functioning financial markets

In this section, we study the effects that various government interventions could have on aggregate outcomes once the economy has entered a "crisis" state. We focus on the case when financial markets continue to function normally during this crisis. We derive several theoretical statements about the impact of policy interventions on aggregate outcomes, which we illustrate with numerical examples based on the calibration of the previous section. A common thread is that policy interventions primarily influence firms' decisions via their effect on the *effective cost of capital*, in other words the expected return that a firm needs to pay to claim-holders.

6.1 An irrelevance result for funding programs

Suppose that the government provides emergency funding to businesses during the crisis. Crucially, we assume that the funding comes *at the same price as private markets'* — a feature shared by many business funding programs used in the US (the CCF), the Eurozone (the CSPP) and the UK (the CBPS) in the recent past, as discussed in Section 2. Finally, we assume that the intervention does not affect investors' SDF.

Imagine for example that the government buys up to $\iota_g^d(x_t)k_t$ (per unit of time) notional of bonds issued by a firm that has capital k_t and leverage x_t.[40] Since those purchases are done at private market prices, firms are indifferent as to whether the funding is extended by the government or the private sector. The equity value $\underline{e}$ then satisfies

$$0 = \max\left[\alpha_k \underline{e}\left(\frac{\alpha_b}{\alpha_k}x\right) - \underline{e}(x)\,;\max_{\iota,g}\Big[-(R_e(x) - g + \chi)\underline{e}(x) + \underline{a} + \left(\iota_g^d(x) + \iota\right)\underline{d}(x) - \Phi(g) \right.$$
$$\left. - (\kappa + m)\,x - \Theta\left(\underline{a} - \kappa x\right) + \chi e(x) + \left[\iota_g^d(x) + \iota - (g+m)\,x\right]\underline{e}'(x) + \frac{\sigma^2 x^2}{2}\underline{e}''(x)\Big]\right] \quad (18)$$

Investment and default policies for this problem are identical to those in the laissez-faire environment. Instead, firms adjust their capital structure policy so as to "undo" the government intervention: private market debt issuances ι are tilted downwards, but total debt issuances $\iota + \iota_g$ (public and private) are identical to those in the laissez-faire environment. This reasoning can easily be generalized, leading to the following proposition.

Proposition 1 *Suppose that financial markets continue to operate without any friction during the crisis period. Any policy intervention (a) under which funding is provided to firms at market prices and (b) that does not affect investors' SDF (relative to the laissez-faire), does not have any impact on aggregate investment and growth.*

Proposition 1 captures interventions in the form of debt funding, equity funding, or in the form of hybrid instruments. It encapsulates any corporate credit guarantee scheme implemented by the government, so long as such scheme entails a fee payment by the corporate issuer that fairly compensates the government for the default risk underwritten. The intervention can also be conditional on firm's leverage. In the end, so long as the intervention is executed at market prices and so long as it does not alter firm's effective cost of debt and equity capital, it does not influence aggregate outcomes, as firms adjust their capital structure decisions so as to "undo" the government intervention.

Importantly, Proposition 1 assumes that the intervention does not affect market participants' SDF. At first glance, this assumption might appear unrealistic: in general equilibrium,

[40] This formulation encapsulates interventions in which the government can condition the funding extended on the leverage of the firm.

when the SDF arises from consumption-savings decisions made by a representative household who holds all the debt and equity claims in the economy, one might expect the purchase of corporate securities by the central bank to reduce the supply of such risky assets to the private sector, thereby affecting the SDF. However, if such public-sector purchases are financed via taxes that are levied onto households in lump-sum, Ricardian equivalence ensures that the representative investors' equilibrium consumption process (and thus risk-free rates and risk-price) are unaffected by the intervention. This logic rationalizes our choice to first focus on interventions that do not alter investors' SDF.

6.2 Subsidized loan programs

We next focus on interventions where the government purchases loans at subsidized prices from businesses. As highlighted in Section 2, direct or indirect subsidies to debt prices were a feature of many of the business programs implemented in 2020.

Description of the intervention The intervention consists in providing debt funding to the corporate sector at a required rate of return that is strictly below the market rate. Either the monetary or the fiscal authority can implement such policy via a "whatever it takes" announcement, telling the market it stands ready to purchase an unlimited amount of corporate bonds at some target price above the market. One of two outcomes can then occur: either (a) the announcement is credible, and credit market prices adjust immediately to the levels suggested by the government, or (b) the announcement is not credible, and the government is left acquiring the entire stock of debt securities issued by firms. Irrespective of whether (a) or (b) occurs, the effective cost of debt capital $R_d(x)$ faced by firms decreases upon announcement. Importantly, we assume that the intervention does not impact conditions in *equity* markets —- in other words, risk-free rates r_e and risk-prices ν_e for equity investors are unaffected.

Result: subsidized loan purchases are distortionary As the marginal investor in credit markets (the government) is now different from the marginal investor in equity markets, the intervention creates a wedge between (i) the required return on bonds $R_d(x)$ of the marginal debt investor and (ii) the corresponding return as perceived by shareholders $\tilde{R}_d(x)$. Firms take advantage of the cheaper cost of debt capital to increase their leverage, as suggested in Section 3.3.1 and equation (15), since the optimal debt issuance policy is increasing in the wedge $\tilde{R}_d(x) - R_d(x)$. Since financial conditions in *equity* markets are unchanged, and since firms' investment and default policies only depend on conditions in equity markets (as explained in Section 3.3.1), those policies are identical to those in the laissez-faire. Only the debt issuance policy is distorted by the intervention. The resulting higher future leverage then affects negatively future aggregate investment and growth rates through the debt overhang channel, as our next proposition concludes.

Proposition 2 *Suppose that financial markets continue to operate without any friction during the crisis period. Any policy intervention that leads to a decreases in the effective cost of debt capital $R_d(x)$, but that leaves the risk-free rate r_e and the price of risk ν_e prevalent in equity markets unchanged, has a negative impact on expected future aggregate investment and output.*

Proposition 2 not only captures central bank purchases of corporate bonds at above market prices, but also credible commitments to make such purchases, with corresponding adjustments to risk prices ν_d or risk-free rates r_d in credit markets. It encapsulates government-provided subsidized corporate credit guarantee schemes. So long as firms' effective cost of debt capital decreases, and so long as financial conditions in equity markets are unaffected, future investment and growth will be lower than without intervention.

As a practical example, consider the MSLP implemented by the Federal Reserve and under which debt funding was extended to eligible firms at a fixed yield of LIBOR + 3%. Such program allows firms with private market debt returns above LIBOR + 3% to obtain credit at a subsidized rate. According to Proposition 2, such program, if implemented when financial markets function normally, and if it does not alter equity market conditions, leads to strictly worse future aggregate outcomes relative to the laissez-faire.

A quantitative example To quantify the magnitude of the distortion induced by subsidized loan programs, we provide an illustration of our proposition when the government announces that it will be purchasing corporate debt at an implied risk-free rate that is 200bps below the equity market risk-free rate throughout the crisis.

Figure 5 shows aggregate bond issuances in the economy with the policy intervention (in yellow) and in the laissez-faire environment (in blue). While investment and default policies are identical in both environments, all firms, faced with a lower effective cost of debt capital, increase debt issuances and pay relatively more dividends than in the laissez-faire (middle right panel), as firms effectively distribute proceeds from the "arbitrage" opportunity created by the government intervention. This high debt issuance activity in response to the debt purchase program implies that, one year after the start of the crisis, the aggregate debt to capital ratio is expected to be 0.5% below steady state, versus 2% in the laissez-faire. Thus aggregate leverage is persistently *higher* with the debt purchase program. This depresses aggregate investment via the debt overhang channel, even though the magnitude of the effect is relatively small — an observation we will come back to in Section 7.1. Finally, we note that neither credit nor equity market prices react *on impact*, as equity market conditions are unchanged and as bond prices are pinned down via equation (9). This observation can be viewed as an empirical test for whether our assumptions in this section hold.

6.3 Grants programs

We next turn to interventions where the government provides grants to businesses, similar to the PPP described in Section 2. We first consider programs where firms are not subject to restrictions in the use of funds. Motivated by the discussion of Section 2, we then highlight the effects of imposing dividend payment restrictions for firms receiving public funds.

6.3.1 Unrestricted grants

Description of the intervention We focus on transfer programs under which a firm with capital k_t receives $\phi(a - \underline{a})k_t$ per unit of time during the crisis. $\phi = 0$ is the laissez-faire, while $\phi = 100\%$ makes firms *pre-tax* cash-flows identical to those in the pre-crisis environment. Since we assume the grant receipts are non-taxable, this government intervention is isomorphic to an increase in the marginal product of capital, from $\underline{a}$ to $\underline{a} + \phi(a - \underline{a})/(1 - \Theta)$. The related expected present value of aggregate fiscal costs is detailed in Appendix A.2.11.1.

Results Given our crisis modeling as a cash-flow *and* risk-price shock, even if the government grants were sized so as to make firms' *after-tax* cash-flows equal to their pre-crisis level, economic aggregates still deteriorate (in comparison to the balanced-growth path) due to the increase in Sharpe ratios during the crisis. Figure 5 illustrates this point. It suggests that the financial market shock contributes significantly more to our economy's response than the cash-flow shock. One can then study the efficiency of such grant program, by calculating the magnitude of the improvement in aggregate ebitda (measured as $\mathbf{E}[Y_t]$ at different time horizons) as a function of the size of its fiscal cost (by varying ϕ).

Figure 8 demonstrates the modest "bang for the buck" of grant programs. Even when the government spends the equivalent of 10% of aggregate pre-crisis ebitda to inject funds to firms, expected future output 5 years after the start of the crisis is barely 0.8% above what it would be in the laissez-faire. Grant receipts are mostly used by firms to pay additional dividends, as the bottom right panel of Figure 5 indicates, reducing the stimulative effects of the intervention onto investment. This suggests that restrictions on the use of grant proceeds might improve the efficacy of such programs.

6.3.2 Grants with strings attached

Description of the intervention Consider a grant program identical to the one described in Section 6.3.1, but that includes a restriction on *dividend payments* during the crisis. We focus on dividend restrictions since they should improve the efficacy of such interventions, and since they were commonly used in the programs implemented in 2020 (see Section 2).

Results Payout restrictions add a friction in the problem of shareholders. As described in Section 3.3.2, whenever the dividend restriction binds, the first order condition $\underline{e}'(x) + \underline{d}(x) = 0$ no longer holds and corporate investment decisions are distorted *upwards*.

Figure 6 shows how this restriction changes the economic effects of the grants program. In order to isolate the impact of the dividend restriction, we compare the case $\phi = 0$, in which the government does not provide any funds and only imposes a moratorium on dividend distributions, to the case $\phi = 100\%$, where both grants and the moratorium are active. A moratorium on dividends without government funding does not improve aggregate outcomes significantly, as the restriction binds for only a small fraction of firms. Instead, when the government provides grants, the fraction of constrained firms is larger, and those firms not only increase investment but also use some of the grant proceeds to buy back bonds from private market investors, proactively reducing leverage and thus increasing investment via the debt overhang channel. Since the effects of a shift in the leverage distribution are persistent, grants with "strings attached" end up having a larger impact, increasing the expected aggregate future capital stock 5 years after the shock by 1.2 p.p., relative to the unrestricted grants program (reported in Figure 5).

The two middle panels of Figure 8 illustrate how the efficacy (defined as the improvement in expected future output per unit of fiscal cost) of the program with the restriction varies with the size of the grants. Compared to the top panel (which reports the effects of the program without restrictions), for a given fiscal cost, adding dividend restrictions improves the long-run effects of the program on aggregate ebitda.

A constraint to limit shareholder distributions indirectly weakens the commitment problem faced by firms' management. Dampening the incentive to take on debt leads to lower corporate leverage and higher future corporate investment. This begs the question of whether temporary or conditional limits on shareholder distributions might have positive effects in situations beyond those studied in this paper, a point we come back to in the conclusion.

6.4 Expansionary announcement effects

In Proposition 2, we assumed that the government, via an announcement or actual debt purchases, only affects the pricing of risk in debt markets. In practice, though, interventions of this type may also have an impact on equity markets, as the events following the Spring 2020 Fed announcements suggest. This can lead to expansionary effects, as we now discuss.

Result We now assume an intervention that reduces the effective cost of equity capital $R_e(x)$, either via a reduction in risk-free rates r_e or risk prices ν_e. As discussed in Section 3.3.1, such a reduction causes an increase in Tobin's q for all firms, leading to an immediate stimulative effect on corporate investment, as our next proposition concludes.

Proposition 3 *Suppose that financial markets continue to operate without any friction during the crisis period. Any policy intervention that decreases the effective cost of equity capital $R_e(x)$ of firms will, on impact, increase Tobin's q and thus aggregate investment.*

Importantly, Proposition 3 only relies on the assumption that the policy intervention affects risk-free rates r_e or risk-prices ν_e prevalent in equity markets, since the equity value $\underline{e}$ (and thus Tobin's q and corporate investment) does not depend on conditions prevalent in credit markets. When these markets are integrated, the decrease in Sharpe ratios does not create adverse debt financing incentives, as $R_d(x)$ and $\tilde{R}_d(x)$ decrease by identical amounts. However, if Sharpe ratios in credit markets decrease more than Sharpe ratios in equity markets following the policy intervention, its effects on future aggregate outcomes will be ambiguous. While *on impact*, aggregate investment will be stimulated by the decrease in equity markets' risk prices, *over time* firms' increased incentive to issue debt (due to the wedge $\tilde{R}_d(x) - R_d(x)$) will lead to higher leverage, which will put downward pressure on capital growth via the debt overhang channel, as already illustrated in Section 6.2.

Quantitative effects The model can be used to go beyond the qualitative prediction of Proposition 3, and allows us to quantify the impact of the Spring 2020 Fed announcements on economic aggregates. To do so, we solve a version of the model with integrated debt and equity markets (i.e. $r_e = r_d = r$ and $\nu_e = \nu_d = \nu$) where the crisis-time Sharpe ratio $\underline{\nu}$ is calibrated so as to match the more moderate equity market price decline observed between (i) February 19, 2020 (i.e. just before the shock) and (ii) April 30, 2020, a date a few weeks after the Fed announcements.[41]

Figure 7 plots the response of the economy (i) in the laissez-faire (as in Figure 4), and (ii) when the Fed's announcements result in lowered equity risk prices. The announcement effects are very large: while aggregate capital suffers a permanent decline of 4.8 p.p. in the laissez-faire, the corresponding decline in an economy with improved financial market conditions is only 1.9 p.p. — i.e. a 60% reduction in the magnitude of the shock. Most

[41] We do not take into account the staggered timing of the two events —– that is, the delay between the initial economic shock, and the Fed announcements. Instead, we pick the crisis-time Sharpe ratio $\underline{\nu} = 20\%$ that matches a 14.0% initial decline in the total market value of equity in the model, which corresponds to the decline in S&P 500 from the pre-recession peak on February 19, 2020 (3,386) to April 30, 2020 (2,912), a few weeks after the Fed's intervention. Thus, given a laissez-faire crisis-time Sharpe ratio of 85%, one can view the Fed announcement as dampening the increase in Sharpe ratio, from 85% to 20%. See Table 3 for additional details.

of this effect stems from the differential response in gross investment, which declines by 4 percentage points in the laissez-faire environment, versus only 1.6 percentage points with the intervention. Contrary to the thought experiment of Section 6.2, both equity and bond prices react immediately upon announcement, with credit spreads (respectively equity risk-premia) only 69bps (respectively 10.1 p.p.) higher than before the crisis, as opposed to 323bps (respectively 45.8 p.p.) absent intervention.

Figure 8 shows how expected future aggregate ebidta at different time horizons varies under different decreases in risk-prices induced by the government announcement. Each percentage point reduction in Sharpe ratio is associated with a 0.05% increase in expected aggregate ebitda 5 years after the shock. Importantly, this analysis excludes any potential costs — due to moral hazard, or any other distortion — that the market perception of a permanent "Fed put" would have on firms' choices, as well as on investment decisions; we come back to this point in our conclusion.

Our results shed light on the effectiveness of the *announcement* of the different Fed corporate lending facilities at the height of the Spring 2020 crisis. Given the small amount of corporate bonds actually purchased under the CCF and the low take-up of loans by SMEs under the MSLP, it was often concluded that those Fed emergency actions had been a failure. By contrast, the counter-factual calculation above suggests that, without the programs, aggregate investment and growth would have suffered more substantially. Importantly, the model indicates that the success of the programs should not be measured by quantity of funding deployed, but rather by the impact of their announcement on the effective cost of capital of firms, which is the ultimate driver of investment decisions in the model.

7 Interventions during a sudden stop

In the previous section, we studied a variety of government interventions when financial markets function normally during the crisis. We now study the other extreme scenario — a "sudden stop", in which firms lose access to both credit and equity markets.

7.1 Subsidized loan programs

We start by discussing interventions where government participations take the form of loans.

7.1.1 Description of the intervention

Consider a loan program structured as follows: between t and $t + dt$, the government advances up to $\iota_g(a - \underline{a})d_g k_t dt$ to any firm with capital k_t during the crisis period, in exchange for $\iota_g(a - \underline{a})k_t dt$ notional of debt. ι_g represents the size of the intervention, whereas $d_g \leq 1$

is the implied price at which the government advances debt funding. Our modeling choice allows us to capture important features of loan programs implemented during 2020: fixed interest rates (captured by d_g) and borrowing limits, as emphasized in Section 2.

When the government offers to acquire debt issued by firms at a fixed price d_g, firms decide whether to take advantage of this funding facility, and to what extent. This choice leads to a *negative* selection effect. As shown in Appendix A.2.10.1, firms whose equity sensitivity to leverage $-\underline{e}'(x)$ is greater than the fixed price d_g will decline the government funding, while firms with $-\underline{e}'(x) < d_g$ will maximize the amount of public funding they can get.Thus, for any policy parameter d_g, there is a cutoff x_g^* so that firms with $x_t < x_g^*$ do not use public funding, while firms with $x_t > x_g^*$ do. For the latter firms, the funding extended is thus subsidized by an amount equal to $\iota_g(a - \underline{a})k_t \left(d_g - \underline{d}(x_t)\right) dt$ at each time period.

7.1.2 An untargeted loan program with full income replacement

We first consider the case $\iota_g = 100\%$ (so that the funding extended is at most equal to the lost business revenues due to the crisis) and $d_g = 1$ (so that the government buys debt at par). As debt in our model trades at a discount[42], all firms accept the government support and there is thus no negative selection effect.

Results Figure 9 illustrates the (very large) impact of this intervention. The aggregate capital stock suffers a permanent shock of 6.4% 5 years after the start of the crisis, rather than 19.8% in the laissez-faire economy. The government intervention not only allows a number of firms to avoid bankruptcy during the crisis (since the default boundary $\bar{x}$ is pushed towards higher leverage levels), but it also helps stimulate investment (as Tobin's q increases, and as the investment rate cap $\bar{g}(x)$ is raised due to the additional government-provided funds).

The downside of the intervention is clear, when focusing on the trajectory of the investment to capital ratio: firms emerge from the crisis with greater debt burdens (compared to the laissez-faire), which lowers investment during the recovery (top right panel of Figure 9). Crucially, however, the effect is small. It takes 1.7 years for the aggregate capital stock to return to positive growth without the intervention, versus 2.0 years with the intervention, and differences in growth rates during the recovery phase are small: three years out, aggregate gross investment is 11.5% p.a. with the intervention, compared to 11.7% without it.

Debt overhang thus seems to have only a moderate impact on the depth of the crisis and the speed of the recovery, despite the persistent effects of the program on leverage. As a result, the short-run "level" effect (the reduction in deadweight losses associated with bankruptcies) far outweighs any long-run "growth" effects of debt overhang.

[42]In our model, $d(x) \leq 1$ since we calibrate $\kappa = r$.

Why are the effects of debt overhang so small? Heuristically, in the data as well as in our model, average net debt-to-ebitda is equal to 2.1. Moreover, the real effects of our crisis are modeled as a 25% decline in firms' ebitda for one year on average. Thus, a credit market intervention with full income replacement is expected to raise the average debt-to-ebitda ratio by 0.25 (from 2.1 to 2.35) at the start of the recovery period. Since the average sensitivity of gross investment to leverage (in both our model and the data) is equal to -1.04, our credit market intervention will, on average, reduce aggregate investment by $-1.04 \times 0.25 = -0.26\%$ per year, which is small compared to the average gross investment rate of approximately 12%. This can also be seen in Figure 2a, which shows that the "modal" firm occupies a part of the state-space where investment is only moderately sensitive to leverage. This observation, in combination with the relatively modest — but persistent — increase in leverage induced by the program, suggests that the "marginal" debt overhang generated by the program is small.

7.1.3 An untargeted loan program with partial income replacement

As noted in Section 2, many BFPs put explicit restrictions on loan amounts. These restrictions reduce the overall implicit subsidies to firms but they also blunt the benefits of these programs. We can quantify these effects by looking at aggregate outcomes for different values of the policy parameter ι_g, keeping the debt pricing parameter $d_g = 1$ fixed.

The top two panels of Figure 10 illustrate our results, by plotting various aggregate outcomes (vertical axis) vs. the fiscal cost of the intervention (horizontal axis) for varying policy parameters ι_g. $\iota_g = 0$ is the laissez-faire, while $\iota_g = 100\%$ corresponds to a full income replacement program.[43] The top left panel shows that increasing the replacement rate from 50% to 100% approximately doubles the total fiscal costs, but only improves 5-year expected gains in aggregate ebitda (relative to the laissez-faire) from 13% to 16.7%. Similarly, the fraction of total capital lost to initial defaults is 4% with 50% income replacement, vs. 1.1% with full replacement (compared to 14% in the laissez-faire). Thus the marginal gains of an untargeted intervention decrease with the size of its fiscal costs, as more and more public funds end up being used to pay dividends rather than being invested in productive capital.

7.1.4 A targeted program

Changing the (fixed) pricing d_g of program loans can induce a negative selection effect, since firms whose shadow debt valuation is above d_g (which are also firms with lower leverage) do not participate in the program. Thus the choice of the fixed interest rate (or, in our model,

[43] In order to compare the outcomes with our baseline case, we set $d_g = 1$ for the comparative statics reported in the top panel of Figure 10.

the choice of d_g) creates a trade-off for program design. A more targeted program, with a higher fixed interest rate (that is, lower d_g), will only include more levered firms; this reduces its fiscal costs, but also its aggregate effects, since fewer firms end up participating.

The middle panels of Figure 10 trace out the frontier associated with different pricing parameters d_g. They report expected aggregate ebitda $\mathbb{E}[Y_t]$ at different time horizons, as well as initial default rates, as a function of the fiscal cost of each intervention, varying d_g.

The main insight from this exercise is that the aggregate effects of the program are highly non-linear with respect to the degree of targeting. For example, a policy parameter $d_g = 60\%$ has a modest fiscal cost of 0.2% of pre-crisis ebitda, and only targets firms closest to default. But it also has large aggregate effects, with 5-year expected future ebitda 11% higher than in the laissez-faire. Targeting the program towards highly levered firms has a modest fiscal cost, as they don't represent a large share of capital in the model, but keeping them alive prevents a large amount of deadweight bankruptcy costs and helps maintain corporate investment, while it has only limited future debt overhang costs. Moreover, broadening the intervention to lower leverage firms (that is, increasing d_g) achieves relatively small improvements with respect to default and output, at the expense of much larger fiscal costs.

Comparing the top and bottom panels of Figure 10 also sheds light on the efficacy of targeted vs. untargeted programs. For a given fiscal cost, targeted programs offer a higher "bang for the buck", and particularly so when the fiscal cost is small. They focus support on firms that are closer to default, and whose exit is responsible for most of the deadweight losses in the absence of intervention, instead of transferring resources to lower-leverage firms that remain far from default throughout the crisis.

The MSLP program did involve this form of targeting, since the loans extended via the commercial banking sector all carried a fixed LIBOR + 3% coupon, guaranteeing that only firms with private sector debt funding costs above such levels would be willing to participate. From the cost/benefit analysis of Figure 10, this fixed loan pricing is desirable.[44]

7.1.5 Robustness

Alternative model calibrations could threaten our conclusion that the positive short term effects of loan-based government funding programs exceed the long run debt overhang costs.

Crisis duration $1/\chi$ If the length of the crisis (and thus the size of the program loans needed) is materially longer than in our current calibration ($1/\chi = 1$ year in expectation), it is possible that debt-based interventions saddle firms with so much additional leverage

[44] As described in Section 2, the MSLP also involved other eligibility criteria, such as a limit on borrowers' debt to ebitda ratio. Within the model, this restriction would have the effect of barring precisely those firms most in need of emergency funding (the highest-leverage firms) from participating. Our model suggests that such eligibility criteria should be avoided, as they might diminish the program's efficacy.

that eventually, the overhang costs can no longer be neglected. In Figure A-2, we plot aggregate outcomes in the laissez-faire (featuring the sudden stop) as well as the full income replacement loan intervention when assuming a longer crisis duration $1/\chi = 3$ years. As anticipated, the additional leverage induced by the intervention is higher than in our benchmark calibration (bottom left panel). While the full income replacement program is still a desirable policy (compared to the laissez-faire), its effects are much more muted.

Adjustment cost γ While the parameter $\gamma = 7.16$ — governing adjustment costs and an important determinant of the slope of investment to leverage — is estimated using information from the cross-section of firms, we also consider an alternative calibration $\gamma = 5$, which exacerbates debt overhang. With this alternative calibration, the model-implied moment $\hat{\Gamma}$ (that is, the weighted average sensitivity of investment to the debt to ebitda ratio) is equal to -1.60, rather than -1.04 — a more than 50% increase in the degree of debt overhang. Instead, the ergodic density $\hat{f}_0$ as well as the steady-state financing and default policies are similar to our base case calibration. As illustrated in Appendix Figure A-3, the impact of the crisis with a sudden stop on aggregate outcomes, as well as the effects of a loan-based intervention program, are substantially muted. The crisis with sudden stop leads to significantly lower initial defaults (than in our base case calibration) as firms can cut investment more aggressively with a lower parameter γ. The benefits of the loan-based intervention are then reduced, but not sufficiently to be outweighed by the debt overhang costs.

Bankruptcy costs α_k Since the key benefit of a government loan program stems from its ability to delay bankruptcies and reduce the amount of deadweight losses from default, one might ask what happens in versions of our model where capital destructions from default are smaller. We solve our model with a capital recovery rate α_k that is 50% larger than in our base case calibration. While this calibration preserves the degree of debt overhang in the model, it leads to a counter-factual steady state leverage density $\hat{f}_0$ — as the cost of default is smaller, shareholders take advantage of it to default sooner, leading to an ergodic debt-to-ebitda ratio that is approximately half what we measure in the data. As expected, with lower capital destructions, a crisis with a sudden stop leads to more muted effects on aggregate outcomes, as depicted in Appendix Figure A-4. But once again, the loan program does not lead to overhang effects that are large enough to offset their benefits.

7.2 Grant programs

We next focus on grants programs similar to those analyzed in Section 6.3: firms receive grants worth $\phi(a - \underline{a})k_t$ per unit of time. ϕ governs the size of the program; $\phi = 100\%$ corresponds to full income replacement, while $\phi = 0$ is the laissez-faire.

When financial markets continue to function during the crisis, Section 6.3 showed that those interventions improve aggregate outcomes, but that their efficacy is limited. By varying the parameter ϕ, we can trace the frontier of (a) fiscal cost of the intervention vs. (b) expected future aggregate ebitda $\mathbb{E}[Y_t]$ at various horizons. The two bottom panels of Figure 10 show that a grants program, during a sudden stop, indeed leads to significant improvement. For low fiscal costs, every incremental percent point of aggregate pre-crisis ebitda spent on such intervention improves $\mathbb{E}[Y_t]$ 5 years after the crisis by more than 2 p.p.

Figure 10 also shows that the "bang for the buck" of this type of program is substantially smaller than that of the subsidized loan programs. A grant program with $\phi = 25\%$ achieves aggregate outcomes comparable to those of the untargeted, full-replacement loan program, but has a fiscal cost that is about twice as large. Intuitively, the sudden stop that occurs during the crisis starves firms of needed cash to service debt and support investment; any cash advanced by the government during such period of stress will be useful in postponing default and reducing deadweight losses; during the recovery period, as financial markets reopen, firms can afford to repay their obligations — hence why a loan program ends up significantly reducing the fiscal costs incurred by the public sector, as compared to a grants program. Moreover, the downside of the loan program — the increase in corporate leverage and related reduction in corporate investment during the recovery — is too small to meaningfully alter its efficacy.

As with the loan programs of Section 7.1, the efficacy of transfer programs can in principle be enhanced by targeting highly levered firms. To the extent that size and leverage are negatively correlated (Crouzet and Mehrotra, 2020), limiting program participation depending on size, as the PPP did, could be a way to partially achieve this targeting.

7.3 Other program designs

Our study of government business credit programs focuses mostly on implementations in the form of loans or grants, with potential announcement effects. Our primary objective was to analyze programs that have been used over the past few years, as described in Section 2. However, one could also consider alternatives designs, as we discuss next.

Equity injections With debt purchase programs, the government injects funds into firms in exchange for debt claims. While such interventions reduce the magnitude of the initial wave of defaults, surviving firms end up taking on more debt, which depresses investment during the recovery. To mitigate the resulting debt overhang effects, one could instead design a policy with identical expected fiscal costs, but according to which the government obtains either (i) shares of firms receiving such emergency funding, or (ii) convertible debt.

However, the small magnitude of the debt overhang effects suggests that such alternative program design, while improving on credit market interventions, might only have small additional benefits. Moreover, these benefits have to be weighed against the un-modeled costs of such intervention: it may be challenging to convince existing firm owners to cede partial ownership to the government, making the implementation more complex and potentially slower, and delaying the intervention at a time when firms' need for funds is urgent.

Loan forbearance The government could also consider broad-based debt forbearance policies, under which firms are allowed to delay their debt interest payments. Such delayed payments are then capitalized, and remain payable as markets reopen when the crisis subsides. This type of intervention has been used in the US agency mortgage market as a tool to mitigate the impact of the crisis on households.[45] While a forbearance program, applied to the corporate credit market, is unlikely to have large benefits if financial markets remain open (the case of Section 6), it may have positive effects on investment and defaults when firms lose access to external sources of funding, given that it allows them to save cash for investment purposes.

Such policy would however be complex to implement. Indeed, it would impose potential costs onto debt investors. Since unpaid interest is capitalized, (a) creditors do not receive current interest on their debt, and (b) firms' leverage increases, potentially worsening default risk. These severely adverse consequences for existing creditors could make such an intervention hard to implement in practice.

8 Conclusion

Business Funding Programs (BFPs) are a novel feature of the public policy response to the crisis. In this paper, we developed a model to quantify the potential impact of this new kind of intervention on corporate leverage and investment. The model stresses debt overhang as a potential long-run distortion created by BFPs.

Our main findings are as follows. First, if, during a downturn, capital markets continue to function normally, BFPs have ambiguous aggregate effects. A BFP offering debt at a subsidized rate (without affecting equity market conditions) is a bad policy idea, as it simply depresses investment via a debt overhang channel. On the other hand, a BFP that ends up reducing market Sharpe ratios is a good idea, as it stimulates investment via a Tobin's q channel. Second, if the downturn is accompanied by a sudden stop in financial markets, a

[45]Under the CARES act, home mortgages that were purchased by one of the mortgage agencies after origination (that is, close to 2/3 of outstanding residential mortgages in the US) could benefit from a temporary suspension of their required monthly payments. Note however that those payments are not forgiven; instead, they are owed later on, either as a lump-sum payment or smoothed over a certain time period.

debt-based BFP offers a substitute source of funding for firms, and thereby helps avoid large output losses driven by forced liquidations. This "level" effect is substantially larger than the negative "growth" effect of the overhang of debt created by the BFP. The key quantitative insight leading to this conclusion is that in the estimated model, most firms operate in a part of the state-space where investment is not very sensitive to leverage, so that the incremental BFP debt has little impact on their investment decisions. Additionally, because debt overhang effects are small, the efficacy of debt-based BFPs can be improved by targeting firms most at risk of defaulting. Finally, we show that these insights are robust various alternative model calibrations (some decreasing the benefit of debt-based interventions, some increasing the cost of debt overhang).

While our focus is primarily on US BFPs, our results are also useful speak to the effects of corporate lending programs in other contexts. More generally, they challenge the view that a high amount of corporate debt issuance in response to credit interventions (particularly among rated, publicly traded firms) portends a slow recovery and lackluster investment.

One important additional message of our paper is that credit market interventions can operate not only through the quantity of funding they provide to the corporate sector, but also through their impact on risk free rates and Sharpe ratios that firms face when making investment and financing decisions. We thus add support to a growing empirical literature that documents that the pass-through of unconventional monetary policies to the real economy, via the corporate sector, occurs via "announcement" effects. One critical piece missing in our analysis is the potentially negative consequences (due, for instance, to moral hazard) that the perception of a permanent "Fed put" would have on the choices of firms. We leave such analysis for future work.

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	Government participation	Borrower eligilibity	Interest rate	Restrictions on borrowers	Funding cap
CCF	Debt (any seniority)	Based on credit rating	Prevailing market rates	None	None
MSLP	Debt (senior)	Size + Debt/ebitda $\leq$ 4 or 6	LIBOR + 3%	Dividends + debt repurchases	Up to debt/ebitda $\leq$ 4 or 6 + max loan size
PPP	Debt convertible to grant	Size	1%	Funds must be used to cover op. costs	Based on past wage costs

Table 1: Summary of key features of the 2020 Business Funding Programs (BFPs) in the United States. For the CCF, only investment-grade or firms downgraded after March 22nd, 2020 were eligible for direct purchases; ETFs with high-yield exposure were also eligible. The CCF could only purchase up to 25% of individual issuances or 10% of the total debt outstanding of particular firms, but there were no caps on total issuance by the borrower. Size restrictions for the MSLP were based on 2019 revenue or employment. Funding caps depended on the facility used (new loan, priority loan, or expanded loan). For the PPP, grant convertibility is based on the firm using loan proceeds to cover payroll costs, rent, and payments on utilities. For more details, see the main text; see also, on the CCF, Boyarchenko et al. (2020); on the MSLP, Crouzet and Gourio (2020); and on the PPP, Granja et al. (2020).

Panel A. Calibrated parameters

Parameter	Description	Value	Source
r	risk-free rate	0.05	Crouzet and Eberly (2020) (Figure A3)
m	debt amortization rate	0.10	Saretto and Tookes (2013) (Table I, Panel B)
δ	depreciation rate	0.10	Hennessy and Whited (2005) (Table III)
Θ	corporate income tax rate	0.35	OECD (2020) (Table II.1)
α_k	1−deadweight losses	0.33	Kermani and Ma (2020) (Table I, Panel A)
α_b	1−debt haircut	0.15	Bris, Welch and Zhu (2006) (Table XIII)

Panel B. Estimated parameters

Parameter	Description	Point estimate	Standard error	$[5,95]$ normal confidence interval
a	average product of capital	0.24	0.01	$[0.22, 0.25]$
σ	volatility of capital quality shock	0.31	0.03	$[0.27, 0.35]$
γ	curvature of capital adjustment cost	7.16	0.35	$[6.59, 7.74]$

Panel C. Model fit

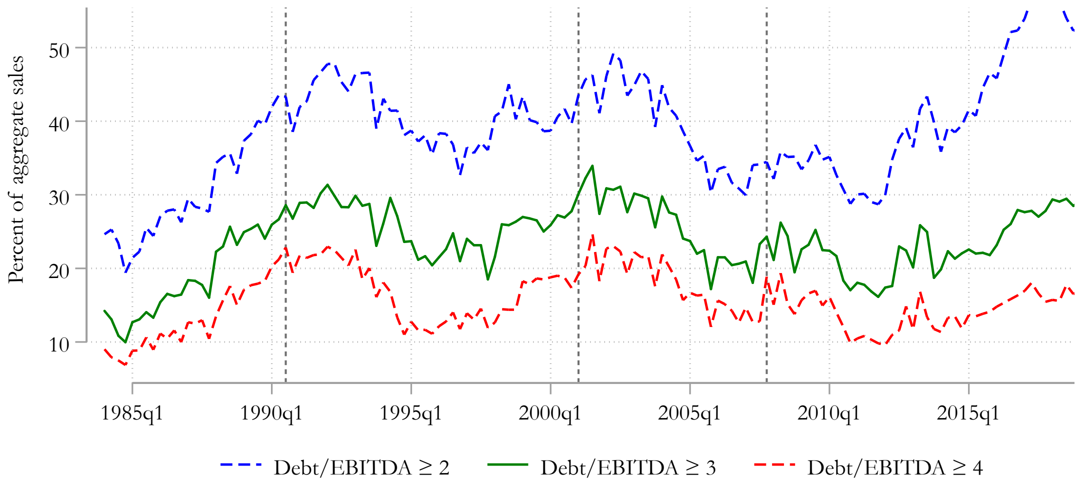
Moment	Description	Source	Targeted?	Data	Model
$100 \cdot \hat{\Phi}$	gross investment rate	Compustat sample	✓	11.28	11.28
$\hat{z}$	debt/ebitda		✓	2.13	2.14
$\hat{\Gamma}$	slope of inv. wrt debt/ebitda		✓	−1.04	−1.04
$100 \cdot \kappa\hat{z}$	inverse interest cov. ratio	Compustat sample	✗	11.3	10.7
$100 \cdot \widehat{\pi/e}$	equity payout rate		✗	4.6	3.0
$100 \cdot \widehat{\iota/x}$	debt issuance rate		✗	25.7	17.9
$100 \cdot \widehat{x}$	book leverage		✗	25.6	51.0
$100 \cdot \hat{\sigma}_e$	equity volatility	Choi and Richardson (2016)	✗	43.6	50.7
$100 \cdot \hat{cs}$	average credit spreads	Feldhütter and Schaefer (2018)	✗	0.87/4.17	4.98
$100 \cdot d(\overline{x})$	debt recovery rate	Ou, Chiu and Metz (2011)	✗	29.3	12.1
$100 \cdot \lambda_d$	default rate	S&P (2019)	✗	1.0	1.5
$100 \cdot G$	aggregate growth rate	BEA Fixed Asset Tables	✗	1.9	0.5

Table 2: Baseline estimation. Panel A reports the sources for the model parameters which we calibrate. The tax rate is the US pre-2017 statutory corporate income tax rate, drawn from the OECD's Tax Database. Additionally, we set $\kappa = r$, so that the price of risk-free debt is normalized to 1. Panel B reports values for the three estimated parameters. We use a two-step feasible GMM approach, and data on non-financial public firms in 2019, as described in Section 4. Panel C reports model and data moments: targeted moments (first three lines); non-targeted moments drawn from the same sample used for the estimation (lines 4 through 7); and non-targeted moments drawn from other sources (lines 8 through 12). For credit spreads, the two numbers reported correspond to average corporate bond yield spreads for investment-grade and high-yield firms, respectively. Details on Compustat data is reported in Appendix A.3.1, and details on sources for key moments are reported in Appendix A.3.5.

	Ebitda jump $100 \cdot (Y_{0+}/Y_0 - 1)$	Equity jump $100 \cdot (E_{0+}/E_0 - 1)$	Credit spread jump $100 \cdot (\overline{cs}_{0+} - \overline{cs}_0)$
A. Data			
Before Fed announcement	-25.0	-34.0	3.57 / 7.30
After Fed announcement	-25.0	-14.0	1.51 / 4.06
B. Model			
Cash flow shock	-25.0†	-5.2	0.12
Cash flow + risk premium	-25.0†	-34.0†	3.23
Cash flow + risk premium + announcement	-25.0†	-14.0†	0.69
Cash flow + risk premium + sudden stop	-25.0†	-39.9	8.73

Table 3: Calibration of the crisis state. In the data, we measure the pre-intervention crisis impact on market prices by focusing on the change of various market indices between February 19, 2020 and March 23, 2020 (the date of the Fed facility announcements); the S&P 500 index declined from 3,386 to 2,237, while the option-adjusted spread of the ICE BofA BBB (resp. high yield) US corporate index increased from 131bps p.a. to 488bps p.a. (resp. from 357bps p.a. to 1,087bps p.a.). We measure the post-intervention crisis impact on market prices by focusing on these same market indices, between February 19, 2020 and April 30, 2020, so as to encapsulate *both* (i) the real effects of the pandemic (ii) the Fed announcement effects: over that time period, the S&P 500 index decreased from 3,386 to 2,912, while the option-adjusted spread of the ICE BofA BBB (resp. high yield) US corporate index increased from 131bps p.a. to 282bps p.a. (resp. from 357bps p.a. to 763bps p.a.). Instead, the pre- (and post-) intervention crisis impact on ebitda is measured via the change in quarterly ebitda of our sample of firms in Compustat, from Q4 2019 to Q2 2020. Moments marked with † are those targeted by the calibration. We pick the two parameters $\underline{a}$ (the marginal product of capital in the crisis state) and $\underline{\nu}$ (the Sharpe ratio in the crisis state) in order to match the drop in aggregate ebitda and in aggregate stock market capitalization reported in the first two columns of Panel A.

(a): Corporate leverage prior to the 2020 recession



(b): Effects of the Secondary Market Corporate Credit Facility

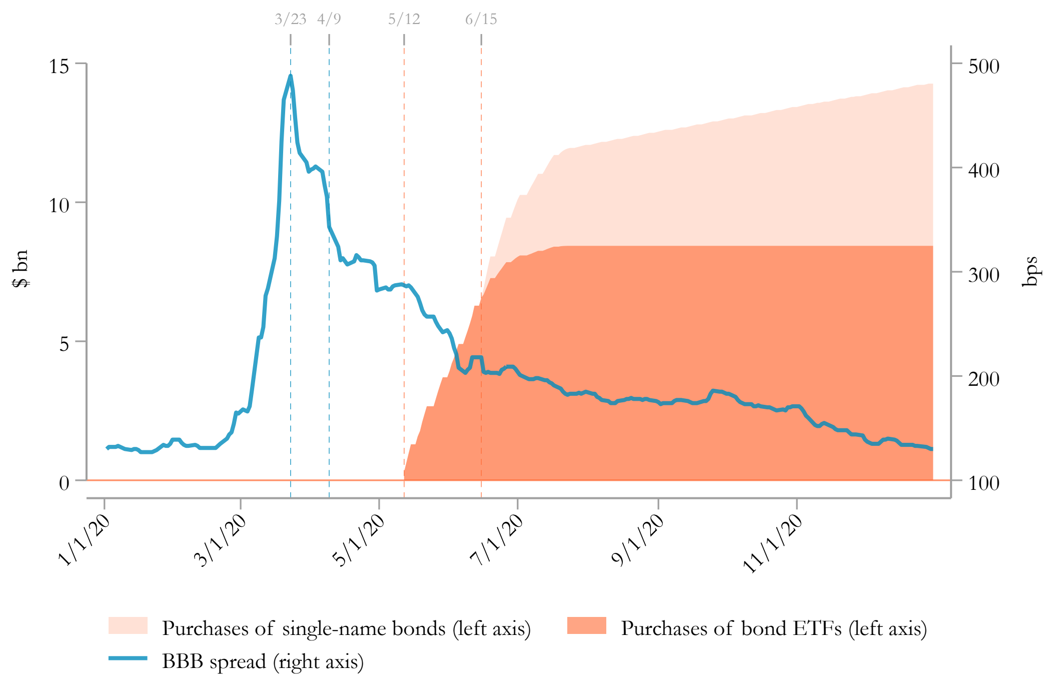


Figure 1: Corporate leverage prior to the 2020 recession (top panel), and effects of the Secondary Market Corporate Credit Facility (bottom panel). The top panel reports the percentage of total sales of public firms accounted for by firms with debt-to-EBITDA ratios above certain thresholds; sample selection and variable definitions are reported in Appendix A.3. In the bottom panel, the shaded areas report the cumulative gross purchases of bond ETFs (dark orange) and single-name bonds (light orange) realized under the SMCCF until its closure, on 12/30/2020. The blue line reports the ICE/BofA BBB corporate option-adjusted spread, which measures the option-adjusted spread of an index of corporate securities rates BBB over the corresponding spot Treasury rates (FRED series BAMLC0A4CBBB). The two dates highlighted with blue lines correspond to the Fed's announcement of the Corporate Credit Facilities, on March 23rd, 2020, and of their extension, on April 9th, 2020. The two dates highlighted in orange correspond to the start of the purchases of ETFs (May 12th, 2020) and single-name bonds (June 16th, 2020).

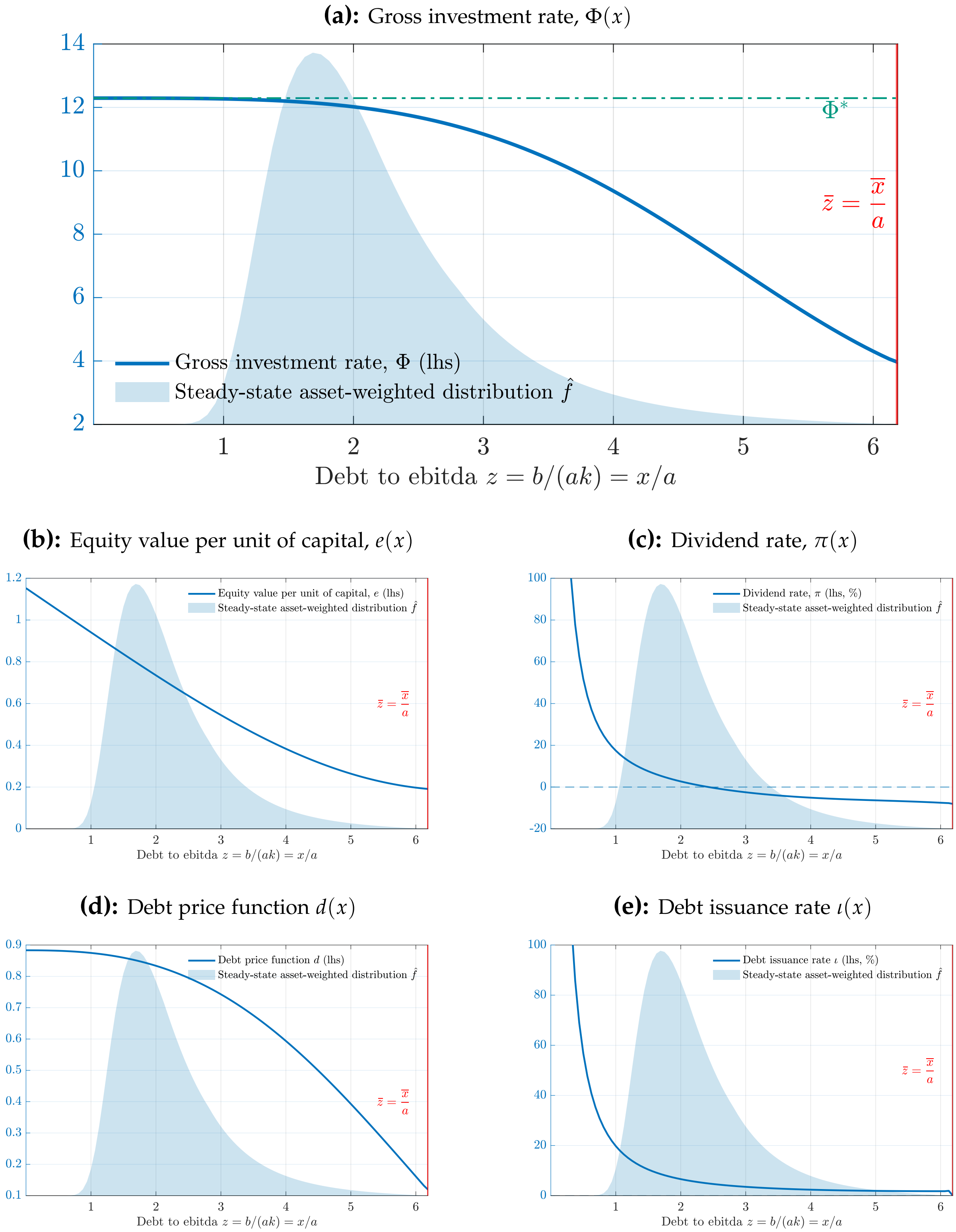


Figure 2: Policy functions in the steady-state of the model of Section 3. In all graphs, the shaded blue area represents the steady-state asset-weighted (or distorted) distribution $\tilde{f}$, and the red line to the right of the graph represents the default threshold. Φ^* refers to the gross investment rate in the no-debt model. All graphs are plotted with $z = x/a$ on the horizontal axis, instead of x, in order to facilitate interpretation of the units. The calibration used is reported in Table 2.

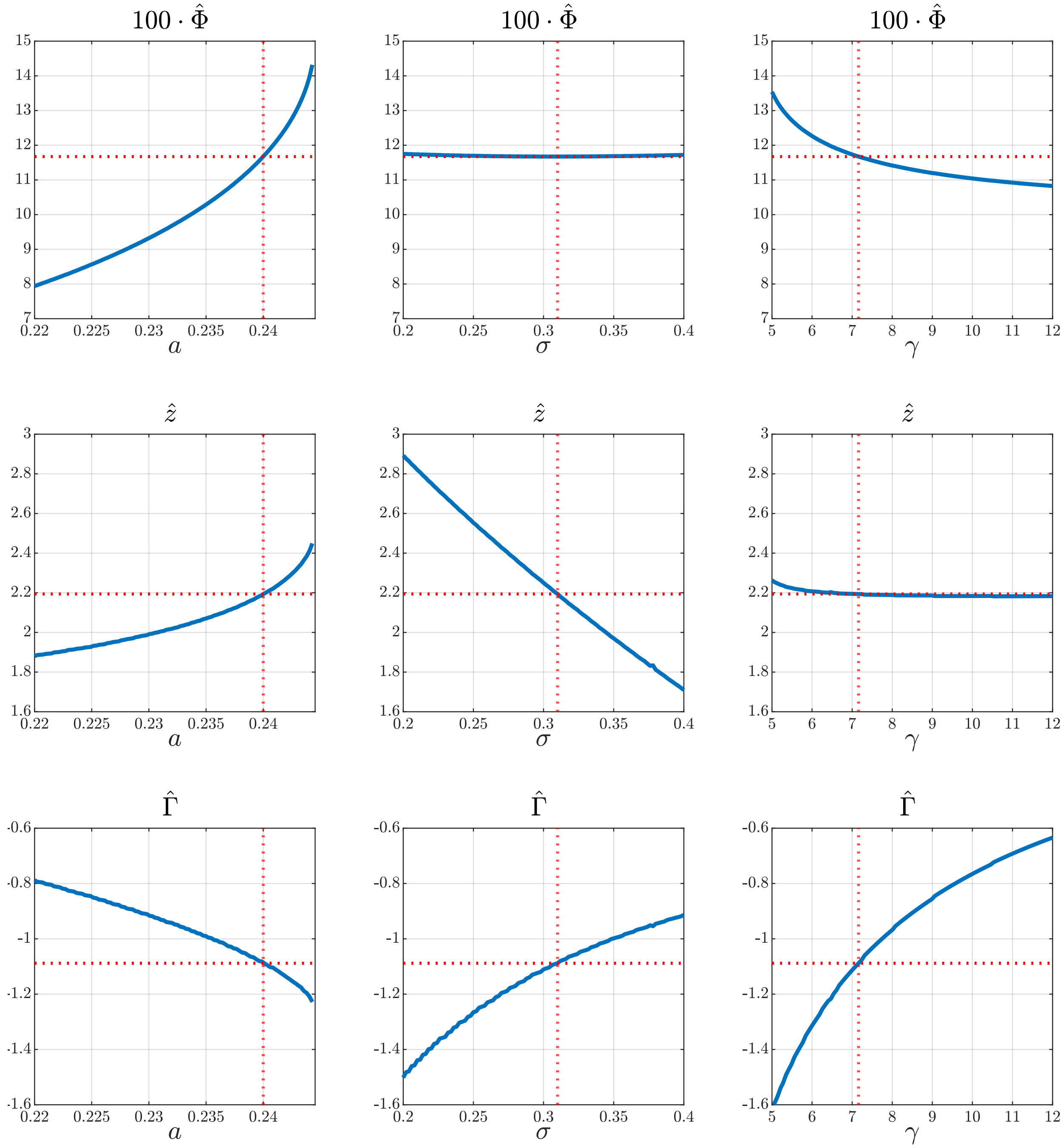


Figure 3: Comparative statics of the model of Section 3, for the three moments targeted in the baseline estimate. Each row of panels corresponds to a different moment: the top row corresponds to $100 \cdot \hat{\Phi}$, the average gross investment rate; the middle row corresponds to $\hat{z}$, the average debt/ebitda ratio; the bottom row corresponds to $\hat{\Gamma}$, the sensitivity of gross investment with respect to leverage. Each column corresponds to a different parameter: the first column corresponds to a, the average return to productive capital; the second column correspond to σ, the volatility of capital quality shocks; and the third column corresponds to γ, the parameter governing the convexity of investment adjustment costs. In each graph, the solid blue line is the value of the moment reported in the graph, and the two red lines highlight the value of the parameter and the moment at the point estimate in our baseline estimation, which corresponds to $a = 0.24$, $\sigma = 0.31$, and $\gamma = 7.16$ for the parameters, and $100 \cdot \hat{\Phi} = 11.28$, $\hat{z} = 2.13$, and $\hat{\Gamma} = -1.04$ for the moments.

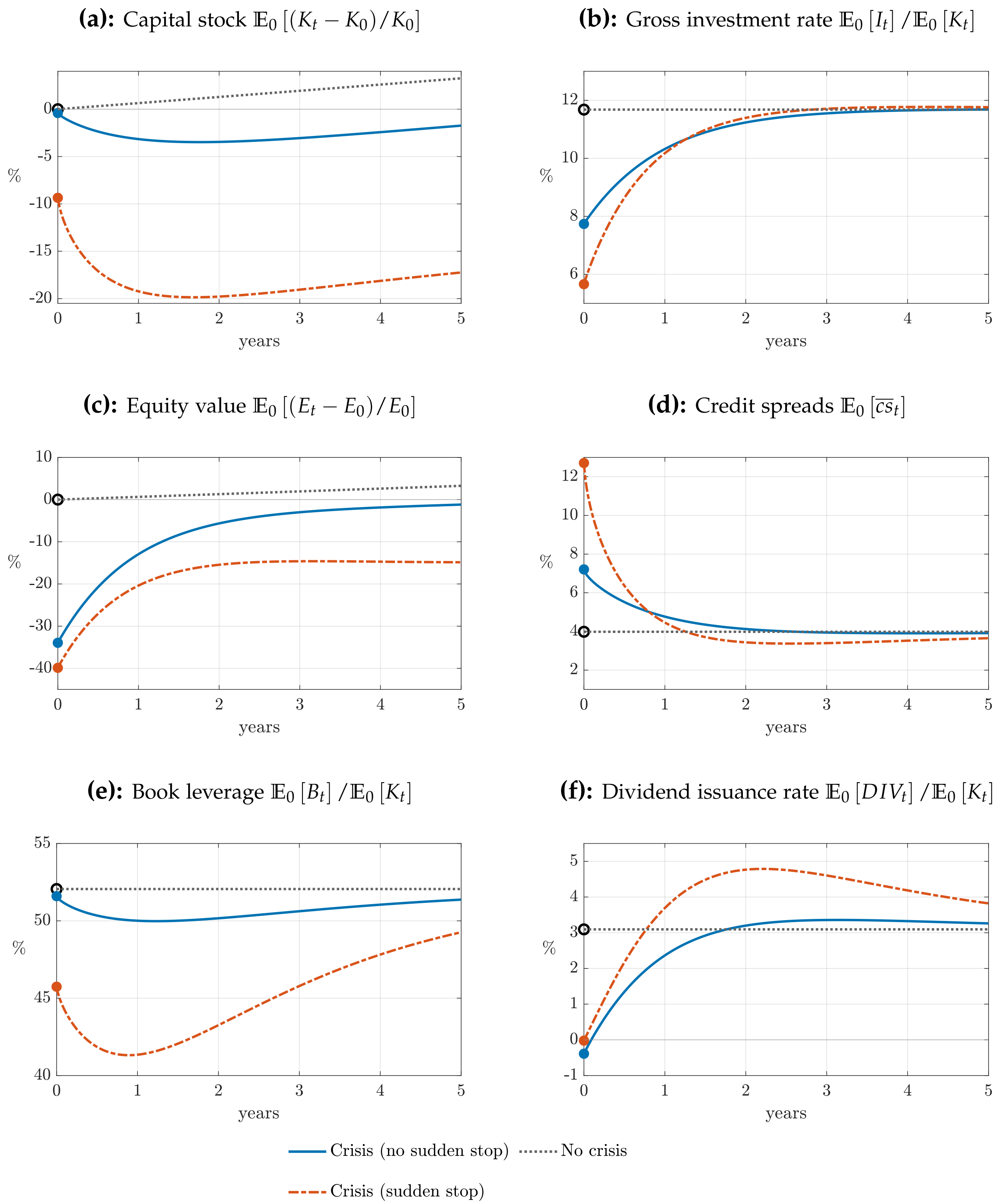


Figure 4: Crisis dynamics, with and without sudden stop. The solid blue lines report the expected path of economic aggregates following an unexpected crisis, if financial markets continue to function normally (no sudden stop). The long dashed orange lines report the expected path of economic aggregates following an unexpected entry into the crisis state, if financial markets shut down (sudden stop). In both cases the average duration of the crisis is one year, the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

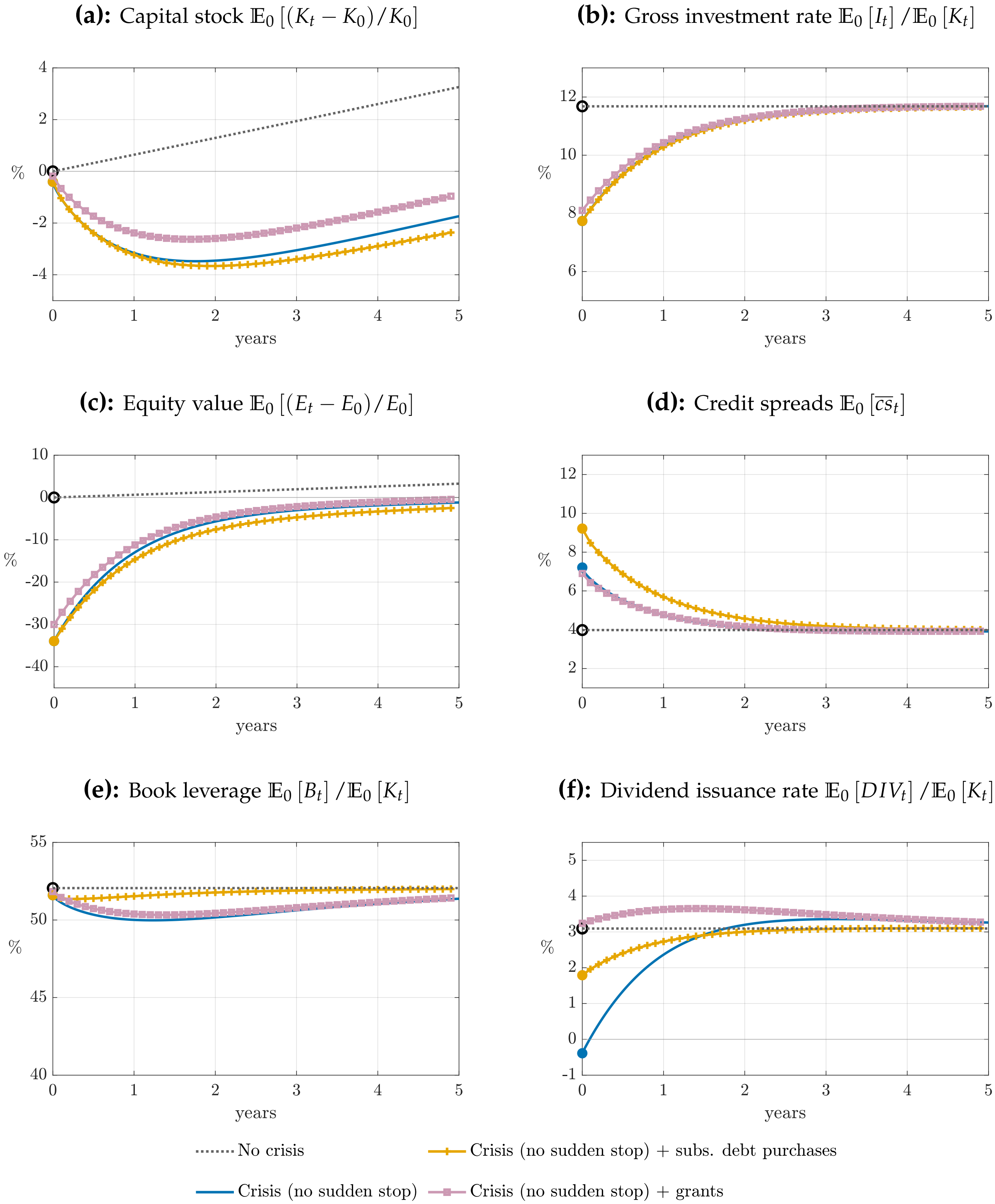


Figure 5: Crisis without sudden stop, with and without government interventions. The solid blue lines report the expected path of economic aggregates following an unexpected crisis, if financial markets continue to function normally (no sudden stop) and the government does not intervene. The dotted yellow lines report responses if the government provides subsidized loans, as described in Section 6.2, while the squared purple lines report responses if the government provides grants, as described in Section 6.3. The crisis absent government intervention is calibrated as described in Section 5.1. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

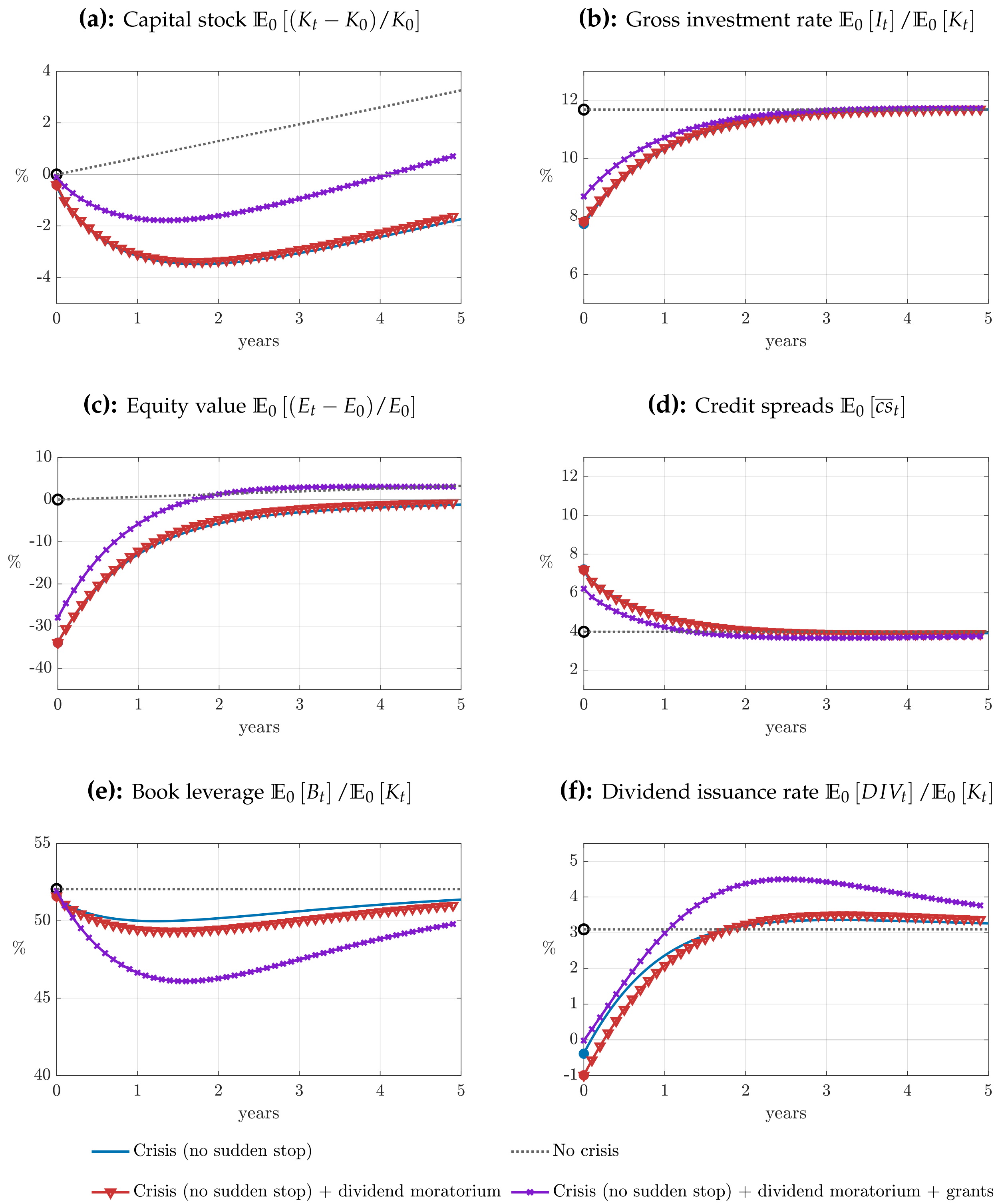


Figure 6: Crisis without sudden stop: adding restrictions on borrowers. The solid blue lines report the expected path of economic aggregates following an unexpected crisis without government intervention. The triangle red lines assume the government implements a temporary moratorium on dividends, and the crossed purple lines assume that the moratorium is combined with a grants program, as in Section 6.3.2. The average duration of the crisis is one year; the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

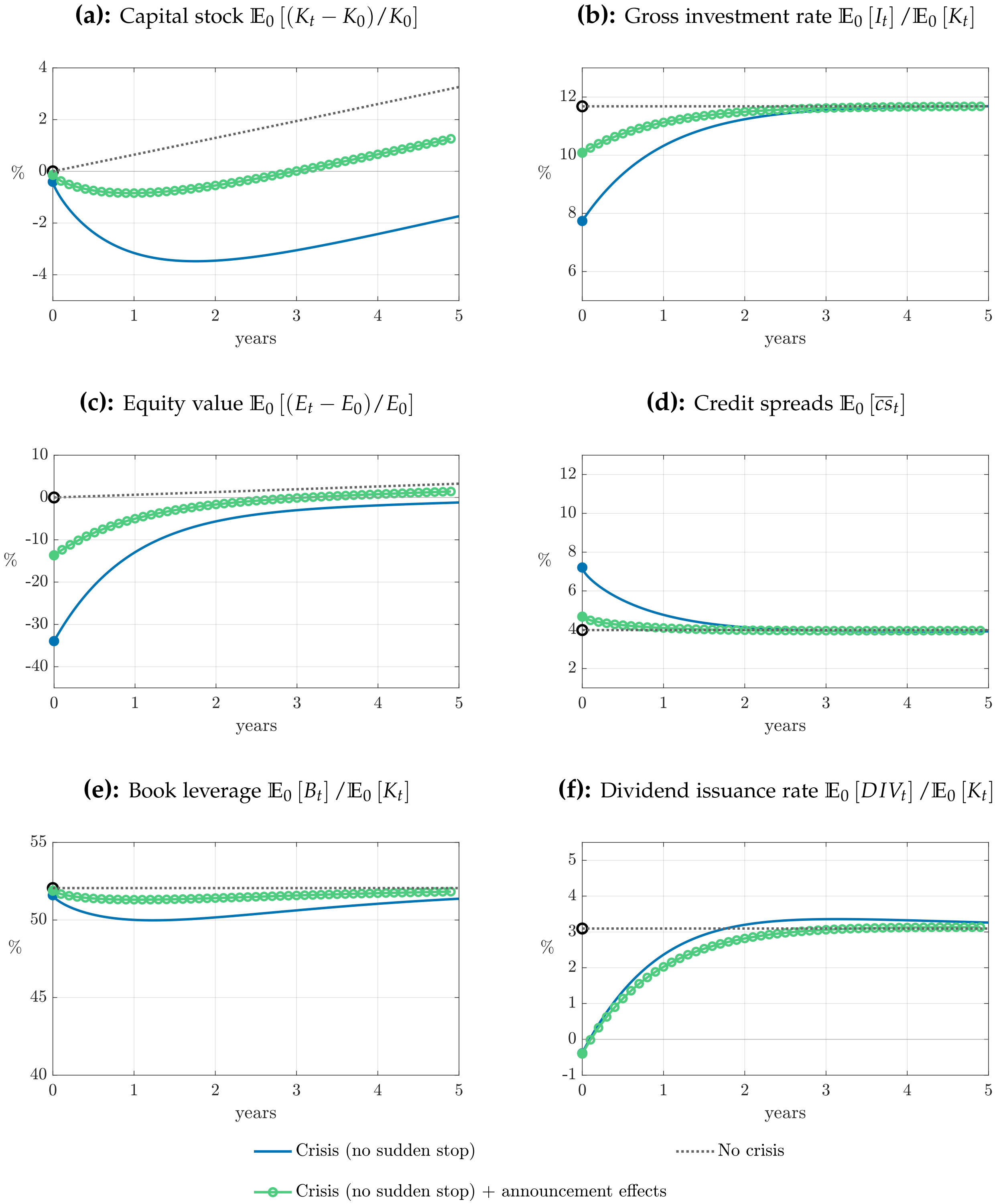


Figure 7: Crisis without sudden stop: announcement effects. The solid blue lines report the expected path of economic aggregates following an unexpected crisis without government intervention. The circled green lines assume report the path of economic aggregates when the government makes an announcement that reduces the Sharpe ratios on debt and equity, as described in Section 6.4. The average duration of the crisis is one year; the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

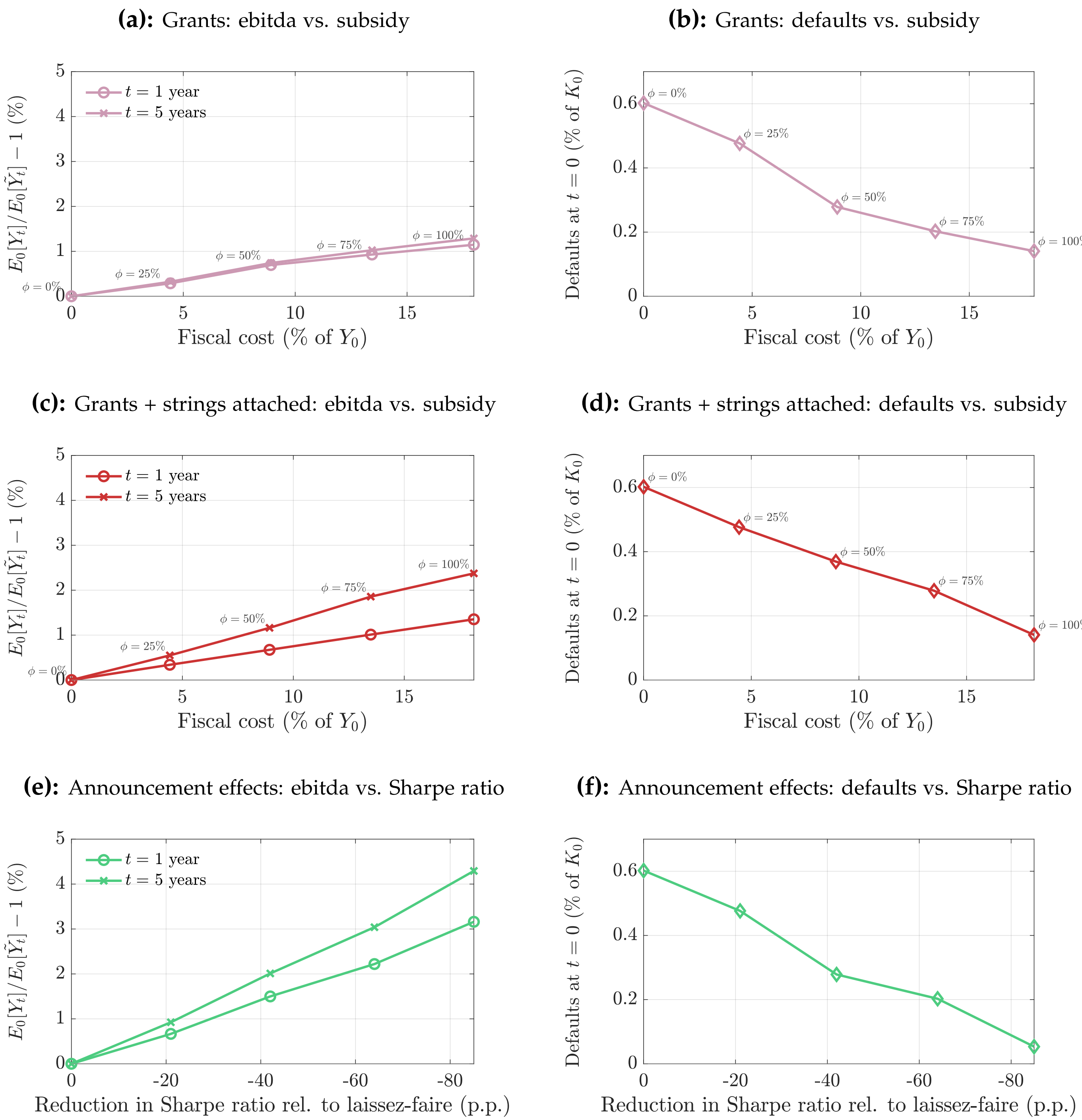


Figure 8: Comparative statics for different interventions in the crisis without sudden stop. The top panels report comparative statics for the grants program described in Section 6.3, for different values of the replacement rate ϕ. The top left panel reports the change in aggregate ebitda relative to laissez faire at different horizons, while the top right panel reports defaults rate on impact. The middle two panels repeat this exercise for the case when the grants program is combined with a dividend moratorium, as in Section 6.3.2. The bottom two panels report the same aggregate moments, for interventions that reduce the Sharpe ratios on equity and debt by different amounts, as described in Section 6.4.

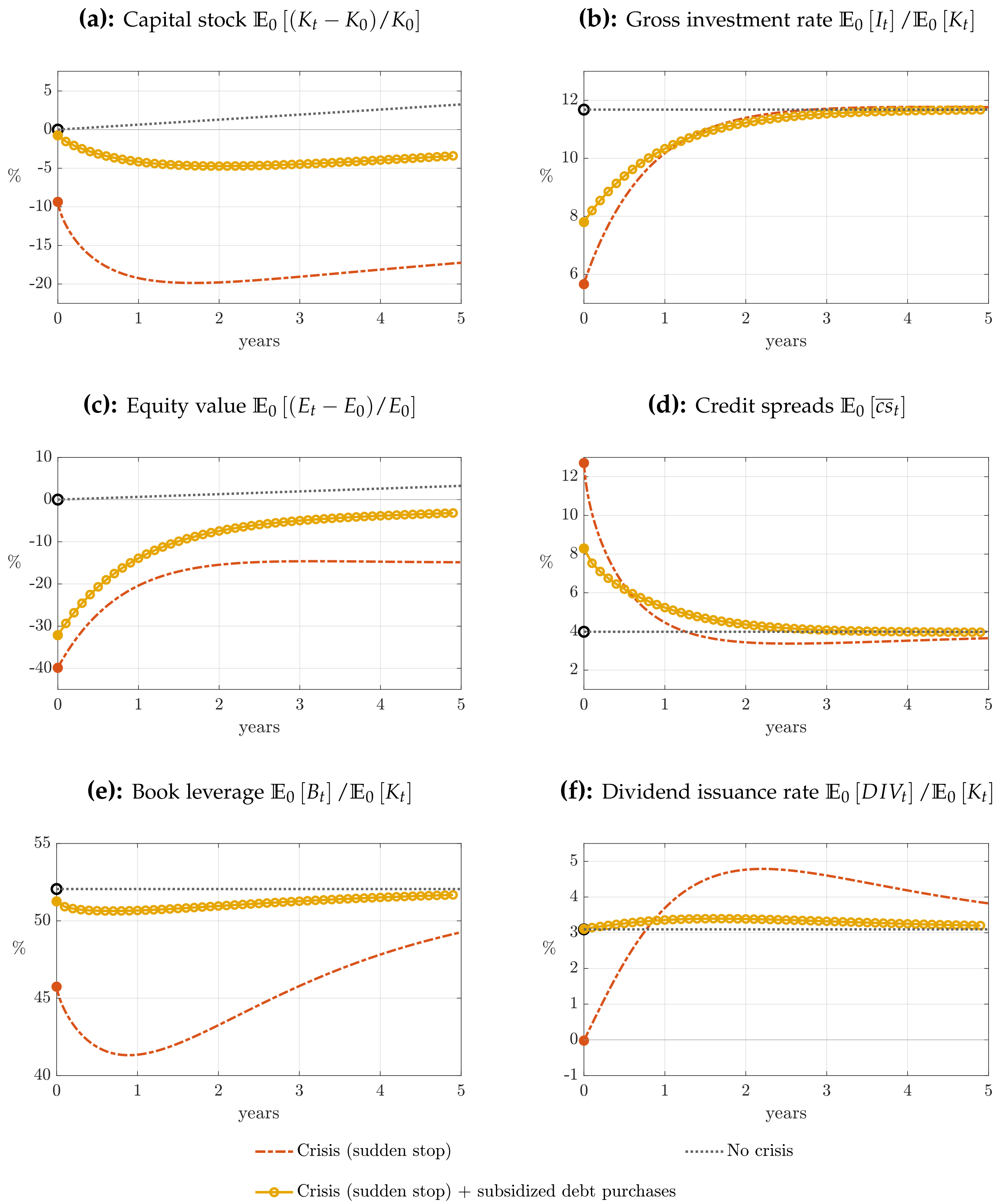


Figure 9: Crisis with sudden stop: the effect of subsidized debt purchases. The long dashed orange lines report the expected path of economic aggregates following an unexpected crisis, if financial markets shut down (sudden stop) and the government does not intervene. The circled yellow lines report the expected path of economic aggregates if the government provides subsidized loans, as described in Section 7.1. The average duration of the crisis is one year; the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

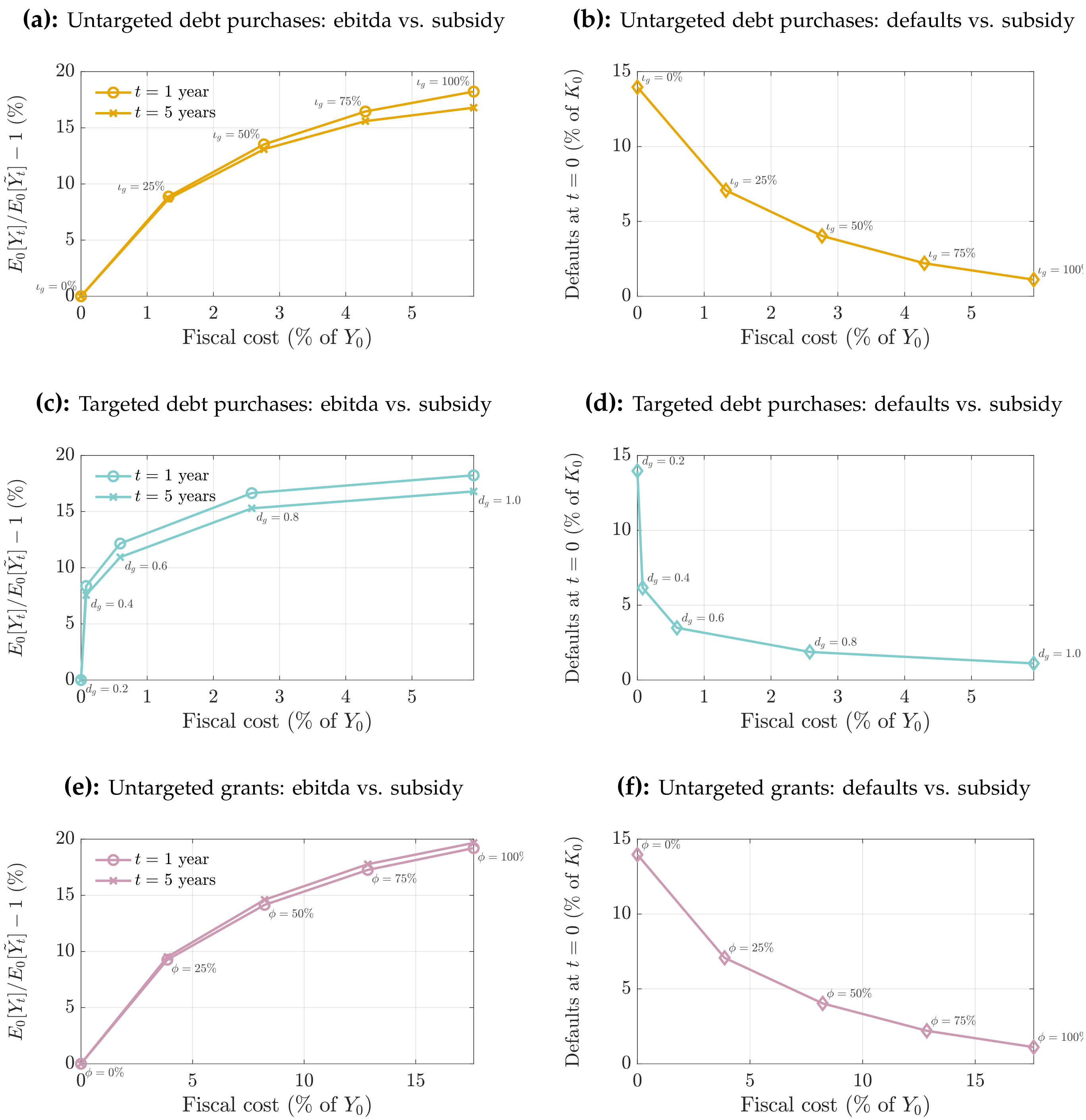


Figure 10: Comparative statics for different interventions in the crisis with sudden stop. The two top panels report comparative statics for the untargeted debt purchases described in Section 7.1.3, for different values of ι_g, the replacement rate of the program. The top left panel reports the change in aggregate ebitda relative to laissez-faire at different horizons, while the top right panel reports defaults rate on impact. The middle two panels report results for the targeted program described in Section 7.1.4, where d_g determines the degree of targeting. The bottom two panels report the results for untargeted grants with different replacement rates, as described in Section 7.2.

Internet Appendix

Nicolas Crouzet and Fabrice Tourre

A.1 More details on BFPs

A.1.1 Details on US BFPs

We briefly summarize the key features of the three main business funding programs that were implemented in the US in 2020: the Corporate Credit Facilities (CCF), the Main Street Lending Program (MSLP), and the Paycheck Protection Program (PPP). The CCF and MSLP were initially scheduled to operate until September 30th, 2020. The date was eventually extended to December 31st, 2020, when both programs stopped new purchases. New applications to the PPP stopped on August 8th, 2020.

The Corporate Credit Facilities (CCF) Two Corporate Credit Facilities (CCF) were initially announced on March 23rd, 2020.[46] The Primary Market Corporate Credit Facility (PMCCF) was created to allow the Fed to purchase corporate bonds and syndications on the primary market, while the Secondary Market Corporate Credit Facility (PMCCF) was meant to allow the Fed to participate in the secondary markets for single-name corporate bond and bond ETFs. Both were created under section 13(3) of the Federal Reserve Act.

The PMCCF and the SMCCF were each initially funded with a \$10bn equity investment from the Treasury's Exchange Stabilization Fund (ESF), with the Fed providing an additional \$90bn, allowing for total purchases of up to \$200bn. After the passage of the Coronavirus Aid, Relief and Economic Security (CARES) act, on March 27th, the ESF was expanded to \$454bn. On April 9th, some of these additional funds were earmarked to allow the total scale of the facilities to reach up to \$750bn. The announcement is available here.

The CCF targeted firms with access to bond markets.[47] Initially, the program was restricted to investment-grade rated firms, but the April 9th announcement expanded eligibility to firms that had been downgraded since March 22nd, and also allowed purchases of ETFs with exposure to high-yield bonds. Other than credit ratings, neither the PMCCF nor the SMCCF had other eligibility criteria. All Fed purchases were done at market prices, without caps on the size of individual purchases, and participation was not subject to restrictions on the use of funds. The CCF facilities could not hold more than 25% of an individual issuance (for the PMCCF) or 10% of total issuances (for the SMCCF) of a particular firm. Note that the PMCCF allowed new issuances to be used to refinance existing debt. Official term sheets for the Corporate Credit Facilities are available here (for the PMCCF) and here (for the SMCCF). Boyarchenko et al. (2020) provides a more detailed discussion of the program.

[46] They were subsequently expanded through additional Treasury funding authorized by the Coronavirus Aids, Relief and Economic Stimulus (CARES) act.

[47] As is well known, rated firms make up a small fraction of the firm population, but a large fraction of corporate assets and corporate investment; see, e.g., Faulkender and Petersen (2006) or Crouzet and Mehrotra (2020).

The Main Street Lending Program (MSLP) The Main Street Lending Program (MSLP) was first announced on April 9th, at the same time as the expansion of the CCF. The announcement is available here. The aim of this program was to provide loans to small and medium-size firms. Participating borrowers had to meet certain size criteria (fewer than 15,000 employees, or less than $5bn revenue in 2019).

Three facilities were created: two of them allowed the Fed to buy newly issued loans, while the third allowed it to fund the upsizing of existing loans. Two additional facilities, for non-profit entities, were added on June 15th, 2020; the announcement is available here. The facilities were funded with a $75bn equity contribution from the Treasury's ESF, allowing, in principle, total purchases to reach $600bn. As of January 31st, 2021, the average size of loans outstanding was $9.54mm, with a minimum of $0.10mm and a maximum of $300mm. Loan-specific information on MLSP purchases is available here.

The low take-up relative to the potential scale of the program may partly reflect the requirements imposed on both lenders and borrowers. Under the terms of the program, all MSLP loans had to be be originated by banks, in exchange for a fee. Originating lenders had to retain a 5% participation in the loan. Loan size and leverage were subject to caps. The minimum loan size, initially set at $0.5mm, was eventually lowered to $0.1mm; the maximum loan size varied from $30mm to $300mm across facilities. Borrower eligibility was also subject to a leverage constraint: firms could borrow up to no more than four times 2019 EBTIDA (or 6, in the case of the Priority facility), effectively excluding firms that had high debt to EBITDA ratios in 2019 from participating. Interest rates on MSLP loans are set at LIBOR plus 3%, regardless of the underlying financial conditions of the borrower. All loans made were five-year maturity, with a two-year grace period for principal payment. MSLP loans also had to be contractually senior to other forms of debt, though security was not explicitly required. Finally, borrowers' use of funds was tightly restricted. Borrowers were prevented from using proceeds from MSLP loans to refinance existing debt, and they must also follow the limits on distributions to shareholders outlined in the CARES act. Borrowers had to make "commercially reasonable efforts" to maintain employment for the time during which the MSLP loan is outstanding. Official information on the MSLP is available here; see also Crouzet and Gourio (2020) for a discussion of the program.

The Paycheck Protection Program (PPP) The CARES act, adopted on March 27th, included the creation of the Paycheck Protection Program (PPP). This program is meant to allow Treasury to lend to small firms (those with less than 500 employees) via the Small Business Administration. PPP loans were made through private banks, but were fully guaranteed by the SBA. The Fed's involvement in the program was minimal: it only provided funding to banks originating the loans, through its Paychek Protection Program Liquidity Facility

(PPPLF). The CARES act initially endowed the SBA with $349bn to fund loans to businesses, eventually expanding the sum to $669bn on April 24th, given the high demand by small businesses for funds under that program. A total of approximately $525bn of loans across approximately 5mm lenders were made through the first wave of the program. On January 11, 2021, the PPP was re-activated, following passage of the Consolidated Appropriations act of 2021. The act included an additional $284bn of loans convertible to grants, following the model of the first wave. As of March 28th, 2021, an additional $211bn of loans had been extended as part of this second wave. Official information on the program is available here.

The main goal of the program is to make loans to small businesses, with loan proceeds that can be used by firms to cover interest, payroll, rent, and utilities. Loans have maturity of up to five years and an interest rate of 1%. Loan caps are calculated on the basis of past payroll costs of the borrower. Most importantly, loans can be partially or totally forgiven, provided the borrower meets certain criteria, particularly regarding employee retention. Granja et al. (2020), Hubbard, Strain et al. (2020), and Lutz et al. (2020) also discuss in detail the conditions, take-up, and impact of the program.

A.1.2 BFPs in other advanced economies

In many other advanced economies, while BFPs were an important part of the policy response to the 2020 events, they had already been in use prior to those events.

Programs implemented following the 2020 crisis In response to the events of 2020, a number of European economies developed and implemented ad-hoc programs in order to provide credit support to firms. In the five largest economies European economies (Germany, the United Kingdom, France, Italy, and Spain), *government participations* under these programs generally involved loan guarantees, as opposed to direct lending.[48] The guarantees were generally underwritten by public financial institutions (such as Germany's KfW or France's BPI) with implicit backing from their respective fiscal authorities. *Eligibility* to specific programs was determined on the basis of firm size, with countries offering programs designed for SMEs (such as the Bounce Back Loan Scheme in the UK) and for large firms (such as the Coronavirus Large Business Interruption Loan Scheme, or CLBILS, in the UK). Access to the guarantee schemes were also restricted by a European Union directive, adopted in March 2020, which barred firms that already were "in difficulty" as of 12/31/2019 from participating, with the notion of "difficulty" defined in, particular, in terms of existing leverage. Both *pricing* and *constraints on participating firms* varied across programs and countries.

[48] Anderson, Papadia and Véron (2021) estimate that 92% of the funds initially committed by these five countries to business funding programs were tied to loan guarantee schemes, though some grant programs were also implemented. For instance, Germany provided €50bn in loan guarantees and €50 in direct grants.

For instance, in France, the loan guarantee program constrained interest rates to be very low (specifically, tied to banks' liquidity costs, which was close to zero during the period), and it also restricted participating firms from engaging in share buybacks or dividend payments. By contrast, the UK's CLBILS allowed for market pricing of loans and put no restrictions on participating borrowers.[49]

Overall, the business funding programs implement by European fiscal authorities were generally closer to the Fed's CCF and MSLP. The public guarantee advantages were passed through to banks and borrowers, either in terms of higher volumes, riskier portfolios, or lower interest rates, resulting in what amounted to subsidized lending programs subject to eligibility and participation restrictions (like the MSLP and the CCF), as opposed to grants programs (like the PPP).

Take-up in these programs was generally much larger than in the MSLP or CCF. Anderson, Papadia and Véron (2021) estimate a total take-up €503bn (or about 4.2% of 2019 GDP) for the five economies mentioned above as of the end of 2020.

Programs pre-dating the 2020 crisis In the Eurozone and in the UK, programs similar to the CCF had been implemented by monetary authorities prior to the 2020 crisis. The ECB's Corporate Sector Purchase Program (CSPP) was launched in 2016; it involved secondary market purchases of corporate bonds issued by investment-grade firms and meeting certain eligibility criteria. One of the explicit goals of the program is to reduce the cost of capital for issuers of eligible bonds (De Santis et al., 2018). Consistent with the large announcement effects of the CCF, the announcement of the CSPP lead to an increase in bond prices for both ineligible and eligible issuers and bonds, at least in part through a reduction in risk premia in corprorate bond markets, as documented by Pegoraro and Montagna (2020); the risk premium effect is one of the key channels our model will emphasize. In the UK, the Bank of England launched the Corporate Bond Purchase Scheme (CBPS) in August of 2016; purchases of eligible bonds ran from September 2016 to April 2017. D'Amico and Kaminska (2019) discuss the effects of the announcement on bond prices, and find that it lead to a reduction in credit spreads. They also document an increase of corporate bond issuances in the wake of the program's announcement. Thus, similar to the CCF, the EU and the UK bond purchase programs had substantial announcement effects on yields and issuance, separate from implementation effects.[50]

[49] Funding caps also varied across countries and programs; for instance, the French program was capped on the basis of past annual turnover, while there were no explicit caps for the CLBILS.

[50] Additionally, the Bank of Japan (BoJ) conducted annual purchases of corporate bonds ranging from 0.5 to 3.2 trillion yen from February of 2009 to October of 2012 (see Suganuma, Ueno et al. 2018); over this period of time, Arai (2017) shows that corporate bond yields typically declined following BoJ announcements, though the announcements were not necessarily specific to the corporate bond-buying scheme.

A.2 Model

A.2.1 No leverage firm

The average and marginal value e^* of a firm that cannot take on debt satisfies

$$0 = \max_g -(r-g)e^* + (1-\theta)\,a - \Phi\,(g)$$

The optimal investment rule can thus be written

$$g^* = \left(\Phi'\right)^{-1}(e^*) = \frac{e^*-1}{\gamma},$$

where the last equality follows from the functional form assumption we made for the adjustment cost function Φ. The value (per unit of capital) e^* can then be computed via

$$e^* = \mathbb{E}\left[\int_0^{+\infty} e^{-(r-g^*)t}\left[(1-\theta)a - \Phi\,(g^*)\right]dt\right]$$

Thus g^* and e^* solve a system of 2 equations in 2 unknown

$$e^* = \frac{(1-\theta)a - \Phi\,(g^*)}{r-g^*} \qquad\qquad e^* = \Phi'\,(g^*) \tag{A1}$$

This system can be re-written

$$r = g^* + \frac{1}{\Phi'\,(g^*)}\left[(1-\theta)a - \Phi\,(g^*)\right]$$

Since the function Φ is increasing and convex (for $g > -1/\gamma$, where $-1/\gamma$ is the minimum achievable capital growth rate), the right hand side of this equation is a decreasing function of g^* (so long as the steady state cash-flow rate $(1-\theta)a - \Phi\,(g^*) > 0$). In order for a solution $g^* < r$ to this non-linear equation to exist, we must impose $\Phi\,(r) > (1-\theta)a$. In such case, the right hand side of the equation above, evaluated at $g^* = r$, is strictly less than r, whereas the right hand side of the equation above, evaluated at $g^* \to -1/\gamma$, diverges to $+\infty$ under the assumption that $\Phi\,(-1/\gamma) < (1-\theta)a$. Thus, under the parameter condition

$$\Phi\,(r) > (1-\theta)a > \Phi\,(-1/\gamma)\,,$$

there is a unique stationary investment rule g^* and a unique equity value (per unit of capital) e^* satisfying the system of equations (A1). Our function form for Φ allows us to determine

g^* analytically, as the smallest solution to the quadratic equation

$$\frac{\gamma}{2}(g-r)^2 + (1-\theta)a - \Phi(r) = 0$$

This yields

$$g^* = r - \left[\frac{2}{\gamma}\left(\Phi(r) - (1-\theta)a\right)\right]^{1/2}$$

A.2.2 Rescaling

Let N_t be the counting process for default events. The effective capital k_t satisfies:

$$k_t = k_0 \exp\left(\int_0^t \left(g_s - \frac{\sigma^2}{2}\right) ds + \sigma Z_t + \ln \alpha_k \int_0^t dN_s\right) = k_0 \tilde{M}_t \exp\left(\int_0^t g_s ds + \ln \alpha_k \int_0^t dN_s\right).$$

In the above, we have introduced the martingale $\tilde{M}_t := e^{\sigma Z_t - \frac{1}{2}\sigma^2 t}$. This defines the change-of-measure $\tilde{\mathbb{P}}(A) = \mathbb{E}[\tilde{M}_t 1_A]$. We then have

$$E(k,b) = \sup_{g,\iota,\boldsymbol{\tau}} \mathbb{E}^{k,b}\left[\int_0^{+\infty} e^{-rt}\pi_t k_t dt\right] = k \sup_{g,\iota,\boldsymbol{\tau}} \tilde{\mathbb{E}}^x\left[\int_0^{+\infty} e^{-\int_0^t (r-g_u)du + \int_0^t \ln \alpha_k dN_u}\pi_t dt\right] = ke(x)$$

Under $\tilde{\mathbb{P}}$, $\tilde{Z}_t := Z_t - \sigma t$ is a standard Brownian motion, and x_t evolves according to

$$dx_t = \left[\iota_t - (g_t + m)\, x_t\right] dt - \sigma x_t d\tilde{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1\right) x_{t-} dN_t$$

A.2.3 Optimal financing

When we use equation (9) and the firm's optimal growth policy $g(x)$ in the variational inequality (6) satisfied by the equity value, we then have, for $x \in [0, \bar{x}]$:

$$(r - g(x))\, e(x) = a - \Phi\left(g(x)\right) - (\kappa + m)\, x - \Theta\left(a - \kappa x\right) - \left(g(x) + m\right) x e'(x) + \frac{\sigma^2 x^2}{2} e''(x) \quad \text{(A2)}$$

Equation (A2) is the HJB equation for shareholders of a firm that can commit never to issue bonds ever again. If one were to differentiate equation (A2) w.r.t. x, substract equation (7), and use the first order condition (9), one can back out the issuance policy

$$\iota(x) = \frac{\Theta\kappa}{-d'(x)}$$

Equation (A2) makes it clear that in our setup, the equity value of a firm with leverage x can be computed as if such firm was allowing its bonds to amortize, and as if it could commit to never issuing bonds in the future.

A.2.4 Credit spreads

We define the credit spread $cs(x)$ of a firm with leverage x as the spread over the risk-free benchmark at which risk-free bond cashflows need to be discounted in order to recover the bond price $d(x)$:

$$d(x) := \int_0^{\infty} e^{-(r+m+cs(x))t}(\kappa+m)dt = \frac{\kappa+m}{r+m+cs(x)} \qquad \Rightarrow \qquad cs(x) = \frac{\kappa+m}{d(x)} - (r+m)$$

Since the debt price is decreasing in x, the credit spread is an increasing function of x. Lastly, we note that the bond price and credit spreads, when the firm has no leverage, satisfy

$$d(0) = \frac{\kappa+m}{r+m} - \frac{\Theta\kappa}{r+m} \qquad \Rightarrow \qquad cs(0) = \frac{(r+m)\Theta\kappa}{\kappa+m-\Theta\kappa} > 0$$

Credit spreads are thus bounded from below by $cs(0) > 0$ since creditors anticipate the fact that the firm will be issuing bonds in the future, therefore increasing corporate default risk. The lower bound $cs(0)$ is increasing in θ and in κ, since bond issuances are increasing in the flow tax benefits of debt.

A.2.5 Convexity

The convexity of e can be seen from the fact that shareholders always have the option to issue a non-zero measure of bonds. Indeed, take two arbitrary leverage ratios x_1, x_2, and $\lambda \in [0,1]$, with $x_\lambda = \lambda x_1 + (1-\lambda)x_2$. Consider feasible debt policies that make the firm's leverage jump from x_1 to x_λ, or from x_2 to x_λ. Then we have

$$e(x_1) \geq e(x_\lambda) + (x_\lambda - x_1)d(x_\lambda)$$
$$e(x_2) \geq e(x_\lambda) + (x_\lambda - x_2)d(x_\lambda)$$

Take a weighted average of these inequalities to obtain $\lambda e(x_1) + (1-\lambda)e(x_2) \geq e(x_\lambda)$.

A.2.6 Default boundary $\bar{x}$

Since the case with "reinjections" is more subtle to treat, in this section we analyze the simpler environment in which, at the time of default, shareholders are totally wiped out – i.e. $e(\bar{x}) =$

0 at the default boundary. Call $e^*(x)$ the equity value (per unit of capital) for a firm that uses the no-leverage optimal investment rule g^*:

$$
\begin{aligned}
e^*(x) :&= \sup_{\tau} \mathbb{E}^x \left[\int_0^{\tau} e^{-(r-g^*)t} \left(a - \Phi\left(g^*\right) - (\kappa + m)x_t - \Theta\left(a - \kappa x_t\right)\right) dt \right] \\
dx_t &= -\left(g^* + m\right) x_t dt - \sigma x_t dZ_t
\end{aligned}
\tag{A3}
$$

Denote $\bar{x}^*$ the default boundary for the above problem. One can show that

$$
\bar{x}^* = \frac{\xi}{\xi - 1} \left(\frac{r+m}{\kappa(1-\theta)+m} \right) \left(\frac{(1-\Theta)a - \Phi\left(g^*\right)}{r - g^*} \right),
$$

where ξ is the positive root of the characteristic equation

$$
\frac{\sigma^2}{2}\xi^2 - \left(g^* + m + \frac{\sigma^2}{2} \right) \xi - \left(r - g^*\right) = 0
$$

Since the investment policy g^* and the default policy that solved (A3) are feasible for managers of a firm that can adjust its investment policy freely, we must have $e(x) \geq e^*(x)$. In particular, at the boundary $\bar{x}^*$, we must have $e(\bar{x}^*) \geq e^*(\bar{x}^*) = 0$. Thus, it must be the case that $\bar{x} \geq \bar{x}^*$.

A.2.7 Aggregation

Remember that the dynamics of the aggregate capital stock are as follows:

$$
\begin{aligned}
dK_t &= \int_0^1 k_t^{(j)} g\left(x_t^{(j)}\right) dj dt + \int_0^1 \sigma k_t^{(j)} dZ_t^{(j)} dj - (1 - \alpha_k) \int_0^1 k_t^{(j)} dN_t^{(j)} dj \\
&= \left(\int_x \int_{\omega} g\left(x\right) \omega f_t(x,\omega) d\omega dx \right) K_t dt - (1-\alpha_k)\hat{\lambda}_t^d K_t dt,
\end{aligned}
$$

where $\hat{\lambda}_t^d$ is the capital-share-weighted default rate, computed as follows:

$$
\hat{\lambda}_t^d := \frac{1}{dt} \int_0^1 \omega_t^{(j)} dN_t^{(j)} dj
$$

We introduce $\hat{f}_t(x) := \int_{\omega} \omega f_t(x,\omega) d\omega$, which represents the percentage of the total capital stock at firms with leverage x. Then,

$$
dK_t = \left(\hat{g}_t - (1-\alpha_k)\hat{\lambda}_t^d \right) K_t dt := \mu_{K,t} K_t dt,
$$

where $\hat{g}_t := \int_x g(x) \hat{f}_t(x) dx$ is the average investment rate. The law of motion for an individual firm's capital share is:

$$d\omega_t^{(j)} = \left(g\left(x_{t-}^{(j)}\right) - \hat{g}_t \right) \omega_{t-}^{(j)} dt + \sigma \omega_{t-}^{(j)} dZ_t^{(j)} - (1-\alpha_k) \omega_{t-}^{(j)} \left(dN_t^{(j)} - \hat{\lambda}_t^d dt \right)$$

The firm's capital share increases or decreases depending on whether its capital growth rate is greater or less than the weighted-average growth rate in the economy (the first term in the stochastic differential equation above). The firm's capital share also jumps down with default, due to bankruptcy costs. Introduce the coefficients $\mu_{\omega,t}, \sigma_\omega$, such that:

$$d\omega_t^{(j)} := \mu_{\omega,t} \left(x_{t-}^{(j)}, \omega_{t-}^{(j)} \right) dt + \sigma_\omega \left(\omega_{t-}^{(j)} \right) dZ_t^{(j)} - (1-\alpha_k) \omega_{t-}^{(j)} dN_t^{(j)}$$

Remember that a firm's leverage follows:

$$\begin{aligned} dx_t^{(j)} &= \left[\iota\left(x_t^{(j)}\right) - \left(g\left(x_{t-}^{(j)}\right) + m - \sigma^2 \right) x_{t-}^{(j)} \right] dt - \sigma x_{t-}^{(j)} dZ_t^{(j)} + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dN_t^{(j)} \\ &:= \mu_{x,t} \left(x_{t-}^{(j)} \right) dt + \sigma_x \left(x_{t-}^{(j)} \right) dZ_t^{(j)} + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dN_t^{(j)} \end{aligned} \tag{A4}$$

The Kolmogorov forward equation for the density $f_t(x, \omega)$ can be written, for $x \in (0, \bar{x})$, $x \neq \frac{\alpha_b}{\alpha_k} \bar{x}$:

$$\begin{aligned} \partial_t f_t(x,\omega) = &-\partial_x \left[\mu_{x,t}(x) f_t(x,\omega) \right] - \partial_\omega \left[\mu_{\omega,t}(x,\omega) f_t(x,\omega) \right] \\ &+ \frac{1}{2} \partial_{xx} \left[\sigma_x(x)^2 f_t(x,\omega) \right] + \frac{1}{2} \partial_{\omega\omega} \left[\sigma_\omega(\omega)^2 f_t(x,\omega) \right] + \partial_{x\omega} \left[\sigma_x(x) \sigma_\omega(\omega) f_t(x,\omega) \right] \end{aligned} \tag{A5}$$

Multiplying equation (A5) by ω, integrating over $\omega \in [0, +\infty)$ and using integration by parts, one can prove that the capital-share-weighted ergodic leverage density $\hat{f}_t(x)$ is solution to the following differential equation, for $x \in (0, \bar{x})$, $x \neq \frac{\alpha_b}{\alpha_k} \bar{x}$:

$$\partial_t \hat{f}_t(x) = g(x) \hat{f}_t(x) - \partial_x \left[\left(\iota(x) - \left(g(x) + m \right) x \right) \hat{f}_t(x) \right] + \frac{\sigma^2}{2} \partial_{xx} \left[x^2 \hat{f}_t(x) \right] - \mu_{K,t} \hat{f}_t(x) \tag{A6}$$

$$:= \mathcal{L}^* \hat{f}_t(x) - \mu_{K,t} \hat{f}_t(x) \tag{A7}$$

In the above, the operator $\mathcal{L}^*$ is a linear second-order differential operator, which we will later relate to the expected future capital stock of a firm. In addition,

$$\hat{\lambda}_t^d = -\frac{1}{2} \sigma^2 \bar{x}^2 \partial_x \hat{f}_t(\bar{x}), \qquad \hat{g}_t = \int g(x) \hat{f}_t(x) dx, \qquad \hat{f}_t(\bar{x}) = 0, \qquad \mu_{K,t} = \hat{g}_t - (1-\alpha_k) \hat{\lambda}_t^d$$

Since the endogenous default and growth rates $\hat{\lambda}_t^d$ and $\hat{g}_t$ depend on moments of the density $\hat{f}_t(x)$, equation (A6) is a nonlinear integro-differential equation.

A.2.8 Priced Aggregate Shocks

A.2.8.1 Individual firm problem

We assume that the capital stock is now exposed to aggregate risk, via

$$dk_t^{(j)} = k_t^{(j)} \left(g_t^{(j)} dt + \sigma \left(\rho dZ_t + \sqrt{1-\rho^2} dZ_t^{(j)} \right) \right)$$

In particular this means that all firms' efficiency units of capital share the same exposure to aggregate risk Z_t. We assume that the risk free rate (resp. price of risk) faced by equity investors is r_e (resp. ν_e), whereas the risk free rate (resp. price of risk) faced by debt investors is r_d (resp. ν_d). In particular, the stochastic discount factor $M_{n,t}$ for investor $n \in \{e, d\}$ satisfies

$$\frac{dM_{n,t}}{M_{n,t}} = -r_n dt - \nu_n dZ_t$$

We introduce $Z_t^{\mathbb{Q}_n} = Z_t + \nu_n t$, which will be a Brownian motion under the measure induced by investor n stochastic discount factor. Under such measure, we must have

$$dk_t^{(j)} = k_t^{(j)} \left(\left(g_t^{(j)} - \rho \nu_n \sigma \right) dt + \sigma \left(\rho dZ_t^{\mathbb{Q}_n} + \sqrt{1-\rho^2} dZ_t^{(j)} \right) \right)$$

In other words, priced aggregate risk leads to a downward drift adjustment of the capital stock under the investor n's risk-neutral measure. The equity and debt values in such case are equal to

$$E(k,b) = \sup_{g,\iota,\tau} \mathbb{E}\left[\int_0^{+\infty} \frac{M_{e,t}}{M_{e,0}} \pi_t k_t dt \right] = \sup_{g,\iota,\tau} \mathbb{E}^{\mathbb{Q}_e} \left[\int_0^{+\infty} e^{-r_e t} \pi_t k_t dt \right]$$
$$D(k,b) = \mathbb{E}\left[\int_0^{+\infty} \frac{M_{d,t}}{M_{d,0}} e^{-mt} \alpha_b^{N_t} (\kappa + m) dt \right] = \mathbb{E}^{\mathbb{Q}_d} \left[\int_0^{+\infty} e^{-(r_d+m)t} \alpha_b^{N_t} (\kappa + m) dt \right]$$

One can then show that the re-scaled equity value (per unit of capital) satisfies

$$e(x) = \sup_{g,\iota,\tau} \hat{\mathbb{E}} \left[\int_0^{+\infty} e^{-\int_0^t (r_e - g_u + \rho \nu_e \sigma) du} \alpha_k^{N_t} \pi_t dt \right]$$
$$dx_t = \left[\iota_t - (g_t + m - \rho \sigma \nu_e) x_t \right] dt - \sigma x_t d\hat{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dN_t$$

Similarly, we can compute debt prices as follows

$$d(x) = \tilde{\mathbb{E}}\left[\int_0^{+\infty} e^{-(r_d+m)t}\alpha_b^{N_t}(\kappa+m)ds\right]$$
$$dx_t = \left[\iota(x_t) - \left(g(x_t)+m-\sigma^2-\rho\sigma\nu_d\right)x_t\right]dt - \sigma x_t d\tilde{Z}_t + \left(\frac{\alpha_b}{\alpha_k}-1\right)\bar{x}dN_t$$

Equity and debt prices satisfy the following pair of HJB equations on $[0,\bar{x})$:

$$0 = \max_{\iota,g}\Big[-(r_e-g+\rho\nu_e\sigma)e(x) + a - \Phi(g) - (\kappa+m)x + \iota d(x) - \Theta(a-\kappa x) \\ + [\iota - (g+m-\rho\sigma\nu_e)x]e'(x) + \frac{\sigma^2}{2}x^2 e''(x)\Big] \quad \text{(A8)}$$

and

$$(r_d+m)d(x) = \kappa + m + \left[\iota(x) - \left(g(x)+m-\sigma^2-\rho\sigma\nu_d\right)x\right]d'(x) + \frac{\sigma^2}{2}x^2 d''(x) \quad \text{(A9)}$$

A.2.8.2 Comparative static w.r.t. ν_e and r_e

Since the case with "reinjections" is more subtle to treat, in this section we analyze the simpler environment in which, at the time of default, shareholders are totally wiped out – i.e. $e(\bar{x}) = 0$ at the default boundary. Since the equation $d(x) + e'(x) = 0$ still holds, the equity value is solution of:

$$(r_e - g(x) + \rho\nu_e\sigma)e(x) = a - \Phi(g(x)) - (\kappa+m)x - \Theta(a-\kappa x) \\ - (g(x)+m-\rho\sigma\nu_e)xe'(x) + \frac{\sigma^2}{2}x^2 e''(x)$$

By differentiating this equation w.r.t. ν_e and using the envelop theorem, we can obtain a PDE and boundary condition satisfied by $\partial_{\nu_e}e$:

$$(r_e - g(x) + \rho\nu_e\sigma)\partial_{\nu_e}e(x) = -\rho\sigma q(x) - (g(x)+m-\rho\sigma\nu_e)x\partial_{\nu_e,x}e(x) + \frac{\sigma^2}{2}x^2\partial_{\nu_e,xx}e(x)$$
$$\partial_{\nu_e}e(\bar{x}) = 0$$

Using the integral representation of the above equation, we can conclude that the sensitivity of the price of equity w.r.t. the price of risk ν_e takes the following form:

$$\partial_{\nu_e}e(x) = -\rho\sigma\hat{\mathbb{E}}^x\left[\int_0^\tau e^{-\int_0^t(r_e-g(x_u)+\rho\sigma\nu_e)du}q(x_t)dt\right] < 0$$

Note then that the total firm value (per unit of capital) $v(x)$ satisfies $v(x) = e(x) + xd(x) = e(x) - xe'(x) = q(x)$, which satisfies

$$(r_e - g(x) + \rho\nu_e\sigma)\, q(x) = (1-\theta)a - \Phi\left(g(x)\right) - \left(g(x) + m - \rho\sigma\nu_e\right) xq'(x) + \frac{\sigma^2}{2}x^2 q''(x)$$

The boundary condition satisfied by q is $q(\bar{x}) = 0$. Then notice that

$$\Phi'\left(g(x)\right) = q(x) \Rightarrow \partial_{\nu_e} g(x) = \frac{\partial_{\nu_e} q(x)}{\Phi''\left(g(x)\right)}$$

Thus, the sensitivity of q w.r.t. the equity market's price of risk ν_e solves

$$\zeta(x)\partial_{\nu_e} q(x) = -\rho\sigma\left(q(x) - xq'(x)\right) - \left(g(x) + m - \rho\sigma\nu_e\right) x \left(\partial_{\nu_e} q(x)\right)' + \frac{\sigma^2}{2}x^2\left(\partial_{\nu_e} q(x)\right)'',$$

where we have introduced, for notational convenience, the implicit discount rate

$$\zeta(x) := r_e - g(x) + \rho\nu_e\sigma - \frac{x^2 e''(x)}{\Phi''(g(x))}$$

Then note that at the boundary, since $q(\bar{x}) = 0$, we must have

$$\partial_{\nu_e} q(\bar{x}) = -\left(\partial_{\nu_e}\bar{x}\right) q'(\bar{x}) < 0,$$

since $\bar{x}$ is a decreasing function of ν_e and since $q'(x) = -xe''(x) < 0$. $\partial_{\nu_e} q$ admits the integral representation

$$\begin{aligned}\partial_{\nu_e} q(x) &= -\hat{\mathbb{E}}^x\left[\rho\sigma\int_0^\tau e^{-\int_0^t \zeta(x_u)du}\left[q(x_t) - x_t q'(x_t)\right]dt - e^{-\int_0^\tau \zeta(x_u)du}\partial_{\nu_e} q(\bar{x})\right] \\ &= -\hat{\mathbb{E}}^x\left[\rho\sigma\int_0^\tau e^{-\int_0^t \zeta(x_u)du}\left[q(x_t) + x_t^2 e''(x_t)\right]dt - e^{-\int_0^\tau \zeta(x_u)du}\partial_{\nu_e} q(\bar{x})\right] < 0\end{aligned}$$

Thus, a sudden decrease in the price of risk ν_e is expected to lead to an increase in $q(x)$, and thus to a boom in investment. Using a similar method, one can show that $\partial_{r_e} q$ admits the integral representation

$$\partial_{r_e} q(x) = -\hat{\mathbb{E}}^x\left[\int_0^\tau e^{-\int_0^t \zeta(x_u)du} q(x_t)dt - e^{-\int_0^\tau \zeta(x_u)du}\partial_{r_e} q(\bar{x})\right] < 0$$

Thus, a sudden decrease in the risk free rate r_e is expected to lead to an increase in $q(x)$, and thus to a boom in investment.

A.2.8.3 Optimal financing

We also want to recover the optimal debt issuance policy. We differentiate the ODE satisfied by e w.r.t. x and use the optimality condition w.r.t. investment to obtain

$$(r_e + m)\, e'(x) = \Theta\kappa - (\kappa + m) - \left(g(x) + m - \sigma^2 - \rho\sigma\nu_e\right) x e''(x) + \frac{(\sigma x)^2}{2} e'''(x)$$

Add this equation to the pricing equation (A9), use the identity $e'(x) + d(x) = 0$ to obtain

$$(r_d - r_e) d(x) = \Theta\kappa + \iota(x) d'(x) + \rho\sigma x(\nu_d - \nu_e) d'(x)$$

Thus, we can back out the issuance policy:

$$\iota(x) = \frac{\Theta\kappa}{-d'(x)} + \frac{(r_e - r_d) d(x)}{-d'(x)} + \rho\sigma x(\nu_e - \nu_d) = \frac{\Theta\kappa}{-d'(x)} + \frac{\left(\tilde{R}_d(x) - R_d(x)\right) d(x)}{-d'(x)}$$

In the above, the expected returns (from the point of view of equity and debt investors) are

$$R_d(x) = r_d - \rho\sigma x\nu_d \frac{d'(x)}{d(x)} \qquad \tilde{R}_d(x) = r_d - \rho\sigma x\nu_d \frac{d'(x)}{d(x)}$$

A.2.8.4 Risk premia

It is worthwhile noting that in the presence of aggregate shocks, equity risk premia are non-zero. In fact, the expected equity return is equal to

$$\mathbb{E}\left[\frac{\pi_t k_t dt + dE\left(k_t, b_t\right)}{E\left(k_t, b_t\right)}\right] = \left[r_e + \rho\nu_e\sigma\left(1 - \frac{x_t e'(x_t)}{e(x_t)}\right)\right] dt := R_e(x) dt$$

A.2.8.5 Aggregation with aggregate shocks

The dynamics of the aggregate capital stock in the presence of the aggregate shock Z_t are as follows:

$$dK_t = K_t \Bigg[\underbrace{\int_0^1 \omega_t^{(j)} g\left(x_t^{(j)}\right) dj dt}_{= \hat{g}_t dt} + \underbrace{\sigma\sqrt{1-\rho^2} \int_0^1 \omega_t^{(j)} dZ_t^{(j)} dj}_{=0}$$
$$+ \underbrace{\sigma\rho \int_0^1 \omega_t^{(j)} dZ_t dj}_{= \sigma\rho dZ_t} - (1 - \alpha_k) \underbrace{\int_0^1 \omega_t^{(j)} dN_t^{d,(j)} dj}_{= \hat{\lambda}_t^d dt} \Bigg]$$

In particular, the aggregate capital stock is now exposed to aggregate shocks:

$$\frac{dK_t}{K_t} = \left(\hat{g}_t - (1-\alpha_k)\hat{\lambda}_t^d\right) dt + \rho\sigma dZ_t$$

The density $f_t(x,\omega)$ is now being subject to aggregate shocks. Derivations similar to those in section (A.2.7) show that the capital-weighted density $\hat{f}_t(x)$ satisfies the following SDE, for $x \in (0,\bar{x})$, $x \neq \frac{\alpha_b}{\alpha_k}\bar{x}$:

$$d\hat{f}_t(x) = (g(x) - \mu_{K,t})\,\hat{f}_t(x)dt - \partial_x\left[(\iota(x) - (g(x)+m)\,x)\,\hat{f}_t(x)\right]dt \\ + \frac{(1-\rho^2)\,\sigma^2}{2}\partial_{xx}\left[x^2\hat{f}_t(x)\right]dt + \rho\sigma\partial_x\left[x\hat{f}_t\,(x)\right]dZ_t,$$

and

$$\hat{\lambda}_t^d = -\frac{1}{2}\sigma^2\bar{x}^2\partial_x\hat{f}_t(\bar{x}), \qquad \hat{g}_t = \int g\,(x)\,\hat{f}_t(x)dx, \qquad \hat{f}_t(\bar{x}) = 0, \qquad \mu_{K,t} = \hat{g}_t - (1-\alpha_k)\hat{\lambda}_t^d$$

The equation above makes clear that the capital-weighted density $\hat{f}_t$ is also subject to the aggregate shock process Z_t.

A.2.9 A crisis state

A.2.9.1 Partial equilibrium calculations

We model the crisis as an unanticipated change, for all firms, in (a) the productivity parameter, which decreases from a to $\underline{a} < a$, and (b) the price of risk, which increases from ν to $\underline{\nu} > \nu$. The crisis lasts an exponentially distributed time, with parameter χ. Equity (per unit of capital) $\underline{e}$ satisfies, during such crisis time:

$$0 = \max\left[\alpha_k\underline{e}\left(\frac{\alpha_b}{\alpha_k}x\right) - \underline{e}(x), \max_{\iota,g}\left[-(R_e(x)+\chi-g)\underline{e}(x) + \underline{a} - \Phi\,(g) - (\kappa+m)\,x \right.\right. \\ \left.\left. -\Theta\,(a-\kappa x) + \iota\underline{d}(x) + \chi e(x) + [\iota - (g+m)\,x]\,\underline{e}'(x) + \frac{\sigma^2}{2}x^2\underline{e}''(x)\right]\right] \quad \text{(A10)}$$

This equation is the counterpart to equation (6), adjusted to reflect Poisson transitions (at rate χ) back to the "normal" state of the economy. Instead, debt prices $\underline{d}$ satisfy, during such crisis time:

$$(R_d(x)+m+\chi)\underline{d}(x) = \kappa + m + \chi d(x) + \left[\underline{\iota}(x) - \left(\underline{g}(x) + m - \sigma^2\right)x\right]\underline{d}'(x) + \frac{(\sigma x)^2}{2}\underline{d}''(x) \quad \text{(A11)}$$

$$\underline{d}(\underline{\bar{x}}) = \alpha_b \underline{d}\left(\frac{\alpha_b}{\alpha_k}\underline{\bar{x}}\right). \tag{A12}$$

In the above, we have used notation $\underline{\iota}(x), \underline{g}(x)$ for the optimal issuance and growth policies followed by the firm during the crisis. Similarly, $\underline{\bar{x}}$ is the optimal default barrier of the firm during such crisis. A reasoning identical to what was previously discussed allows us to derive the optimal investment and issuance policies during such crisis time:

$$\Phi'\left(\underline{g}(x)\right) = \underline{e}(x) - x\underline{e}'(x) := \underline{q}(x)$$
$$\underline{\iota}(x) = \frac{\Theta\kappa}{-\underline{d}'(x)}$$

A.2.9.2 Expected future aggregates

In this section, we discuss our computation of expected future macroeconomic aggregates upon the occurrence of a transient aggregate shock. If T is the exponentially distributed crisis length, the expected aggregate capital stock then satisfies

$$\begin{aligned}
\mathbb{E}\left[K_t\right] &= \int_j \mathbb{E}\left[k_t^{(j)}\right] dj \\
&= \int_j k_0^{(j)} \tilde{\mathbb{E}}\left[\alpha_k^{N_t^{(j)}} \exp\left(\int_0^{t\wedge T} \underline{g}\left(x_u^{(j)}\right) du + \int_{t\wedge T}^{t} g\left(x_u^{(j)}\right) du\right)\right] dj \\
&= K_0 \iint \omega \underline{h}_k\left(x,t\right) f_0\left(x,\omega\right) d\omega dx \\
&= K_0 \int \underline{h}_k\left(x,t\right) \hat{f}_0\left(x\right) dx
\end{aligned}$$

In the above, we have implicitly defined the function $\underline{h}_k(\cdot,\cdot)$ as follows

$$\begin{aligned}
\underline{h}_k(x,t) :&= \tilde{\mathbb{E}}\left[\alpha_k^{N_t} \exp\left(\int_0^{t\wedge T} \underline{g}\left(x_u\right) du + \int_{t\wedge T}^{t} g\left(x_u\right) du\right) | x_0 = x\right] \\
dx_u &= \left[\underline{\iota}(x_u) - \left(\underline{g}(x_u) + m\right) x_u\right] du - \sigma x_u d\tilde{Z}_u + \left(\frac{\alpha_b}{\alpha_k} - 1\right) x_{u-} dN_u, \qquad u \le T \\
dx_u &= \left[\iota(x_u) - \left(g(x_u) + m\right) x_u\right] du - \sigma x_u d\tilde{Z}_u + \left(\frac{\alpha_b}{\alpha_k} - 1\right) x_{u-} dN_u, \qquad u \ge T
\end{aligned}$$

The function $\underline{h}_k$ satisfies the PDE

$$\begin{aligned}
&\partial_t \underline{h}_k(x,t) = \underline{\mathcal{L}} \underline{h}_k(x,t) + \chi\left(h_k(x,t) - \underline{h}_k(x,t)\right) \\
&\underline{h}_k(x,0) = 1 && \forall x \le \underline{\bar{x}} \\
&\underline{h}_k(x,t) = \alpha_k^{\underline{n}(x)} \underline{h}_k\left(\left(\frac{\alpha_b}{\alpha_k}\right)^{\underline{n}(x)} x, t\right) && \forall t, x > \underline{\bar{x}}
\end{aligned}$$

In the above, the operator $\underline{\mathcal{L}}$ is defined, for $x \leq \underline{\bar{x}}$, $t \geq 0$, via:

$$\underline{\mathcal{L}}\varphi(x,t) = \underline{g}(x)\varphi(x,t) + \left[\underline{\iota}(x) - \left(\underline{g}(x) + m\right) x\right] \partial_x \varphi(x,t) + \frac{\sigma^2 x^2}{2} \partial_{xx}\varphi(x,t),$$

where φ is an arbitrary smooth function. Similarly, the function $h_k(\cdot,\cdot)$ is defined as follows

$$\begin{aligned} h_k(x,t) :&= \tilde{\mathbb{E}}\left[\alpha_k^{N_t} \exp\left(\int_0^t g\left(x_u\right) du\right) | x_0 = x\right] \\ dx_u &= \left[\iota(x_u) - \left(g(x_u) + m\right) x_u\right] dt - \sigma x_u d\tilde{Z}_u + \left(\frac{\alpha_b}{\alpha_k} - 1\right) \bar{x} dN_t, \qquad x_0 = x \end{aligned}$$

The function h_k satisfies the PDE

$$\begin{aligned} \partial_t h_k(x,t) &= \mathcal{L} h_k(x,t) \\ h_k(x,0) &= 1 && \forall x \leq \bar{x} \\ h_k(x,t) &= \alpha_k^{n(x)} h_k\left(\left(\frac{\alpha_b}{\alpha_k}\right)^{n(x)} x, t\right) && \forall t, x > \bar{x} \end{aligned}$$

The operator $\mathcal{L}$ is defined, for $x \leq \bar{x}$, $t \geq 0$, and for an arbitrary smooth function φ, via:

$$\mathcal{L}\varphi(x,t) = g(x)\varphi(x,t) + \left[\iota(x) - \left(g(x) + m\right) x\right] \partial_x \varphi(x,t) + \frac{\sigma^2 x^2}{2} \partial_{xx}\varphi(x,t)$$

$n(x)$, defined for $x > \bar{x}$, represents the number of consecutive debt restructuring, i.e.:

$$n(x) := 1 + \lfloor \frac{\ln\left(x/\bar{x}\right)}{\ln\left(\alpha_k/\alpha_b\right)} \rfloor$$

We then notice that the PDE satisfied by h_k can be expressed using the operator $\mathcal{L}$, which is exactly the adjoint of the operator $\mathcal{L}^*$ defined in equation (A7):

$$\partial_t h_k(x,t) = \mathcal{L} h_k(x,t) \tag{A13}$$

Using a similar reasoning, we can compute the aggregate future output, investment, growth, debt issuance, profits, and bonds outstanding, via:

$$\begin{aligned} \mathbb{E}\left[Y_t\right] &= \int_j \mathbb{E}\left[a_t k_t^{(j)}\right] dj = K_0 \int \underline{h}_y\left(x,t\right) \hat{f}_0\left(x\right) dx \\ \mathbb{E}\left[G_t\right] &= \int_j \mathbb{E}\left[g\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_g\left(x,t\right) \hat{f}_0\left(x\right) dx \\ \mathbb{E}\left[I_t\right] &= \int_j \mathbb{E}\left[\Phi\left(g\left(x_t^{(j)}\right)\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_\Phi\left(x,t\right) \hat{f}_0\left(x\right) dx \end{aligned}$$

$$\mathbb{E}\left[\mathcal{I}_t\right] = \int_j \mathbb{E}\left[\iota\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_\iota\left(x,t\right) \hat{f}_0\left(x\right) dx$$
$$\mathbb{E}\left[DIV_t\right] = \int_j \mathbb{E}\left[\pi\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_\pi\left(x,t\right) \hat{f}_0\left(x\right) dx$$
$$\mathbb{E}\left[B_t\right] = \int_j \mathbb{E}\left[b_t^{(j)}\right] dj = K_0 \int \underline{h}_b\left(x,t\right) \hat{f}_0\left(x\right) dx$$
$$\mathbb{E}\left[E_t\right] = \int_j \mathbb{E}\left[e\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_e\left(x,t\right) \hat{f}_0\left(x\right) dx$$

The functions $\underline{h}_y, \underline{h}_g, \underline{h}_\Phi, \underline{h}_\iota, \underline{h}_\pi, \underline{h}_b, \underline{h}_e$ can be computed in a similar way to $\underline{h}_k$: they all satisfy the same equation (A13), and only differ in their initial condition at $t = 0$:

$$\begin{aligned}
\underline{h}_y\left(x,0\right) &= \underline{a} \\
\underline{h}_g\left(x,0\right) &= g(x) \\
\underline{h}_\Phi\left(x,0\right) &= \Phi\left(g(x)\right) \\
\underline{h}_\iota\left(x,0\right) &= \iota(x) \\
\underline{h}_\pi\left(x,0\right) &= \pi(x) \\
\underline{h}_b\left(x,0\right) &= x \\
\underline{h}_e\left(x,0\right) &= e(x)
\end{aligned}$$

A.2.10 Market frictions

In this section, we discuss several market environments that feature market frictions: (i) when all financial markets suffer a sudden-stop and (ii) when firms are prevented from paying dividends.

A.2.10.1 Sudden stop

Imagine now that there is a sudden stop in all markets. For simplicity, we assume that firms cannot buy back their own debt, i.e. we set $\iota = 0$. Optimal policies then solve

$$\max_g \quad g\underline{e}(x) - \Phi\left(g\right) - xg\underline{e}'(x)$$
$$\text{s.t. } \underline{a} - (\kappa + m)x - \Phi\left(g\right) - \Theta\left(\underline{a} - \kappa x\right) \geq 0$$

The constraint yields a maximum possible investment rate, equal to

$$\bar{g}(x) = \Phi^{-1}\left(\underline{a} - (\kappa + m)x - \Theta\left(\underline{a} - \kappa x\right)\right)$$

The optimal investment policy is thus

$$g(x) = \min\left[\bar{g}(x), (\Phi')^{-1}(q(x))\right]$$

When the government offers up to ι_g notional amount of loans at a price of d_g, the decision for a firm to accept such loans is based on:

$$\max_{\iota\in[0,\iota_g]} \iota\left(d_g + \underline{e}'(x)\right)$$

Thus, the firm's optimal public-sector borrowing decision is $\iota^* = \iota_g 1_{\{d_g > -\underline{e}'(x)\}}$.

A.2.10.2 Dividend payment restriction

Imagine that there is a restriction on dividend payments during the crisis. Optimal investment and debt issuance policies must now satisfy

$$\begin{aligned}\max_{\iota,g} \quad & g\underline{e}(x) + \underline{d}(x)\iota - \Phi(g) + [\iota - xg]\,\underline{e}'(x) \\ \text{s.t. } & \underline{a} - (\kappa+m)x + \underline{d}(x)\iota - \Phi(g) - \Theta(\underline{a} - \kappa x) \leq 0\end{aligned}$$

Let $\lambda(x) \geq 0$ be the Lagrange multiplier on the constraint, the first order conditions for this optimization problem are

$$\lambda(x) = 1 + \frac{\underline{e}'(x)}{\underline{d}(x)} \qquad\qquad \Phi'(g(x)) = \frac{\underline{e}(x) - x\underline{e}'(x)}{1-\lambda(x)},$$

with complementary slackness conditions

$$\begin{aligned}&0 \leq \lambda(x),\\ &0 \geq \underline{a} - (\kappa+m)x + \underline{d}(x)\iota(x) - \Phi(g(x)) - \Theta(\underline{a} - \kappa x),\\ &0 = \lambda(x)\left[\underline{a} - (\kappa+m)x + \underline{d}(x)\iota(x) - \Phi(g(x)) - \Theta(\underline{a} - \kappa x)\right]\end{aligned}$$

A.2.11 Fiscal cost calculations

A.2.11.1 Grant programs

In the case of a grant program, the government injects $\phi(a - \underline{a})k_t dt$ during each time period $[t, t+dt]$. For a given firm with initial capital k_0 and leverage x_0, this fiscal cost is equal to

$$S_g(x_0)k_0 := \mathbb{E}^{x_0}\left[\int_0^T e^{-rt}\phi(a - \underline{a})k_t dt\right],$$

where $S_g(x)$ is the expected fiscal cost, per unit of capital, for a firm with initial leverage x. $S_g(x)$ solves the Feynman-Kac equation

$$(R_e(x) - g(x) + \chi)\, S_g(x) = \phi(a - \underline{a}) + [\iota(x) - (g(x) + m)\, x]\, S_g'(x) + \frac{\sigma^2}{2} x^2 S_g(x)\Big]$$
$$S_g(\bar{x}) = \alpha_k S_g\left(\frac{\alpha_b}{\alpha_k}\bar{x}\right)$$

The total fiscal cost of the intervention is then equal to $K_0 \int S_g(x)\hat{f}_0(x)dx$.

A.2.11.2 Subisidized debt purchases

The subsidy received by a firm with initial leverage x_0 and capital k_0 is equal to

$$S_d(x_0)k_0 := \mathbb{E}^x\left[\int_0^T e^{-rt}\iota^g(x_t)k_t \max\left(0, d_g(x_t) - d(x_t)\right) dt\right]$$

The maximum operator inside the integral is meant to encode firm's decisions: when the funding obtainable from private credit markets is "cheaper" (i.e. has a higher price) than public sector funding, firms elect not to take the funding offered by the government. It is then straightforward to show that the subsidy per unit of capital $S_d(x)$ satisfies the following Feynman-Kac equation:

$$\begin{aligned}(r - g(x) + \chi)\, S_d(x) &= \iota^g(x) \max\left(0, d_g(x) - d(x)\right) \\ &\quad + \left[\iota^g(x)1_{\{d_g(x) > d(x)\}} + \iota(x) - (g(x) + m)\, x\right] S_d'(x) + \frac{\sigma^2}{2}x^2 S_d(x)\end{aligned}$$

The total tax subsidy is then equal to $K_0 \int S_d(x)\hat{f}_0(x)dx$.

A.3 Estimation

A.3.1 Data

Compustat variable definitions and mapping to model variables We define the following variables from Compustat items (denoted by their mnemonic, in bold font):

$$\text{ebitda} = \texttt{oibdp} \qquad \text{(operating income before depreciation)}$$

$$\text{gross debt} = \texttt{dlc} + \texttt{dltt} \qquad \text{(short- plus long-term debt)}$$

$$\text{net debt} = \text{gross debt} - \texttt{che} \qquad \text{(gross debt minus cash)}$$

$$\text{market value of equity} = \texttt{mkvalt} \qquad \text{(market value of common shares)}$$

$$\text{equity payouts} = \texttt{dv} + \texttt{prstkc} - \texttt{sstk} \qquad \text{(cash dividends plus net stock purchases)}$$

$$\text{gross issuance of debt} = \texttt{dltis} + \texttt{dlcc} \qquad \text{(LT debt issuance plus change in ST debt)}$$

In the definitions above, `oibdp` is operating income before depreciation (from income statements), `dlc` and `dltt` are short and long-term debt outstanding (from balance sheets), `che` are cash and cash equivalents, `dv` are cash dividend payments (from cash flow statements), `prstkc` are repurchases of stock (from cash flow statements), `sstk` are sales of stock (from cash flow statements), `mkvalt` is the market value of common shares (replaced by `prcc_f` × `csho` if missing), `dltis` is the gross issuance of long-term debt (from cash flow statements), and `dlcc` is the net change in short-term debt (from cash flow statements).[51]

Our baseline empirical measure for the investment rate in the model, $\Phi(x)$, is:

$$\Phi(x) \equiv \frac{\texttt{capx}}{\Pi_K \times (l.\texttt{ppegt})} \quad \text{(gross investment rate).}$$

Here, l. indicates that we use beginning-of-period values (that is, values from the close of the preceding fiscal year). Additionally, `capx` is capex (from cash flow statements), `ppegt` is gross property, plant and equipment at book value (from balance sheets), and Π_K is the gross change in a price index for property, plant and equipment, defined below. We map key financial ratios from the model to the data as follows:

$$\begin{aligned}
z(x) &\equiv \frac{\text{net debt}}{\text{ebitda}} && \text{(debt/ebitda ratio)} \\
\kappa z(x) &\equiv \frac{\texttt{xint}}{\text{ebitda}} && \text{(inverse interest coverage ratio)} \\
\pi(x)/e(x) &\equiv \frac{\text{equity payouts}}{l^{(1/2)}.\text{(market value of equity)}} && \text{(payout rate on equity)} \\
\iota(x)/x &\equiv \frac{\text{gross issuance of debt} - \texttt{chech}}{l^{(1/2)}.\text{(net debt)}} && \text{(gross issuance rate of debt)} \\
x &\equiv \frac{\text{net debt}}{\texttt{at}} && \text{(book leverage)}
\end{aligned}$$

[51] We treat missing variable fields as follows: if one of `dv`, `prstkc` or `sstk` is missing but at least one of the two others is not, we replace the missing value by 0. We do the same for `dltis` and `dlcc`, and for `dlc` and `dltt`.

Here, xint are interest and other related expenses (from income statements), at are total book assets (from balance sheets), and chech is change in cash and cash equivalents (from cash flow statements). Additionally, we define $l^{(1/2)}.x \equiv (1/2)(\Pi \times l.x + x)$, the average of prior-period (appropriately reflated by the inflation rate Π) and current period values for variable x. (We use this average in order to reduce the influence of observations with extremely large gross debt issuance rates due to very low 2018 debt stocks.)

In mapping data and model variables, we made certain choices that impact the values of the moments used in the estimation, aside from those that were already discussed in the main text (the use of net debt to measure the ratio of debt to ebitda, and the use of gross PP&E to measure investment rates). We briefly highlight these choices here, and discuss their impact on key moments in the robustness appendix.

First, we do not substract dltr (reduction in LT debt, from cash flow statements) from the computation of the gross issuance of debt in levels. The Compustat manual defines this cash flow item as "a reduction in long-term debt caused by its maturation, payments of long-term debt, and the conversion of debt to stock". Thus, this item contains principal payments on maturing debt, $mb_t dt$ in our model. Our goal is to construct a measure of gross issuance, $n_t dt = \iota_t k_t dt$, so we omit this term from our empirical measure.

Second, we define the equity payout rate as a fraction of the market value of equity, $\pi(x)/e(x) = \pi(b/k)k/E(k,b)$, instead of the book value of productive capital, $\pi(x) = \pi(b/k)k/k$. Likewise, we measure the rate of gross issuance of debt as a fraction of existing debt, $\iota(x)/x = \iota(b/k)k/b$, instead of the book value of productive capital, $\iota(x) = \iota(b/k)k/k$. This is because, to the extent possible, we prefer not to rely too much on a particular measure of the book value of productive capital k, since it is not obvious how to measure productive capital k from accounting data. We however acknowledge that our baseline measure of investment rates does take a stance on what productive capital is (PP&E), and so we also provide a robustness check that defines investment including intangible capital.

Finally, we also report measures of book leverage x defined as the ratio of book debt to book assets. Conceptually, the analog in our model is $x = b/k$. Our empirical measure uses book assets in the denominator, thus implicitly assuming that book assets is a measure of productive capital, at odds with the definition of k used in investment rates. We report this moment, despite this tension with our other measures, in order to allow comparisons with other papers using similar data. We do not target this moment in our estimation.

Compustat sample selection criteria We apply sequentially the following sample selection criteria:

1. drop firm-year observations not incorporated in the USA (fic = "USA")

2. drop firm-year observations whose two-digit SIC code (`sic`) is between 60 and 69 (financials), between 91 and 99 (multinationals), or equal to 49 (utilities);

3. drop observations whose name contains the strings "-REDH", "PRE FASB", "PRO FORMA", "INDEX", "-ADR", "-ADS", or has a non-missing and strictly positive value for `adrr`[52];

4. drop firm-year observations with negative entries for one of the following variables: `sale` (total revenue, from income statements) and `che`;

5. drop firm-year observations with strictly negative ebitda in 2019;

6. drop firm-year observations with zero short and long-term debt `dlc` and `dltt` in 2018 or 2019;

7. keep observations for fiscal year (`fyear`) 2019;

8. after computing the key ratios of interest, keep observations such that $\Phi(x)$, $z(x)$, $\kappa z(x)$, $\pi(x)/e(x)$, $\iota(x)/x$, and x are non-missing.

Unweighted summary statistics after winsorizing all moments at the top 99% and bottom 1% are reported in Appendix Table A-1.

Comparison of key moments to existing evidence We next compare some of the moments reported in Table A-2 to existing estimates in the literature. Our estimates may differ from the literature for at least two reasons: first, we select a sample of firms with positive ebitda; second, moments are weighted by ebitda. We make both of these data choices in order to remain consistent with the model we use. Both of them imply that our moments will tend to reflect the characteristics of the larger firms in our sample.

Average book leverage in our sample $(\hat{x})$ is somewhat more elevated than existing estimates. For instance, in our sample, average *gross* book leverage (gross debt divided by gross book assets; Column (3) in Table A-2) is approximately 38%, whereas Lemmon, Roberts and Zender (2008) document that *gross* leverage is 27% in their sample of nonfinancial firms between 1965 and 2003 (see their Table I). Aside from weighting and sample selection, mentioned above, two other reasons explain why our measures of leverage are somewhat more elevated than existing estimates.[53] First, book leverage has trended up over the past two

[52] A positive value for this variable indicates that the observation corresponds to an American Depositary Receipt. The other filters used similarly indicate either foreign entities, or stale or redundant observations.

[53] In our sample, value-weighting pushes up average leverage: for instance, gross leverage is 33% on average in our unweighted sample. Measures of aggregate leverage also tend to be higher: for instance, using aggregate data from the Flow of Funds, Gomes, Jermann and Schmid (2016) document an aggregate book leverage of 42%.

decades, and particularly so after 2010. Second, FASB Accounting Standards Update 2017-02 mandated the inclusion of operating leases in measures of book debt, creating an upward jump in book values of debt in Compustat starting in the first quarter of 2019. The same factors likely explain why our measures of net book leverage (gross debt minus cash, divided by book assets) are more elevated than existing estimates. For instance, Hennessy and Whited (2007) target a net debt to assets ratio of 12.04% (Table I) for their baseline estimation, and 14.52% (Table III) in their restricted large firm sample, whereas our baseline estimate in Table A-2 is 25.58%. Our estimates of debt/ebitda are likewise elevated, though other evidence report similar magnitudes: in a sample of rated firms, Baghai, Servaes and Tamayo (2014) report an average gross debt to ebitda ratio of 3.7 for 2009 (our sample average is 2.13).

Our baseline estimates of physical investment rate are consistent with previous research using similar data sources. In particular, in his sample of non-financial public firms, Hennessy (2004) documents an average gross investment rate of 12% per year, as a fraction of capital at replacement cost (Table I), while Hennessy and Whited (2005) report a gross investment rate of 7.9% per year, as a fraction of book assets, in their baseline sample. The total (physical plus intangible) investment rates we consider in our robustness section are also similar to (though slightly smaller than) those reported by Peters and Taylor (2017). Compared to aggregate investment rates, we find somewhat smaller numbers than estimates of gross investment rates that would be obtained using the Fixed Assets Table. For instance, Crouzet and Eberly (2020), Figure A8, report an aggregate gross physical investment rate of approximately 9% for 2017. The difference between the two may be driven by sample selection and by weighting, as Crouzet and Eberly (2020) show that aggregate gross investment rates in the Fixed Assets Table match those of Compustat data, in levels, when the latter are computed using gross property, plant and equipment in the denominator (see their Internet Appendix IA.2).

The average equity payout rate, in our baseline sample, is approximately 4.6% per year. This is in the range of the moments used by Hennessy and Whited (2005) and Hennessy and Whited (2007), who, respectively, report an equity issuance rate of 4.2% (Table II) and 8.9% (Table I), and Frank and Goyal (2003) (Table 2) report equity issuance rates in the order of 5% per year in the 1975-1990 period, and 10% per year in the 1990-2000 period. These equity issuance rates are expressed as a fraction of book assets; equity issuance rates defined as a fraction of book assets are somewhat lower in our data, approximately 4% per year. We choose to express them as a fraction of the market value of equity for the reasons discussed above.

Our estimate of the average debt issuance rate in our baseline sample is also relatively high. As a point of comparison, Frank and Goyal (2003) define debt issuance rates as the ratio of net issuance of long-term debt to book assets $(\texttt{dltis} - \texttt{dltr})/\texttt{at}$, an average debt issuance rate of 3.4% for 1998 (Table 2). Defined in the same way as Frank and Goyal (2003), the weighted average debt issuance rate in our sample is 3.7%, in line with their estimates.

Growth rates of prices of capital goods We construct a rate of inflation in capital goods prices, $\Pi_{K,t}$, using data from the BEA's Fixed Assets tables, as follows. First, we define the gross rate of change in the quantity of equipment and structures for the non-financial corporate sector as:

$$G_t = \left(\frac{K_{\text{struct},t}}{K_t} G^{-1}_{\text{struct},t} + \frac{K_{\text{equip},t}}{K_t} G^{-1}_{\text{equip},t} \right)^{-1}.$$

Here, $K_{\text{struct},t}$ is the current-cost net stock of non-residential structures in the non-financial corporate sector (Fixed Assets Table 4.1, line 39), $G_{\text{struct},t}$ is the growth rate in the chain-type quantity index for the net stock of non-residential structures in the non-financial corporate sector (Fixed Assets Table 4.2, line 39), $K_{\text{equip},t}$ and $G_{\text{equip},t}$ are similarly defined, but for equipment, and $K_t = K_{\text{struct},t} + K_{\text{equip},t}$ is the current-cost total stock of equipment and structures. The reason why we need to construct this index from underlying Fixed Assets data is that the gross rate of change in the quantity of capital for the non-financial corporate sector (Fixed Assets Table 4.2, line 37) includes intellectual property products, which our baseline measures of investment rate, which is limited to PP&E, does not capture. When including intangibles in our investment measure, we use the deflator $\Pi_{tot,t} = K_{tot,t}/K_{tot,t-1}G^{-1}_{tot,t}$, where $K_{tot,t}$ is the current-cost net stock of capital in the non-financial corporate sector (Fixed Assets Table 4.1, line 37), and $G_{tot,t}$ is the growth rate in the chain-type quantity index for the net stock of capital in the non-financial corporate sector (Fixed Assets Table 4.2, line 37).

We then define the rate of inflation in capital prices Π_K as:

$$\Pi_{K,t} = \frac{K_t}{K_{t-1}} G_t^{-1}.$$

Finally, we define the inflation rate Π as:

$$\Pi_t = \frac{Y_{\text{nom},t}/Y_{\text{real},t}}{Y_{\text{nom},t-1}/Y_{\text{real},t-1}},$$

where $Y_{\text{nom},t}$ is gross value added of of the non-financial corporate business sector in current dollars (NIPA table 1.14, line 17), and $Y_{\text{real},t}$ is gross value added of of the non-financial corporate business sector in chained 2012 dollars (NIPA table 1.14, line 41).

A.3.2 Estimation method

Let Y be the vector of estimated structural parameters, of size $N_p \times 1$. Let $\{X_i\}_{i \in [1,\ldots,N]}$ be a set of data vectors, each of size $N_X \times 1$. Additionally, each observation is attached a particular empirical weight w_i, a scalar, which satisfies: $\sum_{i=1}^{N} w_i = N$, where N is the total number of observations in the cross-section. The empirical construction of these weights has been described above. Next, define the mapping $\Xi(.) : \mathbb{R}^{N_X} \times \mathbb{R} \to \mathbb{R}^{N_m}$. Here, N_m is the number of moments to be matched from the data, and the function Ξ describes how these moments are computed from data observations. Finally, define the mapping $\Xi_m : \mathbb{R}^{N_p} \to \mathbb{R}^{N_m}$. Here, N_p is the number of structural parameters in the model, and Ξ_m describes how the theoretical moments are computed from these structural parameters.

For instance, $X_i = \{x_i\}_{i=1}^{N}$ could be a vector of observed leverage ratios x_i in the cross-section, weighted by their relative share of total ebitda: $w_i = \frac{ebitda_i}{\sum_j ebtida_j / N}$. To match weighted average leverage, on would use $\Xi(X_i, w_i) \equiv w_i x_i$ and $\Xi_m(\Upsilon) = \int_{x \leq \bar{x}(\Upsilon)} \hat{f}(x; \Upsilon) x dx$, where $\hat{f}(.; \Upsilon)$ is the stationary distorted density and $\bar{x}(.)$ is the default threshold.

Define the functions:

$$g(\Upsilon, X, w) \equiv \Xi(X, w) - \Xi_m(\Upsilon), \quad G(\Upsilon, \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N}) \equiv \frac{1}{N} \sum_{i=1}^{N} g(\Upsilon, X_i, w_i).$$

We obtain an initial point estimate for Υ, $\tilde{\Upsilon}$, as:

$$\tilde{\Upsilon} = \arg\min_{\Upsilon} G(\Upsilon; \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N})' G(\Upsilon; \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N}).$$

We then compute an estimate of the optimal weighting matrix, $\hat{W}$, as:

$$\hat{W} \equiv \left[\frac{1}{N} \sum_{n=1}^{N} g(\tilde{\Upsilon}, X_i, w_i) g(\tilde{\Upsilon}, X_i, w_i)' \right]^{-1}.$$

Finally, we compute the point estimate ofr Υ, $\hat{\Upsilon}$, as:

$$\hat{\Upsilon} = \arg\min_{\Upsilon} G(\Upsilon, \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N})' \hat{W} G(\Upsilon, \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N}).$$

The asymptotic distribution of $\hat{\Upsilon}$ is given by:

$$\sqrt{N} \left(\hat{\Upsilon} - \Upsilon_0 \right) \sim N(0, \Omega),$$

where an estimate of the variance-covariance matrix Ω is given by:

$$\hat{\Omega} = \left\{ \left(\frac{\partial G}{\partial \Upsilon} \left(\Upsilon, \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N} \right) \right)' \hat{W} \left(\frac{\partial G}{\partial \Upsilon} \left(\Upsilon, \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N} \right) \right) \right\}^{-1}.$$

The Jacobian of $G(.)$ must be approximated using numerical differentiation. To assess model fit, a test statistic for over-identifying restrictions in the case $N_m > N_p$ is:

$$J = G(\mathrm{Y}, \{\mathrm{X}_i\}_{i=1}^N, \{w_i\}_{i=1}^N)' \hat{W} G(\mathrm{Y}, \{\mathrm{X}_i\}_{i=1}^N, \{w_i\}_{i=1}^N),$$

which is distributed as a χ-squared with $N_m - N_p$ degrees of freedom under the null that the over-identifying restrictions hold.

We use 101 gridpoints in x in our numerical solution routine, with maximum leverage set to $x^{max} = 3.0$; we check that this maximum is never binding when solving the model. In order to estimate the model, we use 10 random starting points, and report estimation results from the starting point achieving the lowest value for the GMM objective. We use Matlab's patternsearch algorithm to find minima in the two-step GMM procedure, and we constraint the search so that the unconstrained optimal investment rate is well-defined, that is, (a, γ) such that:

$$a \leq (1 - \Theta)^{-1} \left(r + \delta + \frac{1}{2} \gamma r^2 \right).$$

A.3.3 Calibrated parameters

Values for the six calibrated parameters, along with the sources for these values, are reported in the top panel of Table 2. We note two brief comments about their values.

First, we use a value of $r = 0.05$. This value is higher than standard measures of the risk-free rate, though it is also lower than recent measures of the weighted average cost of capital, or the cost of equity capital (Frank and Shen, 2016).[54] Though our baseline model does not explicitly include equity risk premia, our choice of a value of r that is above the risk-free rate can be thought of as capturing positive average equity risk premia.

Second, we use a recent estimate of deadweight losses on capital from Kermani and Ma (2020). This estimate is close to the median post- to pre- liquidation value of book assets of 38% reported by Bris, Welch and Zhu (2006) in a sample of 61 chapter 7 liquidations. These authors find creditor recovery rates in liquidation (defined as the ratio of amount recovered to par value of debt owed) ranging from 5.4% to 27.4%, depending on assumptions about collateral recovery by secured creditors. We use the intermediate value of $\alpha_b = 0.15$ in our calibration.

A.3.4 Intuition for identification

We use an exactly identified approach to estimate the remaining three parameters, a (the average product of capital), σ (the volatility of capital quality shocks), and γ (the convexity of

[54] The value of 5% is consistent with estimates of weighted average cost of capital obtained in aggregate data for the non-financial corporate sector in Karabarbounis and Neiman (2019), Barkai (2020), and Crouzet and Eberly (2020).

capital adjustment costs). We match three data moments: the average gross investment rate $100 \cdot \hat{\Phi}$; the average ratio of debt to ebitda $\hat{z}$; and the cross-sectional sensitivity of investment to the ratio of debt to ebitda, $\hat{\Gamma}$.

Figure 3 reports how each of these moments vary with each of the three parameters. The main intuition for this graph is that identification in this model is "almost recursive". Put differently, Figure 3 is "almost" lower-diagonal, in the sense that the sensitivity of moments to parameters in the upper triangular portion of Figure 3 is relatively small.

The top row of Figure 3 shows that variation in the model's average gross investment rate primarily identifies the average product of capital a. The slope of the average investment rate with respect to γ is negative but relatively small, while the average investment rate is almost insensitive to the volatility of capital quality shocks. An increase in the volatility of capital quality shocks shifts the default boundary to the right, because the option value of continuing for equityholders increases. This tends to increase average leverage and lower investment rates. At the same time, a higher volatility increases (in absolute value) the slope of the debt price function with respect to leverage, so that debt issuance is lower with higher volatility. This tends to lower average leverage and increase investment. Quantitatively, these effects approximately offset each other. Investment adjustment costs γ also only have small effects on the level of investment (consistent with the assumption that they depend on the square of the expected net growth rate of the capital stock).

Given a value for a, the second row of Figure 3 shows that the average debt to ebitda ratio falls with the volatility of capital quality shocks, σ, while the sensitivity of investment with respect to leverage varies very little with σ. The average debt to ebitda ratio therefore helps identify the parameter σ. A higher volatility leads to lower average leverage because the debt price function becomes steeper as volatility rises. The debt price function is relatively insensitive to capital adjustment costs γ because those have only a small (local) impact on the average net growth rate of capital.

Finally, given values for a and σ, the third row of of Figure 3 shows that the slope of investment with respect to leverage helps identify the adjustment cost parameter γ. This slope is negative everywhere, as debt overhang causes investment to decline with leverage. Moreover, the magnitude of the slope decreases (in absolute value) with the magnitude of adjustment costs. All else equal, in response to a capital quality shock that increases their leverage, firms cut back investment less when investment adjustment costs are higher. Thus, the marginal effect of leverage on investment helps identify adjustment costs γ.

A.3.5 Non-targeted moments

Table A-2, Panel C compares model and data values for non-targeted moments. Interest coverage ratios in the model are close to their data counterparts. The average equity payout rate is somewhat lower than in the data, though it is closer if distributions are measured only as cash dividends (see Table A-2, Column 1). The average debt issuance rate is also somewhat smaller in the model than in the data. The empirical dispersion of the debt issuance rates is very large (see Appendix Table A-1), making the empirical average potentially imprecise.

The ratio of debt to capital, x, is approximately 50% on average in the model, whereas in our sample, the ratio of book debt to book assets is only half that. As noted above, given the definition of capital in our baseline approach (PP&E), the correct empirical counterpart for x is the ratio of debt to PP&E, which is 64%, on average, in our sample. Thus, whether the model understates or overstate book leverage depends on one's choice of empirical proxy for productive assets. This is why we do not target this ratio in our baseline approach.

Table A-2, Panel C also compares the model's implications for default rates, recovery rates, and aggregate growth, to the data. The model somewhat overstates default rates, and understates aggregate growth, relative to the data. Default rates are somewhat higher than empirical default rates for 2018 among rated firms (1.5% vs. 1.0%), though we also note that they are lower than default rates among non-investment grade firms (which are 2% in the data).[55] We define debt recovery rates, in the model, as the value of debt claims at the default boundary, $d(\bar{x})$. The data provided in Table A-2 computes recovery rates in a similar fashion, as enterprise value divided by total debt, or $e(\bar{x})/\bar{x} + d(\bar{x})$ in our model.[56] The main difference is that, as discussed above, the resolution of default in our model assumes APR violations, so that $e(\bar{x}) > 0$. This may partly explain why recovery rates in our model are low relative to those in the data, though this is also likely due to the low value of α_b used in our calibration. Finally, we note that the model under-estimates the aggregate growth rate of the total capital stock, where the empirical counterpart is computed using the Fixed Assets tables. This reflects, in part, the relatively high default rate implied by the calibration compared to the data.

In Table A-2, Panel C reports estimates of two important asset pricing moments from the model: the volatility of equity returns, and credit spreads. For equity volatility, our data counterpart is taken from Choi and Richardson (2016). These authors estimate monthly equity volatilities across quantiles of market leverage. In Table 2, we use the estimate of asset volatility for the second quintile of their Table 1, which we annualize by multiplying it by $\sqrt{12}$. The other annualized values for equity volatilities for the bottom and top two quintiles in their Table 1 are 64.0%, 46.4%, 47.0%, and 63.3%. Our model thus implies an average equity volatility that is between the median volatility in their sample, and the higher volatilities of high- and low-leverage firms. We also compare credit spreads in the model

[55] Default rates are from S&P (2019) Table 1. These only include rated firms, which are presumably less likely to default than non-rated firms in our sample.

[56] Recovery rates are from Ou, Chiu and Metz (2011), Exhibit 9, for subordinated debt. Moody's recovery rates are measured as enterprise value divided by total debt owed at time of resolution.

and in the data. We use estimates of credit spreads reported in Feldhütter and Schaefer (2018), who estimates average yield spreads to the swap rate of non-callable bonds issued by industrial firms, by credit rating, for the 1987-2012 period. For investment-grade bonds, the average spread is 92bps, while for high-yield bonds, the spread is 544bps (see their Table 9). Focusing more specifically on the maturities corresponding to calibration (7-13 y), they find an average spread of 87bps for investment-grade bonds, and 417bps for high-yield bonds. For the three ratings groups that are likely to be most important in our sample (A,BBB, and B), average yield spreads are 61bps, 141bps, and 290bps. Thus, in general, our model tends to overestimate yield spreads, relative to the data. This discrepancy arises because, as a result of the commitment problem, the model produces a frequency and size of debt issuances that is high, relatively to the frequency at which rated firms issue bonds. In the model, the limit as $x \to 0$ of credit spreads can be solved in closed form; it is given by:

$$\underline{cs} = \frac{\kappa\theta}{1 - \theta(\kappa/(\kappa + m))}.$$

In our baseline calibration, this minimum credit spread is equal to $198bps$. This result also indicates which features of the calibration would need to change in order to match credit spreads more closely. In particular, a lower coupon rate would help reduce spreads; so would lower corporate income tax rates, which would reduce the incentive to increase bond issuance.

Finally, Figure A-1 reports information on the higher moments of the data and the model. Specifically, the figure reports the cumulative share of different variables of interest (assets, k_t; EBITDA, ak_t; gross investment, $\Phi(g_t)$; and dividends, $\pi_t k_t$) by level of debt-to-ebitda, both in the model and the sample used for model estimation. These empirical CDFs are not targeted in our calibration. They suggest that the model under-predicts cross-sectional dispersion in ebitda and investment or assets, relative to the data. Note that the model under-predicts the importance of both very high- and very low-leverage firms, so that this does not bias the effects of debt overhang on investment in one particular direction.

The bottom right panel reports the empirical CDF for dividend issuance. The model-implied CDF rises above 100% because some firms in the model issue negative dividends.[57] The model thus tends to over-predict the frequency with which firms (particularly those with high leverage) use equity issuances as a way to smooth revenue and continue debt payments. The lack of equity issuance costs, as well as the lack of ability for firms to hoard liquidity (discussed in Section 3.2) contributes to this implication of the model. In our view, while counterfactual, this implication of the model is useful, because it magnifies the real

[57]The empirical CDF also rises above 100%, for the same reason, though this is not clearly visible in the graph.

effects of credit market shutdowns, and thus provides a form of upper bound on what the effects of these shutdowns (and the benefits of credit interventions) might be.

A.3.6 Robustness

Key moments obtained under alternative variable definitions or sample selection criteria are reported in Table A-2. The implied estimates for parameter values are reported in Appendix Table A-3. We next discuss in more detail our different robustness exercises.

Column (1) reports moments when equity payouts are measured using only dividends (as opposed to dividends plus net stock repurchases). This only affects the equity payout rate, which falls to 2.33%, closer to historical estimates of the dividend-price ratio for the S&P500 of around 2% (Shiller, 2015). This does not change the estimated values of structural parameters because equity payout rates are not a target in our estimation.

Column (2) reports moments when adjusting for changes in the treatment of operating leases in 2019. FASB Accounting Standards Update 2017-02 mandated the inclusion of operating leases in measures of book debt, creating an upward jump in book values of debt in Compustat starting in 2019.[58] In order to correct for the change in accounting rules, we subtract rental commitments (variable `mrct` in Compustat), when they are reported in 2019, from our definition of total debt. When they are not reported, but are reported in either 2018 or 2017, we subtract the lagged values, applying a 10% annual growth rate. (The value of 10% is somewhat arbitrary; we choose it to minimize the discontinuity in median book leverage over the 2010-2019 sample.) This lowers the debt-to-ebitda ratio by about 10%, and book leverage by about 3 percentage points. The sensitivity of investment to leverage is almost unchanged, and parameter estimates also change very little, except for the slightly higher dispersion of idiosyncratic shocks.

Column (3) reports moments when gross debt is used to define leverage ratios and issuance rates. By construction, leverage ratios are higher. The sensitivity of investment to leverage remains negative, but its magnitude falls by about half. As a result, the point estimate of σ is lower, and that of γ is larger. This would mitigate debt overhang effects of an increase in leverage.

Column (4) reports moments when the weight of an observation is defined as its total book assets relative to average book assets in the sample. This definition is only consistent with the model to the extent that book assets, `at`, are a good proxy for a firms' productive assets. With this weighting, leverage ratios are higher, investment rates are somewhat lower, and the sensitivity of investment to leverage is less than half of our baseline estimate, though it remains negative.

[58] For details on the effect of the change on book leverage in Compustat, see Palazzo and Yang (2019).

Column (5) reports moments after excluding short-term debt and the change in short-term debt from all the definitions of debt-related variables, since the model only allows for long-term debt of fixed maturity. This lowers leverage ratios, but leaves the sensitivity of investment rates to leverage approximately unchanged.

Column (6) reports moments when we replace ppegt by a perpetual inventory estimate of the current cost stock of property, plant and equipment for each firm in our sample. We compute this estimate using the following recursion:

$$\tilde{k}_{j,t+1} \quad = \quad \Pi_{K,t}(1-\delta_{j,t})\tilde{k}_t + \texttt{capx}_{j,t}$$

where j indexes a firm, $\tilde{k}_{j,t+1}$ is the end of year t, current cost estimate of the stock of property, plant and equipment, $\Pi_{K,t}$ is the gross rate of change in capital prices derived above, $\texttt{capx}_{j,t}$ are capital expenditures reported by firm j in year t, and $\delta_{j,t}$ is the rate of economic depreciation of the stock of property, plant and equipment.

Given an initial estimate of a firm's capital stock $\tilde{k}_{j,t_0(j)}$, we iterate on the relationship above. For the depreciation component, we set: $\delta_{j,t} = \delta = 10\%$, consistent with our calibration of the model, described below.[59]

We compute an initial value for the capital stock $\tilde{k}_{j,t_0(j)}$ as follows. Assuming that the rate of inflation in capital prices and the growth rate of investment are constant for $t \leq t_0(j)$, we obtain:

$$\tilde{k}_{j,t_0(j)} \quad = \quad \frac{1+g_{j,I,t_0(j)}}{g_{I,t_0(j)} + \delta - \left(\Pi_{K,t_0(j)} - 1\right)(1-\delta)} \texttt{capx}_{j,t_0(j)-1}. \tag{A14}$$

We then set $g_{j,I,t_0(j)}$ equal to the 10-year backward-looking moving average of $g_{I,t}$, where:

$$g_{I,t} \equiv \frac{I_{\text{struct},t} + I_{\text{equip},t}}{I_{\text{struct},t-1} + I_{\text{equip},t-1}} - 1. \tag{A15}$$

Here, $I_{\text{struct},t}$ is investment in non-residential structures in the non-financial corporate sector (Fixed Assets Table 4.7, line 39), and $I_{\text{equip},t}$ is similarly defined, but for equipment.[60] Finally, for each firm, we define $t_0(j)$ as the earliest year, before 2019, such that $\texttt{capx}_{t,j}$ is observed

[59] Unlike Hennessy and Whited (2007), Riddick and Whited (2009), Belo et al. (2019) and Falato et al. (2020), among others, we do not use reported accounting depreciation in Compustat, dp to measure depreciation expenses, i.e. $dp_{j,t} = \delta_{j,t}\Pi_{K,t}\tilde{k}_t$. First, because firms may use accelerated accounting depreciation, estimates of $\delta_{j,t}$ based on accounting depreciation may overstate the true rate of economic depreciation. Second, in our sample, there are frequent occurences of firms reporting depreciation in excess capital expenditures despite a growing net PP&E stock. The difference is due to acquisitions, but it implies that perpetual inventory method estimates of $\tilde{k}_{j,t}$ obtained this way can be negative. This is the case for instance with Amazon, which reports depreciation in excess of capex for 6 of the 25 years that it is present in Compustat.

[60] A alternative approach for initializing the perpetual inventory method consists of using, for $g_{j,I,t_0(j)}$, the sample average growth rate of $\texttt{capx}_{j,t}$ for $t \leq t_{t_0(j)}$ for firm j. Empirically, the relative small sample and the highly variable investment rates imply that the measure of $g_{j,I,t_0(j)}$ obtained this way will be noisy, and can lead to values that violate the transversality condition needed for condition (A14) to hold.

continuously from year $t_0(j) - 1$ to 2019, and we only keep firms for which $t_0(j)$ is lower than or equal to 2010, so that at least 10 years of data is used to compute the estimate of the capital stock. The sample is therefore smaller than in our baseline (1241 observations, instead of 1589).

The approach in column (6) results in an average investment rate that is higher by about 3 percentage points (14.86% instead of 11.28%), and a somewhat larger slope of investment with respect to leverage, in absolute value (1.19 instead of 1.09). The resulting estimate of the marginal product of capital, a, is higher, but the overall calibration remains close to the baseline.

Columns (7) to (9) deal with adjustments related to intangible capital and intangible investment. In order to adjust for intangibles, we follow the approach developed by Peters and Taylor (2017). The data for these adjustments is only available up to 2017. In Column (7), we therefore start by computing the moments of interest in the 2017 sample, with no adjustments for intangibles. Additionally, we require that firms in the sample have all the data required to make the adjustments for intangibles in Columns (8) and (9), so that the sample used in Columns (7)-(9) can be kept constant.

Column (7) indicates that even without intangible adjustments, leverage is lower, and debt issuance rates higher, in the 2017 sample.[61] Column (8) then adjusts for intangible investment in R&D. Specifically, we adjust our ebitda measure as:

$$\tilde{\text{ebitda}} = \text{ebitda} + \texttt{xrd},$$

where `xrd` are R&D expenditures (from income statements), which are treated as operating expenditures for accounting purposes. (We additionally impose that `xrd` be weakly smaller than `xsga`, sales, general and administrative expenses (from income statements), of which `xrd` is a subcomponent, and we replace missing values of `xrd` by zero.) Additionally, we make the following adjustments before computing investment rates:

$$\begin{aligned} \tilde{\text{ppegt}} &= \texttt{ppegt} + \texttt{k\_int\_know} \\ \tilde{\text{capx}} &= \texttt{capx} + \texttt{xrd} \end{aligned}$$

Here, `k_int_know` is the capitalized value of past R&D expenditures, as computed by Peters and Taylor (2017).[62] Note that, with this adjustment, the proxy for productive capital k used in the definition of investment rates and in the computation of dividend and debt issuance ratios is the same; productive capital is defined as $\texttt{ppegt} + \texttt{k\_int\_know}$.[63] Finally,

[61] The difference between 2017 and 2019 leverage is partly due to the upward trend in leverage, and partly to the changes implemented in 2019Q1 of the FASB rules for capitalizing leases mentioned above.

[62] We obtain its value from the latest download of the Total Q file on WRDS.

[63] This adjustment is correct if intangibles and physical capital are perfect substitutes, but it may not be

in this case, we use the deflator $\Pi_{tot,t}$ defined above in the computation of investment rates. This rate of price change is appropriate because the BEA Fixed Assets tables define private nonresidential fixed assets inclusive of intellectual property products which, in the Fixed Assets tables, primarily consist of R&D capital.

In Column (8), note first that debt to ebitda ratios are lower than in Column (7). This is because adjusting ebitda for R&D expenditures increases ebitda. Additionally, note that the investment rate is higher than in the baseline case. Finally, the resulting sensitivity of investment to leverage is substantially higher than in the baseline.[64]

Column (9) repeats the same exercise, but using an adjustment for both R&D capital and organization capital. In this adjustment, we define total productive capital as $\texttt{ppegt} + \texttt{k\_int\_know} + \texttt{k\_int\_org}$, where the estimate of the value of organization capital, `k_int_org`, is again obtained from Peters and Taylor (2017). We use the same series $\Pi_{tot,t}$ defined above in order to deflate the lagged capital stock. This does not allow for different inflation rates of organization vs. R&D capital, but we are not aware of good estimates for the rate of change in the price of organization capital. Following the definition of that paper, we measure total investment as $\texttt{capx} + \texttt{xrd} + 0.3 \times (\texttt{xsga} - \texttt{xrd})$. Relative to the case with only R&D, leverage ratios are even lower, investment rates even higher, and the sensitivity of leverage to investment rates even higher (in absolute value).

In order to limit the amount of results reported, we only estimate the model with intangible capital using the intermediate adjustment reported in column (8) of Table A-2. The results of table (4) indicate that this leads to a much lower value of the adjustment cost parameter γ, less than half of our baseline estimate. This would potentially magnify the effects of debt overhang following a shock.

Finally, Column (10) restricts the sample to firms with a credit rating. To retrieve the credit rating, we use Capital IQ's `wrds_erating` file on WRDS. After applying the different sample selection criteria, we link the remaining 2019 Compustat observations to the history of their ratings using the `wrds_gvkey` file on WRDS, which maps Capital IQ/S&P ratings to Compustat `gvkey`. We designate a firm in our sample as rated when (a) the `gvkey` received at least one rating (variable `ratingsymbol`) between 2017 and 2019, and (b) the rating was different from "NR" (the value which corresponds to unrated securities in the `wrds_erating` file). This merge leaves 443 firms in sample, compared to 1589 in our baseline sample.[65]

correct if they are not; in that case, not straightforward notion of total capital or total investment rate exists, as highlighted by Crouzet and Eberly (2020).

[64] Some of the moments not directly affected by the modification of ebitda also change relative to Column (7), but this is because adjustments to ebitda also affect each observation's weight.

[65] This number somewhat higher than other estimates of the relative importance of rated firms, who make up approximately 20% of the population of public firms (see, e.g. Faulkender and Petersen 2006). This is due to our sample selection criteria, which tend to eliminate smaller firms. Additionally, firms in the rated sample account for approximately 50% of aggregate book assets, relative to the baseline sample.

With the exception of leverage (which is somewhat higher), moments in the rated sample are very close to the baseline, with only higher investment and debt issuance rates. This is intuitive, given the fact that rated firms tend to be larger, and that moments are weighted by ebitda.

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Variable	Description	mean	s.d.	p25	p50	p75	N
$100 \cdot \Phi(x)$	gross investment rate	11.07	9.93	5.42	8.36	13.34	1589
$z(x)$	debt/ebitda	3.22	5.34	0.86	2.44	4.44	1589
$100 \cdot \kappa z(x)$	inverse interest coverage ratio	25.61	42.24	7.02	14.00	26.65	1589
$100 \cdot \pi(x)/e(x)$	payout rate on equity	2.43	6.23	0.07	1.61	4.53	1589
$100 \cdot \iota(x)/x$	gross issuance rate of debt	45.75	230.45	2.82	26.41	87.43	1589
$100 \cdot x$	book leverage	27.83	28.44	9.98	27.52	44.03	1589
w_i	ebitda (rel. to average)	1.00	3.50	0.04	0.17	0.60	1589

Table A-1: Summary statistics in the baseline sample. Variable definitions, sample selection, and other adjustments are described in Section 4 and Appendix A.3. All moments are unweighted. The data are for 2019. The sample is restricted to include only firms with strictly positive ebitda in 2019 and strictly positive total gross debt in 2018 and 2019. Some variables are scaled by a factor of 100 in order to facilitate interpretation.

Moment	Description	Baseline	(1) Only cash dividends	(2) Leasing adjustment	(3) Gross debt	(4) Asset-weighting	(5) Only LT debt	(6) k from PIM	(7) 2017 sample	(8) 2017 intan 1	(9) 2017 intan 2	(10) Rated firms
			Robustness									
$100 \cdot \hat{\Phi}$	gross investment rate	11.28	11.28	11.28	11.28	10.89	11.28	14.86	11.71	17.95	19.70	10.91
$\hat{z}$	debt/ebitda	2.13	2.13	1.90	2.93	2.70	1.79	2.03	1.61	1.38	1.13	2.42
$\hat{\Gamma}$	inv. to debt/ebitda slope	−1.04	−1.04	−1.09	−0.62	−0.40	−1.13	−1.19	−1.05	−2.89	−3.02	−1.12
$100 \cdot \kappa\hat{z}$	inverse interest cov. ratio	11.26	11.26	11.26	11.26	14.02	11.26	10.60	10.89	9.36	7.65	12.68
$100 \cdot \widehat{\pi/e}$	equity payout rate	4.62	2.33	4.62	4.62	4.30	4.62	4.82	3.91	3.89	3.90	4.58
$100 \cdot \widehat{\iota/x}$	debt issuance rate	25.74	25.74	33.22	28.38	31.82	32.50	25.96	66.87	72.10	72.45	28.28
$100 \cdot \hat{x}$	book leverage	25.58	25.58	22.53	37.52	27.03	21.45	24.80	20.17	17.81	18.15	27.38
N	nr. of obs.	1589	1589	1589	1589	1589	1589	1241	1826	1826	1826	443

Table A-2: Moments used in the robustness checks. The sample used is all Compustat non-financial firms (both rated and unrated). The moments reported are defined in the text. The third column, marked "Baseline", reports the moments used in the estimation of the model in our baseline sample from 2019; the baseline ebitda weights are used to compute them. Columns marked (1)-(10) report the values of these moments using alternative sample selection criteria and variable definitions. Column (1) defines equity payouts as only cash dividends. Column (2) adjusts for the changes in the accounting treatment of leases that occured in 2019Q1. Column (3) uses gross debt, instead of net debt, in the definition of leverage ratios and debt issuance rates. Column (4) weighs observations by book assets instead of ebitda. Column (5) excludes short-term debt from the definition of leverage ratios and debt issuance rates. Column (6) uses a measure of the physical capital stock obtained using the perpetual inventory method. Columns (7) to (9) deal with adjustments for intangibles. Data for adjustments related to intangible is only available up to 2017, so Column (7) first reports the moments constructed in the same way as the baseline (with no intangible adjustment), but for the 2017 sample. Column (8) then adjusts for intangible investment in R&D, while Column (9) adjusts for intangible investment in both R&D and organization capital. Column (10) reports moments obtained when restricting the sample to firms with a credit rating.

Panel A. Estimated parameters

Parameter	Baseline	Robustness (2) Leasing adj.	(3) Gross debt	(4) Asset-wgt.	(5) LT debt	(6) k from PIM	(8) Intan. adj.	(10) Rated firms
a	0.24 [0.22, 0.25]	0.24 [0.22, 0.25]	0.24 [0.22, 0.27]	0.24 [0.20, 0.29]	0.24 [0.22, 0.25]	0.25 [0.23, 0.26]	0.31 [0.29, 0.32]	0.24 [0.21, 0.26]
σ	0.31 [0.27, 0.35]	0.35 [0.31, 0.40]	0.21 [0.12, 0.29]	0.24 [0.09, 0.38]	0.37 [0.34, 0.41]	0.39 [0.34, 0.44]	0.36 [0.30, 0.42]	0.26 [0.18, 0.33]
γ	7.16 [6.59, 7.74]	6.24 [5.70, 6.79]	15.00 [13.31, 16.69]	14.39 [13.87, 16.12]	5.82 [5.28, 6.32]	10.35 [10.25, 10.47]	3.33 [3.32, 3.36]	7.52 [6.52, 8.52]

Panel B. Model fit

Moment	Baseline		Robustness (2) Leasing adj.		(3) Gross debt		(4) Asset-wgt.		(5) LT debt		(6) k from PIM		(8) Intan. adj.		(10) Rated firms	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
$100 \cdot \hat{\Phi}$	11.28	11.28	11.28	11.28	11.28	11.28	10.89	10.96	11.28	11.28	14.86	14.12	17.95	16.98	10.91	10.91
$\hat{z}$	2.13	2.14	1.90	1.90	2.93	2.92	2.70	2.63	1.79	1.79	2.03	2.06	1.38	1.37	2.42	2.42
$\hat{\Gamma}$	−1.04	−1.04	−1.09	−1.10	−0.62	−0.70	−0.40	−0.61	−1.13	−1.13	−1.19	−0.79	−2.89	−2.70	−1.12	−1.12
$100 \cdot \kappa\hat{z}$	11.3	10.7	11.3	10.7	11.3	14.6	14.0	13.2	11.3	9.0	10.6	10.3	9.4	6.9	12.7	12.1
$100 \cdot \widehat{\pi/e}$	4.6	3.0	4.6	3.1	4.6	3.1	4.3	3.3	4.6	3.1	4.8	0.6	3.9	2.7	4.6	3.4
$100 \cdot \widehat{\iota/x}$	25.7	17.9	33.2	23.2	28.4	14.9	31.8	15.4	32.5	83.2	26.0	25.6	72.1	97.0	28.3	15.9
$100 \cdot \hat{x}$	25.6	51.0	22.5	45.2	37.5	71.5	27.0	63.8	21.5	42.6	24.8	51.4	17.8	42.9	27.4	57.5
$100 \cdot d(\bar{x})$	29.3	12.1	29.3	10.5	29.3	16.8	29.3	15.4	29.3	9.9	29.3	10.4	29.3	10.6	29.3	12.9
$100 \cdot \lambda_d$	1.0	1.5	1.0	1.4	1.0	1.9	1.0	1.8	1.0	1.3	1.0	1.1	1.0	1.2	0.0	1.6
$100 \cdot G$	1.9	0.5	1.9	0.3	1.9	−0.1	1.9	−0.3	1.9	0.3	1.9	2.7	2.3	1.0	1.9	−0.2

Table A-3: Robustness checks on baseline estimate. The top panel reports point estimates, and the bottom panel reports measures of model fit. In each panel, columns report estimates of the model when different moments are targeted than in the baseline. The targeted moments are the first three reported in Panel B ($100 \cdot \hat{\Phi}, \hat{z}, \hat{\Gamma}$). Columns are indexed with the same numbers as in Table A-2. Compustat sample moments reported in the columns marked "Data" are the same as in that table. Non-Compustat moments are the same as in Table 2, except for Column (8), where the growth rate of the quantity index for the net stock of capital $G_{tot,t}$ is used, and in Column (10), where the default rate for investment-grade firms from S&P (2019) is used. We omit the case of cash dividends because targeted moments are the same, and we only include one of the two robustness checks on intangibles to save space. Calibrated parameters are identical across all estimations, and equal to those reported in Table 2, Panel A, with the exception of column (8), where we set $\delta = 15\%$ in order to account for the higher depreciation rates of intangible capital.

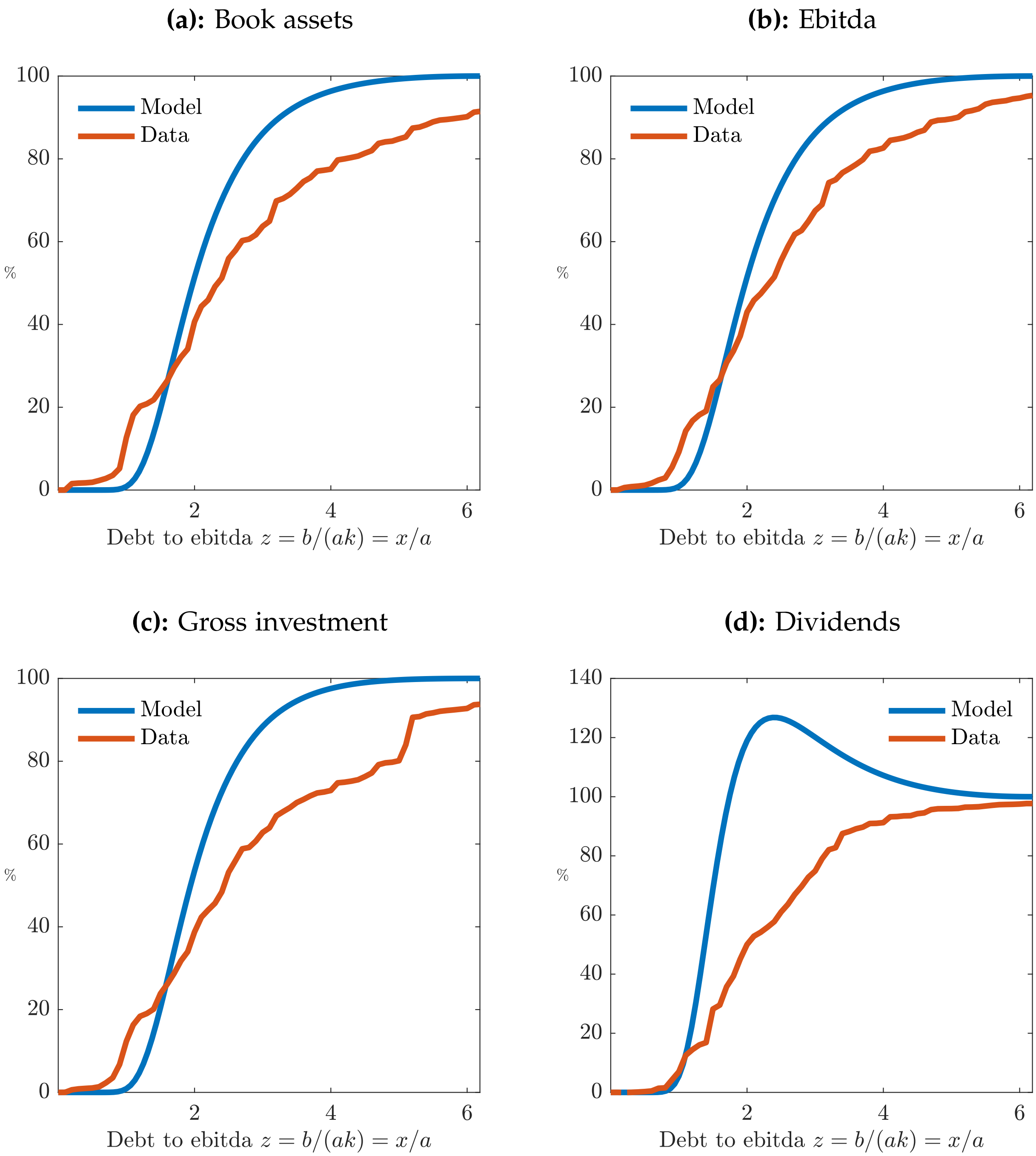

Figure A-1: Comparison of empirical and model-implied cumulative distribution functions (CDF) for different variables. Each panel plots the cumulative share of a variable of interest (as a fraction of the aggregate value of that variable), as a function of the debt-to-ebitda ratio. The plots are constructed using the sample used in the estimation of the model.

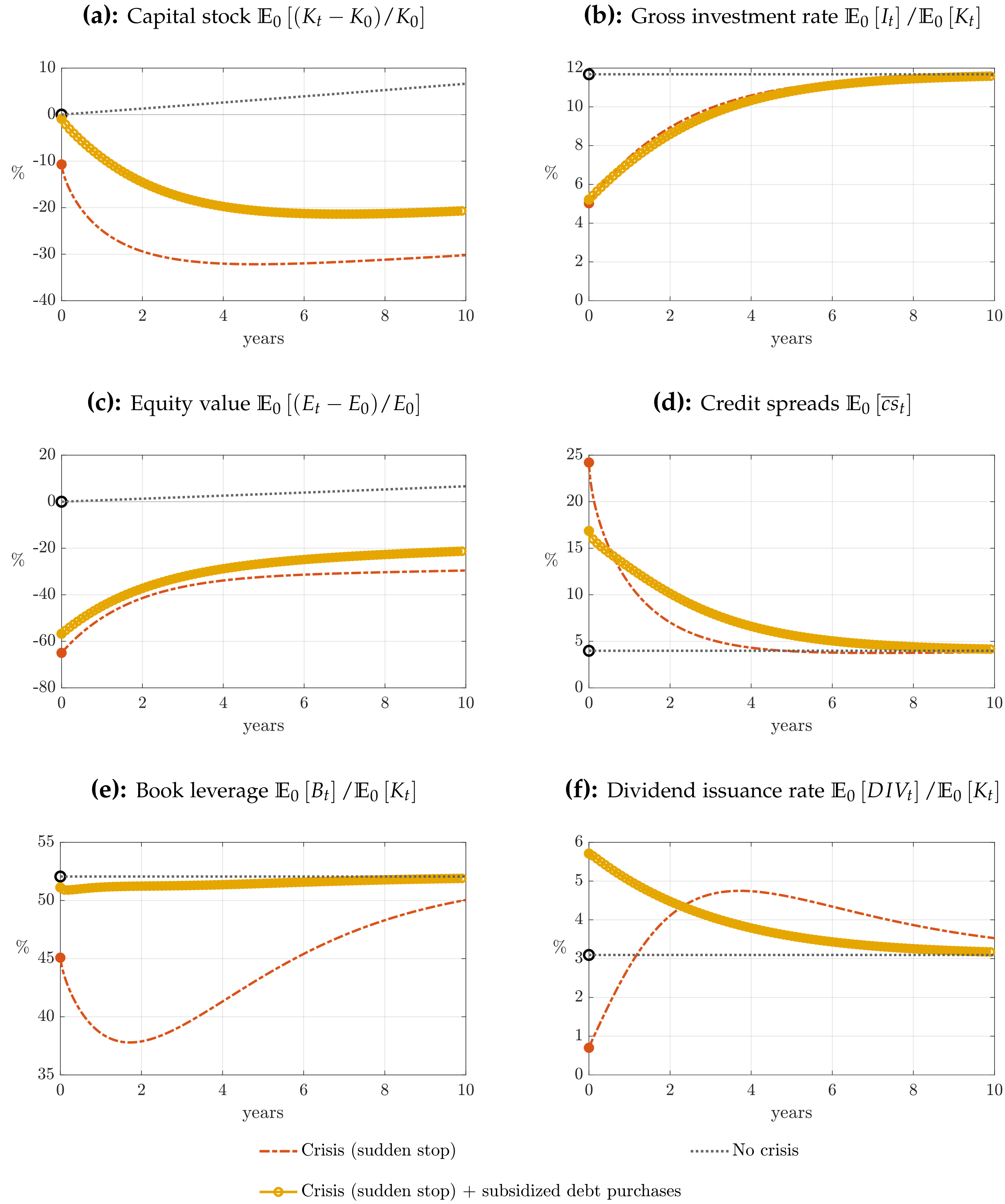

Figure A-2: Robustness checks: crisis duration χ. The impulse responses reported are for an average crisis duration of $1/\chi = 3$ instead of $1/\chi = 1$ year in our baseline calibration. The long dashed orange lines report the expected path of economic aggregates following an unexpected crisis, if financial markets shut down (sudden stop) and the government does not intervene. The circled yellow lines report the expected path of economic aggregates if the government provides subsidized loans, as described in Section 7.1. The average duration of the crisis is one year; the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

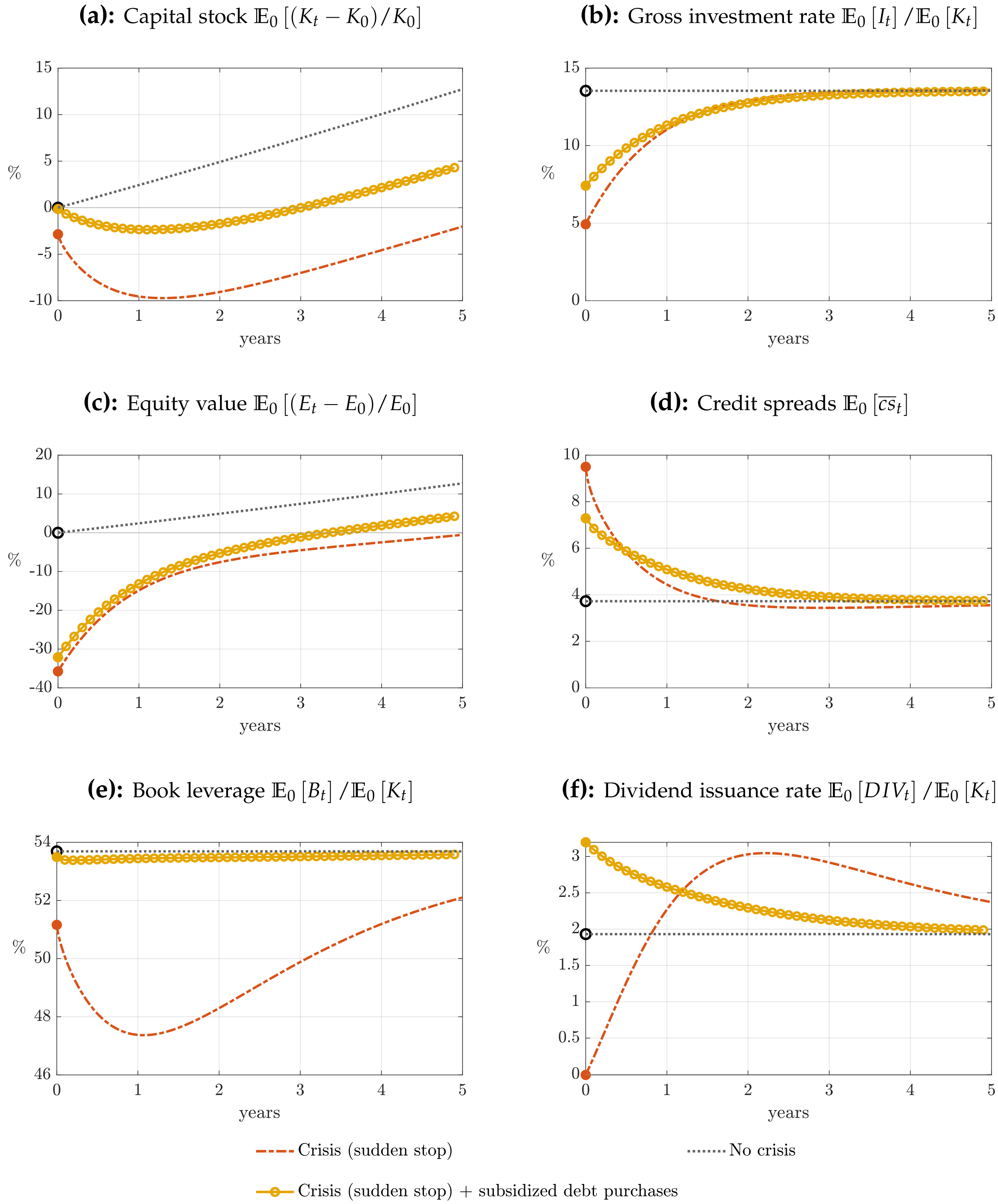

Figure A-3: Robustness checks: curvature of investment adjustment costs γ. The impulse responses reported are for adjustment costs curvature of $\gamma = 5$ instead of $\gamma = 7.16$ year in our baseline calibration. The long dashed orange lines report the expected path of economic aggregates following an unexpected crisis, if financial markets shut down (sudden stop) and the government does not intervene. The circled yellow lines report the expected path of economic aggregates if the government provides subsidized loans, as described in Section 7.1. The average duration of the crisis is one year; the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.

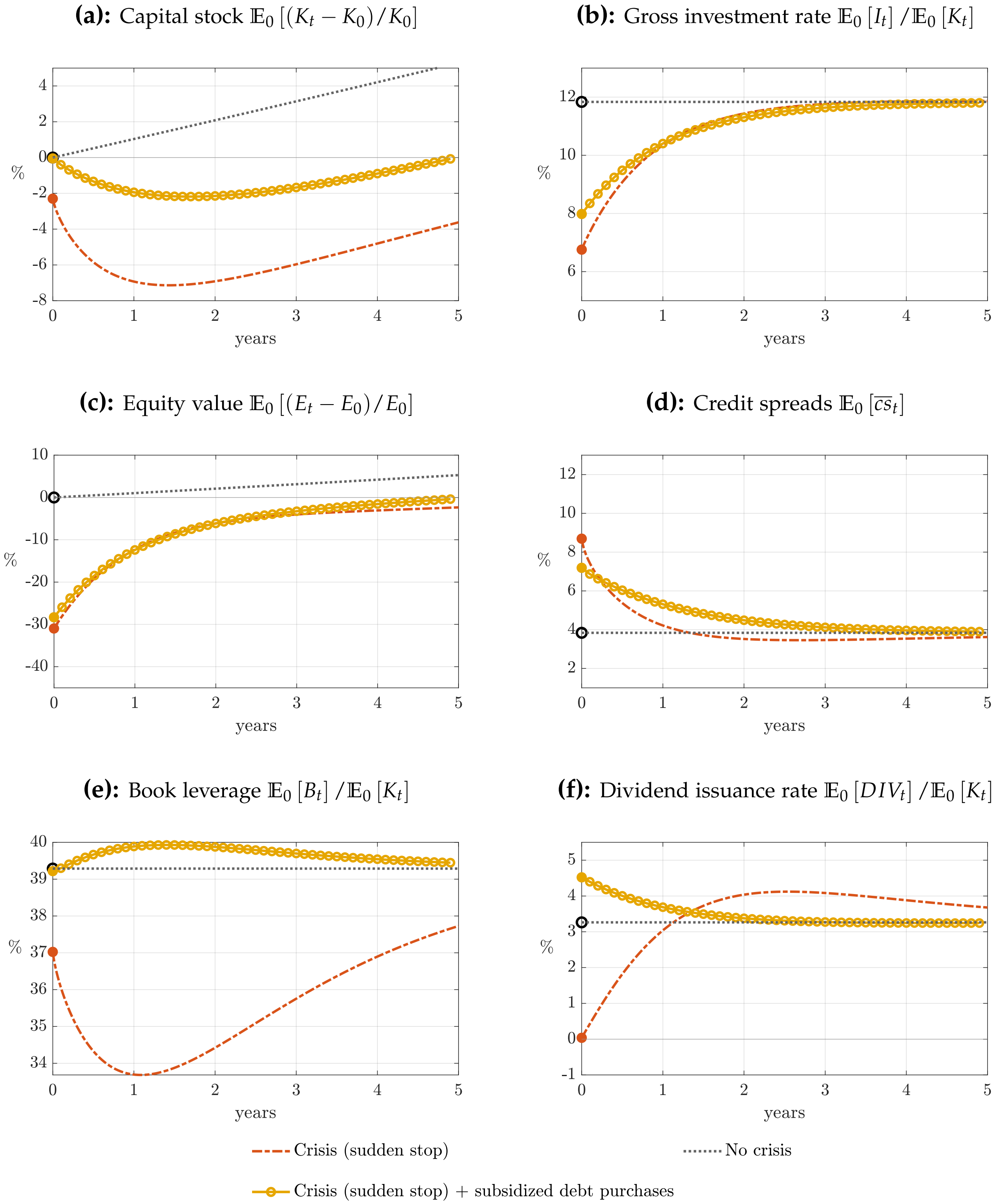

Figure A-4: Robustness checks: deadweight losses in bankruptcy α_k. The impulse responses reported are for deadweight losses of $\alpha_k = 0.495$ instead of $\alpha_k = 0.33$ in our baseline calibration. The long dashed orange lines report the expected path of economic aggregates following an unexpected crisis, if financial markets shut down (sudden stop) and the government does not intervene. The circled yellow lines report the expected path of economic aggregates if the government provides subsidized loans, as described in Section 7.1. The average duration of the crisis is one year; the marginal product of capital falls to $\underline{a} = 0.75a$, and the Sharpe ratios for debt and equity jump to $\underline{\nu}_d = \underline{\nu}_e = 85\%$ during the crisis; see Section 5.1 for details. The dashed gray lines are the balanced growth path. Appendix A.2.9.2 reports the exact definitions of expected aggregates in terms of model objects.