Fiscal Regimes and the Exchange Rate

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Abstract

In this paper, we argue that the effect of monetary and fiscal policies on the exchange rate depends on the fiscal regime. A contractionary monetary (expansionary fiscal) shock can lead to a depreciation, rather than an appreciation, of the domestic currency if debt is not backed by future fiscal surpluses. We look at daily movements of the Brazilian *real* around policy announcements and find strong support for the existence of two regimes with opposite signs. The unconventional response of the exchange rate occurs when fiscal fundamentals are deteriorating and markets' concern about debt sustainability is rising. To rationalize these findings, we propose a model of sovereign default in which foreign investors are subject to higher haircuts and fiscal policy shifts between Ricardian and non-Ricardian regimes. In the latter, sovereign default risk drives the currency risk premium and affects how the exchange rate reacts to policy shocks.

JEL classifications: E52, E62, E63, F31, F34, F41, G15
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1 Introduction

Standard international macroeconomic models predict that a monetary policy tightening leads to an appreciation of the domestic currency. A higher interest rate makes domestic assets more attractive vis-à-vis foreign assets and increases the demand for domestic currency. The empirical evidence for advanced economies supports this prediction. See, for example, Eichenbaum and Evans (1995) for the US and Kim and Roubini (2000) and Zettelmeyer (2004) for other countries.[1] For emerging markets, the evidence is more mixed. Hnatkovska, Lahiri, and Vegh (2016) show that developing country currencies tend to depreciate in response to a monetary tightening, while Kohlscheen (2014) finds no effects of monetary policy surprises on the exchange rates of Brazil, Mexico and Chile.

Similarly, an expansionary fiscal surprise leads to an appreciation of the domestic currency in a large class of models. Higher government spending or lower taxes increase aggregate demand and raise prices, inducing the central bank to tighten. The empirical evidence for advanced economies provides little support for this prediction. Monacelli and Perotti (2008), Kim and Roubini (2008) and Enders, Müller, and Scholl (2011) find that a US expansionary fiscal policy shock decreases the relative price of imports and depreciates the real exchange rate. Ravn, Schmitt-Grohé, and Uribe (2012) confirm these findings in a panel VAR from four industrialized countries. For emerging economies, the empirical evidence is scarce. Ilzetzki, Mendoza, and Végh (2013) show that in developing countries the real exchange rate appreciates, on impact, in response to an increase in government consumption.

In this paper, we study how the exchange rate responds to domestic policies using a different point of view. While most of the literature focuses on the unconditional response of the exchange rate, we emphasize its contingent behaviour. In particular, we highlight how the backing of government bonds, or lack thereof, determines how monetary and fiscal policy affect the exchange rate and, ultimately, domestic macroeconomic variables. Following the terminology set forth by Sargent (1982) and Aiyagari and Gertler (1985), we distinguish between Ricardian and non-Ricardian fiscal regimes. In the former, the fiscal authority provides full backing for its debt; at every point in time, it commits to levying a stream of future taxes with a present discounted value equal to the current value of its obligation. In a non-Ricardian regime, by contrast, the fiscal authority does not fully finance its debt. In this case, either debt is monetised, i.e. the central bank accommodates fiscal deficits with current and future money creation, or the fiscal authority is forced to default. The main conclusion of our paper is that the response of the exchange rate to monetary and fiscal policy shocks changes depending on the fiscal regime. In a Ricardian regime, contractionary monetary or expansionary fiscal shocks tend to appreciate the exchange rate, while they tend to depreciate it if the fiscal regime is non-Ricardian.

Our analysis is both empirical and theoretical. First, we look at the recent history of Brazil and identify two periods in which the fiscal regime was likely perceived by the financial market participants to be non-Ricardian. We show that the covariance between monetary policy surprises and exchange rate variations is positive during these periods, while it is negative during conventional times. Similarly, in these periods the covariance between the exchange rate changes and fiscal policy surprises is positive, while their covariance is zero at all other times. To demonstrate that the cause of the differential behaviour is indeed fiscal, we then take a more agnostic approach to the underlying source of variations. We estimate a Markov-switching regression model in which the

[1] However, as originally documented by Engel and Frankel (1984), there are many days in which a Fed's tightening leads to a depreciation of the dollar. More recently, Stavrakeva and Tang (2018) study the appreciation of the dollar in response to the Fed's easing during the Great Recession and propose an explanation based on information effects and the exorbitant duty. Gürkaynak, Kara, et al. (2021) argue that for the USD/EUR exchange rate information effects cannot fully explain the unconventional response.

parameters are allowed to vary according to an unobserved 2-state Markov chain. The estimated probabilities show that periods in which the slope coefficient is likely to be positive coincide with periods in which fiscal fundamentals were deteriorating in Brazil and/or investors' concern about debt sustainability was raising. Exactly those periods which we identified as non-Ricardian using the narrative evidence.

To rationalize these findings, we develop a small open economy model in which fiscal policy switches stochastically between a Ricardian and non-Ricardian regime and the government can default on its debt. The key feature of our model, and its main departure from the rest of the literature, is that upon default, foreign investors are subject to higher haircuts than domestic investors.[2] This assumption implies that the credit spread on government bonds is not sufficient to compensate foreign investors for the overall risk they face. This, in turn, has two important consequences. First, the excess return required to compensate them for the additional risk must be generated through exchange rate movements. Hence, the probability of a sovereign default enters into the uncovered interest parity condition of the model and drives the currency risk premium. Second, the effective interest rates used to discount future primary surpluses are decreasing in the probability of default. Therefore, the path of default probability is determined endogenously by the government intertemporal budget constraint.

We use the model to characterize the response of the exchange rate to monetary and fiscal shocks. Consistent with the evidence presented in the empirical part, an unexpected increase in the domestic policy rate or in government expenditures leads to an appreciation of the domestic currency when fiscal policy is Ricardian, but leads to a depreciation, or a smaller appreciation, when fiscal policy is in the non-Ricardian regime. The increase in debt raises default risk and the currency expected excess return. Hence, the value of the domestic currency falls.

Finally, we consider the case in which, when fiscal policy is non-Ricardian, the central bank monetises the fiscal deficit and inflates away the debt. This is the typical situation studied in the fiscal theory of the price level literature (see for example Leeper (1991), Sims (1994) and Woodford (2001)). We show that in this case the exchange rate's response to monetary and fiscal shocks is ambiguous and depends on the share of debt denominated in foreign currency and the monetary policy rule. An unexpected increase in the domestic policy rate, or in government expenditures, leads to a depreciation of the domestic currency if debt is mostly denominated in local currency and/or the Taylor coefficient in the monetary policy rule is sufficiently low. Vice versa, when most of the debt is denominated in foreign currency and/or the Taylor coefficient is close to one, the same shocks lead to an appreciation which is larger than in the Ricardian regime.

Our model is related to two broad streams of literature: the literature on currency risk premia and the sovereign default literature. The literature on the determinants of currency risk premia has mostly focused on complete markets (Backus, Kehoe, and Kydland (1992); Pavlova and Rigobon (2007); Verdelhan (2010); Colacito and Croce (2011)) and, less so, on incomplete markets (Chari, Kehoe, and McGrattan (2002); Corsetti, Dedola, and Leduc (2008)). A smaller, but growing, literature focuses on exchange rate modelling in the presence of financial frictions (Bacchetta and Van Wincoop (2010); Gabaix and Maggiori (2015); Engel (2016)). Our model is conceptually related to the framework proposed by Blanchard (2004) which focuses on default risk and heterogeneous risk aversion between domestic and foreign investors. We document that foreign investors risk attitude plays no role in driving the unconventional response of the exchange rate and propose a theory based on heterogeneous recovery rates instead. The literature on sovereign default can be divided into two streams. The strategic default approach, pioneered by Eaton and Gersovitz (1981), which

[2] Other theoretical works that feature differential treatment among creditors are Guembel and Sussman (2009), Broner, Martin, and Ventura (2010) and Broner, Erce, et al. (2014)

focuses on the sovereign's incentive to repay its debt (Aguiar and Gopinath (2006), Arellano (2008), Mendoza and Yue (2012)), and the fiscal limit approach which instead emphasizes the ability to do so (Uribe (2006), Bi (2012), Schabert and Wijnbergen (2014)). Our model fits in the second stream. Similarly to Uribe (2006), we propose a model in which the probability of default is determined endogenously by the government intertemporal budget constraint. But in our model, default risk, rather than default itself, restores the equilibrium by changing the factor used to discount future primary surpluses. Schabert and Wijnbergen (2014) follow a similar approach, but in their model default risk restores debt sustainability by increasing inflation, like in models of the fiscal theory of the price level. In fact, in Schabert and Wijnbergen (2014) an equilibrium with default exists only if monetary policy is passive, that is, subordinated to fiscal policy. In our model an equilibrium with default arises only if monetary policy is active, that is, if the central bank raises the policy rate more than one-for-one with inflation.

The rest of the paper is organised as follows. In Section 2, we present the empirical evidence. In Section 3, we develop the theoretical model, and in Section 4, we prove the main results of the paper. Section 5 concludes.

2 Empirical evidence: the case of Brazil

In this section, we investigate how the exchange rate reacts to monetary and fiscal policy shocks in Brazil. The combination of a flexible exchange rate regime, an independent central bank and a history of recurrent debt crises make Brazil the ideal case study to test our hypothesis.

As many other countries in Latin America, Brazil has a long track record of procyclical fiscal policies (see, for example, Alberola et al. (2016) and Ayres et al. (2019)). In the 1980s and until the mid-1990s, fiscal profligacy led to sovereign debt crises and bouts of hyperinflation. The Brazilian government defaulted on its domestic debt three times (1986, 1987 and 1990) and experienced three technical defaults on its foreign debt (1982, 1986 and 1990). Between 1981 and 1994, the primary deficit and interest payment on domestic and foreign debt averaged 3.1% and 2.3% of GDP per annum, respectively, while yearly inflation averaged 450%.

Since the mid-1990s, Brazil has significantly improved its monetary and fiscal policy frameworks. In March 1999, the central bank of Brazil changed its exchange and monetary regime, abandoning a crawling peg in favour of a floating regime and inflation targeting. Simultaneously, the government took important measures to improve the conduct of fiscal policy, including the announcement of fiscal targets and the enactment of the Fiscal Responsibility Law, which imposed significant constraints on both the federal and local governments. These measures stabilized inflation and led to prolonged periods of fiscal surpluses. Between 1995 and 2016, inflation averaged 8% per annum, and the fiscal balance averaged 0.1% of GDP.[3]

However, while in the past two decades fiscal discipline improved markedly, fiscal issues have not disappeared completely and fiscal concerns resurface periodically. Two episodes, in particular, have characterized the recent fiscal history of Brazil. The first episode coincides with the runoff to the 2002 general election. In March 2002, Luiz Ińacio Lula da Silva, "Lula", was nominated presidential candidate for the left-wing Worker's Party (*Partido dos Trabalhadores*) for the fourth consecutive time. In the previous races, Lula, a former union leader and one of the founders of the party, had advocated for the abandonment of the free-market economic model and for the renegotiation of Brazil's external debt, indicating the possibility of an outright default. Despite moderating his program, when the first pools revealed that Lula was favourite to win the presidential election,

[3]The data in this paragraph comes from Ayres et al. (2019).

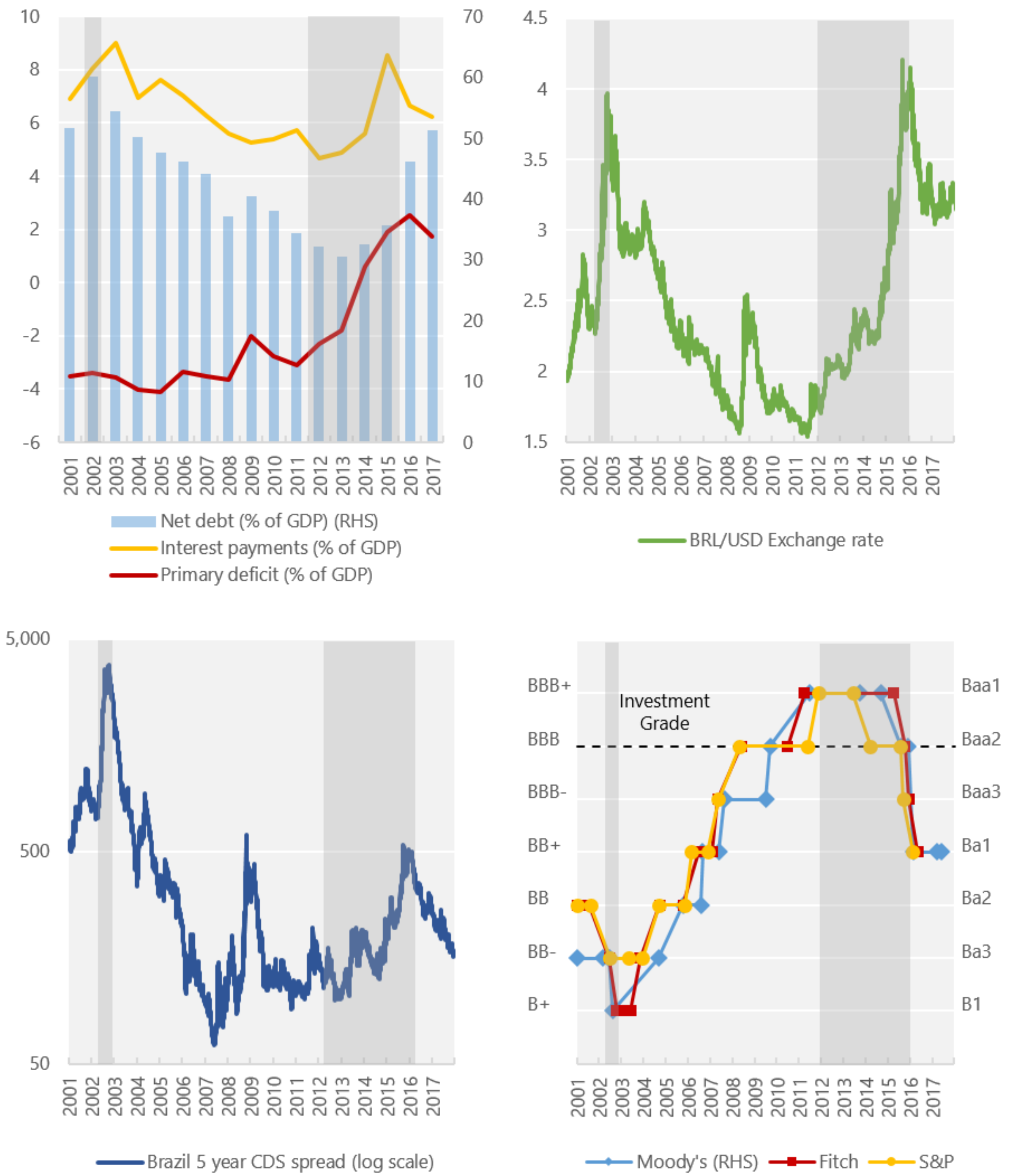

Figure 1: **Fiscal variables, exchange rate, credit ratings and CDS spread.** The figure shows Brazil's public sector net debt, interest payments and primary deficit (top left panel), BRL/USD exchange rate (top right panel), Brazil 5-year CDS spread (bottom left panel) and sovereign credit ratings (bottom right panel). Shaded areas denote periods which we identify as non-Ricardian fiscal regimes.

investors' confidence in the stability of Brazil's debt collapsed and the rate of interest on both domestic and external debt increased sharply.

Between April and October 2002, the Brazil 5-year Credit Default Swap (CDS) spread increased by a factor of five, reaching an all-time high of 3,750 basis points in mid-October (Figure 1, bottom left panel). In June, Fitch downgraded Brazil's debt rating to "highly speculative", while Standard & Poor's and Moody's followed suit in July and August, respectively (Figure 1, bottom right panel). In September, the International Monetary Fund (IMF) stepped in and granted Brazil a record \$30.4 billion loan. At the end of October, after two runoff rounds, Lula was finally elected president. After the election, Lula's announcement that Brazil would honour its agreements with the International Monetary Fund and would continue to make payments on its debt convinced financial markets that the fiscal outlook was better than feared. The CDS spread fell below 2,600 basis points by the end of November and below 2,000 basis points in early January 2003, when Lula took office. Over the following months, Lula kept his promises and markets slowly returned to normality. By June 2003, the CDS spread was back to its pre-crisis level.

The second episode, begins in the aftermath of the global financial crisis, at the end of the commodity supercycle, and culminates in the fiscal crisis of 2015. Like in many other countries, the Brazilian government responded to the 2008-2009 crisis by adopting a countercyclical fiscal policy to prevent a major recession (see Vegh and Vuletin (2014)). Initially, the policy seemed to be very successful. Real GDP grew 7.5% in 2010, and by early 2011, the government was ready to embark on a fiscal consolidation plan. However, in 2012 the recovery appeared to be weaker than expected, and the Brazilian government returned to using fiscal incentives in an attempt to restart the economy. The primary surplus fell to 2.3% of GDP in 2012, the first year in which the target was missed, and to 1.8% in 2013 (Figure 1, top left panel).[4] Despite these efforts, growth slowed to an average of 2.9% per year between 2011 and 2013.

In June 2013, Standard & Poor's revised Brazil's debt outlook from stable to negative and downgraded its rating a few months later. The fall in commodity prices in mid-2014 made the situation even worse. The fiscal deterioration accelerated, while the economy started to contract. The primary surplus turned into a deficit which rose to 1.9% of GDP in 2015. In the second half of the year, a new round of downgrades led Brazil to lose its investment grade rating and brought the debt-servicing cost to 8.5% of GDP in 2015, almost twice as much as its 2012 level. Brazil's CDS spread started rising in early 2012 and peaked in December 2015, before declining throughout 2016 and 2017.[5]

While different in their duration and severity, these two episodes share a common cause. A fiscal policy, either actual or expected, that was deemed by market participants as being unsustainable. Indeed, both episodes have been labelled as periods of fiscal dominance. According to Blanchard (2004), "in 2002, the level and the composition of Brazilian debt, together with the general level of risk aversion in world financial markets" were such as to imply that Brazil was in a regime of fiscal dominance. He argues that, under these circumstances, "the increase in real interest rates would probably have been perverse, leading to an increase in the probability of default, to further depreciation, and to an increase in inflation". Similarly, in 2015, de Bolle argued that,

[4] In those years the government started to implement budget maneuvers to hide deficit figures, a practice known as *contabilidade criativa* (creative accounting). These fiscal maneuvers led to the impeachment in 2015 of former president Dilma Rousseff, who had replaced Lula in 2010 and was reelected in 2014. See Holland (2019) for details.

[5] During this period, the implementation and subsequent withdrawal of quantitative easing in advanced economies led to substantial spillover effects for emerging markets (see Fratzscher (2012), Fratzscher, Duca, and Straub (2016), and Aizenman, Binici, and Hutchison (2016)). To isolate the country specific risk, we estimate the principal component of the CDS spreads of various emerging economies and extract its orthogonal component from the Brazilian CDS spread. Figure 5 in Appendix A Brazil's sovereign risk rises steadily from early 2012 and accelerates sharply in 2015.

since Brazil was "suffering from fiscal dominance", the central bank should "temporarily abandon the inflation targeting framework in favour of a crawling exchange rate regime".[6] Following this narrative evidence, we argue that during the period that goes from March 2002 to October 2002 and the period that goes from January 2012 to December 2015, the fiscal regime in Brazil was non-Ricardian. In the next sections, we study whether during these periods the response of the exchange rate to monetary and fiscal policy surprises is different from the rest of the sample.

Monetary policy

The test our hypothesis, we estimate the following regression:

$$\Delta e_t = \alpha_t + \beta_t \xi_t + \boldsymbol{\gamma} \Delta \mathbf{X}_t^\top + \varepsilon_t \tag{1}$$

where Δe_t, is the daily log change of the BRL/USD exchange rate, ξ_t is our proxy for Brazilian monetary policy shocks and X_t is a vector of additional control variables. Our focus is on the sign of the slope coefficient β_t and its evolution across time. A negative sign means that a tightening shock appreciates the *real* vis-à-vis the dollar. This is the conventional sign predicted by most economic models. On the other hand, a positive sign implies that an unexpected increase of the Brazilian policy rate depreciates the *real*.

Following the event study approach pioneered by Cook and Hahn (1989) and Kuttner (2001), we focus on the daily change of the BRL/USD exchange rate around monetary policy decisions. We consider all the decisions made by the Monetary Policy Committee (Copom) of the Central Bank of Brazil from November 2001 to December 2017. During this period, the frequency of the regular Copom meetings changed from monthly, until 2005, to every 45 days, from 2006 onward. In total, our sample includes 147 monetary policy decisions: 42 decisions to increase the Selic rate; 55 decisions to lower the rate; and 50 in which the rate was left unchanged. Most Copom decisions were announced in the evening after markets closure, while a few were announced in the early afternoon.[7] For this reason, we use the daily BRL/USD exchange rate measured at 13:15 GMT, obtained from the BIS foreign exchange statistics and look at the change the day after the announcement. Since the relevant time zone for Brazil is GMT-3, by measuring the exchange rate close to market opening, its variation should be dominated by the news regarding the monetary policy decision.

We identify monetary policy shocks using survey data obtained from the Central Bank of Brazil Market Expectation System. The database collects daily survey conducted by the Central Bank of Brazil among professional forecasters regarding the main macroeconomic variables, including the end-of-month Selic rate target (see Marques (2013) and Carvalho and Minella (2012) for a detailed description of the survey). We construct the monetary policy surprise series by taking the difference between the new announced Selic rate target and the average rate that was expected by market participants the day before the announcement. Selic target announcements higher than expected constitute a contractionary shock. In the sample we identify 71 contractionary shocks and 59 expansionary shocks. The average (median) shock is 3 (zero) basis points and its standard deviation is 33 basis points. Figure 2 shows the time series of the monetary policy shocks (left panel), and their scatter plot with the exchange rate changes (right panel).

Two features of the data are immediately evident. First, there is no clear relation between exchange rate changes and monetary policy surprises. The scatter plot reveals a large dispersion

[6] Peterson Institute for International Economics blog post, available at: https://www.piie.com/blogs/realtime-economic-issues-watch/brazil-needs-abandon-inflation-targeting-and-yield-fiscal

[7] This occurred between May 2002 and August 2003, for a total of 12 Copom meetings.

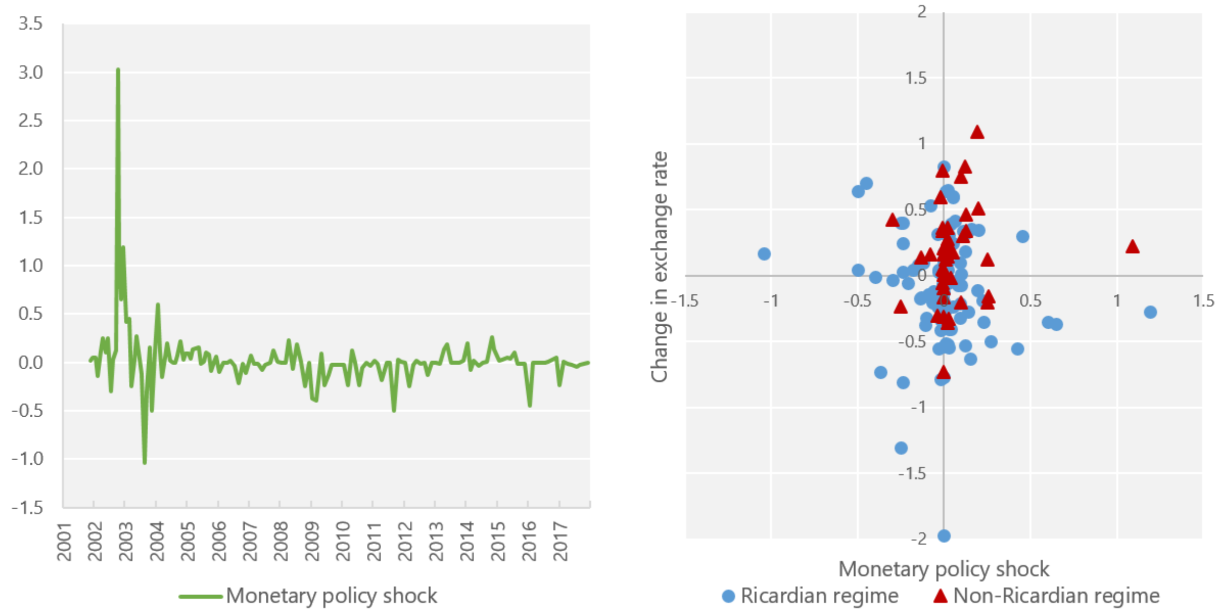

Figure 2: **Monetary policy shocks and exchange rate changes.** The figure shows the time series of monetary policy surprises (left panel) and the associated exchange rate changes (right panel, excluding 14/10/2002 observation).

of the observations, especially along the vertical dimension. Second, there is one particularly large realization of the monetary policy shock. This data point is associated with the Copom decision of 14 October 2002. As described in the previous section, the confidence crisis induced by the presidential campaign reached its peak in the middle of October, between the first (6 October) and second (27 October) round of the general election. The fall of the *real*, which from April to August lost 30% of its value vis-à-vis the dollar, accelerated in September (Figure 1, top right panel). The depreciation reached 50% in mid-September and peaked at 70% in early October, threatening to breach the ominous 4 BRL/USD barrier, amid much trepidation in financial and political circles.

To stop the slide, on 14 October the Copom called an extraordinary meeting during which it decided to raise the Selic target rate by 300 basis points, from 18% to 21%. The following day, the *real* lost almost 90 basis points vis-à-vis the dollar. The central bank's decision caught markets by surprise. Both the timing and the size of the hike were unprecedented. The meeting was the first, and to this date only, extraordinary Copom meeting since the adoption of inflation targeting. The decision to call an extraordinary meeting was even more surprising considering that a regular one was already scheduled to take place just a week later. Furthermore, from April to October, despite the continuous slide of the *real*, the central bank of Brazil had only changed its policy rate target once, reducing it by 50 basis points in July. The hike of 14 October was the single largest interest rate change since 1999. When the Copom raised the Selic target rate by 300 basis points again two months later in December 2002, markets were better prepared and were already expecting an increase of 200 basis points. While there is no fundamental reason to discard this observation, one might wonder whether it drives all the results. Therefore, to test their robustness we perform the empirical analysis with and without this data point.

As a preliminary step in our analysis, we estimate equation (1) assuming that the intercept and the slope coefficients are constant across the whole sample. The first column in Table 1 reports the estimation result when no controls are included, whereas the second column repeats

Table 1: Exchange rate response to monetary policy shocks

	Unconditional		Fiscal regimes			
	(1)	(2)	(3)		(4)	
			R	N	R	N
Constant	-0.02	0.01	-0.09**	0.14**	-0.05	0.16***
	(0.03)	(0.03)	(0.04)	(0.06)	(0.04)	(0.06)
$\boldsymbol{i - \mathbb{E}[i]}$	**0.14**	**0.14**	**-0.22**	**0.25*** **	**-0.25** **	**0.27*** **
	(0.12)	**(0.12)**	**(0.13)**	**(0.04)**	**(0.12)**	**(0.04)**
Δ VIX		0.06*			0.06*	
		(0.03)			(0.03)	
Δ Comm. Prices		-0.07***			-0.07***	
		(0.03)			(0.03)	
Δ 2 year T-note		0.18			0.08	
		(0.68)			(0.64)	
Constant (diff.)			0.23***		0.21***	
			(0.07)		(0.07)	
$\boldsymbol{i - \mathbb{E}[i]}$ **(diff.)**			**0.46*** **		**0.52*** **	
			(0.14)		**(0.12)**	
R^2	0.01	0.11	0.11		0.21	
No. of observations	147	147	147		147	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

the exercise including controls. In line with the rest of the literature, the unconditional regressions yield positive but insignificant βs. The controls in $\mathbf{X}_t$ intend to capture changes in three factors that can independently affect the BRL/USD exchange rate: global risk sentiment, international commodity prices, and foreign monetary conditions. We proxy changes in global risk aversion with daily variations in the VIX index. Changes in international commodity prices are captured by daily variations in the CRB index, a commodity price index that is calculated on a daily basis by the Commodity Research Bureau. Finally, changes in foreign monetary conditions are measured by daily changes in the 2-year US Treasury yield. As shown by De Pooter et al. (2021) this measure captures not only surprise changes in the federal funds rate, which occur twice in our sample,[8] but also variations in its expected path.[9]

To test our main hypothesis, we estimate equation (1) allowing α and β to vary between Ricardian and non-Ricardian fiscal regimes, as identified in the previous section. We allow the intercept α_t to vary together with β_t to capture shifts in trend depreciation that might occur across periods. Formally, we assume that $\alpha_t = (1 - \mathbf{1}_t)\,\alpha_R + \mathbf{1}_t \alpha_N$ and $\beta_t = (1 - \mathbf{1}_t)\,\beta_R + \mathbf{1}_t \beta_N$, where $\mathbf{1}_t$ is an indicator function that takes value 1 if t is between March 2002 and October 2002 or between January 2012 and December 2015. The third and fourth columns in Table 1 report the result of the

[8]The Federal Open Market Committee (FOMC) and the Copom decision were announced on the same day on 29 April 2009 and 29 April 2015. On both dates, the FOMC left the federal funds target rate unchanged.

[9]Our control variables attain the expected sign. Increases in global risk aversion and in the US interest rate, and decreases in international commodity prices lead to a depreciation of the *real*. However, only variations in the VIX rate and the CRB index are statistically significant. Throughout, as expected, these extra variables only add explanatory power to the regression, but do not modify the estimated coefficient on the monetary policy shock.

Table 2: Exchange rate response to monetary policy shocks (excluding 14/10/2002 observation)

	Unconditional		Fiscal regimes			
	(1)	(2)	(3)		(4)	
			R	N	R	N
Constant	-0.02	0.01	-0.09**	0.14**	-0.05	0.16***
	(0.03)	(0.03)	(0.04)	(0.06)	(0.04)	(0.06)
$\boldsymbol{i-\mathbb{E}\left[i\right]}$	**-0.07**	**0.10**	**-0.22**	**0.21**	**-0.25****	**0.23**
	(0.13)	**(0.12)**	**(0.13)**	**(0.19)**	**(0.12)**	**(0.18)**
Δ VIX		0.06*			0.06*	
		(0.03)			(0.03)	
Δ Comm. Prices		-0.07***			-0.07***	
		(0.03)			(0.03)	
Δ 2 year T-note		0.06			0.08	
		(0.65)			(0.65)	
Constant (diff.)			0.23***		0.21***	
			(0.07)		(0.07)	
$\boldsymbol{i-\mathbb{E}\left[i\right]}$ **(diff.)**			**0.43***		**0.49****	
			(0.23)		**(0.21)**	
R^2	0.00	0.11	0.08		0.18	
No. of observations	146	146	146		146	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

estimation. The slope coefficients in the two regimes are significantly different and have opposite signs. In a Ricardian regime, an unexpected monetary tightening of 100 basis points on impact appreciates the *real* between 21 and 24 basis points. Vice versa, during periods in which fiscal policy is perceived to follow a non-Ricardian regime, the same shock on impact depreciates the *real* between 25 and 27 basis points. Table 2 reports the results of the estimation performed excluding the 14 October 2002 observation. The results are largely unchanged. The slope coefficient in the non-Ricardian regime falls only slightly, even though it becomes marginally insignificant, and the difference between the two regimes remains strongly significant, with and without control variables.

These results suggest that, while during normal times, the exchange rate unambiguously appreciate following a positive monetary policy surprise, during periods of fiscal distress its response changes sign. However, it could be the case that sign change occurs also during other periods, and that the underlying cause has nothing to do with the fiscal regime. To test whether the differential behaviour of the exchange rate is indeed linked to fiscal policy, in the second step of our empirical analysis we estimate (1) under a more agnostic assumption regarding the evolution of α_t and β_t. Rather than imposing ex-ante the dates in which the parameter changes, we assume that they are a function of an underlying, unobservable, state which evolves according to a 2-state Markov process. Formally, we assume $\alpha_t = \alpha(s_t)$ and $\beta_t = \beta(s_t)$, where $s_t \in \{1, 2\}$ is the state of the system which evolves according to a Markov chain with constant transition matrix $\mathbf{P}$. We estimate the Markov-switching dynamic regression model by maximizing the full log-likelihood function and back out the implied probabilities of being in one state or the other.[10] The first four columns of

[10]The estimation is performed with the Stata command *mswitch* which uses the expectation-maximization (EM) algorithm. See Hamilton (1994) for details.

Table 3: Markov-switching regression model estimation results

		Monetary policy				Fiscal policy			
		(1)		(2)		(3)		(4)	
		State 1	State 2	State 1	State 2	State 1	State 2	State 1	State 2
Transition	State 1	0.95	0.05	0.96	0.04	0.95	0.05	0.97	0.03
matrix	State 2	0.06	0.94	0.06	0.94	0.07	0.93	0.08	0.92
Constant		-0.11	0.09	-0.06	0.14**	-0.12**	0.01	-0.07	-0.01
		(0.18)	(0.17)	(0.05)	(0.06)	(0.05)	(0.07)	(0.05)	(0.08)
policy shock		**-0.14**	**0.19**	**-0.21***	**0.23****	**-0.02**	**0.08*****	**-0.01**	**0.09*****
		(0.43)	**(0.39)**	**(0.13)**	**(0.09)**	**(0.02)**	**(0.02)**	**(0.02)**	**(0.02)**
Δ VIX				0.06*				0.13***	
				(0.03)				(0.03)	
Δ Comm. Prices				-0.07***				-0.04	
				(0.03)				(0.03)	
Δ 2 year T-note				0.02				1.37**	
				(0.72)				(0.70)	
Volatility		0.40		0.37		0.44		0.40	
		(0.05)		(0.03)		(0.03)		(0.03)	
Obs.		147				177			

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

Table 3 report the output of the estimation, performed with and without controls. By convention, we label the regime associated with the lowest β regime 1, and the regime associated with the highest β regime 2. In both specifications, the response of the exchange rate to a monetary policy surprise changes sign across regimes and the estimates are very close to those reported in Table 1.

Figure 3 (left panel) shows the time series of the estimated probability of being in state 2. Remarkably, periods in which β is more likely to be positive correspond closely to the periods in which fiscal policy was non-Ricardian. The fact that the Markov-switching model does not identify other periods in which the exchange rate is likely to covary positively with monetary policy surprises, seem to rule out alternative explanations. For example, it is striking that the probability of being in state 2 is close to zero during the global financial crisis of 2008-2009, despite the large depreciation of the *real* and the surge in the CDS spread (see Figure 1). This suggests that the origin of the differential behaviour of the exchange rate is indeed domestic, and not linked to external conditions.

To assess the robustness of these conclusions, we perform two additional exercises. First, we test whether our results depend on the choice of proxy for monetary innovations. An alternative and popular approach in the event study literature is to proxy monetary policy surprises with market interest rate variations. Therefore, we re-estimate our empirical models using 1-day changes in the 30 day interbank rate (*Deposito Interbancario*) swap around monetary policy announcements. The results are reported in Appendix A. On the whole, the results are very similar. For the simple regressions, the β is negative in the Ricardian regime, while it is positive in the non-Ricardian one, and they are significantly different from each other. Regarding the Markov-switching model, without controls the model has a hard time identifying the two regimes and attributes to state 1 only two observations. When control variables are included the estimated βs are close to those obtained with the simple regressions, and periods in which the probability of state 2 is high correspond closely

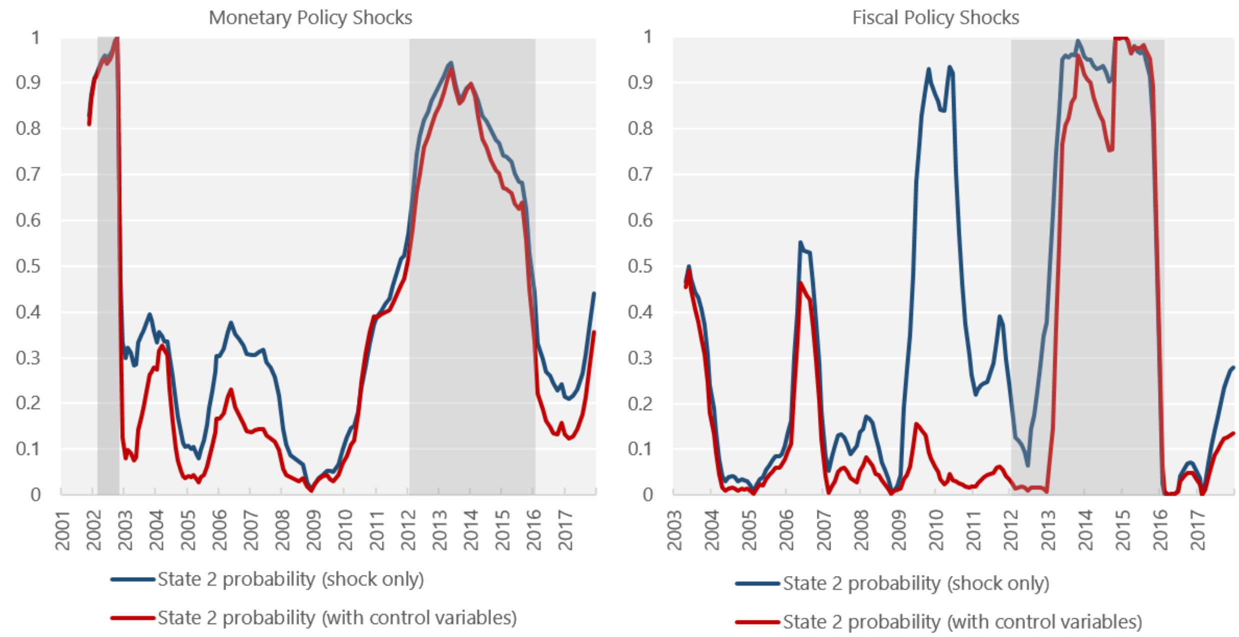

Figure 3: **Markov-switching regression state 2 probability.** The figure shows the probability of state 2 estimated by the Markov-switching regression model using monetary policy shocks (left panel) and fiscal policy shocks (right panel). Shaded areas denote periods which we identify as non-Ricardian fiscal regimes.

to periods of non-Ricardian fiscal policy. Finally, we check whether the unconventional behaviour of the exchange rate is due to information revealed by the decision of the central bank, or inferred by market participants.[11] Following Gürkaynak, Sack, and Swansonc (2005), we quantify the multi-dimensional aspect of monetary policy announcements using changes in the expected future path of policy rates that are uncorrelated with changes in the current policy target. We compute path surprises by orthogonalising the change in the one-year interbank swap rate with our measure of monetary policy shocks, and taking the residual. The results of the regressions estimated including this additional control, reported in Appendix A, show that path surprises play no role in explaining the behaviour of the exchange rate in our sample.[12]

Fiscal policy

In this section, we ask whether the differential response of the exchange rate to monetary policy surprises, and its link with the fiscal regime, holds also for fiscal policy shocks. Following the approach used in the previous section, we identify fiscal policy surprises as the difference between the announced primary deficit and its expected value, obtained from survey data. A higher announced primary deficit represents a positive fiscal shock. The policy announcement is the official monthly release of the Brazilian public sector primary deficit, published by the Central Bank of Brazil on the last Friday of the month. Since that data are published in the morning, and to avoid computing

[11] A recent strand of the literature on monetary policy surprises has proposed central bank information-based explanations to reconcile asset prices behaviour that are puzzling from the perspective of standard models. See Nakamura and Steinsson (2018), Jarociński and Karadi (2020) and Cieslak and Schrimpf (2019), among others.

[12] To properly account for the use of generated regressors, following Gilchrist, López-Salido, and Zakrajšek (2015) we estimate the first and second regression jointly by nonlinear least squares.

exchange rate changes over the weekend, we use the last price reported by Bloomberg for the day, instead of using prices at 13:15 GMT of the following day. In other words, we look at the variation of the exchange rate between the day of the announcement and the day before.

The expected primary deficit is computed as the average forecast obtained from Bloomberg survey of professional forecasters. Both realized and expected series are expressed in current monetary units, therefore we transform them into 2010 *reals* using the core Consumer Price Index. The data spans from April 2003 to December 2017 and includes 177 announcements. Unfortunately, no survey data is available between November 2001 and March 2003, which excludes the 2002 event from our analysis. In the sample, we identify 79 positive shocks and 98 negative shocks. The average (median) shock is -0.32 (-0.12) billions of 2010 *reals* and its standard deviation is 3.63. Brazil 2010 GDP is 3.9 trillion *reals*, therefore one unit of shock is equivalent to 0.026% of 2010 GDP. Figure 4 shows the time series of the fiscal policy surprises (left panel) and their scatter plot with the exchange rate changes (right panel).

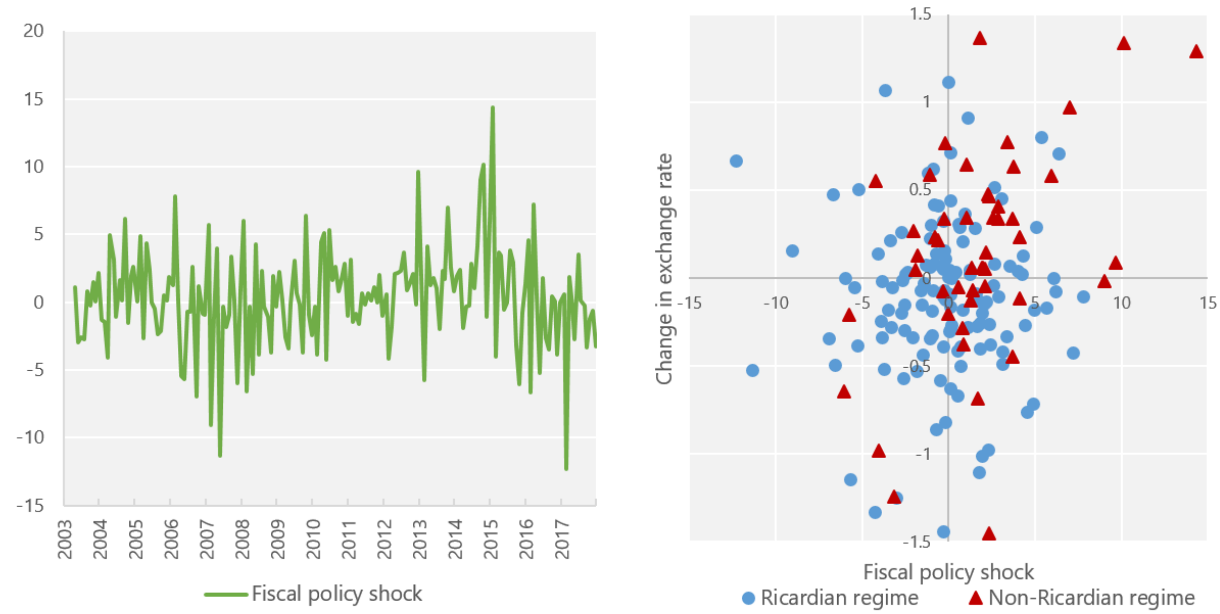

Figure 4: **Fiscal policy shocks and exchange rate changes.** The figure shows the time series of fiscal policy surprises (left panel) and the associated exchange rate changes (right panel).

Table 4 reports the result of the simple regressions. The slope coefficient estimated across the whole sample is positive, but it turns insignificant when control variables are included. Once we allow the coefficients to vary across fiscal regimes, we obtain a different picture. While in the Ricardian regime, the exchange rate does not respond significantly to fiscal surprises, in the non-Ricardian regime, β is positive and strongly significant. The estimates imply that an unexpected increase in the primary deficit worth 0.1% of GDP on impact depreciates the *real* between 21 and 26 basis points.

The estimation of the Markov-switching regression model confirms these findings. The last four columns in Table 3 report the estimated parameters, while Figure 3 (right panel) plots the implied probability of being in state 2. The slope coefficient is not significantly different from zero in state 1, while it is positive and strongly significant in state 2. Furthermore, periods in which the system is more likely to be in the second state correspond quite closely to periods in which we identify fiscal policy as being non-Ricardian. Without controls, the model identifies two main periods in which

Table 4: Exchange rate response to fiscal policy shocks

	Unconditional		Fiscal regimes			
	(1)	(2)	(3)		(4)	
			R	N	R	N
Constant	-0.05	-0.04	-0.10***	0.03	-0.10***	0.03
	(0.04)	(0.04)	(0.04)	(0.08)	(0.04)	(0.08)
$\mathbf{pd} - \mathbb{E}\,[\mathbf{pd}]$	**0.03****	**0.02**	**0.00**	**0.07*****	**-0.01**	**0.05****
	(0.01)	**(0.01)**	**(0.01)**	**(0.02)**	**(0.01)**	**(0.02)**
Δ VIX		0.12***			0.12***	
		(0.03)			(0.03)	
Δ Comm. Prices		-0.04			-0.04	
		(0.02)			(0.02)	
Δ 2 year T-note		1.20			1.24*	
		(0.73)			(0.70)	
Constant (diff.)			0.14		0.13	
			(0.09)		(0.08)	
$\mathbf{pd} - \mathbb{E}\,[\mathbf{pd}]$ **(diff.)**			**0.07*****		**0.06****	
			(0.02)		**(0.02)**	
R^2	0.05	0.22	0.13		0.29	
No. of observations	177	177	177		177	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

the covariance between fiscal innovations and exchange rate changes is more likely to be positive: from June 2009 to August 2010 and from February 2013 to November 2015. However, only the latter is robust to the inclusion of control variables.

Overall these results suggest that, like for monetary policy shocks, the response of the exchange rate to fiscal policy surprises depends on the fiscal regime. An unexpected increase in the fiscal deficit depreciates the *real* if the fiscal regime is non-Ricardian, while it has no effect if the fiscal regime is Ricardian.

3 A Small Open Economy Model

In this section, we develop a theoretical model that can rationalize our empirical findings. Our model departs from the rest of the literature along two dimensions. First, fiscal policy shifts stochastically between a Ricardian and a non-Ricardian regime. As a consequence, the government can default on its debt. Second, domestic and foreign investors evaluate government bonds differently. Upon default, foreign investors are subject to higher haircuts than domestic investors. Before delving into the equations, it is worth discussing this assumption in more details.

It is widely recognized that the nationality of creditors is one of the main inter-creditor issue in sovereign debt restructuring (see Gelpern and Setser (2006) and Brooks et al. (2015) for a discussion).[13] This issue has become increasingly relevant since financial globalization has severed

[13] Inter-creditor discrimination can arise not only across domestic and foreign creditors, but also within them. See Schlegl, Trebesch, and Wright (2019) for an example.

the link between domestic and external debt with residents and foreign creditors. As argued by Diaz-Cassou, Erce, and Vázquez-Zamora (2008), there are a number of reasons why sovereigns might want to discriminate between domestic and foreign creditors. One the one hand, since residents are subject to the domestic legal and regulatory system, they might be easier to persuade or coerce into participating in a debt exchange. Furthermore, a sovereign may have an incentive to honour its obligations with foreign investors in order to retain access to international capital markets. On the other hand, a sovereign may want to treat residents more favourably, especially banks and businesses, in order to mitigate the domestic financial fallout that could result from restructuring their claims. Finally, domestic residents may have more influence than foreigners over their governments' decision making and, thus, a greater ability to shape outcomes that favour them. While these reasons point in different directions, the evidence seems to suggest that the incentive of sovereigns evolve over time as debt problem unfolds. Erce (2013) shows that domestic investors are more likely to be coerced into further accumulating debt prior to a default, in order to provide the sovereign with breathing space, but once restructuring becomes inevitable, or when a default is consummated, sovereigns tend to give preferential treatment to residents in order to limit the impact of the restructuring on the domestic economy, or for political economy reasons.

Discrimination against foreign investors can take many different forms, including the imposition of capital controls, or the use of "sweeteners" that are particularly attractive to domestic residents. In the 1998 Ukraine exchange, domestic commercial banks and nonresident holders were offered different exchange options.[14] By comparing the net present value of old and new debt, Sturzenegger and Zettelmeyer (2008) (SZ henceforth) estimate that domestic investors endured an average haircut of 7% while nonresident investors were treated significantly worse and endured an average haircut of 56%. In Russia's 1998 default, the offer to exchange ruble-denominated debt for cash and new longer-term instruments was open to all investors. However, unlike domestic investors, foreigners had to deposit all proceeds in restricted accounts preventing them from converting the proceeds into foreign currency and taking them abroad. SZ estimate that through this exchange residents recovered 54% of their credits while nonresidents only 41%. Furthermore, many domestic investors obtained much better deals. Russian banks and Russian depositors that had invested in the defaulted securities indirectly through the banking system, were able to exchange their ruble debt holdings for dollar-denominated bonds, central bank paper, and cash in full.[15] Similarly, in the 2001 Argentinian default all investors were offered to tender their dollar-denominated bonds in exchange for longer-term dollar loans issued under Argentinian law. However, the exchange was uniquely attractive to domestic banks and institutions since they could value the new instrument at par instead of its market price. Nearly all of the bonds held by Argentine financial institutions were tendered in the exchange. The new loans were redenominated in local currency a few months later, the so-called "pesification". Non-resident investors refused the exchange and tendered in 2005 for a different set of instruments.[16] SZ estimate that investors who tendered in the Phase 1 of the exchange, including the pesification, endured an average haircut of 66%, while nonresident

[14] The object of the exchange were treasury bills, domestic-currency securities issued under Ukrainian law. Domestic banks were offered to exchange T-bills into longer-term domestic currency bonds of 3-6 years maturity discounted at the prevailing T-bill rate of about 60%. The interest rate on the new bonds was set at 40% for the first year and a floating coupon equal to the future six-month T-bill yield plus 1 percentage point for the remainder of the period. Nonresident holders, on the other hand, were given the chance to exchange their t-bills for a domestic currency bond with a 22% hedged annual yield, or to receive a two-year zero-coupon dollar denominated Eurobond with a yield of 20% (see Sturzenegger and Zettelmeyer (2008)).

[15] The exchange included GKOs, short-term zero-coupon ruble-denominated treasury securities governed by Russian law and OFZs, coupon-bearing ruble-denominated bonds governed by Russian law (see Gelpern and Setser (2006) for further information on the 1998 Russia's default episode).

[16] See SZ for further details.

investors who exchanged in 2005 endured an average haircut of 73%.

Our model builds on the canonical small open economy framework of Gali and Monacelli (2005) but is developed in continuous time, as in Cavallino (2019). The world economy is composed of a continuum of countries, indexed by $v \in [0, 1]$. The focus of this paper is on the equilibrium of a single economy which we call "Home" and can be thought of as a particular value of $H \in [0, 1]$. To simplify the analysis, we assume that all foreign countries are identical at all points in time.[17] We treat them as a unique country, which we call "Foreign", and denote its variables with a star superscript. Home is inhabited by a measure one of households that consume and work for domestic firms producing tradable goods. The public sector is composed of a monetary authority, which we call central bank, that sets the interest rate on the domestic-currency riskless bond and a fiscal authority, which we call government, that taxes, borrows and spends.

In the next subsections, we describe the problems faced by households and firms located in Home. Unless noted otherwise, the problems faced by Foreign agents are symmetric. We then describe the decision of foreign investors and domestic policies. We conclude this section by characterizing the equilibrium of the model and its log-linear dynamics around the steady state.

Households

Home is inhabited by a measure one of identical households. The representative household maximizes

$$\mathbb{E}\left[\int_0^{\infty} e^{-\rho t}\left(\ln C(t) - \frac{L(t)^{1+\varphi}}{1+\varphi}\right) dt\right] \tag{2}$$

where C is consumption and L is the amount of labour supplied. The parameter $\rho > 0$ is the time discount factor, and φ the inverse of the Frisch elasticity of labour supply. The consumption index C is a composite of Home and imported goods, given by $C(t) \equiv C_H(t)^{1-\alpha} C_F(t)^{\alpha} (1-\alpha)^{-1+\alpha} \alpha^{-\alpha}$, where $\alpha \in [0, 1]$ is the degree of Home bias in consumption. The imported goods index C_F is itself an aggregator of goods produced in different countries and it is defined by $C_F(t) \equiv \exp \int_0^1 \ln C_v(t)\, dv$. The optimal allocation of expenditure across domestic and foreign goods yields the following demand function

$$C_H(t) = (1-\alpha)\left(\frac{P_H(t)}{P(t)}\right)^{-1} C(t)$$

where P_H is the domestic Producer Price Index (PPI). P is the domestic Consumer Price Index (CPI), which is given by $P(t) \equiv P_H(t)\, \mathcal{S}(t)^{\alpha}$ where $\mathcal{S}(t) \equiv P_F(t)/P_H(t)$ denotes the Home terms of trade.

Home households have access to a zero-net-supply riskless bond that pays the Home monetary policy rate i.[18] Households can also save in domestic and foreign currency bonds issued by the Home government which pay the rate of return i_H and i_F, respectively, but are subject to default risk. Let B_H denote the amount of Home-currency government bonds, in units of the Home good, and B_F the amount of Foreign-currency government bonds, in units of the Foreign good, held by the representative Home household. Finally, we assume that Home households can hold bonds issued by Foreign households but they are subject to a friction that delays portfolio adjustments.

[17] The former assumption allows us to abstract from foreign disturbances, while the latter allows us to keep track of only one set of international prices rather than a continuum of bilateral prices.

[18] The central bank affects the interest rate on the riskless nominal bond by changing the growth rate of money supply, through a no-arbitrage condition. This can be modelled formally by introducing money in the utility function or through a cash-in-advance constraint. Here we directly focus on the cashless limit of such economies. To be clear, the central bank does not issue the riskless asset. This would be inconsistent with the assumption that debt issued by the fiscal authority is subject to default risk.

Due to this friction, Home household holdings of foreign assets have only second order effects on the equilibrium of the model and therefore disappears in its log-linearised version.[19] We denote with A_F the value, in units of the Foreign good, of the portfolio of foreign bonds held by the representative Home household.

Let A denote the value of the portfolio of assets held by Home households, W the wage rate and Υ profits received from domestic firms, all in units of the domestic good. Then, the dynamic budget constraint of the representative household is

$$\begin{aligned} dA(t) = & \left[A(t)(i(t)-\pi_H(t)) + W(t)L(t) + \Upsilon(t) - C(t)\mathcal{S}(t)^{\alpha} - T(t)\right]dt \\ & + B_H(t)\left[dB_H(t)/B_H(t) - (i(t)-\pi_H(t))\,dt\right] \\ & + B_F(t)\mathcal{S}(t)\left[d(B_F(t)\mathcal{S}(t))/(B_F(t)\mathcal{S}(t)) - (i(t)-\pi_H(t))\,dt\right] \\ & + A_F(t)\mathcal{S}(t)\left[d(A_F(t)\mathcal{S}(t))/(A_F(t)\mathcal{S}(t)) - (i(t)-\pi_H(t))\,dt\right] \end{aligned} \tag{3}$$

where $\pi_H(t) \equiv dP_H(t)/P_H(t)$ is PPI inflation and T are lump-sum taxes. The second and third lines describe the excess return of the household's portfolio of government bonds, where $dB_H(t)/B_H(t)$ is the return of the domestic-currency bond and $d(B_F(t)\mathcal{S}(t))/(B_F(t)\mathcal{S}(t))$ is the domestic-currency return of the foreign-currency bond. Their laws of motion together with the optimal portfolio decision of the households will be described later.

The problem of the representative household is to choose consumption, savings, and labour to maximize (2) subject to the budget constraint (3) and the no-Ponzi game condition $\lim_{k\to+\infty}\mathbb{E}\left[e^{\int_0^k (i(t)-\pi_H(t))dt}A(k)\right] \geq 0$. Her optimal consumption/saving policy is described by the Euler equation

$$\mathbb{E}\left[\frac{dC(t)}{C(t)}\right] = (i(t) - \pi(t) - \rho + h.o.t.)\,dt \tag{4}$$

where $\pi(t) \equiv dP(t)/P(t)$ is CPI inflation and $h.o.t.$ denotes higher-order terms which vanish in the log-linearisation and are therefore omitted for simplicity. The complete equation is reported in the appendix. Finally, her labour supply schedule is $W(t) = L(t)^{\varphi}C(t)\mathcal{S}(t)^{\alpha}$.

Foreign households have identical preferences and solve a symmetric problem. Their Euler equation is $dC^*(t)/C^*(t) = (i^*(t) - \pi^*(t) - \rho^* + h.o.t.)\,dt$ while their demand function for the Home good is $C_H^*(t) = \alpha(P^*(t)/P_H^*(t))\,C^*(t)$, where P_H^* is the Foreign-currency price of the Home good. We assume that there is full exchange rate pass-through to both import and export prices such that $P_F(t) = \mathcal{E}(t)P^*$ and $P_H^*(t) = P_H(t)/\mathcal{E}(t)$, where $\mathcal{E}$ is the nominal exchange rate between the Home country and the rest of the world defined as the Home currency price of one unit of Foreign currency. A decrease in $\mathcal{E}$ corresponds to an appreciation of the domestic currency. The real exchange rate is defined as $\mathcal{Q}(t) \equiv \mathcal{E}(t)P^*(t)/P(t)$.

[19] This friction can take the form of an adjustment cost or infrequent adjustments. It is meant to capture the attrition involved in trading in international financial markets. To simplify the algebra, we directly assume that the strength of the friction is maximal and Home households hold a fixed portfolio of Foreign bonds which is equal, both in size and composition, to the steady-state portfolio of Home government bonds held by Foreign investors. This assumption allows us to solve for a symmetric steady state, that is with a zero net foreign asset position, in which a fraction of the Home government debt is held by foreign investors. Furthermore, it prevents unexpected time-zero shocks to have first-order country-wide wealth effects. While solving the model around a symmetric steady state is not necessary for our results, it allows us to directly compare our model with standard ones in the literature which are typically solved around a symmetric steady state (see for example Galí and Monacelli (2005)). Similarly, preventing first-order wealth effects is not crucial and it actually weakens our results. For example, assume that all foreign assets held by Home households are denominated in Foreign currency. Then, a depreciation (appreciation) of the exchange rate would generate a negative (positive) wealth effect for the Home country which would further depreciate (appreciate) the exchange rate.

Firms

The Home production sector is composed of intermediate firms and retailers. Intermediate firms hire labour from domestic households to produce a continuum of differentiated goods, indexed by $j \in [0,1]$. Retailers combine intermediate goods to produce the Home good purchased by domestic and foreign households.

The retail sector is competitive and is composed of a measure one of homogeneous firms. Their aggregate production function is described by the constant elasticity of substitution aggregator $Y(t) \equiv \left[\int_0^1 Y_j(t)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$, where the parameter $\epsilon > 1$ measures the elasticity of substitution across intermediate goods. Thus, their demand function for variety $j \in [0,1]$ is given by

$$Y_j(t) = \left(\frac{P_{H,j}(t)}{P_H(t)}\right)^{-\epsilon} Y(t) \tag{5}$$

while the domestic PPI is $P_H(t) \equiv \left(\int_0^1 P_{H,j}(t)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$.

Intermediate good firms are monopolistically competitive. While each of them produce a differentiated good, they all use the same technology described by the production function

$$Y_j(t) = L_j(t) \tag{6}$$

Each firm faces an identical isoelastic demand schedule for its own good, given by (5), and set prices infrequently a la Calvo (1983). Each firm is allowed to reset its price only at stochastic dates determined by a Poisson process with intensity θ. A firm that resets at time t chooses its price $\hat{P}_{H,j}$ to maximize the present discounted value of its stream of profits

$$\mathbb{E}\left[\int_t^\infty \theta e^{-(\rho+\theta)(k-t)} \frac{C(t)\,P(t)}{C(k)\,P(k)} \left\{\hat{P}_{H,j}(t) - (1-\tau(t))\,W(k)\,P_H(k)\right\} Y_j(k|t)\,dk\right]$$

where $Y_j(k|t) = \left(\hat{P}_{H,j}(t)/P_H(k)\right)^{-\epsilon} Y(k)$ and τ is a labour subsidy which is set by the policymaker to maximize welfare in the flexible price equilibrium.[20] The firms optimal price-setting behaviour implies that PPI inflation evolves as

$$\mathbb{E}[d\pi_H(t)] = [(\epsilon-1)\,\pi_H(t) - \theta]\left\{\pi_H(t) + \frac{P_H(t)\,Y(t)}{P(t)\,C(t)}\left[\mathcal{M}(1-\tau(t))\frac{W(t)}{\mathcal{U}(t)} - \frac{1}{\mathcal{V}(t)}\right] + h.o.t.\right\} dt$$

where $\mathcal{U}$ and $\mathcal{V}$ are the present discounted values of future costs and revenues, respectively. Their equations and laws of motion are reported in the appendix.

Foreign investors and no-arbitrage conditions

Foreign investors, like domestic households, can invest in both bonds issued by the Home government.[21] Let B_H^* denote the amount of Home-currency bonds, in units of the Home good, and B_F^* the amount of Foreign-currency bonds, in units of the Foreign good, held by Foreign investors.

[20]The optimal flexible price labour subsidy is $\tau(t) = 1 - \frac{\epsilon}{\epsilon-1}\left(1 + \frac{\alpha}{1-\alpha}\frac{\mathcal{Q}(t)C^*(t)}{C(t)}\right)$. To reduce notation, we assume that the tax needed to finance it is levied lump-sum directly from firms.

[21]Similar to what we assume for Home households, Foreign investors can hold bonds issued by Home households and they are also subject to a friction that delays portfolio adjustment. Due to this friction, Foreign investors holding of the Home risk-free asset has only second order effects on the equilibrium and therefore disappears in the log-linearisation. To simplify the algebra, we directly assume that the strength of the friction is maximal and Foreign investors do not hold any Home risk-free bond.

Government bonds are subject to default risk. We model sovereign default as a random event with endogenous probability. Formally, we assume that the time of default is stochastic and distributed according to a Poisson process $\mathcal{P}$ with time-varying intensity $\eta(t)$. This implies that over the interval of time $[t, t+dt)$ the government defaults with probability $\eta(t)\,dt$. The equilibrium default intensity is determined endogenously by the intertemporal budget constraint of the fiscal authority, as will be described in the next subsection.

We assume that default is non-selective, that is it involves all securities issued by the government,[22] but creditors are ex-post treated unequally. Upon default Home households are able to recover a fraction χ of their credit, while Foreign investors can only recover a fraction $\chi^* < \chi$. This assumption implies that the returns of the assets are different for domestic households and foreign investors. Home households face the following return processes

$$
\begin{aligned}
dB_H(t)/B_H(t) &= (i_H(t) - \pi_H(t))\,dt - (1-\chi)\,d\mathcal{P}(t) \\
dB_F(t)/B_F(t) &= (i_F(t) - \pi^*(t))\,dt - (1-\chi)\,d\mathcal{P}(t)
\end{aligned}
$$

while Foreign investors face

$$
\begin{aligned}
dB_H^*(t)/B_H^*(t) &= (i_H(t) - \pi_H(t))\,dt - (1-\chi^*)\,d\mathcal{P}(t) \\
dB_F^*(t)/B_F^*(t) &= (i_F(t) - \pi^*(t))\,dt - (1-\chi^*)\,d\mathcal{P}(t)
\end{aligned}
$$

The portfolio choices of Home households and Foreign investors give rise to the no-arbitrage conditions

$$i_H(t) - i(t) = (1-\chi)\,\eta(t) + h.o.t. \tag{7}$$

$$i_F(t) + \mathbb{E}\left[\frac{d\mathcal{E}(t)}{\mathcal{E}(t)}\right] - i(t) = (1-\chi)\,\eta(t) + h.o.t. \tag{8}$$

and

$$i_H(t) - \mathbb{E}\left[\frac{d\mathcal{E}(t)}{\mathcal{E}(t)}\right] - i^*(t) = (1-\chi^*)\,\eta(t) + h.o.t. \tag{9}$$

$$i_F(t) - i^*(t) = (1-\chi^*)\,\eta(t) + h.o.t. \tag{10}$$

Equation (7) is the Home household's no-arbitrage condition between the domestic riskless asset and the Home-currency government bond. Households require a default premium over the riskless rate to compensate for the risk of default, captured by the term $(1-\chi)\,\eta(t)$, and a risk premium which is given by the covariance between their stochastic discount factor and the default process. The risk premium is second order and therefore it is included in the h(igher) o(rder) t(erm). The complete equations are reported in the appendix. Equation (8) is the no-arbitrage condition between the domestic riskless asset and the Foreign-currency government bond, which again contains a default and a risk premium. However, since the bond is denominated in foreign currency, its return also depends on the behaviour of the exchange rate. Similarly, equations (9) and (10) are the Foreign investors' no-arbitrage conditions between the Foreign riskless asset and the Home government bonds. By combining (7) and (9), or (8) and (10), we obtain the Uncovered Interest rate Parity (UIP) of the model

$$\mathbb{E}\left[\frac{d\mathcal{E}(t)}{\mathcal{E}(t)}\right] = i(t) - i^*(t) - (\chi - \chi^*)\,\eta(t) + h.o.t. \tag{11}$$

[22] While we could allow the government to default independently on Home- and Foreign-currency bonds without altering our results, the assumption of non-selective default is closer to the data. As reported by Mallucci (2015), non-selective defaults are the norm and represents 55% of the sovereign default episodes between 1990 and 2005.

This equation highlights the link between default risk and the exchange rate, which will be at the core of our analysis. In our model, even in its linearised version, the exchange rate dynamic is not only driven by the interest rate differential, but also by sovereign default risk. When the probability of default increases, bond yields rise to compensate investors for the higher risk of default. However, since the risk faced by domestic and foreign creditors is different, yields move asymmetrically and therefore cannot simultaneously satisfy all their no-arbitrage conditions. In particular, the Home-currency bond yield increases less, to compensate domestic investors, while the foreign-currency bond yield increases more, to compensate foreign investors. This implies that Home-currency bonds become less attractive for foreign investors, while Foreign-currency bonds become more attractive for domestic investors. Hence, the demand for domestic-currency relative to foreign currency assets fall. To restore the equilibrium, the domestic currency weakens, giving rise to an expected appreciation that makes domestic-currency bonds more attractive for foreign investors and foreign-currency bonds less attractive for domestic investors.

This mechanism is consistent with the empirical evidence on the dynamics of international portfolios in response to changes in sovereign risk. Converse and Mallucci (2019) show that changes in yields do not fully compensate foreign investors for additional sovereign risk, and that international bond mutual funds reduce their exposure to a country's assets when the sovereign default risk increases. Andritzky (2012) and Broner, Erce, et al. (2014) document how foreign investors reduced their holding of government securities during the global financial crisis and the European debt crisis, while domestic investors increased them.

Public sector

The Home public sector is composed of a monetary authority and a fiscal authority. The fiscal authority must finance a stream of expenditure given by $G(t) = \varepsilon_g(t)$, where ε_g follows an autoregressive stochastic process. To finance its expenditure, the government levies taxes on domestic households and borrows. Let $B(t)$ denote the total amount of government debt outstanding expressed in units of domestic output. Market clearing requires $B(t) = B_H(t) + B_H^*(t) + \mathcal{S}(t)(B_F(t) + B_F^*(t))$. Therefore, the budget constraint of the fiscal authority is:

$$dB(t) = (G(t) - T(t))\,dt + dB_H(t) + dB_H^*(t) + d(\mathcal{S}(t)\,B_F(t)) + d(\mathcal{S}(t)\,B_F^*(t)) \tag{12}$$

By using the no-arbitrage conditions (7)-(9) we can rewrite the budget constraint of the government as:

$$\mathbb{E}[dB(t)] = [B(t)(i(t) - \pi_H(t) - \xi(t)(\chi - \chi^*)\,\eta(t) + h.o.t.) + G(t) - T(t)]\,dt \tag{13}$$

where $\xi(t) \equiv (B_H^*(t) + \mathcal{S}(t)\,B_F^*(t))/B(t)$ is the share of debt held by foreign investors. By iterating it forward and using the transversality condition $\lim_{k\to+\infty}\mathbb{E}\left[e^{-\int_t^k (i(z)-\pi_H(z)-(\chi-\chi^*)\xi(z)\eta(z)+h.o.t.)dz}B(k)\right] = 0$ we obtain the intertemporal budget constraint:

$$B(t) = \mathbb{E}\left[\int_t^\infty e^{-\int_t^k (i(z)-\pi_H(z)-\xi(z)(\chi-\chi^*)\eta(z)+h.o.t.)dz}\,(T(k) - G(k))\,dk\right] \tag{14}$$

The intertemporal budget constraint requires that, at any point in time, the value of debt outstanding is equal to the present discounted value of future primary surpluses. Whenever the budget constraint is violated one of three things must occur. The fiscal authority can adjust taxes or expenditures to raise expected future surpluses. This is the conventional equilibrium featured in most models. Alternatively, the central bank might cut its policy rate to reduce future real rates and increase the present value of future surpluses. This is the equilibrium analyzed in the fiscal theory of the price level (FTPL) literature. Finally, if neither fiscal policy nor monetary policy can ensure intertemporal solvency then the current value of debt must fall. That is, the government is forced to default instantaneously on a fraction of its debt. This is the equilibrium developed by Uribe (2006) is his fiscal theory of sovereign risk (FTSR) and then used in the fiscal limit literature.

Our model features a fourth mechanism. As it is clear from equation (14), the default probability η affects the effective expected real interest rate used to discount future surpluses. An increase in the default probability increases the discount factor and raises the present value of future primary surpluses. Hence, whenever the budget constraint is violated and policies do not ensure intertemporal solvency, the expected path of the probability of default adjust to make (14) hold. Note that this mechanism is conceptually different from the FTSR approach. In Uribe (2006) default restores the equilibrium by reducing the left-hand side of equation (14). In our model default risk, rather than default itself, restores the equilibrium by increasing the discount factor.[23] The FTPL works in a similar way. A passive monetary policy restores the equilibrium by reducing real rates and increasing the discount factors. Unlike the FTPL, however, our model does not require monetary policy to be passive. Indeed, as we shall see below, in our model an equilibrium with default can arise only if monetary policy is active. When debt is inflated away, the probability of default is always zero.

Note that the default probability enters equation (14) proportionally to the fraction of debt held by foreigners and the gap in the recovery rates of domestic and foreign investors. This is because the default probability affects the discount factor associated with external debt, while it does not affect the one associated with debt held domestically. In other words, the default probability affects the effective interest rate, i.e. the return on debt minus the probability of default, paid by the government on debt held by foreigners, but not the one paid on debt held by domestic agents. When the probability of default rises, the yield on government bonds rises proportionally to compensate domestic investors for the increased risk of default. The two effects offset each other, leaving the effective interest rate on domestic debt unchanged. On the other hand, the additional default risk faced by foreign investors is generated through an expected appreciation of the Home currency, rather than through an increase in yield. Hence, the effective interest rate paid on external debt falls.

To simplify the analysis, we assume that the probability of default depends only on the amount of debt outstanding. This feature is largely supported by the data, and it is consistent with many models of strategic default which predicts that the likelihood of a default is increasing in the real level of debt.[24] Since η drives the currency expected excess return, our assumption is also consistent with the empirical evidence on sovereign risk and currency risk premia documented by Della Corte et al. (2021). Formally, we assume

$$\eta\left(t\right)=\max\left\{0,\bar{\eta}+\eta^{x}\frac{B\left(t\right)-\bar{B}}{\bar{B}}\right\} \tag{15}$$

with $\eta^x \geq 0$ where $x \in \{R, N\}$ denotes the fiscal regime, and $\bar{B}$ denotes the steady-state level of government debt. The parameter $\bar{\eta}$ is assumed to be strictly positive, but negligible, and is

[23] A similar distinction between initial value and discount rates arises in the FTPL literature, but the comparison is misleading. When monetary policy is passive, in discrete-time models both the initial price level and expected inflation rise in response to an increase in debt. However, in a continuous time models only the discount factor effect is present since the price level cannot jump. The different role played by default in our model and in the FTSR literature is not due to the timing convention but, rather, on the assumption of different recovery rates between domestic and foreign bond holders.

[24] See Edwards (1984), Eichengreen and Mody (2000), Uribe and Yue (2006) and Aizenman, Jinjarak, and Park (2013) for the empirical evidence, and Aguiar and Gopinath (2006), Arellano (2008) and Yue (2010) for models of strategic default.

introduced to facilitate the linearisation of (15). This functional form simplifies the determination of the equilibrium default probability since the problem of finding a process $\eta(t)$ that makes (14) hold boils down to a single parameter, η^x. The elasticity of the default probability with respect to debt is the endogenous object that adjusts to satisfy the government intertemporal budget constraint. As such, it depends on the fiscal regime considered.

We close the model by specifying monetary and fiscal policies. The monetary authority sets the interest rate on the domestic-currency risk-free nominal bond which is assumed to follow the simple Taylor rule

$$i(t) = [\rho + (1+\phi_\pi)\pi_H(t)] + \varepsilon_i(t) \tag{16}$$

where the parameter $\phi_\pi > 0$ measures the responsiveness of the policy rate to inflation, and ε_i is an exogenous component of monetary policy that follows an autoregressive stochastic process. When $\phi_\pi > 0$ monetary policy reacts to inflation by raising the real interest rate and, in the characterization by Leeper (1991), is labelled as 'active'. In what follows we assume that the central bank sticks to its inflation stabilisation mandate, regardless of the fiscal regime.

Finally, in the spirit of Bohn (1998), we assume that the government follows a simple tax policy described by the fiscal rule

$$T(t) - \bar{T} = \psi_b^x \left(B(t) - \bar{B}\right) + \psi_\pi^x \phi_\pi \pi_H(t) \bar{B} \tag{17}$$

where $\bar{T}$ denotes the steady-state level of taxes. The parameter $\psi_b^x \geq 0$ measures the responsiveness of taxes to changes in government debt. If ψ_b^x is sufficiently high, an increase in debt leads the government to raise enough taxes to cover the higher debt servicing cost and repay part of the principal. When this is the case, debt tends to return to its steady state level and its dynamic is stable. If on the other hand ψ_b^x is low, the increase in taxes is not sufficient to cover the higher debt servicing cost which might cause debt to grow unboundedly. Following the language set forth by Leeper (1991), in the former case fiscal policy is said to be 'passive' while in the latter it is said to be 'active'.

Proposition (2) below derives the threshold above (below) which fiscal policy is passive (active) and characterizes the possible fiscal regimes of the model. The parameter $\psi_\pi^x \in \{0,1\}$ marks inflation indexation. Note that an increase in expected inflation raises the debt servicing cost at rate $(1+\phi_\pi)\pi_H$ while it reduces the real burden of the nominal stock of debt at rate π_H. Hence, the net effect of inflation on the dynamics of real debt is $\phi_\pi \pi_H$. When $\psi_\pi^x = 1$ this effect is neutralized. We will set $\psi_\pi^x = 0$ or $\psi_\pi^x = 1$ in order to better separate the effects of default and inflation on the dynamics of debt.

Equilibrium

To reduce the number of equilibrium variables, let $\Lambda(t) \equiv C(t) / (\mathcal{Q}(t) C^*(t))$ denote the wedge between the marginal rate of substitution between Home and Foreign consumption and their marginal rate of transformation, i.e. the real exchange rate. Using equations (4) and (11) we can derive its law of motion:

$$\mathbb{E}[d\Lambda(t)] = \Lambda(t)[\rho^* - \rho + (\chi - \chi^*)\eta(t) + h.o.t.]\,dt \tag{18}$$

Let $Y(t) \equiv \left[\int_0^1 Y_j(t)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ be aggregate domestic output. Market clearing in the goods market requires $Y(t) = C_H(t) + C_H^*(t) + G(t)$. Therefore, output evolves as

$$\mathbb{E}[dY(t)] = (Y(t) - G(t))\left[i(t) - \pi_H(t) - \rho - \alpha \frac{\rho^* - \rho + (\chi - \chi^*)\eta(t)}{(1-\alpha)\Lambda(t) + \alpha} + h.o.t.\right] dt + \mathbb{E}[d\varepsilon_g(t)] \tag{19}$$

and the terms of trades are given by $\mathcal{S}(t) = [\alpha + (1-\alpha)\Lambda(t)]^{-1}(Y(t) - G(t))/C^*(t)$. The labour market clearing condition is $L(t) = \int_0^1 L_j(t)\,dj = \Delta(t)Y(t)$ where Δ is an index of price dispersion whose equation and law of motion are reported in the appendix. Hence, the law of motion of PPI inflation can be rewritten as

$$\mathbb{E}[d\pi_H(t)] = [(\epsilon-1)\pi_H(t) - \theta]\{\pi_H(t) + \left(\frac{\alpha}{\Lambda(t)} + 1 - \alpha\right)\left[\frac{\Delta(t)^{\varphi}Y(t)^{1+\varphi}}{(1-\alpha)\mathcal{U}(t)} - \frac{Y(t)/\mathcal{V}(t)}{Y(t)-G(t)}\right] + h.o.t.\Bigg\}dt \quad (20)$$

Finally, let $Z(t) \equiv (B(t) - A(t))/(\mathcal{S}(t)C^*(t))$ be the value of Home net foreign debt, in units of the Foreign good. By combining (3), (13), (11) we obtain the Home country's dynamic budget constraint

$$\mathbb{E}[dZ(t)] = [Z(t)\rho^* + \alpha(\Lambda(t) - 1)]\,dt \quad (21)$$

which is subject to the transversality condition $\lim_{k\to+\infty}\mathbb{E}\left[e^{-\int_0^k(i^*(t)-\pi^*(t))dt}Z(k)\right] = 0$.

We close the model by specifying the laws of motion of the exogenous variables. Let $\mathcal{B}^g$ and $\mathcal{B}^i$ two independent standard Brownian motions.[25] Then, ε_g and ε_i evolve according to the Ornstein-Uhlenbeck processes

$$d\varepsilon_g(t) = -\varrho\varepsilon_g(t)\,dt + \nu d\mathcal{B}_g(t) \quad (22)$$
$$d\varepsilon_i(t) = -\varrho\varepsilon_i(t)\,dt + \nu d\mathcal{B}_i(t) \quad (23)$$

where $\varrho > 0$ governs the speed of their mean-reversion while $\nu > 0$ their volatility.

Definition 1. An equilibrium of the model is a collection $\{\eta^x, \Lambda(t), Y(t), \pi_H(t), B(t), B_H(t), B_F(t), Z(t), \mathcal{U}(t), \mathcal{V}(t), \Delta(t), \varepsilon_g(t), \varepsilon_i(t)\}$ that satisfies (7), (8), (14) and the differential equations (13), (18), (19), (20), (21), (22), (23), (A.45), (A.46), (A.48), subject to (15), (16) and (17), given foreign variables $\{C^*(t), P^*(t), i^*(t)\}$, shocks $\{\mathcal{B}_g(t), \mathcal{B}_i(t), \mathcal{P}(t)\}$ and initial conditions $\{B_H(0), B_F(0), B_H^*(0), B_F^*(0), Z(0), \varepsilon_g(0), \varepsilon_i(0)\}$.

As common in the New Keynesian literature, we approximate the equilibrium dynamics of the model using a log-linear expansion around its deterministic steady state. We focus on a symmetric steady state in which the Home government is a net debtor and only part of its debt is held domestically.[26] We then use the log-linear approximation to study the response of the model to an unexpected increase in government spending and an unexpected tightening of the monetary policy rate. In what follows, a lowercase letter denotes the percentage deviation of the variable from its steady-state value.

Let $\xi \equiv \left(\bar{B}_H^* + \bar{\mathcal{S}}\bar{B}_F^*\right)/\bar{B}$ and $\iota \equiv \left(\bar{B}_F + \bar{B}_F^*\right)\bar{\mathcal{S}}/\bar{B}$ denote the steady-state share of external and foreign-currency debt, respectively. The equilibrium dynamics of the model around its deterministic

[25] The Brownian motions $\mathcal{B}^g$ and $\mathcal{B}^i$, and the Possion process $\mathcal{P}$ are defined on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}(t)\}, \mathbb{P})$. All stochastic processes are assumed to be adapted to $\{\mathcal{F}(t)\}_{t=0}^{\infty}$, the augmented filtration generated by $\{\mathcal{B}^g, \mathcal{B}^i, \mathcal{P}\}$. In what follows, we assume all regularity conditions which ensure that all processes introduced are well defined.

[26] By symmetric we mean a steady state in which the net foreign asset position of the Home country is zero

symmetric steady state are described by the system of differential equations

$$\mathbb{E}\left[d\lambda\left(t\right)\right] = \left[\tilde{\eta}^{x} b\left(t\right)\right] dt \tag{24}$$

$$\mathbb{E}\left[dy\left(t\right)\right] = \left[\phi_{\pi}\pi_{H}\left(t\right) - \alpha\tilde{\eta}^{x} b\left(t\right) - \varrho_{g}\varepsilon_{g}\left(t\right) + \varepsilon_{i}\left(t\right)\right] dt \tag{25}$$

$$\mathbb{E}\left[d\pi_{H}\left(t\right)\right] = \left[\rho\pi_{H}\left(t\right) - \kappa\omega y\left(t\right) + \kappa\varepsilon_{g}\left(t\right)\right] dt \tag{26}$$

$$\mathbb{E}\left[db\left(t\right)\right] = \left[\left(\rho - \psi_{b}^{x} - \xi\tilde{\eta}^{x}\right) b\left(t\right) + \left(1 - \psi_{\pi}^{x}\right)\phi_{\pi}\pi_{H}\left(t\right) + \beta\varepsilon_{g}\left(t\right) + \varepsilon_{i}\left(t\right)\right] dt \tag{27}$$

$$\mathbb{E}\left[dz\left(t\right)\right] = \left[\rho z\left(t\right) + \alpha\lambda\left(t\right)\right] dt \tag{28}$$

and by the initial conditions $b\left(0\right) = \iota e\left(0\right)$ and $z\left(0\right) = 0$, where $\tilde{\eta}^{x} \equiv \left(\chi - \chi^{*}\right)\eta^{x}$, $\kappa \equiv \theta\left(\rho + \theta\right)$ and $\omega \equiv 1 + \varphi$, while $\beta^{-1} \equiv \bar{B}/\bar{Y}$ is the steady-state debt-to-GDP ratio. The exogenous variables evolve as

$$\mathbb{E}\left[d\varepsilon_{g}\left(t\right)\right] = -\varrho\varepsilon_{g}\left(t\right) dt \tag{29}$$

$$\mathbb{E}\left[d\varepsilon_{i}\left(t\right)\right] = -\varrho\varepsilon_{i}\left(t\right) dt \tag{30}$$

with $\varepsilon_{g}\left(0\right) > 0$ and $\varepsilon_{i}\left(0\right) > 0$. We focus on these two shocks since both increase the growth rate of government debt and tend to increase the probability of default, provided $\eta^{x} > 0$. This allows us to avoid the kink featured in equation (15) and approximate it with $\eta\left(t\right) = \eta^{x} b\left(t\right)$.[27]

4 Fiscal regimes and the exchange rate

In this section, we prove the main theoretical results of the paper. We will proceed in steps. We first characterize the response of the exchange rate to monetary and fiscal shocks in the deterministic equilibria of the model, that is in the equilibria in which fiscal policy is always Ricardian or non-Ricardian. Then we characterize the behaviour of the exchange rate in a Markov-switching equilibrium in which fiscal policy shifts exogenously between the two regimes. Note that, while monetary and fiscal shocks are unexpected, the stochastic structure of the regime-shifting is preserved in the log-linearised model. Agents correctly anticipate that the policy regime might change in the future and form expectations accordingly. We close this section by showing that the different behaviour of the exchange rate in the non-Ricardian regime persists if we allow monetary policy to turn passive and inflate debt away. All proofs are reported in Appendix C.

Deterministic fiscal regimes

We start by characterizing the fiscal regimes of the model and the associated equilibria. We then study the response of the nominal exchange rate to monetary and fiscal shocks in each of them.

Proposition 2. *The model described by (24)-(30) has up to two equilibria:*

- *a* ***Ricardian equilibrium****, denoted with R, in which $\psi_{b}^{R} > \rho$ and $\tilde{\eta}^{R} = 0$*
- *a* ***non-Ricardian equilibrium****, denoted with N, in which $\psi_{b}^{N} < \rho$ and $\tilde{\eta}^{N} = \frac{\rho - \psi_{b}^{N}}{\xi - \alpha\left(1 - \psi_{\pi}^{N}\right)}$, provided $\xi > \frac{\alpha}{\rho}\left(1 - \psi_{\pi}^{N}\right)\left(2\rho - \psi_{b}^{N}\right)$ and $\phi_{\pi} > \tilde{\eta}^{N}\left(1 - \psi_{\pi}^{N}\right)\frac{\rho\alpha}{\kappa\omega}$.*

[27] Note that the shocks affect also the initial value of debt and their impact depend on the fraction of foreign-currency debt and on the response of the exchange rate. If, in response to a shock, the exchange rate appreciates and steady-state foreign-currency debt is positive ($\iota > 0$) then debt falls before rising. While this can occur even in a non-Ricardian equilibrium, our approximation remains locally valid since $\bar{\eta} > 0$.

The model has up to two equilibria, each associated with a different fiscal regime. If the fiscal authority commits to raising taxes commensurately to the servicing cost of the newly accumulated debt, then debt is sustainable. Hence, the probability of default is zero. This policy regime gives rise to the classical Ricardian equilibrium. In what follows we say that the fiscal regime is Ricardian if $\psi_b > \rho$. When the fiscal authority is unable or unwilling to commit to raising taxes sufficiently to ensure intertemporal solvency, then debt is unsustainable and default occurs with positive probability, that is $\eta^N(t) > 0$. In what follows we say that the fiscal regime is non-Ricardian if $\psi_b < \rho$.

To understand how the probability of default is determined and how it affects the dynamics of debt, solve (27) forward to obtain

$$\lim_{t\to\infty} \mathbb{E}[b(t)] = \lim_{t\to\infty} \left\{ \int_0^t e^{\left(\rho-\psi_b^x-\xi\tilde{\eta}^x\right)(t-k)} \left[\left(1-\psi_\pi^x\right)\phi_\pi^x \pi_H(k) + \beta\varepsilon_g(k) + \varepsilon_i(k)\right] dk + e^{\left(\rho-\psi_b^x-\xi\tilde{\eta}^x\right)t} b(0) \right\} \quad (31)$$

In the non-Ricardian equilibrium, an increase in debt raises the probability of default, which in turn reduces the expected real interest rates paid by the government on its external liabilities. This reduces the expected growth rate of debt and ensures that its expected path is non-explosive.

The elasticity of default probability with respect to debt is decreasing in ψ_b^N. The more the government raises taxes in response to an increase in debt, the lower the default probability required to stabilize it. On the other hand, $\tilde{\eta}^N$ does not depend on the responsiveness of the monetary policy rate to inflation. However, while ϕ_π does not affect the debt elasticity of the default probability, it does determine the overall default probability through its impact on the path of debt. Similarly, while the currency composition of sovereign debt does not determine the debt elasticity of default, it does affect its Home-currency value and therefore the overall default risk. A depreciation of the domestic currency increases the real burden of Foreign-currency denominated liabilities and raise the value of debt. Hence, for a given $\tilde{\eta}^N$, $\eta(t)$ rises.

Proposition 2 shows that the non-Ricardian equilibrium can arise only under certain parameters restrictions. Specifically, a non-Ricardian equilibrium exists only if the share of externally held debt (ξ) is sufficiently large and if the central bank is sufficiently aggressive in stabilising inflation. Both restrictions are due to our pricing assumption which features complete exchange-rate pass-through to imports and exports which introduces a feedback loop between the probability of default and domestic inflation. An increase in the default probability, as we shall see below, exerts a depreciating pressure on the exchange rate. This, in turn, makes domestic goods relatively cheaper than foreign goods, increasing their demand and raising domestic inflation. Since monetary policy is active, domestic inflation increases the real interest rate paid by the government and therefore the growth rate of debt. Hence, the probability of default must rise even further. If ξ is small, the increase in the default probability required to stabilize debt is too large and this cycle does not converge. Similarly, an increase in the policy rate has a weaker effect on inflation since it increases the default probability which tends to depreciate the exchange rate. To avoid indeterminacy, the central bank must respond to inflation more forcefully than in a Ricardian equilibrium. This formalizes the argument put forward by Blanchard and Galí (2005) on the limit of inflation targeting in a regime of fiscal dominance. These restrictions can be weakened by assuming imperfect exchange-rate pass-through. However, this would complicate the algebra unnecessarily. To keep the algebra tractable and to focus on the role of default risk alone, in what follows we sever the link between debt and inflation by assuming full indexation of taxes, that is, we set $\psi_\pi^N = 1$.[28] Under this assumption, a

[28]The solution for $\psi_\pi^N < 1$ can still be derived analytically, however its equations are very complex, even for an

non-Ricardian equilibrium with default always exists and an active monetary policy is sufficient to guarantee it is determined.

We are now ready to characterize the response of the nominal exchange rate to monetary and fiscal shocks in the two fiscal regimes. Since, $e(t) = s(t) - p^*(t) + p_H(t)$ and prices are pre-determined, the immediate response of the exchange rate to a time-zero shock is given by $e(0) = s(0)$. The next proposition proves our first main theoretical result.

Proposition 3. *The response of the exchange rate to the unexpected time-zero shocks $\varepsilon_i(0) = \varrho\bar{\varepsilon}_i > 0$ and $\varepsilon_g(0) = \varrho\bar{\varepsilon}_g > 0$ in the two equilibria defined in Proposition 2 is:*

- *in the Ricardian equilibrium*

$$\frac{e^R(0)}{\bar{\varepsilon}_i} = -1 + \frac{\kappa\omega\phi_\pi}{\kappa\omega\phi_\pi + \varrho(\rho+\varrho)} \tag{32}$$

$$\frac{e^R(0)}{\bar{\varepsilon}_g} = -\frac{\varrho\kappa\varphi\phi_\pi}{\kappa\omega\phi_\pi + \varrho(\rho+\varrho)} \tag{33}$$

- *in the non-Ricardian equilibrium (with $\psi_\pi^N = 1$)*

$$\frac{e^N(0)}{\bar{\varepsilon}_i} = \frac{e^R(0)}{\bar{\varepsilon}_i} + \frac{\varrho(\rho+\varrho)\left(\rho - \psi_b^N\right)}{\kappa\omega\phi_\pi + \varrho(\rho+\varrho)} \frac{1 - \iota + \frac{\kappa\omega\phi_\pi\frac{1-\alpha}{\rho+\varrho} - \rho}{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2}\frac{\kappa\omega\phi_\pi}{\rho+\varrho}}{\frac{\rho\xi\kappa\omega\phi_\pi}{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2} - \iota\left(\rho - \psi_b^N\right)} \tag{34}$$

$$\frac{e^N(0)}{\bar{\varepsilon}_g} = \frac{e^R(0)}{\bar{\varepsilon}_g} + \frac{\varrho\kappa\varphi\phi_\pi\left(\rho - \psi_b^N\right)}{\kappa\omega\phi_\pi + \varrho(\rho+\varrho)} \frac{\beta\frac{\rho+\varrho}{\kappa\varphi\phi_\pi} - \iota + \frac{\kappa\omega\phi_\pi\frac{1-\alpha}{\rho+\varrho} - \rho}{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2}\frac{1+\varphi}{\varphi}\beta}{\frac{\rho\xi\kappa\omega\phi_\pi}{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2} - \iota\left(\rho - \psi_b^N\right)} \tag{35}$$

To understand the reaction of the exchange rate in the two regimes it is helpful to study its law of motion, given by equation (11). Linearise and solve it forward to obtain

$$e(0) = \int_0^\infty \left(\rho - i(t) + \pi_H(t) + \tilde{\eta}^x b(t)\right) dt \tag{36}$$

where we used the terminal condition $\lim_{t\to\infty} e(t) = \int_0^\infty \left(\pi^*(t) - \pi_H(t)\right) dt$ and the fact that foreign variables are constant. The time-zero response of the exchange rate depends on three components: the interest rate, the path of domestic prices, and the probability of default. Thus, to understand the response of the exchange rate we need to understand how the shocks impact these components.

Consider first a contractionary monetary shock. In a Ricardian regime, the monetary policy rate affects the exchange rate through its impact on the return of domestic assets. An unexpected increase in the policy rate makes Home-currency assets more attractive vis-à-vis Foreign-currency assets. To restore no-arbitrage, the domestic currency strengthens and gives rise to an expected depreciation that increases the expected return of foreign assets.

In a non-Ricardian regime, the shock also affects the exchange rate through its impact on debt. An increase in the policy rate raises the servicing cost of debt and the and therefore the probability of default. This, in turn, tends to depreciate the exchange rate. This effect, which we call the debt channel, is captured by the second term in equation (35).[29] While the presence of the debt channel

appendix. They are available upon request.

[29]While the sign of this term can be negative, this is unlikely to occur in any reasonable calibration of the model. For example, it is sufficient that most of foreign-currency debt is held abroad, $\xi > \iota(1-\alpha)\left(1 + \frac{\alpha\rho}{\rho+\varrho}\right)$, and that the Taylor coefficient is not close to zero, $\phi_\pi > \frac{\rho}{\kappa\omega}\frac{\rho+\varrho}{1-\alpha}$, for the debt channel to be positive

in the non-Ricardian regime tends to reduce the appreciation of the exchange rate in response to a contractionary monetary shock, the net effect depends on its strength relative to the conventional one. When the debt channel is sufficiently strong, in the non-Ricardian regime the exchange rate depreciates in response to a contractionary monetary shock.[30]. It is easy to show that this is more likely to occur the stronger the central bank responds to inflation, that is the higher ϕ_π, and the weaker the fiscal authority attempts to stabilize the dynamic of debt, that is the lower ψ_b^N.

Note that the currency composition of government debt, captured by ι, only affects the magnitude of the response of the exchange rate in the non-Ricardian regime but not its sign. The larger the fraction of debt denominated in Foreign currency, the larger the depreciation. This is because the depreciation increases the value of Foreign-currency denominated liabilities, which in turn raises the default probability and causes the exchange rate to depreciate further.

A similar logic applies when considering a fiscal shock. An unexpected increase in government expenditure raises output but also debt. In a Ricardian regime only the first effect matters for the exchange rate, since the elasticity of the default probability with respect to debt is zero ($\tilde{\eta}^R = 0$). The increase in output puts upward pressure on prices and raises inflation. One the one hand, positive inflation depreciates the exchange rate. On the other hand, the increase in inflation prompts the central bank to hike the policy rate which has the opposite effect. In equilibrium, since $\phi_\pi > 0$, the central bank responds more than one-for-one to inflation and the second effect dominates $(\rho - i(t) + \pi_H(t) = -\phi_\pi \pi_H(t) < 0)$. Hence, the exchange rate appreciates.

In a non-Ricardian regime, the shock also affects the exchange rate through its impact on debt. An expansionary fiscal shock increases debt and the probability of default. This, in turn, tends to depreciate the exchange rate. The debt channel is captured by the second term in equation (35).[31] If the debt channel is sufficiently strong, in the non-Ricardian regime the exchange rate depreciates in response to an expansionary fiscal shock.[32]. As for the monetary shock, this is more likely to occur the stronger the central bank responds to inflation, that is the higher ϕ_π, and the weaker the fiscal authority attempts to stabilize the dynamic of debt, that is the lower ψ_b^N.

Stochastic fiscal regimes

In this section, we characterize the equilibrium of the model and the behaviour of the exchange rate when fiscal policy shifts stochastically between the Ricardian and the non-Ricardian regime. We assume that the probability of switching between is constant over time and described by the transition matrix

$$\Sigma = \begin{bmatrix} -\sigma^N & \sigma^N \\ \sigma^R & -\sigma^R \end{bmatrix} \tag{37}$$

where σ^N is the instantaneous probability of switching to the non-Ricardian regime and σ^R is the instantaneous probability of switching to the Ricardian regime. As explained in the previous sections, solving the model involves computing the elasticity of default $\tilde{\eta}^N$ which ensures that the

[30] A quick back-of-the-envelope calculation suggests that this is the empirically relevant case. Using parameter values typically employed in the literature ($\rho = 0.04$, $\alpha = 0.4$, $\varphi = 3$, $\theta = 0.75$, $\beta = 0.6$, $\phi_\pi = 0.5$, $\psi_b = 0$, and $\varrho \in [0.1, 0.8]$) it is easy to show that (35) is positive, that is the debt channel dominates for values of the external public debt-to-GDP ratio (ξ/β) below 100%. For reference, the average of the per-country maximal external public debt-to-GDP ratio reported in the Institute of International Finance (IIF) database is 51%.

[31] As before, while the sign of this term can be negative, this is unlikely to occur in any reasonable calibration of the model. For example, it is sufficient that most of foreign-currency debt is held abroad, $\xi > \iota \frac{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2}{\kappa\omega\phi_\pi}$, and that the Taylor coefficient is not close to zero, $\phi_\pi > \frac{\rho^2}{\kappa\omega}$, for the debt channel to be positive

[32] As for the monetary shock, a quick back-of-the-envelope calculation shows that (34) is positive for values of the external public debt-to-GDP ratio below 100%.

intertemporal budget constraint of the government holds. Being able to solve the model in closed-form greatly simplifies this step as it avoids the need to solve the fixed-point problem numerically. It turns out that the Markov-switching model can be solved analytically under very mild restrictions. In fact, assuming full indexation of taxes in the Ricardian regime ($\psi_\pi^R = 1$) is sufficient to obtain a closed-form solution. The next proposition derives the equilibrium elasticity of default in the Markov-switching model.

Proposition 4. *Assume that the model described by (24)-(30) switches stochastically between the Ricardian and the non-Ricardian regime according to the transition matrix (37), and $\psi_\pi^R = \psi_\pi^N = 1$. Then, the equilibrium of the model is mean-square stable if*

$$\tilde{\eta}^N = \frac{\rho - \psi_b^N}{\xi} - \frac{\sigma^R}{\xi} \frac{\psi_b^R - \rho}{2\left(\psi_b^R - \rho\right) + \sigma^N} \tag{38}$$

The elasticity of the default probability in the non-Ricardian regime is smaller the higher the probability of switching to the Ricardian one. This is because in forming expectations agents take into account the possibility that taxes will increase in the future to stabilize the dynamics of debt. This reduces the need for a default and therefore its probability. In fact, $\tilde{\eta}^N$ is smaller the more aggressively fiscal policy reduces debt in the Ricardian regime and the higher the persistence of the non-Ricardian regime, that is the lower σ^N. As for the deterministic case, the elasticity of the default probability does not depend on monetary policy parameter ϕ_π. However, in this case the result is due to our assumption of full indexation. In the proof of Proposition (4) reported in the appendix we derive the equation of $\tilde{\eta}^N$ in the more general case $\psi_\pi^N < 1$. When $\psi_\pi^N < 1$ monetary policy does affect the elasticity of default in the Markov-switching model. This has to do with the different behaviour of inflation in the two regimes. In the Ricardian regime inflation is more responsive to monetary policy than in the non-Ricardian one. In the latter, the depreciation induced by the increase in the default probability raises aggregate demand and sustains domestic prices. In the Ricardian regime, on the other hand, a higher ϕ_π leads to a lower inflation path. As a result, expected inflation falls even in the non-Ricardian regime and the real interest rate rises. This implies that the default probability must increase more to stabilize debt. Hence, $\tilde{\eta}^N$ is increasing in ϕ_π.

We are now ready to extend the results of Proposition 3 to the Markov-switching case. While the only restriction required to obtain a closed-form solution is $\psi_\pi^R = 1$, the equations are still very large. Here we focus on a particular calibration that yields a simple solution.

Proposition 5. *Let $\psi_\pi^R = \psi_\pi^N = 1$, $\psi_b^N = 0$, $\psi_b^R \downarrow \rho$, and $\iota = 0$. Then, the response of the exchange rate to the unexpected time-zero shocks $\varepsilon_i(0) = \varrho\bar{\varepsilon}_i > 0$ and $\varepsilon_g(0) = \varrho\bar{\varepsilon}_g > 0$ in the Markov-switching model is given by*

$$\frac{e^R(0)}{\bar{\varepsilon}_i} = \left.\frac{e^R(0)}{\bar{\varepsilon}_i}\right|_{\sigma^N=0} + \sigma^N \Xi$$
$$\frac{e^R(0)}{\bar{\varepsilon}_g} = \left.\frac{e^R(0)}{\bar{\varepsilon}_g}\right|_{\sigma^N=0} + \sigma^N \beta \Xi$$

and

$$\frac{e^N(0)}{\bar{\varepsilon}_i} = \left.\frac{e^N(0)}{\bar{\varepsilon}_i}\right|_{\sigma^R=0} - \sigma^R \Xi$$
$$\frac{e^N(0)}{\bar{\varepsilon}_g} = \left.\frac{e^N(0)}{\bar{\varepsilon}_g}\right|_{\sigma^R=0} - \sigma^R \beta \Xi$$

where $e^x(0)\,\bar{\varepsilon}_j\big|_{\sigma^{-x}=0}$ *is the response of the exchange rate in regime* $x \in \{R, N\}$ *with respect to shock* $j \in \{i, g\}$ *in the deterministic model, and*

$$\Xi \equiv \frac{\varrho\alpha\rho}{\xi\varrho}\frac{\kappa\omega\phi_\pi - (\sigma+\rho+\varrho)(\rho+\varrho)}{[\kappa\omega\phi_\pi + (\sigma+\varrho)(\sigma+\rho+\varrho)][\kappa\omega\phi_\pi + \varrho(\rho+\varrho)]} + \frac{\varrho\alpha\rho}{\xi\varrho}\frac{\rho(\sigma+\rho) - \kappa\omega\phi_\pi}{\kappa\omega\phi_\pi + \sigma(\sigma+\rho)} + \frac{\varrho(1-\alpha)(\sigma+2\rho+\varrho)}{\xi(\rho+\varrho)(\sigma+\rho)(\sigma+\rho+\varrho)} \tag{39}$$

where $\sigma \equiv \sigma^R + \sigma^N$.

Under the assumed parametrisation, the response of the exchange rate to fiscal and monetary shocks in the Markov-switching model takes a particularly intuitive form. The elasticity of the exchange rate in each regime is given by its elasticity in the associated deterministic equilibrium plus a component that is common across regimes and is proportional to the probability of switching. The sign of the common component Ξ determines whether a stochastic fiscal policy reduces or increases the difference between the response of the exchange rate in the two regimes. If $\Xi > 0$ the exchange rate appreciates less in the Ricardian regime and it depreciates less in the non-Ricardian regime. Hence, the difference narrows. Vice versa, if $\Xi < 0$ the exchange rate appreciates more in the Ricardian regime and it depreciates more in the non-Ricardian one. Hence, the difference increases. This case might seem counter-intuitive. After all, the exchange rate is a forward-looking variable and the possibility of switching to other regime with opposite dynamics should counterbalance its response. However, the presence of the other regime affects the exchange rate not only directly, but also indirectly through its impact on the other equilibrium variables. Such impact might actually push the elasticity of the exchange rate in the two regimes further apart. The effect of the two regimes on inflation described above is a case in point.

As is clear from equation (39), the sign of Ξ is ambiguous and depends on the parameters of the model. For most calibrations we expect Ξ to be positive. Since the time discount factor ρ is very small and the degree of openness α is typically below 0.5, the last term in equation (39) should dominate. However, if the degree of openness is sufficiently high then Ξ could be be negative.

The fiscal theory of the price level

When the fiscal authority is unable or unwilling to raise taxes, default is not the only possible outcome. The central bank can reduce the present discounted value of debt by allowing inflation to rise. Following the terminology set forth by Leeper (1991), we call this monetary policy regime 'passive'. The combination of an active fiscal policy and a passive monetary policy is at the core of the FTPL and gives rise to an equilibrium in which the price level is determined by the intertemporal budget constraint of the government, with no direct reference to monetary policy. In this section, we study how the exchange rate responds to monetary and fiscal shocks in such a scenario.

In the next proposition we derive the main features of the equilibrium and show that, when monetary policy is passive, the debt elasticity of default is zero. This implies that the probability of default is constant across regimes.

Proposition 6. *Assume* $\phi_\pi \in \mathbb{R}$. *Then the model described by (24)-(30) has one more* ***non-Ricardian equilibrium****, denoted with* N'*, in which* $\phi_\pi^{N'} < 0$ *and* $\tilde{\eta}^{N'} = 0$*, provided* $\psi_\pi^{N'} < 1$.

Note that even a Taylor coefficient arbitrarily close to, but below, one is sufficient to avert default. The mechanism through which inflation restores debt sustainability is similar to the case of default. A protracted period of high inflation implies low expected real interest rates which, in turn, increase the present discounted value of future surpluses and offset the increase in debt. In our setting this occurs if the fiscal rule does not neutralize the impact of inflation on the dynamics of debt, that is provided $\psi_\pi^{N'} < 1$. In what follows we assume $\psi_\pi^{N'} = 0$, as it is conventional in models with passive monetary policy. The next proposition characterizes the response of the exchange rate to monetary and fiscal shocks.

Proposition 7. *The response of the exchange rate to the unexpected time-zero shocks* $\varepsilon_i(0) = \varrho\bar{\varepsilon}_i > 0$ *and* $\varepsilon_g(0) = \varrho\bar{\varepsilon}_g > 0$ *in the non-Ricardian equilibrium* N' *is given by*

$$\frac{e^{N'}(0)}{\bar{\varepsilon}_i} = \frac{\frac{\varrho(\rho+\varrho)\left(\rho-\psi_b^{N'}\right)}{(\mu+\rho+\varrho)\left(\varrho+\rho-\psi_b^{N'}\right)}}{(1-\iota)\,\mu-\iota\left(\rho-\psi_b^{N'}\right)} \tag{40}$$

$$\frac{e^{N'}(0)}{\bar{\varepsilon}_g} = \frac{\varrho\frac{\beta\omega(\mu+\rho+\varrho)\left(\mu+\rho-\psi_b^{N'}\right)-\varphi\mu(\mu+\rho)\left(\rho-\psi_b^{N'}\right)}{\omega(\mu+\rho+\varrho)\left(\varrho+\rho-\psi_b^{N'}\right)}}{(1-\iota)\,\mu-\iota\left(\rho-\psi_b^{N'}\right)} \tag{41}$$

where $\mu \equiv \left(\sqrt{\rho^2-4\kappa\omega\phi_\pi^{N'}}-\rho\right)/2 > 0$.

When monetary policy is passive, the sign of the exchange rate's response to monetary and fiscal shocks is ambiguous and depends on the response of the policy rate to inflation, $\phi_\pi^{N'}$, and on the share of debt denominated in foreign currency, ι. Following an unexpected monetary tightening, or an unexpected fiscal expansion, the exchange rate depreciates if

$$\phi_\pi^{N'} < -\frac{\iota}{\kappa\omega}\left(\frac{\rho}{1-\iota}\right)^2 \tag{42}$$

while it appreciates if $-\frac{\iota}{\kappa\omega}\left(\frac{\rho}{1-\iota}\right)^2 < \phi_\pi^{N'} < 0$.[33] Assume that condition (42) is satisfied. When monetary policy is passive, an unexpected increase in the policy rate, or in government spending, raises expected inflation and therefore output. This depreciates the terms-of-trade and the domestic currency. The presence of foreign-currency debt amplifies the depreciation, since it increases the value of the outstanding obligations which, in turn, raises expected inflation even further.

The depreciation is larger when $|\phi_\pi^{N'}|$ is low, since inflation must rise more to stabilize debt, and when the fraction of debt denominated in foreign currency is high, since it strengthens the second-round effect. When both these conditions occur, such that (42) is violated, even an infinite depreciation is not sufficient to yield a stable equilibrium. In this case, the exchange rate appreciates instead of depreciating. The appreciation reduces the value of debt and restores debt sustainability. Notice that the price path is still determined by the intertemporal budget constraint of the government. But since the latter is now slack, thanks to the appreciation of the exchange rate, expected inflation and output now fall instead of rising.

To summarize, in an equilibrium with passive fiscal policy the response of the exchange rate to monetary and fiscal shocks can have the conventional sign or an unconventional one. Taking the monetary policy rule as given, the conventional sign is more likely to arise when a large fraction of debt is denominated in foreign currency. When this occurs, the elasticity of the exchange rate to policy shocks has the same sign as in the Ricardian regime but is likely to be larger. In fact, this occurs if $\kappa\omega\phi_\pi^R > (\mu+\rho)(\varrho+\rho)$.

We close this section by studying the Markov-switching equilibrium when fiscal policy shifts between regimes R and N'. Similarly to the case of default, we first derive the endogenous upper

[33]The numerator in 41 is positive if $\beta > \rho$, that is if the steady state debt-to-GDP ratio is smaller than $1/\rho$. This condition is satisfied in any reasonable calibration of the model

bound on $\phi_\pi^{N'}$ that guarantees that the equilibrium is stable and then we compute the elasticities of the exchange rate with respect to the two shocks. Unfortunately, the equations are even more complicated than before. While the stability condition is relatively compact, the equations of the elasticities are unmanageable. In the next proposition we focus on the particular case in which the Ricardian regime is absorbing, i.e. $\sigma^{N'} = 0$. That is, we assume that the system can jump from the non-Ricardian regime to the Ricardian one but not vice versa.

Proposition 8. *Assume that the model described by (24)-(30) switches stochastically from the non-Ricardian regime N' to the Ricardian regime with transition probability σ^R. Let $\psi_\pi^R = 1$ and $\psi_\pi^N = 0$. Then the elasticities of the exchange rate to the unexpected time-zero shocks $\varepsilon_i(0) > 0$ and $\varepsilon_g(0) > 0$ in regime N' are given by*

$$\frac{e^{N'}(0)}{\bar{\varepsilon}_i} = \left.\frac{e^{N'}(0)}{\bar{\varepsilon}_i}\right|_{\sigma^R=0} - \sigma^R \varrho \frac{\sigma^R + \varrho + \frac{\mu\psi_b^{N'} + \rho(\rho+\varrho)}{\mu+\rho+\varrho} + \mu \frac{\mu\left(\psi_b^{N'}+\varrho\right) + \sigma^R(\mu+\rho) + (\rho+\varrho)^2 + \varrho\left(\rho - \psi_b^{N'}\right)}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)}}{\mu\left(\varrho + \rho - \psi_b^{N'}\right)\left(\mu + \rho + \varrho + \sigma^R\right)} \tag{43}$$

$$\frac{e^{N'}(0)}{\bar{\varepsilon}_g} = \left.\frac{e^{N'}(0)}{\bar{\varepsilon}_g}\right|_{\sigma^R=0} - \sigma^R \varrho \frac{\omega\beta + \frac{\varphi\mu\varrho}{\mu+\rho+\varrho+\sigma^R}\left[1 + \frac{\rho-\psi_b^{N'}}{\mu+\rho+\varrho} - \frac{\mu\left(\psi_b^{N'}+\varrho\right) + \sigma^R(\mu+\rho) + (\rho+\varrho)^2 + \varrho\left(\rho-\psi_b^{N'}\right)}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)}\right]}{\mu\omega\left(\rho - \psi_b^{N'} + \varrho\right)} \tag{44}$$

provided that $\mu > \sigma^R/2$, i.e. $\phi_\pi^{N'} < -\sigma^R\left(\sigma^R + 2\rho\right)/4\kappa\omega$.

Note that, while in the deterministic model stability requires $\phi_\pi^{N'} < 0$, in the Markov-switching this condition becomes $\phi_\pi^{N'} < -\sigma^R\left(\sigma^R + 2\rho\right)/4\kappa\omega$. The higher the probability of switching to a Ricardian regime, the more monetary policy must be passive in the non-Ricardian one. The possibility of switching to a Ricardian regime with an active monetary policy decreases expected inflation. Hence, in order to stabilize debt in the non-Ricardian regime monetary policy must be looser.

As in proposition 5, we can decompose the elasticity of the exchange rate to the shocks in two parts: the elasticity in the associated deterministic equilibrium and a component that is proportional to the probability of switching. In the case of a monetary shock, the possibility of switching to a Ricardian regime tends to appreciate the exchange rate in the non-Ricardian one. In the case of a fiscal shock, however, the presence of a Ricardian regime can affect the response of the exchange rate in both directions.

5 Conclusion

Standard international macroeconomic models predict that the response of the exchange rate to domestic monetary and fiscal policies is unambiguous and noncontingent. A monetary tightening or a fiscal expansion leads to an appreciation of the domestic currency. The empirical evidence, especially for emerging economies, does not support these predictions.

In this paper, we argue that the effect of monetary and fiscal policies on the domestic currency depends crucially on the fiscal regime. A contractionary monetary or expansionary fiscal shock can lead to a depreciation, rather than an appreciation, if investors believe that debt is not fully backed by future fiscal surpluses. We provide evidence of this mechanism operating in Brazil during certain periods. By looking at daily movements of the BRL/USD exchange rate around monetary and fiscal policy announcements we find strong support for the existence of two regimes with opposite signs. The unconventional response of the exchange rate is more likely to arise when fiscal fundamentals are deteriorating and investors' concern about debt sustainability is rising. We show how this differential behaviour can arise in a model characterized by stochastic fiscal regimes and asymmetric recovery rates between domestic and foreign investors. In a non-Ricardian regime, policy shocks affect the probability of a sovereign default and the domestic currency risk premium, reversing the response of the exchange rate.

Given the critical importance of the exchange rate for internal and external stability, understanding its response to domestic policies is fundamental for policymakers. Our results suggest that strict inflation targeting and Taylor rules might not be optimal when the fiscal regime is non-Ricardian. Future research should focus on characterizing optimal monetary policy rules under this scenario. Our model provides a natural framework to accomplish this task.

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Appendix A Additional graphs and tables

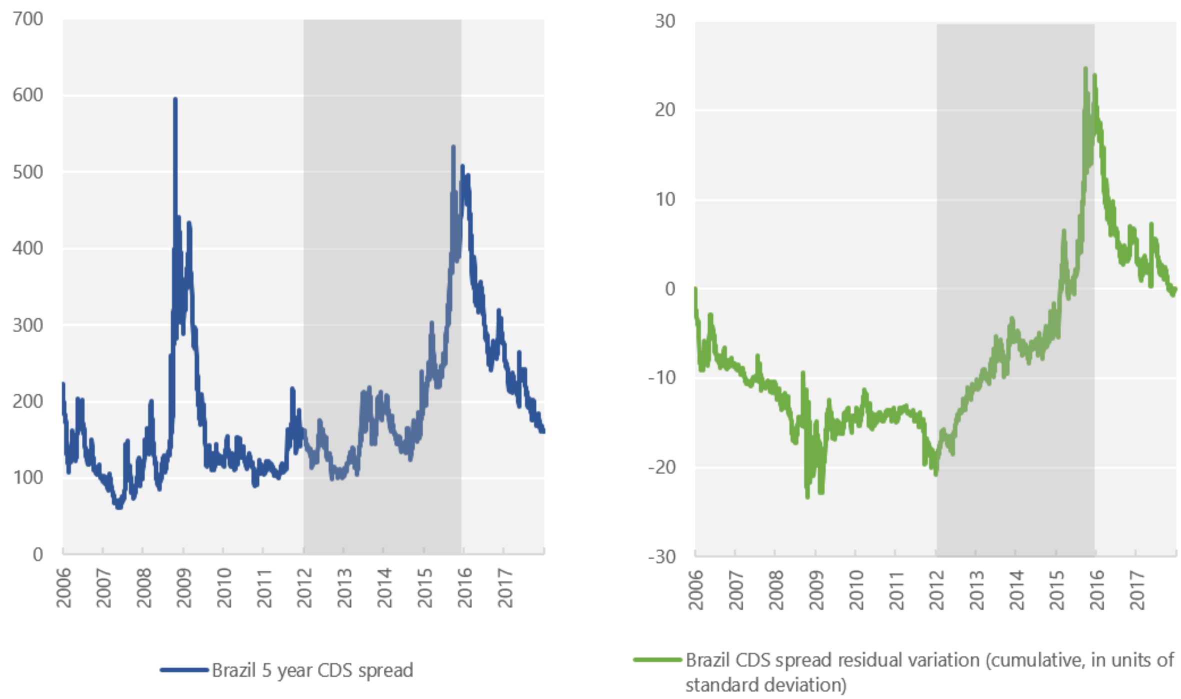

Figure 5: **CDS spread and idiosyncratic component.** The figure shows Brazil 5 year CDS spread (left panel) and its idiosyncratic variation (right panel). The idiosyncratic variation is the (cumulative) residual from regressing the standardized daily change in the Brazilian CDS on the first principal component extracted from a sample of emerging market economies. The sample includes Brazil, Colombia, Mexico, Peru, Poland, Russia, Hungary, Indonesia, Malaysia, Philippines, Thailand, Turkey and South Africa.

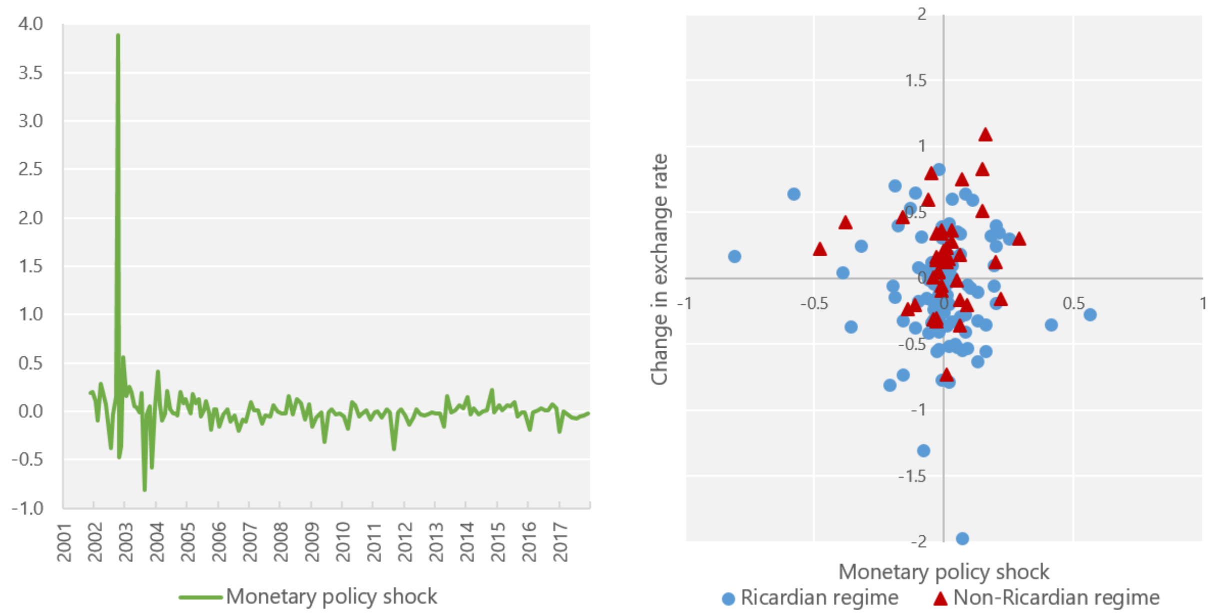

Figure 6: **Monetary policy shocks (alternative) and exchange rate changes.** The figure shows the time series of monetary policy surprises computed as daily change in the 30-day interbank deposit rate (left panel) and the associated exchange rate changes (right panel, excluding 14/10/2002 observation).

Table 5: Exchange rate response to monetary policy shocks

	Unconditional		Fiscal regimes			
	(1)	(2)	(3)		(4)	
Constant	-0.02	0.01	-0.09**	0.15***	-0.06	0.17***
	(0.03)	(0.03)	(0.040)	(0.06)	(0.035)	(0.06)
Δ DI30	**0.16****	**0.16***	**-0.27**	**0.19*****	**-0.35***	**0.21*****
	(0.07)	**(0.09)**	**(0.21)**	**(0.02)**	**(0.19)**	**(0.02)**
Δ VIX		0.058*			0.06*	
		(0.03)			(0.03)	
Δ Comm. Prices		-0.07**			-0.07***	
		(0.03)			(0.03)	
Δ 2 year T-note		0.14			-0.09	
		(0.67)			(0.64)	
Constant (diff.)			0.24***		0.23***	
			(0.07)		(0.07)	
Δ DI30 (diff.)			**0.46****		**0.55*****	
			(0.21)		(0.19)	
$R2$	0.02	0.12	0.11		0.21	
No. of observations	147	147	147		147	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

Table 6: Exchange rate response to monetary policy shocks (excluding 14/10/2002 observation)

	Unconditional		Fiscal regimes			
	(1)	(2)	(3)		(4)	
Constant	-0.02	0.01	-0.09**	0.15***	-0.06	0.17***
	(0.03)	(0.03)	(0.04)	(0.06)	(0.04)	(0.06)
Δ DI30	**-0.16**	**-0.23**	**-0.27**	**0.17**	**-0.35***	**0.14**
	(0.20)	**(0.18)**	**(0.21)**	**(0.39)**	**(0.19)**	**(0.41)**
Δ VIX		0.06*			0.06*	
		(0.03)			(0.03)	
Δ Comm. Prices		-0.07***			-0.07***	
		(0.03)			(0.03)	
Δ 2 year T-note		0.06			-0.07	
		(0.64)			(0.67)	
Constant (diff.)			0.24***		0.23***	
			(0.07)		(0.07)	
Δ DI30 (diff.)			0.44		0.49	
			(0.44)		(0.46)	
R^2	0.00	0.11	0.08		0.18	
No. of observations	146	146	146		146	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

Table 7: Markov-switching regression model estimation results

		Monetary policy shocks			
		(1)		(2)	
		State 1	State 2	State 1	State 2
Transition	State 1	0.63	0.37	0.96	0.04
matrix	State 2	0.01	0.99	0.04	0.96
Constant		-1.63***	0.00	-0.08	0.12
		(0.02)	(0.03)	(0.07)	(0.08)
Δ DI30		**-5.14*** **	**0.16** **	**-0.45** **	**0.19*** **
		(0.50)	**(0.07)**	**(0.19)**	**(0.05)**
Δ VIX				0.05*	
				(0.03)	
Δ Comm. Prices				-0.07***	
				(0.03)	
Δ 2 year T-note				-0.08	
				(0.77)	
Volatility		0.36		0.37	
		(0.02)		(0.03)	
Obs		147		147	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

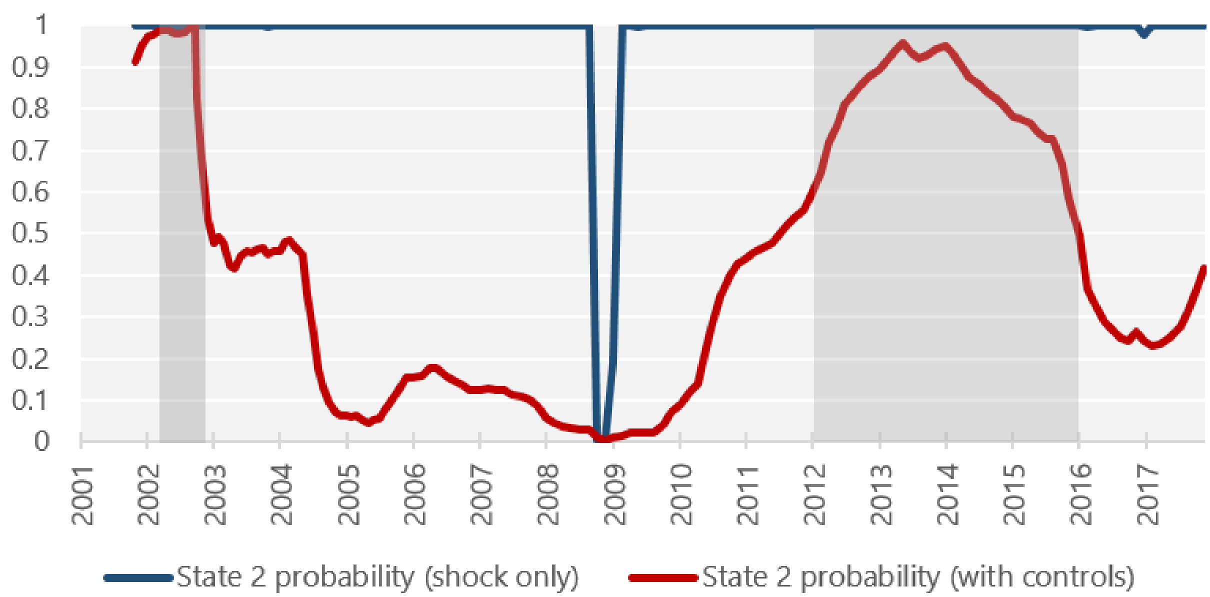

Figure 7: **Markov-switching regression state 2 probability.** The figure shows the probability of state 2 estimated by the Markov-switching regression model using monetary policy shocks. Shaded areas denote periods which we identify as non-Ricardian fiscal regimes.

Table 8: Exchange rate response to monetary policy shocks with path surprises

	All observations				Excluding 14/10/2002			
	(1)		(2)		(3)		(4)	
	R	N	R	N	R	N	R	N
Constant	-0.09**	0.14**	-0.05	0.16***	-0.09**	0.14**	-0.05	0.16***
	(0.04)	(0.06)	(0.04)	(0.06)	(0.04)	(0.06)	(0.04)	(0.06)
$\boldsymbol{i - \mathbb{E}[i]}$	**-0.22**	**0.24*** **	**-0.25** **	**0.26*** **	**-0.22**	**0.19**	**-0.25** **	**0.22**
	(0.13)	**(0.05)**	**(0.12)**	**(0.05)**	**(0.13)**	**(0.18)**	**(0.12)**	**(0.17)**
Path surprises	0.03		0.02		0.03		0.03	
	(0.11)		(0.11)		(0.11)		(0.11)	
Δ VIX			0.06*				0.06*	
			(0.03)				(0.03)	
Δ Comm. Prices			-0.07***				-0.07***	
			(0.03)				(0.03)	
Δ 2 year T-note			0.10				0.10	
			(0.66)				(0.66)	
Constant (diff.)	0.23***		0.21***		0.23***		0.21***	
	(0.07)		(0.07)		(0.07)		(0.07)	
$\boldsymbol{i - \mathbb{E}[i]}$ **(diff.)**	**0.46*** **		**0.51*** **		**0.41** **		**0.47** **	
	(0.15)		**(0.13)**		**(0.23)**		**(0.21)**	
R^2	0.11		0.21		0.08		0.18	
No. of observations	147		147		146		146	

Note: Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***, respectively.

Appendix B Model equations

To simplify the algebra in this appendix we omit time arguments except where strictly necessary. In what follows we will make repeatedly use of Ito's formula for jump-diffusion processes so it is worth it to report it here. Let $\mathbf{S}$ be a generic vector of variables of size J with law of motion

$$d\mathbf{S} = \left(\mathbf{S}\boldsymbol{\mu}^{\mathbf{S}} + \eta \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) dt + \mathbf{S}\boldsymbol{\sigma}_g^{\mathbf{S}} d\mathcal{B}_g + \mathbf{S}\boldsymbol{\sigma}_i^{\mathbf{S}} d\mathcal{B}_i + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}} d\tilde{\mathcal{P}}$$

where $\tilde{\mathcal{P}}$ is a compensated Poisson process which stasifies $d\tilde{\mathcal{P}} = d\mathcal{P} - \eta dt$. Then, the law of motion of $X = F(\mathbf{S})$ is

$$dX = X\left(\mu^X + \eta\delta^X\right) dt + X\sigma_g^X d\mathcal{B}_g + X\sigma_i^X d\mathcal{B}_i + X\delta^X d\tilde{\mathcal{P}}$$

where

$$\begin{aligned}
\mu^X &= \sum_j \frac{F_{S_j}}{F} S_j \mu^{S_j} + \frac{1}{2}\sum_{j,k} \frac{F_{S_j S_k}}{F} S_j S_k \left(\sigma_g^{S_j}\sigma_g^{S_k} + \sigma_i^{S_j}\sigma_i^{S_k}\right) \\
\sigma_g^X &= \sum_j \frac{F_{S_j}}{F} S_j \sigma_g^{S_j} \\
\sigma_i^X &= \sum_j \frac{F_{S_j}}{F} S_j \sigma_i^{S_j} \\
\delta^X &= \frac{F\left(\mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) - F(\mathbf{S})}{F(\mathbf{S})}
\end{aligned}$$

B.1 The households' problem

The representative Home household maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\ln C - \frac{L^{1+\varphi}}{1+\varphi}\right) dt\right]$$

subject to

$$\begin{aligned}
dA = {} & \left[A(i - \pi_H) + WL + \Upsilon - C\mathcal{S}^\alpha - T\right] dt \\
& + B_H\left[\left[i_H - i - \eta(1-\chi)\right] dt - (1-\chi)\, d\tilde{\mathcal{P}}\right] \\
& + B_F\mathcal{S}\left[\left[i_F - i - \eta(1-\chi)\right] dt - (1-\chi)\, d\tilde{\mathcal{P}} + d\mathcal{E}/\mathcal{E}\right] \\
& + A_F\mathcal{S}\left[(i^* - i)\, dt + d\mathcal{E}/\mathcal{E}\right]
\end{aligned}$$

where

$$d\mathcal{E}/\mathcal{E} = \left(\mu^{\mathcal{E}} + \eta\delta^{\mathcal{E}}\right) dt + \sigma_g^{\mathcal{E}} d\mathcal{B}_g + \sigma_i^{\mathcal{E}} d\mathcal{B}_i + \delta^{\mathcal{E}} d\tilde{\mathcal{P}}$$

Her intertemporal problem can be formulated as follows:

$$V(A, \mathbf{S}) = \max_{C,L} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\ln C - \frac{L^{1+\varphi}}{1+\varphi}\right) dt\right]$$

subject to

$$\begin{aligned}
dA/A &= \left(\mu^A + \eta\delta^A\right) dt + \sigma_g^A d\mathcal{B}_g + \sigma_i^A d\mathcal{B}_i + \delta^A d\tilde{\mathcal{P}} \\
d\mathbf{S}/\mathbf{S} &= \left(\boldsymbol{\mu}^{\mathbf{S}} + \eta\boldsymbol{\delta}^{\mathbf{S}}\right) dt + \boldsymbol{\sigma}_g^{\mathbf{S}} d\mathcal{B}_g + \boldsymbol{\sigma}_i^{\mathbf{S}} d\mathcal{B}_i + \boldsymbol{\delta}^{\mathbf{S}} d\tilde{\mathcal{P}}
\end{aligned}$$

with

$$\begin{aligned}
A\mu^{A} &= (i-\pi_H)\,A + WL + \Upsilon - C\mathcal{S}^{\alpha} - T + B_H\,(i_H - i) \\
&\quad + B_F\mathcal{S}\left(i_F - i + \mu^{\mathcal{E}}\right) + A_F\mathcal{S}\left(i^{*} - i + \mu^{\mathcal{E}}\right) \\
A\sigma_g^{A} &= (A_F + B_F)\,\mathcal{S}\sigma_g^{\mathcal{E}} \\
A\sigma_i^{A} &= (A_F + B_F)\,\mathcal{S}\sigma_i^{\mathcal{E}} \\
A\delta^{A} &= (A_F + B_F)\,\mathcal{S}\delta^{\mathcal{E}} - (B_H + B_F)\,(1-\chi)
\end{aligned}$$

where $\mathbf{S}$ is a generic vector of states of size J and $\boldsymbol{\mu}^{\mathbf{S}}$, $\boldsymbol{\sigma}^{\mathbf{S}}$ and $\boldsymbol{\delta}^{\mathbf{S}}$ are a function of the states only. The HJB for this problem is

$$\rho V\,(A,\mathbf{S}) = sup_{C,L,B_H}\left\{\ln C - \frac{L^{1+\varphi}}{1+\varphi} + \mathbb{E}\left[dV\,(A,\mathbf{S})\right]\right\}$$

where

$$\begin{aligned}
\mathbb{E}\,[dV] \quad &= \quad V_A A\mu^{A} + \sum_j V_{S_j} S_j \mu^{S_j} + \eta\left[V\left(A + A\delta^{A}, \mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) - V\,(A,\mathbf{S})\right] \\
&\quad + \frac{1}{2}\left[V_{AA}\left(A\sigma_g^{A}\right)^2 + 2\sum_j V_{AS_j} A S_j \sigma_g^{A}\sigma_g^{S_j} + \sum_j\sum_k V_{S_jS_k} S_j S_k \sigma_g^{S_j}\sigma_g^{S_k}\right] \\
&\quad + \frac{1}{2}\left[V_{AA}\left(A\sigma_i^{A}\right)^2 + 2\sum_j V_{AS_j} A S_j \sigma_i^{A}\sigma_i^{S_j} + \sum_j\sum_k V_{S_jS_k} S_j S_k \sigma_i^{S_j}\sigma_i^{S_k}\right]
\end{aligned}$$

The first order conditions with respect to C is

$$\frac{1}{C} = \mathcal{S}^{\alpha} V_A$$

Apply Ito's lemma to $V_A\,(A,\mathbf{S})$ to obtain its law of motion

$$dV_A = \left(V_A\mu^{V_A} + \eta V_A\delta^{V_A}\right)dt + V_A\sigma_g^{V_A}d\mathcal{B}_g + V_A\sigma_i^{V_A}d\mathcal{B}_i + V_A\delta^{V_A}d\tilde{\mathcal{P}}$$

where

$$\begin{aligned}
V_A\mu^{V_A} &= V_{AA}A\mu^{A} + \sum_j V_{AS_j} S_j \mu^{S_j} \\
&\quad + \frac{1}{2}\left[V_{AAA}\left(A\sigma_g^{A}\right)^2 + 2\sum_j V_{AAS_j} A S_j \sigma_g^{A}\sigma_g^{S_j} + \sum_{j,k} V_{AS_jS_k} S_j S_k \sigma_g^{S_j}\sigma_g^{S_k}\right] \\
&\quad + \frac{1}{2}\left[V_{AAA}\left(A\sigma_i^{A}\right)^2 + 2\sum_j V_{AAS_j} A S_j \sigma_i^{A}\sigma_i^{S_j} + \sum_{j,k} V_{AS_jS_k} S_j S_k \sigma_i^{S_j}\sigma_i^{S_k}\right] \\
V_A\delta^{V_A} &= V_A\left(A + A\delta^{A}, \mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) - V_A\,(A,\mathbf{S}) \\
V_A\sigma_g^{V_A} &= V_{AA}A\sigma_g^{A} + \sum_j V_{AS_j} S_j \sigma_g^{S_j} \\
V_A\sigma_i^{V_A} &= V_{AA}A\sigma_i^{A} + \sum_j V_{AS_j} S_j \sigma_i^{S_j}
\end{aligned}$$

Now derive both sides of the HJB with respect to A to obtain

$$\begin{aligned}
\rho V_A &= V_A (i - \pi_H) + V_{AA} A \mu^A + \sum_j V_{AS_j} S_j \mu^{S_j} + \eta \left[V_A \left(A + A\delta^A, \mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}} \right) - V_A (A, \mathbf{S}) \right] \\
&+ \frac{1}{2} \left[V_{AAA} \left(A \sigma_g^A \right)^2 + 2 \sum_j V_{AAS_j} A S_j \sigma_g^A \sigma_g^{S_j} + \sum_j \sum_k V_{AS_j S_k} S_j S_k \sigma_g^{S_j} \sigma_g^{S_k} \right] \\
&+ \frac{1}{2} \left[V_{AAA} \left(A \sigma_i^A \right)^2 + 2 \sum_j V_{AAS_j} A S_j \sigma_i^A \sigma_i^{S_j} + \sum_j \sum_k V_{AS_j S_k} S_j S_k \sigma_i^{S_j} \sigma_i^{S_k} \right]
\end{aligned}$$

Therefore

$$\mu^{V_A} = \rho - i + \pi_H - \eta \delta^{V_A}$$

Finally, apply Ito's lemma to the first order condition to obtain

$$\begin{aligned}
dC/C = &\left\{ -(\rho - i + \pi) + \sigma_g^{V_A} \sigma_g^{V_A} + \sigma_i^{V_A} \sigma_i^{V_A} + \alpha \frac{1+\alpha}{2} \left(\sigma_g^{\mathcal{S}} \sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}} \sigma_i^{\mathcal{S}} \right) + \alpha \left(\sigma_g^{V_A} \sigma_g^{\mathcal{S}} + \sigma_i^{V_A} \sigma_i^{\mathcal{S}} \right) \right\} dt \\
&+ \left\{ \eta \left[\frac{\left[\left(1 + \delta^S \right)^\alpha - 1 \right]^2}{\left(1 + \delta^S \right)^\alpha \left(1 + \delta^{V_A} \right)} + \delta^{V_A} \frac{\left(1 + \delta^S \right)^\alpha - 1 + \delta^{V_A}}{1 + \delta^{V_A}} \right] \right\} dt \\
&- \left(\alpha \sigma_g^{\mathcal{S}} + \sigma_g^{V_A} \right) d\mathcal{B}_g - \left(\alpha \sigma_i^{\mathcal{S}} + \sigma_i^{V_A} \right) d\mathcal{B}_i + \left[\frac{1}{\left(1 + \delta^S \right)^\alpha \left(1 + \delta^{V_A} \right)} - 1 \right] d\tilde{\mathcal{P}}
\end{aligned}$$

where we used

$$d\mathcal{S}/\mathcal{S} = \left(\mu^{\mathcal{S}} + \eta \delta^{\mathcal{S}} \right) dt + \sigma_g^{\mathcal{S}} d\mathcal{B}_g + \sigma_i^{\mathcal{S}} d\mathcal{B}_i + \delta^{\mathcal{S}} d\tilde{\mathcal{P}}$$

and $\pi \equiv \pi_H + \alpha \mu^S + \eta \left[\left(1 + \delta^S \right)^\alpha - 1 \right]$. Finally, the FOC with respect to L is $W = L^\varphi C \mathcal{S}^\alpha$.

B.2 The firms' problem

A measure one of monopolistic firms (indexed by $j \in [0,1]$) engage in infrequent price setting a la Calvo. Each firm re-optimizes its price $P_{H,j}(t)$ only at discrete dates determined by a Poisson process with intensity θ. A firm that is allowed to re-optimize its price at time t maximizes the present discounted value of future profits[34]

$$max_{\bar{P}_{H,j}(t)} \mathbb{E} \left[\int_t^\infty \frac{P(t) C(t)}{P(u) C(u)} e^{-(\rho+\theta)(u-t)} \left\{ \bar{P}_{H,j}(t) Y_j(u|t) - \mathcal{C}_H \left(Y_j(u|t) \right) \right\} du \right]$$

subject to the demand schedule

$$Y_j(u|t) = \left[\frac{\bar{P}_{H,j}(t)}{P_H(u)} \right]^{-\epsilon} Y(u)$$

where $\mathcal{C}(\cdot)$ is the firms nominal cost function. The first-order condition associated with the problem is

$$\mathbb{E} \left[\int_t^\infty \frac{P(t) C(t)}{P(u) C(u)} e^{-(\rho+\theta)(u-t)} Y_j(u|t) \left\{ \bar{P}_{H,j}(t) - \mathcal{M}MC \left(Y_j(u|t) \right) \right\} du \right] = 0$$

[34]We assume that firms commit to supply whatever quantity demanded at the posted price, even if that implies negative profits

where MC is the nominal marginal cost function and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$. Note that in the limiting case of no price rigidities ($\theta \to \infty$), this condition collapses to the familiar optimal price-setting condition under flexible prices $P_{H,j}(t) = \mathcal{M}MC(Y_j(t))$.

The firm's cost function is $\mathcal{C}(Y_j(u|t)) = Y_j(u|t)(1-\tau)W(u)P_H(u)$, therefore the nominal marginal cost is

$$MC(Y_j(u|t)) = (1-\tau)W(u)P_H(u) \equiv MC(u)$$

The FOC can be rewritten as

$$\bar{P}_{H,j}(t) = P_H(t)\frac{\mathcal{U}(t)}{\mathcal{V}(t)}$$
$$1 = \mathcal{M}(1-\tau)W(u)$$

where

$$\mathcal{U}(t) \equiv \mathbb{E}\left[\int_t^\infty \left\{e^{-\int_t^u[\rho+\theta-\epsilon\pi_H(s)]ds}\frac{\mathcal{M}MC(u)Y(u)}{P(u)C(u)}\right\}du\right]$$
$$\mathcal{V}(t) \equiv \mathbb{E}\left[\int_t^\infty \left\{e^{-\int_t^u[\rho+\theta-(\epsilon-1)\pi_H(s)]ds}\frac{P_H(u)Y(u)}{P(u)C(u)}\right\}du\right]$$

and we used the result that the dynamics of the price level is locally deterministic (see below). Now assume $\mathcal{U}(t) = \mathcal{U}(\mathbf{S})$ and $\mathcal{V}(t) = \mathcal{V}(\mathbf{S})$. We can apply the Feynman-Kac representation formula to obtain

$$\mathbb{E}[d\mathcal{U}/\mathcal{U}] = \rho+\theta-\epsilon\pi_H - \mathcal{M}\frac{YMC}{\mathcal{U}PC}$$
$$\mathbb{E}[d\mathcal{V}/\mathcal{V}] = \rho+\theta-(\epsilon-1)\pi_H - \frac{YP_H}{\mathcal{V}PC}$$

Therefore, their laws of motion are of the form

$$d\mathcal{U}/\mathcal{U} = \left(\rho+\theta-\epsilon\pi_H - \mathcal{M}\frac{YMC}{\mathcal{U}PC}\right)dt + \sigma_g^{\mathcal{U}}dZ_g + \sigma_i^{\mathcal{U}}dZ_i + \delta^{\mathcal{U}}d\tilde{\mathcal{P}} \tag{A.45}$$
$$d\mathcal{V}/\mathcal{V} = \left(\rho+\theta-(\epsilon-1)\pi_H - \frac{YP_H}{\mathcal{V}PC}\right)dt + \sigma_g^{\mathcal{V}}dZ_g + \sigma_i^{\mathcal{V}}dZ_i + \delta^{\mathcal{V}}d\tilde{\mathcal{P}} \tag{A.46}$$

Since all firms resetting prices choose an identical price $\bar{P}_H$ the law of motion of producer price index is PPI is $dP_H/P_H = \pi_H dt$ where PPI inflation is given by (see Cavallino (2019) for a complete proof)

$$dP_H/P_H = \pi_H dt = \frac{\theta}{\epsilon-1}\left[1-\left(\frac{\bar{P}_H}{P_H}\right)^{1-\epsilon}\right]dt$$

By applying Ito's lemma and using the laws of motion for $\mathcal{U}$ and $\mathcal{V}$ we obtain

$$\begin{aligned}\mathbb{E}[d\pi_H] = {}& [(\epsilon-1)\pi_H-\theta]\left\{\pi_H + \mathcal{M}\frac{YMC}{\mathcal{U}PC} - \frac{YP_H}{\mathcal{V}PC} + \frac{\epsilon}{2}\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{U}} + \sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{U}}\right) - (\epsilon-1)\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{V}} + \sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{V}}\right)\right\}dt \\ & + [(\epsilon-1)\pi_H-\theta]\left\{-\frac{2-\epsilon}{2}\left(\sigma_g^{\mathcal{V}}\sigma_g^{\mathcal{V}} + \sigma_i^{\mathcal{V}}\sigma_i^{\mathcal{V}}\right) + \eta\left[\frac{\left(\frac{1+\delta^{\mathcal{U}}}{1+\delta^{\mathcal{V}}}\right)^{1-\epsilon}-1}{\epsilon-1} + \delta^{\mathcal{U}} - \delta^{\mathcal{V}}\right]\right\}dt\end{aligned} \tag{A.47}$$

Finally, let $\Delta \equiv \int_0^1\left[\frac{P_{H,j}}{P_H}\right]^{-\epsilon}dj$ denote the aggregate loss of efficiency induced by price dispersion among firms. Then, its law of motion is

$$d\Delta = \left[\theta\left(1-\frac{\epsilon-1}{\theta}\pi_H\right)^{\frac{\epsilon}{\epsilon-1}} + \Delta(\epsilon\pi_H-\theta)\right]dt \tag{A.48}$$

B.3 Foreign investors and no-arbitrage conditions

The Home stochastic discount factor for payoffs in units of the domestic good is $\mathcal{D} = e^{-\rho t} V_A$ while the return of the Home-currency bond for Home households is

$$dB_H/B_H = \left[i_H - \pi_H - \eta\left(1-\chi\right)\right]dt - \left(1-\chi\right)d\tilde{\mathcal{P}}$$
$$dB_F/B_F = \left[i_F - \pi^* - \eta\left(1-\chi\right)\right]dt - \left(1-\chi\right)d\tilde{\mathcal{P}}$$

The portfolio optimality conditions are $\mathbb{E}\left[d\left(\mathcal{D}B_H\right)\right] = 0$ and $\mathbb{E}\left[d\left(\mathcal{D}B_F\mathcal{S}\right)\right] = 0$. Applying Ito's lemma we obtain the following no-arbitrage equations

$$i_H - i = \eta\left(1-\chi\right)\left(1+\delta^{V_A}\right)$$
$$i_F - i = -\mu^{\mathcal{S}} + \pi^* - \pi_H + \eta\left(1+\delta^{V_A}\right)\left(1-\chi-\chi\delta^{\mathcal{S}}\right) - \sigma_g^{\mathcal{S}}\sigma_g^{V_A} - \sigma_i^{\mathcal{S}}\sigma_i^{V_A}$$

Similarly, let $\mathcal{D}^* = e^{-\rho^* t} V_A^*$ be the Foreign stochastic discount factor for payoffs in units of the foreign good and

$$dB_H^*/B_H^* = \left[i_H - \pi_H - \eta\left(1-\chi^*\right)\right]dt - \left(1-\chi^*\right)d\tilde{\mathcal{P}}$$
$$dB_F^*/B_F^* = \left[i_F - \pi^* - \eta\left(1-\chi^*\right)\right]dt - \left(1-\chi^*\right)d\tilde{\mathcal{P}}$$

Then the foreign investors portoflio optimality conditions, $\mathbb{E}\left[d\left(\mathcal{D}^* B_H^*/\mathcal{S}\right)\right] = 0$ and $\mathbb{E}\left[d\left(\mathcal{D}^* B_F^*\right)\right] = 0$, yield

$$i_H - i^* = \mu^{\mathcal{S}} + \pi_H - \pi^* + \eta\left(1+\delta^{V_A^*}\right)\frac{\delta^{\mathcal{S}} + 1 - \chi^*}{1+\delta^{\mathcal{S}}} + \left(\sigma_g^{V_A^*} - \sigma_g^{\mathcal{S}}\right)\sigma_g^{\mathcal{S}} + \left(\sigma_i^{V_A^*} - \sigma_i^{\mathcal{S}}\right)\sigma_i^{\mathcal{S}}$$
$$i_F - i^* = \eta\left(1+\delta^{V_A^*}\right)\left(1-\chi^*\right)$$

Now, since the Home economy is small, i.e. has a negligible size, $\sigma_g^{V_A^*} = \sigma_i^{V_A^*} = \delta^{V_A^*} = 0$. Hence

$$i_H - i^* = \mu^{\mathcal{S}} + \pi_H - \pi^* + \eta\frac{1-\chi^* + \delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} - \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} - \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}$$
$$i_F - i^* = \eta\left(1-\chi^*\right)$$

and

$$\mu^{\mathcal{S}} = i - i^* - \pi_H + \pi^* - \eta\left[\delta^{\mathcal{S}} + \left(\chi-\chi^*\right) - \left(1-\chi\right)\delta^{V_A} - \delta^{\mathcal{S}}\frac{\left(1-\chi^*\right)+\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\right] + \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}$$

Now use $\mathcal{E} = \mathcal{S}P_H/P^*$ to obtain

$$\mathbb{E}\left[d\mathcal{E}/\mathcal{E}\right] = i - i^* - \eta\left[\chi-\chi^* - \left(1-\chi\right)\delta^{V_A} - \delta^{\mathcal{S}}\frac{\delta^{\mathcal{S}} + \left(1-\chi^*\right)}{1+\delta^{\mathcal{S}}}\right] + \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}$$

B.4 Public sector

Government debt evolves according to

$$dB = \left(G-T\right)dt + dB_H + dB_H^* + d\left(\mathcal{S}B_F\right) + d\left(\mathcal{S}B_F^*\right)$$

Using the equations derived above we can rewrite it as

$$\begin{aligned} dB = {} & B\left[i - \pi_H + \eta\left(1-\chi\right)\delta^{V_A} - \eta\left(\chi-\chi^*\right)\frac{B_H^* + \mathcal{S}B_F^*}{B} + \frac{\mathcal{S}B_F}{B}\eta\delta^{\mathcal{S}}\frac{\chi-\chi^*}{1+\delta^{\mathcal{S}}} + \frac{G-T}{B}\right]dt \\ & + \mathcal{S}\eta\frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\left(\chi B_F + \chi^* B_F^*\right)dt + \left(\mathcal{S}B_F + \mathcal{S}B_F^*\right)\left[\left(\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}\right)dt + \sigma_g^{\mathcal{S}}d\mathcal{B}_g + \sigma_i^{\mathcal{S}}d\mathcal{B}_i\right] \\ & + \left[B + \chi B_H + \chi\mathcal{S}B_F\left(1+\delta^{\mathcal{S}}\right) + \chi^* B_H^* + \chi^*\mathcal{S}B_F^*\left(1+\delta^{\mathcal{S}}\right)\right]d\tilde{\mathcal{P}} \end{aligned}$$

B.5 Equilibrium

Let $\Lambda = \frac{C}{QC^*}$ and use $\mathcal{Q} = \mathcal{S}^{1-\alpha}$ to derive its law of motion

$$d\Lambda/\Lambda = \left\{\rho^* - \rho + \eta\left[\chi - \chi^* - (1-\chi)\,\delta^{V_A} - (1-\chi^*)\,\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}}{1+\delta^{V_A}}\left(\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \delta^{V_A}\right)\right]\right\}dt$$
$$+ \left(\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A} + \sigma_g^{V_A}\sigma_g^{\mathcal{S}} + \sigma_i^{V_A}\sigma_i^{\mathcal{S}}\right)dt - \left(\sigma_g^{V_A} + \sigma_g^{\mathcal{S}}\right)d\mathcal{B}_g - \left(\sigma_i^{V_A} + \sigma_i^{\mathcal{S}}\right)d\mathcal{B}_i$$
$$+ \left[\frac{1}{\left(1+\delta^{\mathcal{S}}\right)\left(1+\delta^{V_A}\right)} - 1\right]d\tilde{\mathcal{P}}$$

Use the market clearing condition $Y = C_H + C_H^* + G = [\alpha + (1-\alpha)\,\Lambda]\,\mathcal{S}C^* + G$ to compute the law of motion of Y

$$dY = (Y-G)\left\{i - \pi_H - \rho + \eta\frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} + (1-\alpha)\,\Lambda\frac{\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A}}{\alpha + (1-\alpha)\,\Lambda} + (Y-G)\,\alpha\frac{\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}}{\alpha + (1-\alpha)\,\Lambda}dt\right\}dt$$
$$+ (Y-G)\,\alpha\frac{\rho - \rho^* - \eta\left[\chi - \chi^* - (1-\chi)\,\delta^{V_A} - (1-\chi^*)\,\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} - \frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\right]}{\alpha + (1-\alpha)\,\Lambda}dt$$
$$+ dG + (Y-G)\,\frac{\alpha\sigma_g^{\mathcal{S}} - (1-\alpha)\,\Lambda\sigma_g^{V_A}}{\alpha + (1-\alpha)\,\Lambda}d\mathcal{B}_g + (Y-G)\,\frac{\alpha\sigma_i^{\mathcal{S}} - (1-\alpha)\,\Lambda\sigma_i^{V_A}}{\alpha + (1-\alpha)\,\Lambda}d\mathcal{B}_i$$
$$+ (Y-G)\,\frac{\alpha\delta^{\mathcal{S}} - \frac{\delta^{V_A}}{1+\delta^{V_A}}\,(1-\alpha)\,\Lambda}{\alpha + (1-\alpha)\,\Lambda}d\tilde{\mathcal{P}}$$

Finally, let $Z = \frac{B-A}{\mathcal{S}C^*}$ and derive its law of motion

$$dZ = [Z\rho^* + \alpha\,(\Lambda - 1)]\,dt - \frac{B_H^*}{\mathcal{S}C^*}\sigma_g^{\mathcal{S}}d\mathcal{B}_g - \frac{B_H^*}{\mathcal{S}C^*}d\mathcal{B}_i - \frac{(1-\chi^*)\left(\mathcal{S}B_F^* + B_H^*\right) + \delta^{\mathcal{S}}\frac{1-(1-\chi^*)}{1+\delta^{\mathcal{S}}}B_H^*}{\mathcal{S}C^*}d\tilde{\mathcal{P}}$$

B.6 Log-linearization

Using the policy rules described in the main text, the equilibrium of the model can be reduced to the following system of equations

$$\mathbb{E}\,[d\Lambda] = \Lambda\left\{\rho^* - \rho + \eta\left[(\chi - \chi^*) - (1-\chi)\,\delta^{V_A} - (1-\chi^*)\,\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}}{1+\delta^{V_A}}\left(\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \delta^{V_A}\right)\right]\right\}dt$$
$$+ \Lambda\left(\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A} + \sigma_g^{V_A}\sigma_g^{\mathcal{S}} + \sigma_i^{V_A}\sigma_i^{\mathcal{S}}\right)dt$$

$$\mathbb{E}\,[dY] = (Y-G)\left[\phi_\pi^x\pi_H + \varepsilon_i + \eta\frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} + (1-\alpha)\,\Lambda\frac{\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A}}{\alpha + (1-\alpha)\,\Lambda} + \alpha\frac{\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}}{\alpha + (1-\alpha)\,\Lambda}\right]dt$$
$$+ (Y-G)\,\alpha\frac{\rho - \rho^* - \eta\left[(\chi - \chi^*) - (1-\chi)\,\delta^{V_A} - (1-\chi^*)\,\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} - \frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\right]}{\alpha + (1-\alpha)\,\Lambda}dt + \mathbb{E}\,[dG]$$

$$\mathbb{E}\,[dB] = B\left[\rho + \phi_\pi^x\pi_H + \varepsilon_i + \eta\,(1-\chi)\,\delta^{V_A} - \eta\,(\chi - \chi^*)\,\frac{Z + \frac{\bar{B}_F}{C^*}\frac{Y-G}{\alpha + (1-\alpha)\Lambda}}{B}\right]dt$$
$$+ \frac{Y-G}{\alpha + (1-\alpha)\,\Lambda}\frac{B_F^*}{C^*}\left[\eta\,(1 - (1-\chi^*))\,\frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}\right]dt$$

$$
\begin{aligned}
&+\left[G-\bar{T}-\psi_b^x\left(B-\bar{B}\right)-\psi_\pi^x\phi_\pi^x\pi_H\bar{B}\right]dt\\
\mathbb{E}\left[dZ\right]&=\left[Z\rho^*+\alpha\left(\Lambda-1\right)\right]dt\\
\mathbb{E}\left[d\pi_H\right]&=\left[\left(\epsilon-1\right)\pi_H-\theta\right]\left\{\pi_H+\left(\frac{\alpha}{\Lambda}+1-\alpha\right)\left[\frac{\Delta^\varphi Y^{1+\varphi}}{(1-\alpha)\mathcal{U}}-\frac{Y/\mathcal{V}}{Y-G}\right]+\frac{\epsilon}{2}\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{U}}+\sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{U}}\right)\right\}dt\\
&+\left[\left(\epsilon-1\right)\pi_H-\theta\right]\left[\left(1-\epsilon\right)\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{V}}+\sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{V}}\right)-\frac{2-\epsilon}{2}\left(\sigma_g^{\mathcal{V}}\sigma_g^{\mathcal{V}}+\sigma_i^{\mathcal{V}}\sigma_i^{\mathcal{V}}\right)\right]dt\\
&+\left[\left(\epsilon-1\right)\pi_H-\theta\right]\eta\left[\frac{\left(\frac{1+\delta^{\mathcal{U}}}{1+\delta^{\mathcal{V}}}\right)^{1-\epsilon}-1}{\epsilon-1}+\delta^{\mathcal{U}}-\delta^{\mathcal{V}}\right]dt\\
\mathbb{E}\left[d\mathcal{U}\right]&=\mathcal{U}\left[\rho+\theta-\epsilon\pi_H-\left(\frac{\alpha}{\Lambda}+1-\alpha\right)\frac{\Delta^\varphi Y^{1+\varphi}}{(1-\alpha)\mathcal{U}}\right]dt\\
\mathbb{E}\left[d\mathcal{V}\right]&=\mathcal{V}\left[\rho+\theta-\left(\epsilon-1\right)\pi_H-\left(\frac{\alpha}{\Lambda}+1-\alpha\right)\frac{Y}{Y-G}\frac{1}{\mathcal{V}}\right]dt\\
\mathbb{E}\left[d\Delta\right]&=\left[\theta\left(1-\frac{\epsilon-1}{\theta}\pi_H\right)^{\frac{\epsilon}{\epsilon-1}}+\Delta\left(\epsilon\pi_H-\theta\right)\right]dt
\end{aligned}
$$

with $G=\varepsilon_g$ and $\eta=\max\left\{0,\bar{\eta}+\eta^x\frac{B-\bar{B}}{\bar{B}}\right\}$, subject to

$$
\begin{aligned}
d\varepsilon_g&=-\varrho_g\varepsilon_g dt+\sigma_g d\mathcal{B}_g\\
d\varepsilon_i&=-\varrho_i\varepsilon_i dt+\sigma_i d\mathcal{B}_i
\end{aligned}
$$

To linearize the model, premultiply $\bar{\eta}$, σ_g, and σ_i by the perturbation parameter ς, such that when $\varsigma=0$ the model is deterministic. Then, in a deterministic steady state we have

$$
\begin{aligned}
\bar{\pi}_H&=0\\
\bar{\Delta}&=1\\
\bar{Z}&=\frac{\alpha}{\rho^*}\left(1-\bar{\Lambda}\right)\\
\bar{\mathcal{U}}&=\frac{\frac{\alpha}{\Lambda}+1-\alpha}{\rho+\theta}\frac{\bar{Y}^{1+\varphi}}{1-\alpha}\\
\bar{\mathcal{V}}&=\frac{\frac{\alpha}{\Lambda}+1-\alpha}{\rho+\theta}\frac{\bar{Y}}{\bar{Y}-\bar{G}}
\end{aligned}
$$

We chose a symmetric steady state with $\Lambda=1$, and we normalize $\bar{Y}=1$ and $\bar{G}=0$. Then, the log-linear dynamics around this points are described by the system of differential equations

$$
\begin{aligned}
\mathbb{E}\left[d\lambda\right]&=\tilde{\eta}^x b dt\\
\mathbb{E}\left[dy\right]&=\left(\phi_\pi^x\pi_H-\alpha\tilde{\eta}^x b-\varrho_g\varepsilon_g+\varepsilon_i\right)dt\\
\mathbb{E}\left[db\right]&=\left[\left(\rho-\psi_b^x-\xi\tilde{\eta}^x\right)b+\left(1-\psi_\pi^x\right)\phi_\pi^x\pi_H+\beta\varepsilon_g+\varepsilon_i\right]dt\\
\mathbb{E}\left[dz\right]&=\left(\rho z+\alpha\lambda\right)dt\\
\mathbb{E}\left[d\pi_H\right]&=\left(\rho\pi_H-\kappa\omega y+\kappa\varepsilon_g\right)dt\\
\mathbb{E}\left[d\varepsilon_g\right]&=-\varrho_g\varepsilon_g dt\\
\mathbb{E}\left[d\varepsilon_i\right]&=-\varrho_i\varepsilon_i dt
\end{aligned}
$$

where we used

$$\rho = \rho^* + \bar{\eta}\,(\chi - \chi^*)$$
$$\bar{T} = [\rho - \bar{\eta}\,(\chi - \chi^*)\,\xi]\,\bar{B}$$

and $\tilde{\eta}^x = (\chi - \chi^*)\,\eta^x$, $\beta = \bar{Y}/\bar{B}$, $\kappa = \theta\,(\rho + \theta)$, $\omega = 1 + \varphi$. We can omit the linearized laws of motion of $\mathcal{U}$, $\mathcal{V}$, and Δ since these variables do not enter the linearized dynamic. Note that, like in most small open economy models, the net foreign asset position of the country z, and therefore λ, are not mean-reverting variables. This well-known problem can be solved by assuming that the cost of borrowing abroad is increasing in net foreign debt. This would introduce a term ϖz with $\varpi > 0$ in the equation for $d\lambda$ and make both variable mean-reverting. Since this is true for any $\varpi > 0$, to simplify the algebra we directly take the limit for $\varpi \downarrow 0$.

Appendix C Proofs

Solution method We solve the model using the method of undetermined coefficients. The system of linear differential equation derived above can be written in matrix form

$$\begin{bmatrix} \mathbb{E}\left[d\mathbf{m}\left(t\right)|x\right] \\ \mathbb{E}\left[d\mathbf{n}\left(t\right)|x\right] \end{bmatrix} = \begin{bmatrix} \mathbf{A}^x & \mathbf{B}^x \\ \mathbf{C}^x & \mathbf{D}^x \end{bmatrix} \begin{bmatrix} \mathbf{m}\left(t\right) \\ \mathbf{n}\left(t\right) \end{bmatrix} dt \tag{A.49}$$

where $\mathbf{m}\left(t\right) \equiv \begin{bmatrix} \lambda\left(t\right) & y\left(t\right) & \pi_H\left(t\right) \end{bmatrix}^\top$, $\mathbf{n}\left(t\right) \equiv \begin{bmatrix} z\left(t\right) & b\left(t\right) & \varepsilon_g\left(t\right) & \varepsilon_i\left(t\right) \end{bmatrix}^\top$ and

$$\mathbf{A}^x \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \phi_\pi^x \\ 0 & -\kappa\omega & \rho \end{bmatrix} \qquad \mathbf{B}^x \equiv \begin{bmatrix} 0 & \tilde{\eta}^x & 0 & 0 \\ 0 & -\alpha\tilde{\eta}^x & -\varrho_g & 1 \\ 0 & 0 & \kappa & 0 \end{bmatrix}$$

$$\mathbf{C}^x \equiv \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 0 & \phi_\pi^x\left(1-\psi_\pi^x\right) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{D}^x \equiv \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \rho - \psi_b^x - \xi\tilde{\eta}^x & \beta & 1 \\ 0 & 0 & -\varrho_g & 0 \\ 0 & 0 & 0 & -\varrho_i \end{bmatrix}$$

for $x \in \{R, N\}$. The transition matrix between the two states is given by

$$\mathbf{\Sigma} = \begin{bmatrix} -\sigma^N & \sigma^N \\ \sigma^R & -\sigma^R \end{bmatrix}$$

We guess a solution of the form $\mathbf{m}\left(t\right) = \mathbf{\Gamma}^x \mathbf{n}\left(t\right) + \boldsymbol{\zeta}^x\left(t\right)$ where

$$\mathbf{\Gamma}^x \equiv \begin{bmatrix} \gamma_{\lambda z}^x & \gamma_{\lambda b}^x & \gamma_{\lambda g}^x & \gamma_{\lambda i}^x \\ \gamma_{yz}^x & \gamma_{yb}^x & \gamma_{yg}^x & \gamma_{yi}^x \\ \gamma_{\pi z}^x & \gamma_{\pi b}^x & \gamma_{\pi g}^x & \gamma_{\pi i}^x \end{bmatrix} \quad \boldsymbol{\zeta}^x\left(t\right) \equiv \begin{bmatrix} \zeta_\lambda^x\left(t\right) \\ \zeta_y^x\left(t\right) \\ \zeta_\pi^x\left(t\right) \end{bmatrix}$$

and apply Ito's lemma to obtain

$$\mathbb{E}\left[d\mathbf{m}\left(t\right)|x\right] = \mathbf{\Gamma}^x \mathbb{E}\left[d\mathbf{n}\left(t\right)|x\right] + \sigma^{-x}\left(\mathbf{\Gamma}^{-x} - \mathbf{\Gamma}^x\right)\mathbf{n}\left(t\right) + \sigma^{-x}\left(\boldsymbol{\zeta}^{-x}\left(t\right) - \boldsymbol{\zeta}^x\left(t\right)\right) + d\boldsymbol{\zeta}^x\left(t\right)$$

By replacing our guess into the equilibrium system of differential equations and matching coefficients we obtain the system of polynomials

$$0 = \mathbf{\Gamma}^R\left(\mathbf{C}^R\mathbf{\Gamma}^R + \mathbf{D}^R\right) + \sigma^N\left(\mathbf{\Gamma}^N - \mathbf{\Gamma}^R\right) - \mathbf{A}^R\mathbf{\Gamma}^R - \mathbf{B}^R \tag{A.50}$$

$$0 = \mathbf{\Gamma}^N\left(\mathbf{C}^N\mathbf{\Gamma}^N + \mathbf{D}^N\right) + \sigma^R\left(\mathbf{\Gamma}^R - \mathbf{\Gamma}^N\right) - \mathbf{A}^N\mathbf{\Gamma}^N - \mathbf{B}^N \tag{A.51}$$

and the system of differential equations

$$\begin{bmatrix} d\boldsymbol{\zeta}^R\left(t\right) \\ d\boldsymbol{\zeta}^N\left(t\right) \end{bmatrix} = \mathbf{U}\begin{bmatrix} \boldsymbol{\zeta}^R\left(t\right) \\ \boldsymbol{\zeta}^N\left(t\right) \end{bmatrix} \tag{A.52}$$

where

$$\mathbf{U} \equiv \begin{bmatrix} \mathbf{A}^R - \mathbf{\Gamma}^R\mathbf{C}^R + \sigma^N\mathbf{I}_{|\mathbf{m}|} & -\sigma^N\mathbf{I}_{|\mathbf{m}|} \\ -\sigma^R\mathbf{I}_{|\mathbf{m}|} & \mathbf{A}^N - \mathbf{\Gamma}^N\mathbf{C}^N + \sigma^R\mathbf{I}_{|\mathbf{m}|} \end{bmatrix} \tag{A.53}$$

and $\mathbf{I}_{|\mathbf{m}|}$ is the identity matrix of size $|\mathbf{m}| \times |\mathbf{m}|$. The pair of matrices $\left\{\mathbf{\Gamma}^R, \mathbf{\Gamma}^N\right\}$ is a solution to A.49 if it satisfies (A.50) and (A.51). The solution is determined (or unique) if $\boldsymbol{\zeta}^R\left(t\right) = \boldsymbol{\zeta}^N\left(t\right) = 0$

is the only stable solution to (A.52). That is, iff all eigenvalues of $\mathbf{U}$ are strictly positive. Finally, define

$$\begin{aligned}
\mathbf{F}^R &\equiv \mathbf{C}^R\mathbf{\Gamma}^R + \mathbf{D}^R \\
\mathbf{F}^N &\equiv \mathbf{C}^N\mathbf{\Gamma}^N + \mathbf{D}^N \\
\mathbf{S} &\equiv \mathbf{\Sigma}^\top \otimes \mathbf{I}_{|\mathbf{m}|^2} + \operatorname{diag}\left(\mathbf{F}^x \otimes \mathbf{I}_{|\mathbf{n}|} + \mathbf{I}_{|\mathbf{n}|} \otimes \mathbf{F}^x\right)
\end{aligned} \tag{A.54}$$

The solution is mean-square stable iff all eigenvalues of $\mathbf{S}$ are strictly negative. For a proof of this result, see Theorem 3.15 in Costa et al. (2013). The endogenous elasticity of default with respect to debt, η^x, is obtained by setting the maximum eigenvalue of A.54 equal to zero.

Given a solution $\left\{\mathbf{\Gamma}^R, \mathbf{\Gamma}^N\right\}$of the model, the response of the nominal exchange rate in regime $x \in \{R, N\}$ to fiscal and monetary shocks can be calculated using $e\left(0\right) = y\left(0\right) - \left(1-\alpha\right)\lambda\left(0\right) - \varepsilon_g\left(0\right)$, which yields

$$e\left(0\right) = \frac{\gamma^x_{yg} - \left(1-\alpha\right)\gamma^x_{\lambda g} - 1}{1 - \iota\left[\gamma^x_{yb} + \left(1-\alpha\right)\gamma^x_{\lambda b}\right]}\varepsilon_g\left(0\right) + \frac{\gamma^x_{yi} - \left(1-\alpha\right)\gamma^x_{\lambda i}}{1 - \iota\left[\gamma^x_{yb} + \left(1-\alpha\right)\gamma^x_{\lambda b}\right]}\varepsilon_i\left(0\right)$$

where we used the initial conditions $b\left(0\right) = \iota e\left(0\right)$ and $z\left(0\right) = 0$.

Proof of Propositions 2-3 and 6-7 To adapt the solution method described above to the deterministic case, set $\sigma^R = \sigma^N = 0$ and let $\tilde{\eta}^R = \tilde{\eta}^N = \tilde{\eta}^x$, $\psi^R_\pi = \psi^N_\pi = \psi^x_\pi$, $\psi^R_b = \psi^N_b = \psi^x_b$, and $\phi^R_\pi = \phi^N_\pi = \phi^x_\pi > 0$. First of all, note that output and inflation do not depend directly on z, but only indirectly through λ which captures the consumption absorption share of the Home economy. Therefore we must have $\gamma^x_{yz} = 0$ and $\gamma^x_{\pi z} = 0$, which implies $\gamma^x_{\lambda z} = -\rho/\alpha$.[35] Now solve for the eigenvalues of A.54. The sign of the largest eigenvalues is determined by

$$\operatorname{Sign}\left[\rho - \psi^x_b - \xi\eta^x + \gamma^x_{\pi b}\left(1-\psi^x_\pi\right)\phi^x_\pi\right] \tag{A.55}$$

Now guess $\tilde{\eta}^x = 0$. Solve (A.50)-(A.51) for $\gamma^x_{yb}$and $\gamma^x_{\pi b}$ (they can be solved independently of the other coefficients). There are 3 solutions. The first one has $\gamma^x_{yb} = \gamma^x_{\pi b} = 0$ which implies that the system is stable iff $\psi^x_b > \rho$. Solving for the eigenvalues of A.53 it's easy to show that they are all positive iff $\phi^x_\pi > 0$. This is the Ricardian equilibrium and its solution is $\gamma^x_{\lambda b} = \gamma^x_{\lambda g} = \gamma^x_{\lambda m} = 0$ and

$$\begin{aligned}
\gamma^x_{yg} &= \frac{\kappa\phi^x_\pi + \varrho(\rho+\varrho)}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)} \\
\gamma^x_{\pi g} &= \frac{\kappa(\omega-1)\varrho}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)} \\
\gamma^x_{ym} &= -\frac{\rho+\varrho}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)} \\
\gamma^x_{\pi m} &= -\frac{\kappa\omega}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)}
\end{aligned}$$

The second solution has

$$\gamma^x_{\pi b} = -\frac{\rho - 2\psi^x_b - \sqrt{\rho^2 - 4\kappa\omega\phi^x_\pi}}{2\left(1-\psi^x_\pi\right)\phi^x_\pi}$$

[35] Note that the model contains a unit root for λ and z which can be eliminated by using a debt-elastic foreign interest rate rule.

which implies that implies that it is unstable since A.55 simplifies to Sign $\left[\frac{\rho+\sqrt{\rho^2-4\kappa\omega\phi_\pi^x}}{2}\right]$ whose real part is always positive. The third solution has

$$\gamma_{\pi b}^x = -\frac{\rho - 2\psi_b^x + \sqrt{\rho^2 - 4\kappa\omega\phi_\pi^x}}{2\left(1 - \psi_\pi^x\right)\phi_\pi^x}$$

which implies that A.55 simplifies to Sign $\left[\frac{\rho-\sqrt{\rho^2-4\kappa\omega\phi_\pi^x}}{2}\right]$. Thus, the solution is stable iff $\phi_\pi^x < 0$. Finally, the minimum eigenvalue of A.53 is $\rho - \psi_b^x$, which means that the solution is determinate iff $\psi_b^x < \rho$. This is the non-Ricardian inflationary equilibrium and its solution is $\gamma_{\lambda b}^x = \gamma_{\lambda g}^x = \gamma_{\lambda m}^x = 0$ and

$$\begin{aligned}
\gamma_{yb}^x &= \frac{-\psi_b + \mu + \rho}{\mu - \mu\psi_\pi} \\
\gamma_{\pi b}^x &= -\frac{\kappa\omega\left(-\psi_b + \mu + \rho\right)}{\mu\left(\psi_\pi - 1\right)\left(\mu + \rho\right)} \\
\gamma_{yg}^x &= \frac{\frac{-\psi_b(\mu+\rho+\omega\varrho)+\omega\varrho(\mu+2\rho+\varrho)+\rho(\mu+\rho)}{\omega(\mu+\rho+\varrho)} - \frac{\beta(-\psi_b+\mu+\rho)}{\mu(\psi_\pi-1)}}{-\psi_b + \rho + \varrho} \\
\gamma_{\pi g}^x &= \frac{\kappa\left(\frac{(\omega-1)\varrho}{\mu+\rho+\varrho} - \frac{\beta\omega(-\psi_b+\mu+\rho)}{\mu(\psi_\pi-1)(\mu+\rho)}\right)}{-\psi_b + \rho + \varrho} \\
\gamma_{ym}^x &= \frac{\psi_b\left(\mu\psi_\pi + \rho + \varrho\right) - \mu\psi_\pi(\mu + 2\rho + \varrho) - \rho(\rho + \varrho)}{\mu\left(\psi_\pi - 1\right)\left(\mu + \rho + \varrho\right)\left(-\psi_b + \rho + \varrho\right)} \\
\gamma_{\pi m}^x &= -\frac{\kappa\omega\left(\left(\mu + \rho\right)\left(\mu\psi_\pi + \rho + \varrho\right) - \psi_b(\mu + \rho + \varrho)\right)}{\mu\left(\psi_\pi - 1\right)\left(\mu + \rho\right)(\mu + \rho + \varrho)\left(-\psi_b + \rho + \varrho\right)}
\end{aligned}$$

where we defined $\mu \equiv \left(\sqrt{\rho^2 - 4\kappa\omega\phi_\pi^N} - \rho\right)/2$.

Now assume $\tilde{\eta}^x > 0$ and set it $\tilde{\eta}^x = \frac{\rho-\psi_b^x+\gamma_{\pi b}^x(1-\psi_\pi^x)\phi_\pi^x}{\xi}$, such that A.55 is equal to zero. Solve (A.50)-(A.51) for $\gamma_{yb}^x$and $\gamma_{\pi b}^x$ (they can be solved independently of the other coefficients) to obtain

$$\begin{aligned}
\gamma_{yb}^x &= \frac{\alpha\rho}{\kappa\omega\phi_\pi^x}\frac{\rho - \psi_b^x}{\xi - \alpha\left(1 - \psi_\pi^x\right)} \\
\gamma_{\pi b}^x &= \frac{\alpha}{\phi_\pi^x}\frac{\rho - \psi_b^x}{\xi - \alpha\left(1 - \psi_\pi^x\right)}
\end{aligned}$$

This implies that the elasticity of the default probability becomes

$$\tilde{\eta}^x = \frac{\rho - \psi_b^x}{\xi - \alpha\left(1 - \psi_\pi^x\right)}$$

which, assuming $\xi > \alpha\left(1 - \psi_\pi^x\right)$, is positive iff $\psi_b^x < \rho$. Finally, under these assumption, the minimum eigenvalue of A.53 is

$$\frac{\xi\rho + \alpha\left(\psi_\pi^x - 1\right)\left(2\rho - \psi_b^x\right) - \sqrt{\left(\alpha\psi_b^x\psi_\pi^x - \alpha\psi_b^x + \xi\rho\right)^2 - 4\kappa\omega\phi_\pi^x\left(\alpha\psi_\pi^x - \alpha + \xi\right)^2}}{2\left(\alpha\psi_\pi^x - \alpha + \xi\right)}$$

which is strictly positive iff

$$\phi_\pi > \frac{\alpha\rho\left(1 - \psi_\pi^x\right)\left(\rho - \psi_b^x\right)}{\kappa\omega\left(\xi - \alpha + \alpha\psi_\pi^x\right)}$$

This is the non-Ricardian default equilibrium and its solution is

$$
\begin{aligned}
\gamma^x_{yg} &= \frac{1}{\omega}\frac{-\alpha\left(\rho\omega(\beta+2\varrho)+\rho^2+\omega\varrho^2\right)+\alpha\psi_\pi\left[\rho^2+\omega\varrho\left(2\rho+\varrho\right)\right]}{\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\left(\rho+\varrho\right)\left\{-\alpha\left(\rho+\varrho\right)+\alpha\left[\psi_\pi\left(-\psi_b+\rho+\varrho\right)+\psi_b\right]+\xi\varrho\right\}} \\
&+\frac{1}{\omega}\frac{\alpha\psi_b\left[\omega\left(\beta+\varrho\right)-\psi_\pi\left(\rho+\omega\varrho\right)+\rho\right]+\xi\omega\varrho\left(\rho+\varrho\right)}{\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\left(\rho+\varrho\right)\left\{-\alpha\left(\rho+\varrho\right)+\alpha\left[\psi_\pi\left(-\psi_b+\rho+\varrho\right)+\psi_b\right]+\xi\varrho\right\}} \\
&+\frac{\kappa^2\omega\phi_\pi^2\left(\alpha\psi_\pi-\alpha+\xi\right)+\alpha\beta\rho(\rho+\varrho)\left(\rho-\psi_b\right)}{\kappa\omega\phi_\pi\left(\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_\pi\left(-\psi_b+\rho+\varrho\right)+\psi_b\right)+\xi\varrho\right)\right)} \\
\gamma^x_{\pi g} &= \frac{\kappa(\omega-1)\varrho\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\alpha\beta(\rho+\varrho)\left(\rho-\psi_b\right)}{\phi_\pi\left(\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_\pi\left(-\psi_b+\rho+\varrho\right)+\psi_b\right)+\xi\varrho\right)\right)} \\
\gamma^x_{ym} &= \frac{\kappa\omega\phi_\pi\left((\alpha-\xi)(\rho+\varrho)+\alpha\psi_\pi\left(\psi_b-2\rho-\varrho\right)\right)+\alpha\rho(\rho+\varrho)\left(\rho-\psi_b\right)}{\kappa\omega\phi_\pi\left(\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_\pi\left(-\psi_b+\rho+\varrho\right)+\psi_b\right)+\xi\varrho\right)\right)} \\
\gamma^x_{\pi m} &= \frac{\alpha(\rho+\varrho)\left(\rho-\psi_b\right)-\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)}{\phi_\pi\left(\kappa\omega\phi_\pi\left(\alpha\psi_\pi-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_\pi\left(-\psi_b+\rho+\varrho\right)+\psi_b\right)+\xi\varrho\right)\right)}
\end{aligned}
$$

For each equilibrium, the response of the nominal exchange rate to fiscal and monetary shocks is then calculated using $e\left(0\right)=y\left(0\right)-\left(1-\alpha\right)\lambda\left(0\right)-\varepsilon_g\left(0\right)$, which yields

$$
e\left(0\right)=\frac{\gamma^x_{yg}-\left(1-\alpha\right)\gamma^x_{\lambda g}-1}{1-\iota\left[\gamma^x_{yb}+\left(1-\alpha\right)\gamma^x_{\lambda b}\right]}\varepsilon_g\left(0\right)+\frac{\gamma^x_{yi}-\left(1-\alpha\right)\gamma^x_{\lambda i}}{1-\iota\left[\gamma^x_{yb}+\left(1-\alpha\right)\gamma^x_{\lambda b}\right]}\varepsilon_i\left(0\right)
$$

where we used the initial conditions $b\left(0\right)=\iota e\left(0\right)$ and $z\left(0\right)=0$.

Proof of Propositions 4-5 Assume $\phi^R_\pi=\phi^N_\pi=\phi_\pi>0$, $\psi^R_b>\rho$, and $\psi^N_b<\rho$, such that $\tilde{\eta}^R=0$. Note that output and inflation do not depend directly on z, but only indirectly through λ which captures the consumption absorption share of the Home economy. Therefore we must have $\gamma^R_{yz}=\gamma^N_{yz}=0$ and $\gamma^R_{\pi z}=\gamma^N_{\pi z}=0$, which implies $\gamma^R_{\lambda z}=\gamma^N_{\lambda z}=-\rho/\alpha$. Now solve for the eigenvalues of A.54 and show that the maximum one is zero iff

$$
\tilde{\eta}^N=\frac{1}{2\xi}\left[2\left(\rho-\psi^N_b\right)+2\gamma^N_{\pi b}\left(1-\psi^N_\pi\right)\phi_{\pi\mathrm{N}}-\sigma^R-\frac{\sigma^N\sigma^R}{2\left(\rho-\psi^R_b\right)-\sigma_N+\frac{2\alpha\rho\sigma^N\left(1-\psi^R_\pi\right)\left(\rho+\sigma^N+\sigma^R\right)}{\xi\kappa\omega\phi_\pi+\xi\left(\sigma^N+\sigma^R\right)\left(\rho+\sigma^N+\sigma^R\right)}}\right] \tag{A.56}
$$

Using (A.56) and assuming $\psi^R_\pi=1$, (A.50)-(A.51) can be solved analytically. Unfortunately the equations are too large to be reported here (they are available upon request). To simplify them, assume also $\psi^N_\pi=1$, $\psi^R_b=\rho$, and $\psi^N_b=0$. Then (A.56) simplyfies to $\tilde{\eta}^N=\rho/\xi$ and the solution of (A.50)-(A.51) is

$$
\begin{aligned}
\gamma^R_{yb} &= \frac{\alpha\rho\sigma^N\left(\rho\left(\sigma^N+\rho+\sigma^R\right)-\kappa\omega\phi_\pi\right)}{\kappa\xi\omega\phi_\pi\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R\right)\left(\sigma^N+\rho+\sigma^R\right)\right)} \\
\gamma^R_{\pi b} &= \frac{\alpha\rho\sigma^N\left(\sigma^N+\rho+\sigma^R\right)}{\xi\phi_\pi\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R\right)\left(\sigma^N+\rho+\sigma^R\right)\right)} \\
\gamma^R_{\lambda b} &= -\frac{\sigma^N}{\xi\left(\sigma^N+\rho+\sigma^R\right)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{yg}^{R} &= \sigma^{N}\frac{\phi_{\pi}\left((\rho+2\varrho)\left(\kappa-\beta\gamma_{\pi b}^{N}\right)+\sigma^{R}\left(2\kappa-\beta\left(\gamma_{\pi b}^{N}+\gamma_{\pi b}^{R}\right)\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\sigma^{N}\frac{\beta\gamma_{yb}^{N}\left((\rho+\varrho)\left(\rho+\sigma^{R}+\varrho\right)-\kappa\omega\phi_{\pi}\right)+\varrho(\rho+\varrho)\left(\rho+2\sigma^{R}+2\varrho\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{\sigma^{N}\left(\phi_{\pi}\left(\kappa-\beta\gamma_{\pi b}^{N}\right)+(\rho+\varrho)\left(\beta\gamma_{yb}^{N}+\varrho\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{\beta\gamma_{yb}^{R}\left(\kappa\omega\phi_{\pi}\left(\sigma^{N}+\rho+\varrho\right)+(\rho+\varrho)\left(\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{\left(\phi_{\pi}\left(\kappa-\beta\gamma_{\pi b}^{R}\right)+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_{\pi}+\left(\sigma^{R}+\varrho\right)\left(\rho+\sigma^{R}+\varrho\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\pi g}^{R} &= \frac{\sigma^{N}\left(\beta\gamma_{\pi b}^{N}\varrho+\beta\kappa\omega\gamma_{yb}^{N}+\kappa(\omega-1)\varrho\right)+\sigma^{R}\left(\beta\gamma_{\pi b}^{R}\varrho+\beta\kappa\omega\gamma_{yb}^{R}+\kappa(\omega-1)\varrho\right)}{\left(\sigma^{N}+\sigma^{R}\right)(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))}\\
&+\frac{\beta\sigma^{N}\left(\kappa\omega\left(\gamma_{yb}^{R}-\gamma_{yb}^{N}\right)-\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^{N}+\sigma^{R}+\varrho\right)\right)}{\left(\sigma^{N}+\sigma^{R}\right)\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\lambda g}^{R} &= -\frac{\beta\sigma^{N}\left(\sigma^{N}+2\rho+\sigma^{R}+\varrho\right)}{\xi(\rho+\varrho)\left(\sigma^{N}+\rho+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{ym}^{R} &= -\phi_{\pi}\frac{\sigma^{N}\left(\gamma_{\pi b}^{N}\left(\sigma^{N}+\rho+\sigma^{R}+2\varrho\right)+\kappa\omega\gamma_{yb}^{N}\right)-\kappa\omega\gamma_{yb}^{R}\left(\sigma^{N}+\rho+\varrho\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&-\phi_{\pi}\frac{\gamma_{\pi b}^{R}\left(\sigma^{R}\left(\sigma^{N}+\rho+\sigma^{R}+2\varrho\right)+\varrho(\rho+\varrho)\right)+\kappa\omega(\rho+\varrho)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{(\rho+\varrho)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\left(\left(\gamma_{yb}^{N}-1\right)\sigma^{N}+\gamma_{yb}^{R}\left(\sigma^{R}+\varrho\right)-\sigma^{R}-\varrho\right)+\gamma_{\pi b}^{R}(-\kappa)\omega\phi_{\pi}^{2}}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\pi m}^{R} &= \frac{\sigma^{N}\left(\kappa\omega\left(\gamma_{yb}^{N}-1\right)+\gamma_{\pi b}^{N}\varrho\right)+\sigma^{R}\left(\kappa\omega\left(\gamma_{yb}^{R}-1\right)+\gamma_{\pi b}^{R}\varrho\right)}{\left(\sigma^{N}+\sigma^{R}\right)(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))}\\
&+\frac{\sigma^{N}\left(\kappa\omega\left(\gamma_{yb}^{R}-\gamma_{yb}^{N}\right)-\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^{N}+\sigma^{R}+\varrho\right)\right)}{\left(\sigma^{N}+\sigma^{R}\right)\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\lambda m}^{R} &= -\frac{\sigma^{N}\left(\sigma^{N}+2\rho+\sigma^{R}+\varrho\right)}{\xi(\rho+\varrho)\left(\sigma^{N}+\rho+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)}
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_{yb}^{N} &= \frac{\alpha\rho\left(\rho\sigma^{N}\left(\sigma^{N}+\rho+\sigma^{R}\right)+\kappa\omega\phi_{\pi}\left(\rho+\sigma^{R}\right)\right)}{\kappa\xi\omega\phi_{\pi}\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)}\\
\gamma_{\pi b}^{N} &= \frac{\alpha\rho\left(\kappa\omega\phi_{\pi}+\sigma^{N}\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)}{\xi\phi_{\pi}\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)}\\
\gamma_{\lambda b}^{N} &= -\frac{\sigma^{N}+\rho}{\xi\left(\sigma^{N}+\rho+\sigma^{R}\right)}
\end{aligned}
$$

$$\begin{aligned}
\gamma_{yg}^{N} &= \phi_\pi \frac{\beta\gamma_{\pi b}^{N}\left(-\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\varrho\right)-\sigma^N\sigma^R\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\phi_\pi \frac{\sigma^R\left(\kappa\left(-\beta\omega\gamma_{yb}^{R}+2\sigma^N+\rho+2\varrho\right)-\beta\gamma_{\pi b}^{R}\left(\sigma^N+\rho+2\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\phi_\pi \frac{\sigma^R\left(\kappa-\beta\gamma_{\pi b}^{R}\right)+\kappa(\omega+1)\varrho(\rho+\varrho)+\kappa\sigma^N\left(\sigma^N+\rho+2\varrho\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\frac{\kappa\omega\phi_\pi^2\left(\kappa-\beta\gamma_{\pi b}^{N}\right)+\beta\gamma_{yb}^{N}\left(\left(\rho+\varrho\right)\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)+\kappa\omega\phi_\pi\left(\rho+\sigma^R+\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\frac{\left(\rho+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\left(\sigma^R\left(\beta\gamma_{yb}^{R}+\varrho\right)+\varrho\left(\sigma^N+\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\pi g}^{N} &= \frac{\sigma^N\left(\beta\gamma_{\pi b}^{N}\varrho+\beta\kappa\omega\gamma_{yb}^{N}+\kappa(\omega-1)\varrho\right)+\sigma^R\left(\beta\gamma_{\pi b}^{R}\varrho+\beta\kappa\omega\gamma_{yb}^{R}+\kappa(\omega-1)\varrho\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)} \\
&+\frac{\beta\sigma^R\left(\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^N+\sigma^R+\varrho\right)+\kappa\omega\left(\gamma_{yb}^{N}-\gamma_{yb}^{R}\right)\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\lambda g}^{N} &= -\frac{\beta\left(\left(\sigma^N+\rho\right)\left(\sigma^N+\rho+\varrho\right)+\sigma^N\sigma^R\right)}{\xi(\rho+\varrho)\left(\sigma^N+\rho+\sigma^R\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)}
\end{aligned}$$

$$\begin{aligned}
\gamma_{ym}^{N} &= -\phi_\pi \frac{\gamma_{\pi b}^{N}\left(\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\varrho\right)+\sigma^N\sigma^R\right)+\gamma_{\pi b}^{R}\sigma^R\left(\sigma^N+\rho+\sigma^R+2\varrho\right)+\kappa\omega\left(\gamma_{yb}^{R}\sigma^R+\rho+\varrho\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\frac{\gamma_{\pi b}^{N}\kappa\omega\phi_\pi^2+\gamma_{yb}^{N}\left(\left(\rho+\varrho\right)\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)+\kappa\omega\phi_\pi\left(\rho+\sigma^R+\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&-\frac{\left(\rho+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\left(-\left(\gamma_{yb}^{R}-1\right)\sigma^R+\sigma^N+\varrho\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\pi m}^{N} &= \frac{\sigma^N\left(\kappa\omega\left(\gamma_{yb}^{N}-1\right)+\gamma_{\pi b}^{N}\varrho\right)+\sigma^R\left(\kappa\omega\left(\gamma_{yb}^{R}-1\right)+\gamma_{\pi b}^{R}\varrho\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)} \\
&+\frac{\sigma^R\left(\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^N+\sigma^R+\varrho\right)+\kappa\omega\left(\gamma_{yb}^{N}-\gamma_{yb}^{R}\right)\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\lambda m}^{N} &= -\frac{\left(\sigma^N+\rho\right)\left(\sigma^N+\rho+\varrho\right)+\sigma^N\sigma^R}{\xi(\rho+\varrho)\left(\sigma^N+\rho+\sigma^R\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)}
\end{aligned}$$

Finally, compute the eigenvalues of (A.53) and check that they are all positive iff $\phi_\pi > 0$. The response of the nominal exchange rate to fiscal and monetary shocks is then calculated using $e(0) = y(0) - (1-\alpha)\lambda(0) - \varepsilon_g(0)$, which yields

$$e^x(0) = \frac{\gamma_{yg}^{x} - (1-\alpha)\gamma_{\lambda g}^{x} - 1}{1-\iota\left[\gamma_{yb}^{x} + (1-\alpha)\gamma_{\lambda b}^{x}\right]}\varepsilon_g(0) + \frac{\gamma_{yi}^{x} - (1-\alpha)\gamma_{\lambda i}^{x}}{1-\iota\left[\gamma_{yb}^{x} + (1-\alpha)\gamma_{\lambda b}^{x}\right]}\varepsilon_i(0)$$

for $x \in \{R, N\}$, where we used the initial conditions $b(0) = \iota e(0)$ and $z(0) = 0$.

Proof of Proposition 8 Assume $\tilde{\eta}^R = \tilde{\eta}^N = 0$, $\psi_\pi^R = 1$ $\psi_\pi^N = 0$, $\psi_b^R = \rho$, $\psi_b^N = 0$ and $\sigma^N = 0$. Note that output and inflation do not depend directly on z, but only indirectly through λ which captures the consumption absorption share of the Home economy. Therefore we must have $\gamma_{yz}^R = \gamma_{yz}^N = 0$ and $\gamma_{\pi z}^R = \gamma_{\pi z}^N = 0$, which implies $\gamma_{\lambda z}^R = \gamma_{\lambda z}^N = -\rho/\alpha$. Define $\mu \equiv \left(\sqrt{\rho^2 - 4\kappa\omega\phi_\pi^N} - \rho\right)/2$. Then the solution of (A.50)-(A.51) is

$$\begin{aligned}
\gamma_{yb}^R &= 0\\
\gamma_{\pi b}^R &= 0\\
\gamma_{\lambda b}^R &= 0
\end{aligned}$$

$$\begin{aligned}
\gamma_{yg}^R &= \frac{\kappa\phi_\pi^R + \varrho(\rho+\varrho)}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)}\\
\gamma_{\pi g}^R &= \frac{\kappa(\omega-1)\varrho}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)}\\
\gamma_{\lambda g}^R &= 0
\end{aligned}$$

$$\begin{aligned}
\gamma_{ym}^R &= -\frac{\rho+\varrho}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)}\\
\gamma_{\pi m}^R &= -\frac{\kappa\omega}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)}\\
\gamma_{\lambda m}^R &= 0
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{yb}^N &= \frac{\mu+\rho-\sigma^R}{\mu}\\
\gamma_{\pi b}^N &= \frac{\kappa\omega\left(\mu+\rho-\sigma^R\right)}{\mu(\mu+\rho)}\\
\lambda_{\mathrm{bN}} &= 0
\end{aligned}$$

$$\begin{aligned}
\gamma_{yg}^N = {} & \frac{\left(\mu^2(\omega(\beta+\varrho)+\rho) + \mu\left(2\rho\omega(\beta+\varrho) + \omega\varrho(\beta+\varrho) + \rho^2\right) + \beta\rho\omega(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)}{\mu\omega(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)}\\
&+ \sigma^R\frac{\kappa\omega\phi_\pi^R(\mu(\rho+\varrho) - \beta\omega\varrho) + \varrho\left(-\beta\omega\varrho(\rho+\varrho) + \mu^2(\omega-1)\varrho + \mu\left(\rho^2\omega + \rho(3\omega-1)\varrho + \omega\varrho^2\right)\right)}{\mu\omega(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)}\\
&+ \sigma^R\frac{-\beta\kappa\omega^2\phi_\pi^R - \varrho\left(\beta\omega(\rho+\varrho) - \mu^2(\omega-1) + \mu(\rho-\rho\omega)\right)}{\mu\omega(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)}\\
\gamma_{\pi g}^N = {} & \frac{\kappa\left((\mu+\rho)(\mu(\beta\omega + (\omega-1)\varrho) + \beta\omega(\rho+\varrho)) - \sigma^R\left(\beta\omega\varrho + \mu^2 + \mu\rho\right) - \beta\omega\sigma^R\right)}{\mu(\mu+\rho)(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)}\\
&- \frac{\sigma^R\left(\kappa\phi_\pi^R + \varrho(\rho+\varrho)\right)\left(-\kappa\omega\phi_\pi^R + \varrho\sigma^R + \varrho^2\right)}{\phi_\pi^R\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)} + \frac{\varrho\sigma^R\left(\sigma^R+\varrho\right)}{\phi_\pi^R(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)}\\
\gamma_{\lambda g}^N = {} & 0
\end{aligned}$$

$$
\begin{aligned}
\gamma_{ym}^{N} &= \frac{\rho(\rho+\varrho)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)-\sigma^{R}\left(\kappa\omega\phi_{\pi}^{R}+\mu^{2}+\mu\rho+\varrho(\rho+\varrho)\right)}{\mu(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)} \\
&\quad -\sigma^{R}\frac{\kappa\omega\varrho\phi_{\pi}^{R}+\mu^{2}\varrho+\mu\left(\rho^{2}+3\rho\varrho+\varrho^{2}\right)+\varrho^{2}(\rho+\varrho)}{\mu(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)} \\
\gamma_{\pi m}^{N} &= \kappa\omega\frac{(\mu+\rho)(\rho+\varrho)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)-\sigma^{R}\left(\kappa\omega\phi_{\pi}^{R}+\mu^{2}+\mu\rho+\varrho(\rho+\varrho)\right)}{\mu(\mu+\rho)(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)} \\
&\quad -\sigma^{R}\kappa\omega\frac{\kappa\omega\varrho\phi_{\pi}^{R}+\mu^{2}(\rho+2\varrho)+\mu\rho(\rho+2\varrho)+\varrho^{2}(\rho+\varrho)}{\mu(\mu+\rho)(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)} \\
\gamma_{\lambda m}^{N} &= 0
\end{aligned}
$$

Compute the eigenvalues of (A.53) and check that they are all positive iff $\phi_{\pi}^{R}>0$ and $\mu>0$, which implies $\phi_{\pi}^{N}<0$. Finally, compute the eigenvalues of (A.54) and show that the maximum eigenvalue is nonpositive iff $\mu\geq\sigma^{R}/2$. The response of the nominal exchange rate to fiscal and monetary shocks is then calculated using $e\left(0\right)=y\left(0\right)-\left(1-\alpha\right)\lambda\left(0\right)-\varepsilon_{g}\left(0\right)$, which yields

$$
e^{x}\left(0\right)=\frac{\gamma_{yg}^{x}-\left(1-\alpha\right)\gamma_{\lambda g}^{x}-1}{1-\iota\left[\gamma_{yb}^{x}+\left(1-\alpha\right)\gamma_{\lambda b}^{x}\right]}\varepsilon_{g}\left(0\right)+\frac{\gamma_{yi}^{x}-\left(1-\alpha\right)\gamma_{\lambda i}^{x}}{1-\iota\left[\gamma_{yb}^{x}+\left(1-\alpha\right)\gamma_{\lambda b}^{x}\right]}\varepsilon_{i}\left(0\right)
$$

for $x\in\{R,N\}$, where we used the initial conditions $b\left(0\right)=\iota e\left(0\right)$ and $z\left(0\right)=0$.