Financial and Total Wealth Inequality with Declining Interest Rates*

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Abstract

Financial wealth inequality and long-term real interest rates track each other closely over the post-war period. We investigate how much of the increase in measured inequality can be explained by the decline in rates, and what the implications are for inequality in total wealth (lifetime consumption). We first directly estimate the exposure of financial portfolios to interest rates at the household level to show that there is enough heterogeneity in portfolio revaluations to explain the entire rise in financial wealth inequality since the 1980s. A standard incomplete markets model calibrated to these data implies that declining rates are not consumption neutral. Instead, the low-wealth young lose, while the high-wealth old gain.

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1 Introduction

Since discount rates have a direct link to the values of financial assets, a natural hypothesis is that falling interest rates cause a rise in financial wealth inequality, and vice versa. Characterizing this channel, however, requires overcoming two obstacles. First, the ultimate impact on financial wealth inequality depends not on the average effect of discount rates on asset prices, but on the *heterogeneity* of these revaluations across the population, and how they covary with initial levels of financial wealth. Further, to the extent that real rates influence financial wealth inequality, whether the resulting gains and losses occur only "on paper" or actually influence future consumption and hence welfare is far from clear.

In this paper, we rigorously measure the link between real yields and financial wealth inequality, with the goal of answering two research questions. First, what share of the rise in financial wealth inequality displayed can be quantitatively explained by falling interest rates? Second, what are the implications for inequality in total wealth — defined as the present value of lifetime consumption — that actually determines welfare?

To answer these questions, we combine a set of novel empirical estimates with a quantitative structural model. For our first question, we directly measure the exposure of financial wealth portfolios to changes in real interest rates at the household level, allowing us to directly estimate the effect of asset revaluations on financial wealth inequality. For our second question, we use a realistic consumption-savings model to compute the implied exposure of each household's consumption plan to the same changes in rates. We show that the extent to which this exposure aligns or diverges from the exposure of the household's financial wealth portfolio determines the ultimate consequences of the change in rates for consumption and welfare.

We begin by summarizing the exposure of agents' portfolios to a change in interest rates using the cash flow duration of those portfolios. A sufficient condition for whether a fall in rates will increase financial wealth inequality is that the the aggregate (value-weighted) duration of financial wealth exceeds the average (equal-weighted) duration of financial wealth.

To check this condition, and quantify the impact of a fall in rates on financial wealth inequality, we turn to the data. We use microdata from the Survey of Consumer finances to characterize households' portfolio allocations across asset classes. We then use an auxiliary asset pricing model, estimated to fit asset prices and cash flows quarter-by-quarter, to compute the cash flow duration of each asset class. Combined, we are able to characterize the distribution of financial wealth durations across the population. Using these measures, we find that the aggregate duration of financial wealth in our baseline 1980s economy was 25.72, compared to an average duration of 15.43, implying that a fall in interest rates will indeed raise measured financial wealth inequality.

We observe substantial heterogeneity in financial durations by wealth level and age. Low-wealth households have low financial durations, driven by their higher shares of deposit-like assets, the presence of consumer debt, and lower shares of housing, private business, and stock market wealth. The reverse is true for high-wealth households. Conditional on wealth, financial durations are declining in age. This heterogeneity in financial duration is a new empirical finding, and crucial for the response of financial inequality to the decline in long-term real rates.

To study the quantitative implications of these measured durations for financial wealth inequality, as well as future consumption (total wealth) inequality, we turn to a calibrated life-cycle model of the U.S. economy. The model features idiosyncratic income risk, calibrated using Panel Survey of Income Dynamics data, as well as a superstar income state that enables the model to exactly match the financial wealth Gini in the 1980s. To capture our key empirical findings, we calibrate heterogeneity in the duration of financial wealth to flexibly match our empirical estimates by wealth bin and age. For our main experiment, we initialize the model at a long-term real interest rate of 4.82%, the level we estimate to have prevailed in the 1980s, then unexpectedly reduce it permanently to 0.34%, the level we estimate to have prevailed in the 2010s.

First, we answer the positive question: what happens to financial wealth inequality in the calibrated model after rates decline unexpectedly? To do this, we compute the implied financial wealth distribution after revaluing all assets using the new interest rate and our distribution of durations fitted to the data, which we denote the *repriced distribution*. Marking-to-market alone can more than explain the rise in financial wealth inequality between the 1980s and 2010s. Repricing increases the financial wealth Gini from 0.754 to 0.891, compared to an increase from 0.754 to 0.817 in the data. The top-10% share increases by 13.6pp under repricing, compared to an increase of 8.5pp in the data, while the top-1% share increases by 17.7pp under repricing, compared to an increase of either 4.8pp or 10.5pp in the data, depending on the source.

Having established the strength of this mechanism, we turn to our second, normative question: what are the implications of this change for total wealth or consumption inequality? Since wealth gains can occur "on paper" without translating into actual consumption gains and losses, establishing the proper benchmark is essential. We compute this benchmark as the change in financial wealth that would be required under the new, lower interest rate, so that each household would be able to afford their prior consumption plans formed under the old interest rate. We denote this object as the *compensated* financial wealth distribution, and verify that it represents a competitive equilibrium in the low-rate economy.

To be completely hedged and keep its consumption unchanged, a household's financial portfolio duration needs to match the duration of its excess consumption plan, which is defined as its future consumption minus labor income. We find that attaining this compensated distribution requires a rightward shift in the wealth distribution following a decline in rates: Compensation requires 170.8% more fin wealth, 32.1% goes to top 1%. with more than 40% of new financial wealth accruing to the top-1% of the financial wealth distribution. These results imply that large increases in the financial wealth of the richest individuals do not imply any actual change in consumption beyond “paper” gains.

However, we find that the aggregate duration of U.S. financial wealth is smaller than the duration of a typical household’s excess consumption plan. Given that condition, financial wealth inequality *should* decline to ensure that all households can afford the same consumption plan. The compensated distribution is much less unequal than the original or the repriced one.

To summarize, financial wealth inequality would need to *fall* substantially under the compensated distribution, rather than rising as observed in the data, implying that a fall in rates is not consumption neutral. Holding agents’ consumption plans fixed would require the financial wealth Gini to fall from 0.754 to 0.638, and for the top-10% and top-1% financial wealth shares to fall by 14.5pp and 10.4pp, respectively. This large deviation from what is observed under repricing implies that household portfolios provide far from perfect hedging, and that repricing instead meaningfully reallocates consumption possibilities across the population.

This result is largely driven by life cycle dynamics. While financial wealth is equal to the present value of lifetime excess consumption by definition, the exposures of these objects to interest rate changes is differs across households. The young, who plan to save in middle age and dissave in retirement, have a very high duration of excess consumption, because they keep consuming after retirement. This renders their consumption plans much more expensive as rates fall. Young households are forced later to buy financial assets at higher prices.

At the same time, the low levels of financial wealth held by the young, combined with their observed financial wealth durations from the data, fall far short of delivering the repricing gains needed to afford this plan. Despite having little change in financial wealth on impact, the young will struggle to accumulate wealth for retirement under low rates, and face a serious contraction in their consumption possibilities. We find that this degree of under-hedging is decreasing with age, while the oldest agents are actually over-hedged, and see their consumption possibilities expand due to large capital gains on their wealth positions. In part because older agents are wealthier, we find that net consumption gains are also increasing in households’ initial financial wealth position.

To sum up, our paper combines micro-level data on household portfolios with a structural model to measure the impact of declining rates on the distribution of financial and total wealth. We find that heterogeneity in repricing gains across household portfolios is a powerful mechanism through which falling interest rates have increased wealth inequality since the 1980s, more than explaining the entire rise in measured financial wealth inequality over this period. According to our model, these mark-to-market effects will actually influence future consumption outcomes, as younger and less wealthy households are forced to save at lower rates for their retirement by purchasing more expensive assets in the future, while older and wealthier households gain because of the mark-to-market effects.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 shows that the share of the top percentiles tracks the cost of an indexed annuity quite closely in the U.S., U.K., and France. This section then infers the distribution of financial duration and computes the repriced wealth distribution. Section 4 derives theoretical results in a general incomplete markets economy with aggregate uncertainty and idiosyncratic income risk. Section 5 describes a benchmark case in which households are fully hedged against rate declines. Section 6 quantifies the effect of an interest rate change by adding a life-cycle component to the model as well as heterogeneity across demographic groups. Section 7 concludes.

Appendix A provides an auxiliary asset pricing model used to infer real interest rates and durations of the components of financial wealth. Appendix C contains details on data sources and construction. Appendix D contains the proofs of the propositions. Appendix E contains some details of the calibrated model.

2 Related Literature

A large strand of recent literature documents the evolution of income inequality as well as financial wealth inequality over the past century (Piketty and Saez, 2003; Piketty, 2015; Alvaredo, Chancel, Piketty, Saez, and Zucman, 2018b). Most of the evidence suggests that financial wealth inequality has increased in many countries over the past decades. Zucman (2019) reviews the empirical literature on the topic. Benhabib and Bisin (2018) survey economic theories of wealth inequality.

Much of the literature on wealth inequality adopts a backward-looking approach and explores the connection between past returns and current wealth. This literature has argued that high past rates of return and heterogeneity therein helps account for the increase in financial wealth inequality (Piketty and Zucman, 2015; Fagereng, Guiso, Malacrino, and Pistaferri, 2020; Bach, Calvet, and Sodini, 2020; Hubmer, Krusell, and Smith, 2020; Cox, 2020).

But wealth is also the current value of the household's future consumption stream. Human wealth is the value of future labor income and financial wealth is the value of future consumption minus income. We bring an asset pricing perspective to the discussion on inequality. We impute a valuation by discounting future cash flows. When rates declines, households need more wealth to finance the same consumption stream. Households that have mostly human wealth are likely to be better hedged. Households with mostly financial wealth need enough duration in their portfolio in order to finance future consumption. To keep consumption shares unchanged, a decline in real rates needs to entail a reallocation of financial wealth towards those households who rely mostly on their (current and future) financial wealth to finance future consumption.

Discount rates matter. In a simple partial equilibrium model, Moll (2020) explains that small discount rate-induced changes in the wealth distribution may have smaller welfare effects than cash flow-induced changes. We make a related point in a version of the Bewley-style general equilibrium model with aggregate and idiosyncratic risk. Recently, Catherine, Miller, and Sarin

(2020) show that discounting social security transfers at time-varying discount rates has quantitatively important implications for wealth inequality.

Greenwald, Lettau, and Ludvigson (2019) point to increases in the share of output accruing to profits as a key source of the rise in equity values since 1989. While we motivate our main experiment using a drop in the real risk-free rate, the decline in expected returns applies more broadly to other financial assets. This decline could arise either from a highly persistent change in the real risk free rate or to a decrease in risk premia. To the extent that economic forces have varied these quantities across time and across different financial assets, our methodology could be extended to capture these more detailed patterns. The auxiliary asset pricing model in Appendix A indeed shows declines in expected real returns not only on bonds but also on stocks and housing.

Our paper is related to recent work by Auclert (2019), who explores the effect of cross-sectional variation in the duration of households' financial assets for the effectiveness of monetary policy. We consider a setting with aggregate risk, we develop measures of household duration based on a no-arbitrage dynamic asset pricing model and household financial portfolios, and we assess quantitatively the extent to which households have hedged their consumption plan against interest rate innovations. In earlier work, Doepke and Schneider (2006) focus on the distributional consequences of inflation. Our work instead focuses on the distributional effects of changes in long-term real rates. Gomez and Gouin-Bonenfant (2020) study the effects of lower interest rate on the cost of raising new capital for entrepreneurs, linking the decline in interest rates to the rise in wealth inequality through a different channel.

There are important normative implications for fiscal policy. The compensated distribution that allows all households to implement their old consumption plans features less top total wealth inequality than both the old distribution and the actual repriced distribution, but a similar total wealth Gini. This suggests that a tax on top-wealth households may be able to improve on the repriced consumption distribution. In our life-cycle model, we find that young households are hurt most by a reduction in rates. In that respect, our model speaks to the inter-generational distribution of the burden of taxation. A large literature studies optimal labor and capital income taxation in Bewley models with idiosyncratic risk, endogenous labor supply, and capital formation (Aiyagari, 1995; Panousi and Reis, 2017; Heathcote, Storesletten, and Violante, 2017; Krueger and Ludwig, 2018; Boar and Midrigan, 2020). We take labor income as given and do not model capital formation, but instead focus on the distributional implications of lower interest rates.

As an aside, we resolve an outstanding issue in the literature on how to compute an individual's human wealth. A common approach in the literature is to use the individual's own SDF to compute human wealth. Instead, Lustig, Van Nieuwerburgh, and Verdelhan (2013) propose using the same stochastic discount factor (SDF) that prices traded assets to discount an individual's labor income stream. In this paper we show that using individual SDFs results in a wealth measure that does not aggregate. For wealth accounting, the aggregate SDF is more convenient, because the aggregate value of individual wealth is consistent with market valuations.

By emphasizing total wealth (inequality), of which human wealth (inequality) forms a very significant component, our work contributes to the literature on measuring wealth (inequality). Our paper provides new and detailed statistics on the duration of financial wealth for U.S. households. Related, Kuhn, Schularick, and Steins (2020) study how housing and equity portfolio shares differ across the wealth distribution and result in differing financial wealth dynamics for the middle class and the top of the financial wealth distribution. Recent work discusses the measurement of private business income and wealth (Kopczuk, 2017; Saez and Zucman, 2016; Piketty, Saez, and Zucman, 2018; Smith, Yagan, Zidar, and Zwick, Working Papers; Kopczuk and Zwick, 2020). In our theoretical work, we sidestep this issue by recognizing that financial wealth is the present discounted value of the future stream of consumption minus labor income. In our empirical work, we infer the duration of private business wealth from that of small stocks.

Our conclusions regarding the differing behavior of financial and total wealth inequality are not sensitive to the source of the decline in interest rates. The literature has proposed a long list of candidates for such a growth slowdown: demographics (Summers, 2014; Eggertsson and Mehrotra, 2014; Eichengreen, 2015), a productivity slowdown due to a plateau in educational attainment or diminishing technological progress (Gordon, 2017), a global saving glut and/or shortage of safe assets (Bernanke et al., 2005; Caballero, Farhi, and Gourinchas, 2008), government spending that leads to depressed future aggregate demand (Mian, Straub, and Sufi, 2020), a decline in competition (Gutiérrez and Philippon, 2017), a decline in desired investment due to lower relative prices of capital goods (Rachel and Smith, 2017), among others. Lower tax progressivity could lead to more saving by the rich, more aggregate wealth, and lower rates (Hubmer et al., 2020). However, Heathcothe, Storesletten, and Violante (2020) argue that once transfers are considered, the U.S. tax system has not become less progressive.

Alternatively, a rise in income inequality could be the origin of lower interest rates. Mian et al. (2020) argue that the rich have a higher propensity to save than the poor; Fagereng, Blomhoff Holm, Moll, and Natvik (2019) provide empirical evidence consistent with this from Norway. This reduces aggregate demand and the real rate of interest in the wake of an exogenous increase in income inequality, for example, due to skill-biased technological change. In our work, we consider a decline in real rates driven by a decline in the expected growth rate of the economy. While the interest rate is endogenous in the Bewley model of Section 4, our model features standard homothetic preferences. The model in Section 6 keeps labor income inequality constant, in order to isolate the effect of a decline in the long-run growth rate of the economy.[1]

[1] Hubmer et al. (2020) show that a rise in earnings risk actually lowers wealth inequality as it strengthens precautionary savings motives meaningfully for all but the richest households. A rise in top-income inequality, in contrast, can increase wealth inequality.

3 Wealth Inequality and Real Rates: Empirical Evidence

In this section we document a strong time-series correlation between the evolution of long-term real interest rates and wealth inequality. While our focus is on the U.S. in most of the paper, this section documents that this correlation is present also in the United Kingdom and in France. This evidence suggests that households are partially hedged against changes in long real rates.

3.1 Decline in Real Rates

We start by documenting a broad-based decline in expected returns across all major asset classes. To do so, we develop an auxiliary no-arbitrage asset pricing model in Appendix A. The model prices bonds of various maturities, both nominal and real, the aggregate stock market, several cross-sectional stock market factors including small, growth, value, and infrastructure stocks, and households' housing wealth. According to this model, the ten-year real bond yield averaged 4.82% in the 40 quarters of the 1980s decade and 0.34% in the 2010s decade.[2] The asset pricing model shows similarly large declines in expected real returns on the aggregate stock market and on housing wealth, as shown in Table 1. Other stock indices such as value and infrastructure stocks show larger declines, while growth and small stocks show smaller declines. Expected returns on total wealth, measured as a claim to GDP or to aggregate consumption, show large declines around 12pp-13pp. In other words, the decline in expected returns was broad-based.

Table 1: Expected Real Returns Decade Averages

Asset	1980s	2010s	Decline
Ten-year real bond yield	4.82%	0.34%	4.48%
Aggregate stock market	7.98%	2.00%	5.98%
Growth stocks	5.21%	3.53%	1.68%
Value stocks	18.50%	7.19%	11.31%
Infrastructure stocks	11.75%	2.35%	9.40%
Small stocks	3.57%	3.18%	0.39%
Housing wealth	8.24%	4.89%	3.35%
GDP claim	15.90%	2.80%	13.10%
Consumption claim	15.27%	2.84%	12.43%

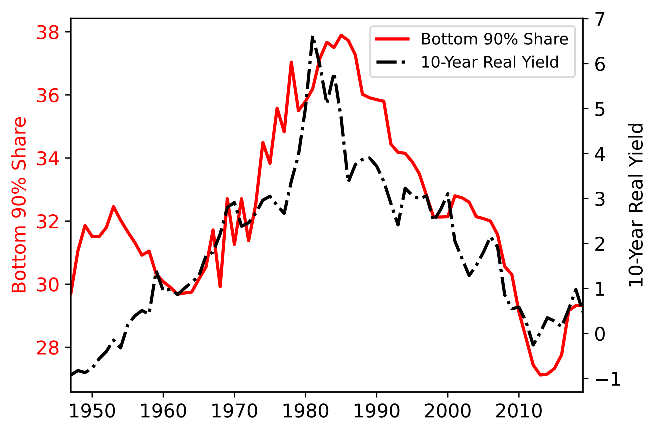
Note: The table reports model-implied real expected real returns and average them over the 40 quarters in the 1980s and the 40 quarters of the 2010s. The model is described in Appendix A.

[2]The asset pricing model matches the available data on Treasury Inflation-Indexed Securities over the period for which they are available. The model-implied yield changes are similar for real bonds of different maturities.

3.2 Increased Wealth Inequality

Financial wealth inequality and long-term real interest rates have displayed a strong negative co-movement in the post-WWII U.S. data. Figure 1 displays the bottom-90% financial wealth share for the U.S., from the World Inequality Database, along with 10-year real yields, obtained from the auxiliary asset pricing model described above. Real yields rose from an average of 0.17% in the 1950s to an average of 4.82% in the 1980s, before falling to an average of 0.34% in the 2010s. Over the same period, the bottom-90% share of financial wealth rose from 31.5% in the 1950s to 37.0% in the 1980s, before falling to 28.1% in the 2010s, displaying the same temporal pattern. Conversely, the top 10% share of financial wealth displays a strong inverse relationship to real interest rates.

Figure 1: Bottom 90% Wealth Share vs. 10-Year Real Yields



Note: The red solid line displays the bottom 90% financial wealth share for the United States, obtained annually from 1947 until 2019 from the World Inequality Database. The black dash-dot line displays an estimate of the 10-year real bond yield, obtained from a dynamic affine term structure model, estimated on quarterly data from 1947.Q1-2019.Q4 (see Appendix A for details).

The inverse relationship between top wealth shares is robust across wealth measures, interest rate measures, and countries. Figure 2 shows the wealth share of the top-10% of the population in the left panels and the wealth share of the top-1% of the population in the right panels. The top panel is for the U.S., the middle panel for the U.K., and the bottom panel is for France. Our main source of wealth inequality data is the World Inequality Database. These data were recently updated so that there is an old and a new WID series. For the U.S., we also plot the wealth shares constructed from the Survey of Consumer Finances (SCF+). For the U.K. (France), we have added wealth shares from the Credit Suisse (CS) Global Wealth report available after 2012 (2014). Each panel also plots the price of a thirty-year real annuity, computed either from nominal yields and inflation or alternatively from our auxiliary asset pricing model. Construction details are in

Appendix C.1. The sample is 1947-2019.[3]

For both inequality measures, there is a strong positive correlation between top wealth shares and the price of a long-term real annuity. Between 1947 and 1982, the top-10% (top-1%) wealth share falls from 70% (29%) to 63% (24%) in the U.S. as the annuity becomes cheaper. From 1982 until 2015, the top-10% (top-1%) wealth share rises from 63% (24%) to 73% (36%). During this period, the cost of the annuity more than doubles. There is a small decline in top wealth shares from 2015 until 2019, which is expected to have reversed again in 2020.

The patterns in both wealth inequality and the evolution of the cost of the annuity are similar in the UK and in France. Rachel and Smith (2017) show that the decline in the real rate has occurred across a broad set of developed and emerging market countries. While many other factors no doubt differ across countries, this shared trend in rates should result in a global rise in financial wealth inequality.

Wealth measures are valuation metrics. From the household budget constraint, it follows that wealth is the present value of future household consumption, and human wealth is the present value of household labor income. Financial wealth is the difference between these two wealth measures. As a result, there is a tight connection between wealth inequality and long rates. When long-term real rates decline and aggregate valuation ratios increase, we expect measures of inequality to increase because wealth is being marked-to-market as long as different households have different exposure to real rates, and even in the absence of news about the distribution of future consumption shares. Wealth inequality measures are not immune to discount rate variation. The evidence in Figure 2 is consistent with this insight.

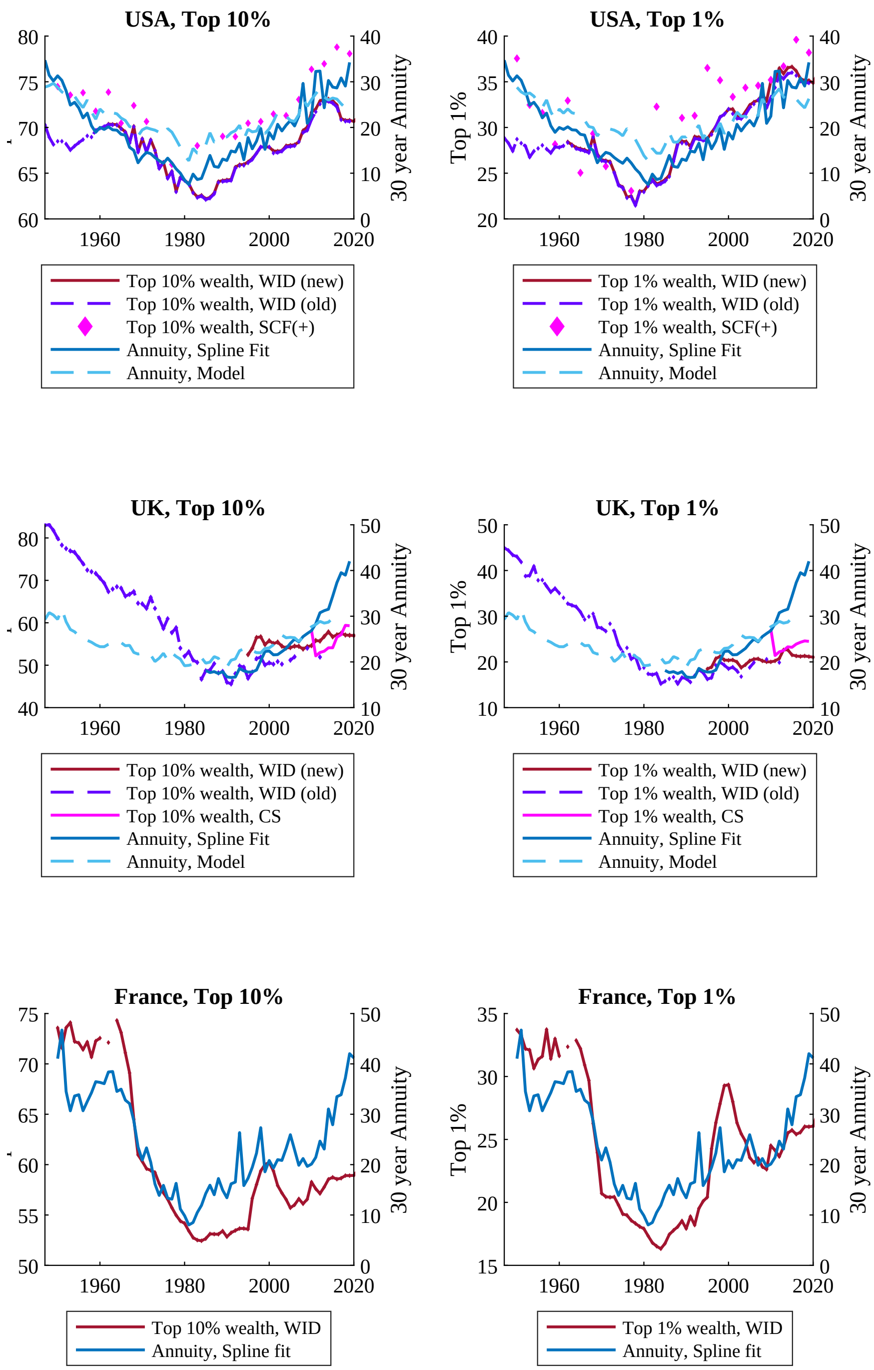
3.3 Household Heterogeneity in Financial Duration

Variation in real rates only matters for wealth inequality if households have heterogeneous portfolios. We find that there is significant heterogeneity in the exposure of household portfolios to real rate variation.

Every asset has its own duration, which measures the sensitivity of the asset's market price to interest rates. By feeding in the actual duration of household portfolios, we can mark the household's portfolios to market. The real world's counter-part to the model's financial asset is a portfolio of various financial and real assets that households own. As Table 2 shows, household assets consist of (i) cash, deposits, and money market instruments, (ii) stocks held directly and indirectly in mutual funds and pension accounts, (iii) real estate, (iv) private business wealth, and (v) fixed income assets (directly and indirectly held). Household liabilities consist of mortgage, student, and consumer debt. The duration of a financial portfolio is the weighted average duration of the components of the financial portfolio, where the weights are the portfolio weights $\omega(k)$ of the

[3] For France we start our sample in 1950 since inflation was very high coming out of WW-II, resulting in implausible real bond yield estimates.

Figure 2: Top Financial Wealth Inequality and Cost of Real Annuity



Note: Each panel plots a financial wealth inequality measure against a measure of the cost of a 30-year real annuity. The inequality measure in the left panels is the share of financial wealth going to the top-10% of the population. The right panels plot the share of the top-1% of the population. The wealth shares are from the World Inequality Database and, the SCF+ (U.S.), the Credit Suisse Global Wealth report (U.K., post 2012; France, post 2014), the U.K. Wealth and Assets Survey (WAS) (U.K., post 2012). Details on annuities and wealth shares in Appendix C.1.

various financial assets k:

$$D_{t,i}^{fin} = \sum_k \omega_t^i(k) D_t(k). \tag{1}$$

Consider an asset that pays a risky payoff stream $\{X_t\}$.To measure the duration of each of the components of financial wealth, we compute each asset's duration as follows:

$$D_t(k) = \frac{\sum_{j=0}^{\infty} M_{t,t+j} X_{t+j} \cdot j}{\sum_{j=0}^{\infty} M_{t,t+j} X_{t+j}},$$

where $M_{t,t+j}$ denotes the pricing kernel for payoffs that accrue at $t+k$. To do so, we use the no-arbitrage asset pricing model, detailed in Appendix A. For these assets, the model provides a McCauley duration in each quarter from 1947.Q1 until 2019.Q4. We use the durations for the 1980s, averaged across the 40 quarters in that decade.

We use the model-implied duration of the aggregate stock market to proxy for the duration of households' directly- and indirectly-held stock market wealth. We use the duration of small stocks to proxy for the duration of household business wealth. We use the duration of owner-occupied housing wealth to measure the duration of households' real estate assets. For cash and deposits, we assume a duration of 0.25 years. For fixed income, we assume a duration of 4 years.[4]

For student debt, we assume a duration of 4.5 years. Student loans are typically 10 year annuities. At an interest rate of 5.8%, the average rate on outstanding student loans in 2017, the duration is 4.56. At higher the interest rates that prevailed in the 1980s, the duration would be slightly smaller. For consumer debt, we assume a duration of 1 year. Much of this debt is revolving debt, while some of it is 24-month personal loans. The personal loans are amortizing.[5] For mortgage debt, we obtain data for the Bloomberg-Barclays Aggregate MBS Index. It is a representative portfolio of all outstanding U.S. pass-through mortgage-backed securities. The average McCauley duration of this representative mortgage portfolio in 1989 and 1990 was 5.2 years. Most mortgage debt in the U.S. is 30-year fixed-rate mortgages. The reasons for this much lower duration than 30 are several: amortization, high interest rates, and prepayment.[6] The resulting durations are reported in the first column of Table 2.

Next, we collect data from the Survey of Consumer Finances (SCF) on household portfolio shares, the $\omega_t^i(k)$ in (1). The wealth-weighted portfolio weights are reported in the last column of Table 2. The details are in Appendix C.3. The aggregate, or wealth-weighted financial duration is

[4]For reference, the maturity of outstanding U.S. Treasury marketable securities averages 62 months between 2000 and 2020. The duration is strictly smaller than the maturity since bonds pay coupons. For example, if the coupon rate is 4.65% and the bond pays semi-annual coupons, then the duration is 4.5 years. Other corporate and international bonds and loans held by U.S. households tends to have somewhat lower duration than U.S. Treasuries because there are fewer long-term bonds and coupons are higher.

[5]We exclude auto debt since we also exclude vehicles from assets. The reason is that our consumption measure includes durable consumption.

[6]The average maturity of the outstanding MBS portfolio in 1989-1990 was 9.8 years and the average coupon rate was 9.35%.

Table 2: Duration of the Household Financial Wealth Portfolio 1980s

	Duration	Portfolio Shares
Assets		
Cash and Deposits	0.25	11.60
Equities	28.78	11.61
Real Estate	14.89	48.75
Private Business Wealth	61.25	24.56
Fixed Income	4.00	17.75
Liabilities		
Mortgage Debt	5.20	12.12
Student Debt	4.50	0.27
Other Debt	1.00	1.88
Aggregate Duration		**25.72**
Average Duration		**15.43**

Note: The column "Duration" reports the duration of the asset, again averaged over all quarters in the 1980s. For Equities, Private Business Wealth, and Real Estate, the durations are computed form the asset pricing model in Appendix A, averaging across the 40 quarters in the 1980s. The column "Portfolio Shares" reports the wealth-weighted average or aggregate portfolio weights. Liabilities receive negative portfolio weights. These weights are based on the 1989 SCF.

the sum-product of the second and third columns of Table 2. It equals 25.72.

We also calculate the duration for each household separately, combining the household-level portfolio weights with the asset-specific durations listed in the first column. The average, or equally-weighted financial duration among all households is 15.43. This value is much lower than the wealth-weighted duration.

The difference between the aggregate and average duration stems from the positive covariance between financial wealth levels and financial durations. Richer households tend to hold more private business wealth, equities, and housing wealth, which are long-duration assets, hold fewer short-duration assets (cash), and hold less debt (negative duration). As a result, the aggregate duration of all wealth in the economy exceeds the average duration. More wealth is created at the macro level when rates decline than at the median household level and financial wealth inequality increases. Conversely, inequality decreases when rates increase.

To quantify the empirical correlation between financial duration and the level of financial wealth, Figure 3 plots the average duration by wealth bin in the SCF (dots). Since higher-wealth agents are more important for aggregate wealth outcomes, Figure 3 displays 5% bins up to the 90th percentile, then 1% bins up to the 99th percentile, and 0.2% bins for the top 1%. The figure shows that wealthier households hold longer-duration financial portfolios.

The second key data pattern is variation in financial duration by age. Figure 4 displays a binscatter of measured duration in our SCF data by age, after controlling for net wealth using dummies for each of the bins constructed in Figure 3. Figure 4 shows that there is a strongly negative relationship between age and duration.

Figure 3: Financial Duration by Net Worth Wealth Percentiles

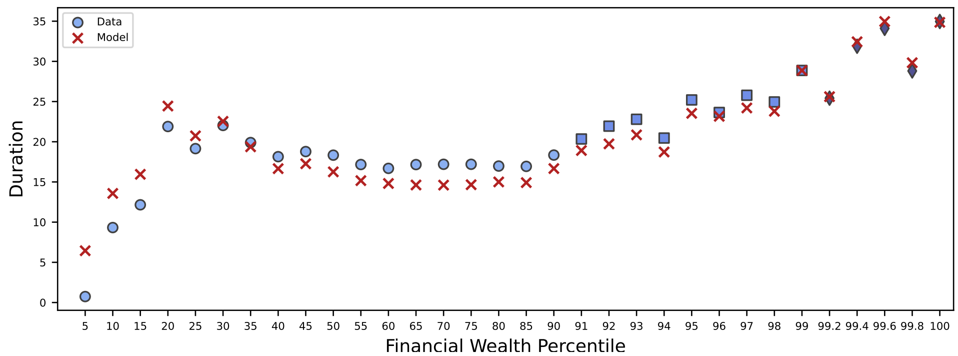
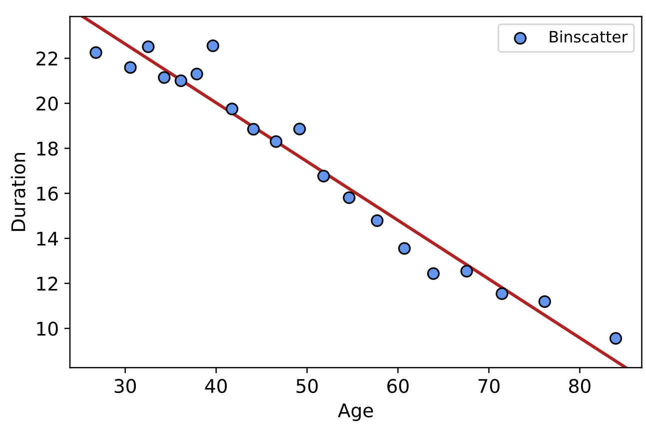


Figure 4: Financial Duration by Age



Our empirical results in Appendix C.3 show that adding other covariates yields little additional power to explain variation in financial duration across households. Therefore, we approximate financial duration using the regression:

$$D_i^{fin} = \alpha + \beta Age_i + \sum_j \gamma_j NetWealthBin_{i,j} + \varepsilon_i, \tag{2}$$

where $NetWealthBin_{i,j}$ is a dummy for whether household i falls in financial wealth bin j. The fitted value is plotted with crosses in Figure 3 and as the solid line in Figure 4. We use this estimated relation to impute durations to households in our structural model below.

To summarize, the key empirical finding is that the aggregate duration exceeds the average household's duration. Given this cross-sectional pattern in duration, a decline in real rates increases wealth inequality as we show next.

3.4 Mark-to-Market Repricing

We can now feed in the actual decline in real rates and evaluate its effects on the wealth distribution. We a consider a one-time unanticipated interest rate decline of the same magnitude as in the data: from 4.8% in 1980s to 0.3% in 2010s. We mark the distribution of wealth to market:

$$\Delta\theta_t^i \approx \Delta R \times D_{t,i}^{fin}, \tag{3}$$

using the cross-sectional distribution of durations $D_{t,i}^{fin}$ in the 1980s, obtained as the fitted value of equation (2). We refer to the resulting wealth distribution as the repriced wealth distribution.

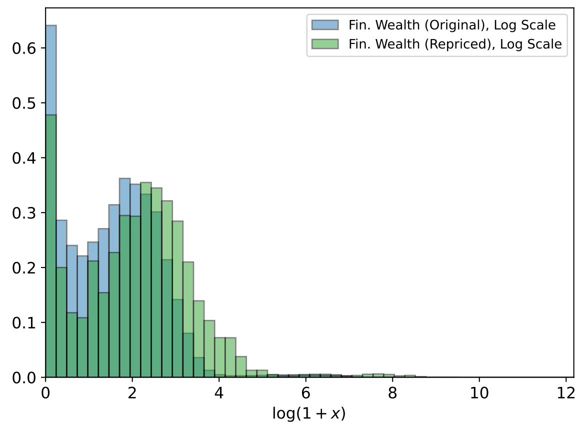
Figure 5a shows the repriced distribution in green, alongside the initial wealth distribution in blue. The decline in interest rates creates 116.2% more financial wealth. All but the poorest agents see large asset valuation gains. As shown in Figure 5b, the proportional gains are increasing in wealth since wealthier households have higher durations.

Table 3 summarizes the results. Each row of the table displays a different statistic measuring inequality. The first two columns display the statistics from the SCF+ data. We take the 1980s to be the period preceding the interest rate decline, and the 2010s to be the period following the interest rate decline. The next three columns display the results from the model. They report results for the initial (pre-shock, high-interest rate) distribution and for the wealth distribution after the interest rate decline. We focus on the repriced distribution here. The effects on the wealth distribution are significant. Repricing increases the U.S. financial wealth Gini by 0.082 compared to 0.063 in data. The top-10% financial wealth share increases by 13.2pp compared to 8.5pp in the SCF data. The top-1% wealth share increases 15.2pp compared to 4.8pp in the SCF and 10.5pp in the WID data.[7]

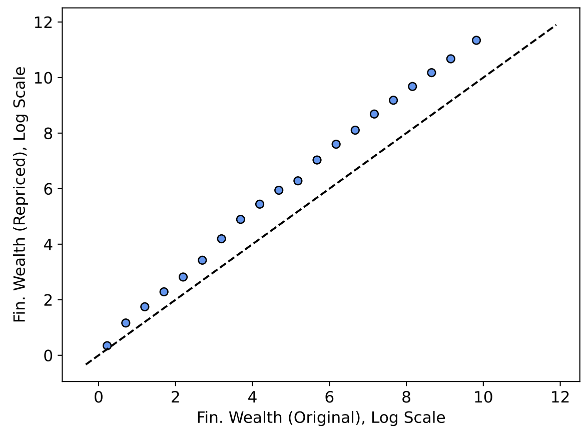
[7] The rise in the top-1% financial wealth share in the United States is even larger, at 12pp, when measured between 1982 and 2015 according to the World Inequality Database. The SCF+ generates an increase in the top-1% financial

Figure 5: Histograms, Repriced Financial Wealth Distribution

(a) Hist: Original vs. Repriced



(b) Binscatter: Original vs. Repriced



If anything, the duration mechanism overpredicts the increase in wealth inequality.

3.5 Robustness to Private Business Durations.

Because we do not directly measure the duration of private business wealth in the data, we approximate them using the duration of a portfolio of small public equities, which our empirical asset pricing model estimates to be 61.25.[8] To explore robustness to this assumption, Table 4 reproduces these results using two alternative assumptions. First, instead of assuming that private business wealth behaves like a portfolio of perpetually small stocks that is continually resorted, we use CRSP microdata on market value and payout to directly estimate the cash flow duration of the smallest public companies, with full details on this procedure provided in Section F. This procedure yields values between 52 and 62, depending on whether we look at the smallest quintile or decile of listed firms. In the interest of robustness, we choose a value of 50.[9] The results under this alternative assumption are displayed in the column $D^{PB} = 50$. Since the share of private business

wealth share of 7.2% between the 1983 and 2016 surveys. The WID generates a 8.9pp increase in the top-10% share between the 1980s and the 2010s, which is nearly identical to the 8.5pp point increase in the top-10% share in the SCF+ over the same period. Hence, the disagreement between data sources is concentrated in the top-1% only.

[8] To understand the high value for the duration of small stocks, consider a back-of-the-envelope calculation based on the Gordon Growth Model where the McCauley duration is $(1+r)/(r-g)$. Under perfect foresight, we can use the average realized real return and average realized real dividend growth rate from 1985–2020 to proxy for the expected real return and expected real dividend growth rate in the 1980s. For the smallest market capitalization decile, we find $r = 8.01\%$ and $g = 6.55\%$. This delivers a duration of 75.8, close to the number we obtain using our more sophisticated SDF model and the bottom quintile of market caps. For comparison, for stocks in the largest decile of market capitalization, we obtain $r = 7.94\%$ and $g = 0.81\%$, resulting in a duration of 15.2. The expected returns on large and small stocks are nearly identical. Thus, the high duration for small stocks arises from its high cash flow growth rate.

[9] For intuition, using the Gordon Growth Model where the McCauley duration is $(1+r)/(r-g)$ with a real discount rate of 8.1%, a duration of 50 implies a growth rate of 5.9% per annum for private businesses.

Table 3: Inequality, Model Comparison

	Data		Model		
	Before	After	Initial	Repriced	Comp
Gini FW	0.754	0.817	0.755	0.837	0.575
Top-10% share FW	64.0%	72.5%	64.0%	77.3%	49.9%
Top-1% share FW	28.8%	33.6%	44.1%	59.3%	35.4%
Gini HW	–	–	0.419	0.452	0.452
Top-10% share HW	–	–	33.9%	31.3%	31.3%
Top-1% share HW	–	–	18.6%	14.2%	14.2%
Gini TW	–	–	0.429	0.484	0.473
Top-10% share TW	–	–	38.2%	39.6%	36.0%
Top-1% share TW	–	–	23.9%	25.7%	20.3%

Note: Top 1%, Top 10% financial wealth shares as well as financial wealth Gini coefficients are estimated using SCF surveys. For the Before period we use the average values in 1983 and 1989. For the After period we use the average values in 2010, 2013 and 2016. More details on the computations are provided in Appendix C.3.8. For model results, the columns represent the pre-shock wealth distribution ("Initial"), the compensated distribution ("Comp"), and the repriced distribution ("Repriced").

wealth in the household's portfolio is increasing in financial wealth, this lower financial duration dampens the positive link between financial wealth and financial duration, reducing the resulting rise in inequality from a fall in rates. Under this calibration, the financial wealth Gini, top-10% share, and top-1% share, rise by 0.063 , 10.4pp, and 11.9pp, compared to 0.063 , 8.5pp, and 4.8pp (10.5%), respectively, in the SCF (WID) data. Thus, these results provide an even closer fit to the growth in wealth inequality observed in the data.

Second, we consider an even more conservative specification in which private business wealth has the same duration as public equity (28.78), with results displayed in the column $D^{PB} = D^{stocks}$.[10] This decreases financial wealth duration for the wealthiest households, reducing the rise in inequality. Under this specification, repricing raises the financial wealth Gini, top-10% share, and top-1% share, by 0.025 , 5.0%, and 6.0%. These results show that while our quantitative findings do vary with the estimated duration of private business wealth, the ultimate conclusion that our duration mechanism explains an economically important share of the rise in financial wealth inequality observed since the 1980s is robust.

4 Incomplete Markets Model with Household Heterogeneity

To analyze the effects of changes in discount rates on the distribution of wealth, we use a standard Bewley (1986) endowment economy in which agents face idiosyncratic and aggregate risk. We al-

[10]Using the Gordon growth formula provided in the previous footnote and a real discount rate of 8.1%, this duration implies cash flow growth of 4.3% per annum for the private businesses.

Table 4: Inequality by Private Business Duration

Fin. Wealth	Data	Repriced, Alternative Duration Specifications: Baseline	$D^{PB}=50$	$D^{PB}=D^{stocks}$
Gini	+0.063	+0.082	+0.063	+0.025
Top-10%	+8.5pp	+13.2pp	+10.4pp	+5.0pp
Top-1%	+4.8pp/+10.5pp	+15.2pp	+11.9pp	+6.0pp

Note: Top 1%, Top 10% financial wealth shares as well as financial wealth Gini coefficients are estimated using SCF surveys. Each cell calculates the difference between the average of the statistic in the 2010s (2010, 2013, 2016 waves) relative to the average of the statistic in the 1980s (1983, 1989 waves). Repriced distributions are computed as in Table 3. The column $D^{PB}=D^{stocks}$ sets the duration of private business duration to be equal to the duration for equity, while the column $D^{PB}=50$ sets the duration of private business wealth equal to 50. More details on the computations are provided in Appendix C.3.8.

low for ex ante heterogeneity. We use an endowment economy to isolate the valuation effects. We first show how to solve this model by transforming the problem into a stationary model without aggregate risk.

4.1 Endowments

Time is discrete, infinite, and indexed by $t \in [0, 1, 2, ...)$. The aggregate endowment e follows the stochastic process:

$$e_t(z^t) = e_{t-1}(z^{t-1})\lambda_t(z_t)$$

where $\lambda(z_t)$ denotes the stochastic growth rate of the aggregate endowment and z_t the aggregate state. The history of aggregate shocks is denoted by $z^t = \{z_t, z_{t-1}, \cdots\}$. A share $\alpha_t(z_t)$ of the aggregate endowment is financial income, the remaining $1-\alpha_t(z_t)$ share represents aggregate labor income.

Households are subject to idiosyncratic income shocks, whose history is denoted by $\eta^h = \{\eta_h, \eta_{h-1}, \cdots\}$. The η_h shocks are i.i.d. across households and persistent over time. The idiosyncratic shock process is assumed to be independent from the aggregate shock process. Labor income y follows the following stochastic process:

$$y_t(z^t, \eta^h) = \widehat{y}_t(z^t, \eta^h)(1-\alpha_t(z_t))e_t(z^t),$$

The ratio of individual to aggregate labor income, which we refer to as the labor income share, is given by $\widehat{y}_t(z^t, \eta^h)$. We use (z^t, η^h) to summarize the history of aggregate and idiosyncratic shocks, and $\pi(z^t, \eta^h)$ to denote the unconditional probability that state s^t will be realized. If the aggregate and idiosyncratic states are independently distributed, then we can decompose state transition probabilities into an aggregate and idiosyncratic component:

$$\pi(z_{t+1}, \eta_{h+1} | z^t, \eta^h) = \phi(z_{t+1} | z^t, x_{t_0}) \varphi(\eta_{h+1} | \eta^h),$$

where x_{t_0} denotes some household-specific characteristics. Below, we suppress dependence of the transition probabilities on initial characteristics. To keep the notation tractable, we assume households die when they reach age H. Below, in the calibrated model, we allow for mortality risk.

4.2 Preferences

A household born at t_0 maximize discounted expected utility over its lifetime:

$$U(c; t_0) = \sum_{j=1}^{H} \beta^j \sum_{(z^{t_0+j}, \eta^j)} \phi(z^{t_0+j}) \varphi(\eta^j) \frac{c(z^{t_0+j}, \eta^j)^{1-\gamma}}{1-\gamma},$$

where the coefficient of relative risk aversion $\gamma > 1$, and the subjective time discount factor $0 < \beta < 1$.

4.3 Technology

Households trade state-contingent bonds $a_t(z^t, \eta^h; z_{t+1})$ for each state z_{t+1} at prices $q_t(z^t, z_{t+1})$ and shares in the Lucas tree $\sigma_t(z^t, \eta^h)$ at price $\nu_t(z^t)$ satisfying the budget constraint:

$$c_t(z^t, \eta^h) + \sum_{z_{t+1}} a_t(z^t \eta^h; z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(z^t, \eta^h) \nu_t(z^t) \leq W_t(z^t, \eta^h).$$

Household cash on hand W evolves according to:

$$\begin{aligned} W_{t+1}(z^{t+1}, \eta^{h+1}) &= a_t(z^t \eta^h; z_{t+1}) + \widehat{y}_{t+1}(z^{t+1}, \eta^{h+1})(1 - \alpha(z_{t+1})) e_{t+1}(z^{t+1}) \\ &+ \left(\alpha(z_{t+1}) e_{t+1}(z^{t+1}) + \nu_{t+1}(z^{t+1})\right) \sigma_t(z^t, \eta^h). \end{aligned}$$

Households are subject to state-uncontingent and state-contingent solvency constraints:

$$\sum_{z_{t+1}} a_t(z^t \eta^h; z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(s^t) \nu_t(z^t) \geq K_t(s^t)$$

$$a_t(z^t \eta^h; z_{t+1}) + \left(\alpha(z_{t+1}) e_{t+1}(z^{t+1}) + \nu_{t+1}(z^{t+1})\right) \sigma_t(s^t) \geq M_t(s^t, z_{t+1})$$

where K and M denote generic borrowing limits. Incomplete risk sharing arises from two sources: the lack of an asset whose payoff depends on the idiosyncratic income shock η^t and the borrowing constraints.

4.4 Transformation into Stationary Economy

We can transform the stochastically growing economy into a stationary economy with a constant aggregate endowment following Alvarez and Jermann (2001); Krueger and Lustig (2010). To that end, define the deflated consumption allocations:

$$\widehat{c}_t(z^t,\eta^h) = \frac{c_t(z^t,\eta^h)}{e_t(z^t,\eta^h)}, \forall (z^t,\eta^h),$$

the deflated transition probabilities and the deflated subjective time discount factor:

$$\begin{aligned}\widehat{\phi}(z_{t+1}|z^t) &= \frac{\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}},\\ \widehat{\beta}(z^t) &= \beta\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}.\end{aligned}$$

Agents in the deflated economy with these preferences:

$$U(\widehat{c})(z^t,\eta^h) = \frac{\widehat{c}(z^t,\eta^h)^{1-\gamma}}{1-\gamma} + \sum_{z_{t+1}}\widehat{\beta}(z_{t+1},z^t)\widehat{\phi}(z_{t+1}|z^t)\sum_{\eta_{h+1}}\varphi(\eta_{h+1}|\eta^h)U(\widehat{c})(z^{t+1},\eta^{h+1}) \tag{4}$$

rank consumption plans identically as in the original economy. These are risk-neutral probabilities. When there is predictability in aggregate consumption growth, shocks to expected growth manifest themselves as taste shocks in the deflated economy. If aggregate growth shocks are i.i.d. over time, then the deflated time discount factor is constant and given by:

$$\widehat{\beta} = \beta\sum_{z_{t+1}}\phi(z_{t+1})\lambda_{t+1}(z_{t+1})^{1-\gamma}. \tag{5}$$

This i.i.d. assumption on aggregate growth shocks is the assumption we will make, noting that it can easily be relaxed. In what follows, we also assume that aggregate factor shares are constant: $\alpha_t(z_t) = \alpha,\ \forall t$. By definition, labor income shares average to one across households:

$$\sum_{t_0\geq 1}\sum_{\eta^h}\varphi(\eta^h|\eta_0)\widehat{y}_t(\eta^h) = 1,\ \forall t.$$

4.5 Equilibrium in the Stationary Economy

In the stationary economy, agents trade a single risk-free bond and a stock. Both securities have the same returns. The stock yields a dividend α in each period. Given initial financial wealth θ_{t_0} for a household born at t_0, interest rates $\widehat{R}_t$ and stock prices $\widehat{v}_t$, households choose consumption $\{\widehat{c}_t(\theta_{t_0},\eta^h)\}$, bond positions $\{\widehat{a}_t(\theta_{t_0},\eta^h)\}$, and stock positions $\{\widehat{\sigma}_t(\theta_{t_0},\eta^h)\}$ to maximize expected utility (4) subject to the budget constraint:

$$\widehat{c}_t(\eta^h) + \frac{\widehat{a}_t(\theta_{t_0}, \eta^h)}{\widehat{R}_t} + \widehat{\sigma}_t(\theta_{t_0}, \eta^h)\widehat{\nu}_t = (1-\alpha)\widehat{y}_t(\eta^h) + \widehat{a}_{t-1}(\theta_{t_0}, \eta^{h-1}) + \widehat{\sigma}_{t-1}(\theta_{t_0}, \eta^{h-1})(\widehat{\nu}_t + \alpha),$$

and subject to borrowing constraints:

$$\frac{\widehat{a}_t(\theta_{t_0}, \eta^h)}{\widehat{R}_t} + \widehat{\sigma}_t(\theta_{t_0}, \eta^h)\widehat{\nu}_t \geq \widehat{K}_t(\eta^h), \quad \forall \eta^h$$

$$\widehat{a}_t(\theta_{t_0}, \eta^h) + \widehat{\sigma}_t(\theta_{t_0}, \eta^h)(\widehat{\nu}_{t+1} + \alpha) \geq \widehat{M}_t(\eta^h), \quad \forall \eta^h.$$

Definition 1. For a given initial distribution of wealth Θ_{t_0} when born, a Bewley equilibrium is a list of consumption choices $\{\widehat{c}_t(\theta_{t_0}, \eta^h)\}$, bond positions $\{\widehat{a}_t(\theta_{t_0}, \eta^h)\}$, and stock positions $\{\widehat{\sigma}_t(\theta_{t_0}, \eta^h)\}$ as well as stock prices $\widehat{\nu}_t$, and interest rates $\widehat{R}_t$ such that each household maximizes its expected utility, and asset markets and goods markets clear.

$$\sum_{t_0 \geq 1} \int \sum_{\eta^h} \varphi(\eta^h | \eta_{t_0}) \widehat{a}_t(\theta_{t_0}, \eta^h) d\Theta_{t_0} = 0,$$

$$\sum_{t_0 \geq 1} \int \sum_{\eta^h} \varphi(\eta^h | \eta_{t_0}) \widehat{\sigma}_t(\theta_{t_0}, \eta^h) d\Theta_{t_0} = 1.$$

$$\sum_{t_0 \geq 1} \int \sum_{\eta^h} \varphi(\eta^h | \eta_{t_0}) \widehat{c}_t(\theta_{t_0}, \eta^h) d\Theta_{t_0} = 1.$$

In the deflated economy, the return on the aggregate stock equals the risk-free rate:

$$\widehat{R}_t = \frac{\widehat{\nu}_{t+1} + \alpha}{\widehat{\nu}_t}. \tag{6}$$

The equilibrium stock price equals the present discounted value of the dividends:

$$\widehat{\nu}_t = \sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{t \to t+\tau} \alpha,$$

discounted at the cumulative gross risk-free rate, defined as: $\widehat{R}_{t \to t+T} = \Pi_{k=0}^{T} \widehat{R}_{t+k}$. Note that $\widehat{R}_{t \to t} = \widehat{R}_t$ and define $\widehat{R}_{t \to t-1} = 1$.

Both of these assets, the stock and the risk-free bond, earn the same risk-free rate of return in the stationary economy. These households are indifferent between these 2 assets, or any other assets with different durations, because the interest rates are deterministic.

In the calibrated version of this economy, in section 6, we will feed the observed heterogeneity in duration into this model in the form of risk-free zero-coupon bonds of maturity k. This is without loss of generality, because these households do not have a preference for one duration over another.

4.6 Equilibrium in the Growing Economy

We can map the equilibrium in the detrended economy into an equilibrium in the stochastically growing economy.

Proposition 4.1. If $\{\widehat{c}_t(\theta_{t_0}, \eta^h), \widehat{a}_t(\theta_{t_0}, \eta^h), \widehat{\sigma}_t(\theta_{t_0}, \eta^h)\}$ and $\{\widehat{\nu}_t, \widehat{R}_t\}$ are a Bewley equilibrium, then $\{c_t(\theta_{t_0}, s^t), a_t(\theta_{t_0}, s^t, z_{t+1}), \sigma_t(\theta_{t_0}, s^t)\}$ as well as asset prices $\{\nu_t(z^t), q_t(z^t, z_{t+1})\}$ are an equilibrium of the stochastically growing economy with:

$$\begin{aligned}
c_t(\theta_{t_0}, z^t, \eta^h) &= \widehat{c}_t(\theta_{t_0}, \eta^h) e_t(z^t) \\
a_t(\theta_{t_0}, z^t, \eta^h; z_{t+1}) &= \widehat{a}_t(\theta_{t_0}, \eta^h) e_t(z^t) \\
\sigma_t(\theta_{t_0}, z^t, \eta^h) &= \widehat{\sigma}_t(\theta_{t_0}, \eta^h) \\
\nu_t(z^t) &= \widehat{\nu}_t e_t(z^t) \\
q_t(z^t, z_{t+1}) &= \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \frac{1}{\widehat{R}_t}.
\end{aligned}$$

The proof is provided in Krueger and Lustig (2010). The last equation implies the following relationship between the interest rate in the growing economy and the stationary economy:

$$R_t = \left(\sum_{z_{t+1}} q_t(z^t, z_{t+1}) \right)^{-1} = \left(\sum_{z_{t+1}} \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \right)^{-1} \widehat{R}_t. \tag{7}$$

5 Wealth Inequality and Heterogeneity in Duration

Next, we let the economy undergo a decline in the interest rate and show that this increases the inequality in financial wealth. In the model, the equilibrium decline in the real rate arises from a slowdown in expected economic growth. We use the model to describe a benchmark equilibrium in which households consume the same share of the aggregate endowment which they had planned to consume before the unanticipated decline in rates. To achieve this perfect hedging, households need to match the duration of their financial assets to that of their excess consumption plan.

To keep the notation tractable, we use a version of the model in which households are ex ante identical and infinitely lived. We relax these assumptions in the calibrated model of the next section.

5.1 Wealth Accounting

What is the right discount rate when measuring household wealth? If we want a wealth measure that can be aggregated, we have to use the same discount rate for all claims.

Proposition 5.1. At time 0, the financial wealth of each household equals the present discounted value of future consumption minus future labor income.

$$\theta_0 = \sum_{\tau=0}^{\infty} \sum_{\eta^\tau} \frac{\varphi(\eta^\tau)}{\widehat{R}_{0\to\tau-1}} \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)$$

As the proof in the appendix shows, the proposition follows easily from iterating forward on the one-period budget constraint. In this iteration, we take expectations over financial wealth in all future states using the objective probabilities of the idiosyncratic events $\varphi(\eta^\tau)$, and discount by the cumulative risk-free rate $\widehat{R}_{0\to\tau-1}$. Aggregate financial wealth in the economy in period 0 is given by:

$$\int \theta_0 d\Theta_0 = \int (\widehat{a}_{-1}(\theta_0) + \widehat{\sigma}_{-1}(\theta_0)\widehat{\nu}_0) d\Theta_0 = 0 + 1\widehat{\nu}_0,$$

where we have used market clearing in the bond and stock markets at time 0.

Aggregating the cost of the excess consumption plan across all households, using the fact that labor income shares average to 1, and imposing goods market clearing at time 0, we get:

$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}\alpha = \widehat{\nu}_0.$$

The aggregate cost of households' excess consumption plan, or households' aggregate financial wealth, exactly equals the stock market value $\widehat{\nu}_0$, the only source of net financial wealth in the economy. This result relies on market clearing:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 = \alpha,$$

at each time t, because $\int \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 = 1$ from market clearing, and the labor income shares sum to one as well.

The choice of the actual probability measure $\varphi(\cdot)$ and rate $\widehat{R}$ to compute an individual's human capital, the expected present discounted value of her labor income stream, may seem arbitrary. After all, claims to labor income are not traded in this model and markets are incomplete. The key insight is that, using any other pricing kernel to discount individual labor income and consumption streams may result in a value of aggregate financial wealth different from the value of the Lucas tree. To see this, consider using a distorted measure $\psi(\eta^\tau)\varphi(\eta^\tau)$ different from the actual measure $\varphi(\eta^\tau)$, where the household-specific wedges satisfy $\mathbb{E}_0[\psi_t] = 1, \forall t$. Under this different measure, the goods markets do not clear and the labor shares do not sum to one, unless the household-specific wedges do not covary with consumption and income shares:

Proposition 5.2. Wealth measures aggregate if and only if the following orthogonality conditions holds for the househehold-specific wedges and household consumption and income:

$$Cov_0\left(\psi_t, \widehat{c}_t\right) = 0, \quad Cov_0\left(\psi_t, \widehat{y}_t\right) = 0.$$

For all other wedge processes $\psi_t(\eta^\tau)$, the resource constraint is violated:

$$\int \sum_{\eta^\tau} \psi(\eta^\tau)\varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 \neq \alpha,$$

It is common in the literature to use the household's own IMRS to compute human capital (e.g., Huggett and Kaplan, 2016). The household's IMRS is a natural choice because it ties the valuation of human wealth directly to welfare. However, this approach does not lend itself to aggregation. The wedges

$$\psi(\eta^{t+1}) = \frac{u'(\widehat{c}(\eta_{t+1}, \eta^t))}{u'(\widehat{c}_t(\eta_0))},$$

do not satisfy the zero covariance restrictions of the proposition. Imperfect consumption insurance implies that:

$$Cov_0(\psi_t, \widehat{c}_t) \leq 0, \quad Cov_0(\psi_t, \widehat{y}_t) \leq 0.$$

Proposition 5.3. If the cross-sectional covariance between the household-specific wedges and consumption is negative ($Cov_0(\psi_t, \widehat{c}_t) \leq 0$), then the aggregate valuation of individual wealth is less than the market's valuation of total wealth.

When aggregating, this pricing functional undervalues human wealth and therefore also total wealth.[11] In sum, while pricing claims to consumption and labor income using the household's IMRS is sensible from a welfare perspective, this approach does not lend itself to wealth accounting and aggregation.

5.2 Interest Rate Decline

We now analyze the main exercise of the paper, which is to let the economy undergo an unexpected and permanent decrease in the interest rate ("MIT shock"). Since interest rates are endogenously determined, we generate this decrease through a decrease in the expected growth rate of

[11] Since the factor shares are constant, the consumption claim is in the span of traded assets. Financial wealth is the value of the Lucas tree, which equals α times the value of a claim to total consumption.

the economy:

$$\mathbb{E}[\lambda] = \sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1}) \to \mathbb{E}[\widetilde{\lambda}] = \sum_{z_{t+1}} \phi(z_{t+1})\widetilde{\lambda}(z_{t+1})$$

where $\mathbb{E}[\widetilde{\lambda}] < \mathbb{E}[\lambda]$. A lower expected growth rate manifests itself as a higher subjective time discount factor in the stationary economy, provided that the coefficient of relative risk aversion $\gamma > 1$, or, equivalently, the elasticity of intertemporal substitution is smaller than one:

$$\widetilde{\beta} = \beta \sum_{z_{t+1}} \phi(z_{t+1})\widetilde{\lambda}_{t+1}(z_{t+1})^{1-\gamma} > \widehat{\beta}.$$

In the transformed incomplete markets economy, the size of the decline in the rate of time preference is governed by the EIS ($1/\gamma$). Just like in a representative agent economy, the larger the EIS, the smaller the effect of a decline in the expected growth rate of aggregate consumption on the risk-free rate.

In the simple case of log-normally distributed aggregate consumption growth, we obtain the following expression for the rate of time preference in the stationary economy:

$$\log \widehat{\beta} = \log \beta - \gamma \mathbb{E}[\log \lambda] - \frac{1}{2}\gamma(1-\gamma)Var[\log \lambda]. \tag{8}$$

Hence, the change in the transformed rate of time preference in response to the growth shock is given by: $\frac{d \log \widehat{\beta}}{d \mathbb{E}[\log \lambda]} = -\gamma$.

It is natural to ask whether we can still implement the equilibrium consumption allocation $\{\widehat{c}_t(\theta_0, \eta^t)\}$ from the economy with high rates in the economy with low rates. Given that the time discount factor of all agents increased by the same amount, there should be no motive to trade away from these allocations. The following proposition shows that the old consumption allocation is indeed still an equilibrium in the low interest rate economy, provided that initial financial wealth is scaled up for every household.

Proposition 5.4. If the allocations and asset market positions $\{\widehat{c}_t(\theta_0, \eta^t), \widehat{a}_t(\theta_0, \eta^t), \widehat{\sigma}_t(\theta_0, \eta^t)\}$ and asset prices $\{\widehat{v}_t, \widehat{R}_t\}$ are a Bewley equilibrium in the economy with $\widehat{\beta}$ and natural borrowing limits $\{\widehat{K}_t(\eta^t)\}$,

$$\widehat{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \widehat{R}^{-1}_{t \to \tau-1} \sum_{\eta^\tau | \eta^t} \varphi(\eta^\tau | \eta^t)(1-\alpha)\widehat{y}_\tau(\eta^\tau),$$

then the allocations and asset market positions $\{\widehat{c}_t(\widetilde{\theta}_0, \eta^t), \widehat{a}_t(\widetilde{\theta}_0, \eta^t), \widehat{\sigma}_t(\widetilde{\theta}_0, \eta^t)\}$ and asset prices $\{\widetilde{v}_t, \widetilde{R}_t\}$ will be an equilibrium of the economy with $\widetilde{\beta}$ and natural borrowing limits $\{\widetilde{K}_t(\eta^t)\}$,

$$\widetilde{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \widetilde{R}^{-1}_{t \to \tau-1} \sum_{\eta^\tau | \eta^t} \varphi(\eta^\tau | \eta^t)(1-\alpha)\widehat{y}_\tau(\eta^\tau),$$

asset prices are given by

$$\widetilde{\beta}\widetilde{R}_t = \widehat{\beta}\widehat{R}_t, \text{ and } \widetilde{v}_t = \sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{t\to t+\tau}\alpha,$$

and every household's initial wealth is adjusted as follows:

$$\widetilde{\theta}_0 = \theta_0 \frac{\sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{0\to\tau} \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)}{\sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{0\to\tau} \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)}.$$

The proof is in the appendix. Aggregate financial wealth undergoes an adjustment equal to the ratio of the price of two perpetuities:

$$\frac{\sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{0\to\tau}}{\sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{0\to\tau}} = \frac{\widetilde{v}_0}{\widehat{v}_0}.$$

Intuitively, with lower interest rates, all asset prices are higher than in the high-rate economy. The Lucas tree becomes more valuable. A fraction $1-\alpha$ of this tree reflects aggregate human wealth, the remaining fraction is aggregate financial wealth. Each individual's financial wealth adjustment differs, and depends on the expected discounted value of the same future excess consumption plan discounted at different rates. The higher one's expected future excess consumption, the larger the initial financial wealth adjustment needed to implement the old equilibrium allocation.

To a first-order approximation, i.e., for a small change in the interest rate, the adjustment in initial financial wealth needed for agents to keep their initial consumption plan is given by the duration of their planned consumption in excess of labor income. This is the duration households will need in their net financial assets in order to be fully hedged against interest rate risk.

Characterizing Interest Rate Sensitivity Using Duration of Excess Consumption Define the duration of a household's excess consumption plan at time 0, following the realization of the idiosyncratic labor income shock η_0, as follows:

$$D^{c-y}(\theta_0,\eta_0) = \frac{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau|\eta_0} \tau \widehat{R}^{-1}_{0\to\tau} \varphi(\eta^t|\eta_0)\left(\widehat{c}_\tau(\eta^\tau|\eta_0) - (1-\alpha)\widehat{y}(\eta^\tau|\eta_0)\right)}{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau|\eta_0} \varphi(\eta^t|\eta_0) \widehat{R}^{-1}_{0\to\tau}\left(\widehat{c}_\tau(\eta^\tau|\eta_0) - (1-\alpha)\widehat{y}(\eta^\tau|\eta_0)\right)}$$

The duration measures the sensitivity of the cost of its excess consumption plan to a change in the interest rate. In our endowment economy, aggregate consumption is fixed. We are interested in the valuation effects of interest rate changes.[12] The duration of the excess consumption claim equals the value-weighted difference of the duration of the consumption claim and that of the

[12]Households in the detrended economy's equilibrium face a deterministic interest rate, and do not anticipate interest rate changes. Auclert (2019) was the first to conduct this type of duration analysis in a model with endogenous labor supply to gauge the effects of monetary policy on consumption.

labor income claim:

$$D^{c-y} \;\; = \;\; \frac{P_0^c}{P_0^{c-y}} D^c - \frac{P_0^y}{P_0^{c-y}} D^y.$$

where $P_0^{c-y} = \theta_0$ is household financial wealth, P_0^y is human wealth, and P_0^c is total household wealth, the sum of financial and human wealth. Households with a high positive duration of excess consumption face a large increase in the cost of their consumption plan when interest rates go down, insofar that this increased cost is not offset fully by the increase in their human wealth.

The duration of the aggregate excess consumption claim, the aggregate duration for short, equals:

$$D^a = \frac{\sum_{\tau=0}^{\infty} \tau \widehat{R}_{0\to\tau}^{-1}}{\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}}$$

This is the duration of a claim to aggregate consumption minus aggregate labor income, or equivalently to aggregate financial income. It is the duration of a perpetuity in the stationary economy. Recall that $\widehat{\nu}_0 = \nu_0 = \alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}$ denotes aggregate financial wealth.

Proposition 5.5. The aggregate duration equals the wealth-weighted average duration of households' excess consumption claims:

$$D^a = \int D^{c-y}(\theta_0, \eta_0) \frac{\theta_0}{\nu_0} d\Theta_0.$$

The proof follows directly from the definition of the household specific duration measure and market clearing.

The next proposition is the main result. It shows that, when households that are richer than average tend to have excess consumption plans of higher duration, then the (equally-weighted) average household's excess consumption plan duration is smaller than the aggregate duration.

Proposition 5.6. If $cov(\theta_0, D^{c-y}(\theta_0)) > 0$ then $\int D^{c-y}(\theta_0, \eta_0) d\Theta_0 \leq D^a$ and lower interest rates increase financial wealth inequality when households are fully hedged.

The proof follows from recognizing the following relationship between (cross-sectional) expectations and covariances:

$$D^a = \mathbb{E}\left[\frac{\theta_0}{\nu_0^a} D^{c-y}(\theta_0, \eta_0)\right] = \mathbb{E}\left[D^{c-y}(\theta_0, \eta_0)\right] + cov\left[\frac{\theta_0}{\nu_0}, D^{c-y}(\theta_0, \eta_0)\right].$$

The proposition says that under the covariance condition, if all households are perfectly hedged in their portfolio, then wealth inequality should increase when rates decline.

Ex Ante Identical Households In this class of Bewley models, if agents are ex ante identical, agents with low financial wealth have encountered a bad history of labor income shocks. If labor income is highly persistent, their labor income is low today and in the near future relative to labor income in the distant future (because of mean-reversion). This pattern makes the duration of their labor income stream high. But since the household is smoothing consumption inter-temporally, $D^c < D^y$. As a result, low-wealth agents tend to have low duration of their excess consumption plan. Conversely, rich agents have high labor income and high excess consumption duration. Consumption smoothing is the force that makes the covariance assumption satisfied in a Bewley model where the only source of heterogeneity is income shock realizations. It follows immediately that, under the stated covariance restriction, the increase in the cost of the excess consumption plan for the average household is smaller than the aggregate (per capita) wealth increase. Put differently, financial wealth inequality should increase when rates go down if households want to afford their old consumption plans.

Low-financial wealth households in a Bewley model have high-duration human wealth, which provides a natural interest rate hedge. High financial-wealth households have low-duration human wealth and need to increase financial wealth by more when rates decline to be able to afford the old consumption plan.

Ex Ante Heterogeneous Households The insights of this normative proposition apply more broadly. The covariance condition applies in the richer model with ex-ante heterogeneity across households.

Proposition 5.7. If $cov(\theta_t, D_t^{c-y}(\theta_{t_0})) > 0$ then the average duration is lower than the aggregate duration, $\sum_{t_0} \int D_t^{c-y}(\theta_0, \eta_0) d\Theta_{t_0} \leq D_t^a$ and lower interest rates increase financial wealth inequality when households are fully hedged.

We check this condition in a calibrated version of the model.

Real-world households may not be fully hedged, unlike the households in the Bewley model. The actual duration of the household's financial assets in the data, denoted D^{fin}, can differ from the duration of the excess consumption claim D^{c-y} in the model where households are fully hedged. We now turn to a calibrated life-cycle version of the Bewley model with overlapping generations to assess how well households are hedged against interest rate risk.

6 Quantitative Implications in Calibrated Economy

The previous section showed that in a Bewley model where agents are fully hedged, a rise in financial wealth inequality is required when interest rates decline. In this section, we aim to quantify this effect in a model with realistic heterogeneity among households. The model introduces overlapping generations of finitely-lived agents, generating heterogeneity by age. We feed in the actual decline in interest rates and investigate how the financial, human, and total wealth distributions change. We refer to these new wealth distributions as the compensated distributions.

We conduct our analysis in the stationary version of this economy, using the mapping described in Proposition 4.1 to go from objects in the stochastically growing economy to objects in the stationary economy. In the stationary economy, agents are indifferent between trading the stock and the one-period risk-free bond, despite their different durations, because both assets earn the risk-free rate in equilibrium and the risk-free rate is constant in the stationary equilibrium.

In the calibrated economy, we allow households to invest in a real, growing perpetuity with duration $k = (1+\widehat{r})/(r-g_k)$, where $\widehat{r} = \widehat{R} - 1$ and coupon stream δ_t^k which grows at rate g_k. We then feed the duration heterogeneity observed in the data into the model: the duration $k(\theta_{t_0}, \eta^h)$ depends on the household characteristics.

Given initial financial wealth θ_{t_0} for a household born at t_0 and bond prices $\widehat{q}_t^k$, households choose consumption $\{\widehat{c}_t(\theta_{t_0}, \eta^h)\}$ and bond positions $\{\widehat{a}_t^{k(\theta_{t_0},\eta^h)}(\theta_{t_0}, \eta^h)\}$ to maximize expected utility (4) subject to the budget constraint:

$$\widehat{c}_t(\eta^h) + \widehat{q}_t^{k(\theta_{t_0},\eta^h)}\widehat{a}_t^{k(\theta_{t_0},\eta^h)}(\theta_{t_0}, \eta^h) = (1-\alpha)\widehat{y}_t(\eta^h) + \widehat{a}_{t-1}^{k(\theta_{t_0},\eta^{h-1})}(\theta_{t_0}, \eta^{h-1}) \left[q_t^{k(\theta_{t_0},\eta^h-1)} + \delta_t^{k(\theta_{t_0},\eta^h-1)} \right]$$

As the previous section explained, the model with aggregate risk in total income maps into a stationary economy without aggregate risk as long as the idiosyncratic and aggregate risk are uncorrelated. The presence of aggregate risk in the growing economy affects the time discount factor and hence the equilibrium risk-free rate in the stationary economy.

6.1 Calibration

Financial Duration We feed in the actual duration of household portfolios into the simulation. We then calibrate financial durations in the model to be equal to the fitted value:

$$\widehat{D}_i^{fin} = \hat{\alpha} + \hat{\beta} Age_i + \sum_j \hat{\gamma}_j NetWealthBin_{i,j}, \tag{9}$$

applied household by household, where hats denote sample estimates. This procedure delivers the close fit between model and data observed in Figure 3, where the small discrepancies are due to slight differences in the relationship between age and net wealth percentile in model and data. The model delivers an equal-weighted duration of 16.9 and a value-weighted duration of 28.5, both close to their empirical counterparts in Table 2.

Aggregate Output Growth Process We assume that aggregate output growth λ follows an i.i.d. log-normal process $\log \lambda \sim N(g, \sigma_\lambda^2)$, where $g = 0.01893$ and $\sigma_\lambda = 0.02319$ are the average annualized growth and volatility of log real per-capita GDP in the U.S. data.

Preferences and Stationarity Households have CRRA preferences with risk aversion γ equal to 2. Substituting into (7), and using the initial risk-free rate 4.82% from the data (see below) implies that the initial risk-free rate in the stationary economy is $\hat{R} = 1.0294$. We set $\widehat{\beta} = 1/\widehat{R}$, implying $\hat{\beta} = 0.9715$. For easier interpretation, converting back to the growing economy using (5) implies that the true preference parameter β is equal to 0.9898.

Size of Decline in Real Yields In the model, the expected growth rate experiences an unanticipated decline, giving rise to decline in the real rates. In the stationary model, interest rates must be adjusted for growth. Using the formula obtained from (7) in the lognormal case, we can back out the rate in the stationary economy as a function of the risk-free rate in the stochastically growing economy:

$$\hat{R}_t = R_t \exp\left\{-g + \left(\gamma - \frac{1}{2}\right)\sigma_\lambda^2\right\}$$

This change in R is the result of an unexpected and permanent decline in the expected aggregate growth rate of the economy (an MIT shock). We model a decline in real rates of 4.48%. The adjusted rates $\widehat{R}$ decline from 2.83% to $\widetilde{R}$ -1.57%. Using our value of $\gamma = 2$, (8) implies that a decline in rates of 4.48% can be generated using a decline in expected growth $\mathbb{E}[\log \lambda]$ of 2.24%. Following Proposition 5.4, the discount factor and the risk-free rate adjustments cancel out to preserve the relation $\tilde{\beta}\tilde{R} = \hat{\beta}\hat{R}$.

Regular Income Component The income process consists of a regular component and a superstar component. The regular income process for household i of age a at time t that is not currently in the superstar state takes the form standard in the literature, given by:

$$\log\left(y_{t,a}^i\right) = m_t + \chi' X_t^i + \eta_t^i, \tag{10}$$

$$\eta_{t+1}^i = \alpha_i + \varepsilon_{t+1}^i + \nu_{t+1}^i, \tag{11}$$

$$\varepsilon_{t+1}^i = \rho\varepsilon_t^i + u_{t+1}^i, \tag{12}$$

where m_t is a year-fixed effect and X_t^i is a vector of household characteristics that includes a cubic function of age.[13] When calibrating the model, we normalize the age profile $\chi' X_t^i$ so that its mean

[13]We have verified that our results are similar if we estimate the year fixed effect and the age profile separately for groups of households that depend on education (college completion or not), race (white or non-white), gender (male or non-male), giving rise to 8 groups in total. Since it makes little difference, we only consider one group here.

is equal to unity during working life.

The stochastic income component η_t^i contains a household-fixed effect α^i, a persistent component ε_{t+1}^i, and an i.i.d. component ν_{t+1}^i. We have: $\mathbb{E}[\eta^i] = \mathbb{E}[\alpha^i] = E[\nu^i] = \mathbb{E}[\varepsilon^i] = 0$ and $Var[\nu^i] = \sigma_\nu^2$, $Var[u^i] = \sigma_u^2$, $Var[\alpha^i] = \sigma_\alpha^2$, and $Var[\varepsilon_0^i] = \sigma_{\varepsilon,0}^2$. Note that the income risk parameters are common across groups. The parameters are estimated by GMM using PSID data from 1970 until 2017, as detailed in Appendix C.2. Figure A9 in that appendix plots the deterministic life-cycle income profile.

The literature typically estimates (10)-(12) on labor income for males between ages 25 and 55. We deviate from this practice in three ways, all of which are important for our purposes. First, we consider a broader income concept. Second, we consider the entire life-cycle from age 18 to 80. Third, we focus on households rather than individuals.

First, from the model's perspective, the relevant notion of income includes transfers. It is the risk in this income that the household is hedging by trading in financial markets (borrowing and saving). To that end, we measure income in the data as income from wages and salaries, the labor income component of proprietor's income, and government transfers (unemployment benefits, social security, other government transfers), and private defined-benefit pension income. Obtaining consistent data on the various components of transfers is involved because successive waves of the PSID use different variable codes for the same concepts. Appendix C.2 provides the details. Catherine et al. (2020) also focuses on after-transfer income.

Second, we are interested in the entire life-cycle. We start at age 18 and go until age 80. Because our income concept includes transfers such as unemployment benefits and retirement income from public or private defined-benefit pension plans, we do not have to model labor force participation decisions or retirement decisions. Our approach captures the average decisions made in the data. For example, we do not need to make the assumption that retirement starts at age 65, that income in retirement is some constant fraction of pre-retirement income, or that income risk disappears in retirement. We can let the data speak on these issues. Since our income concept includes income from part-time work, it captures income earned by students, for example. We assign to students the educational achievement they will attain even before they have completed their education, so that they are classified in the correct group.

Third, we focus on households, aggregating income across its adult members. This absolves us from having to model demographic changes such as getting married, getting divorced, getting widowed. We simply follow households identified by the head of household as designated in the data.

Superstar Income Component To help the model match the level of wealth inequality in the high-interest rate regime, we enrich the income process in (10)-(12) with a superstar income state. This state has a high income level Y^{sup}. Households enter in this state with probability p_{12}^{sup} when they are in the normal income state, and return to the normal state with probability p_{21}^{sup} when they currently are in the superstar income state. The income level Y^{sup} is chosen to match the top-10% wealth share in the 1980s exactly, which requires a value equal to 43.6 times average income. The transition probability parameters $p_{12}^{sup} = 0.0002$ and $p_{21}^{sup} = 0.975$ are taken from Boar and Midrigan (2020). There is about a 1% probability of entering in the superstar income state over one's life-time. Conditional on entering, the state has an expected duration of 40 years.

In the computations, we discretize the stochastic income process η, with the extra superstar state, as a markov chain.

Mortality Risk We assume that households enter at age 18 with zero financial wealth. They enter in an annuity or tontine system in which surviving households receive the assets of households in their gender-age cohort who died, assuring zero wealth at the end of life.[14]

6.2 High Interest-Rate Regime

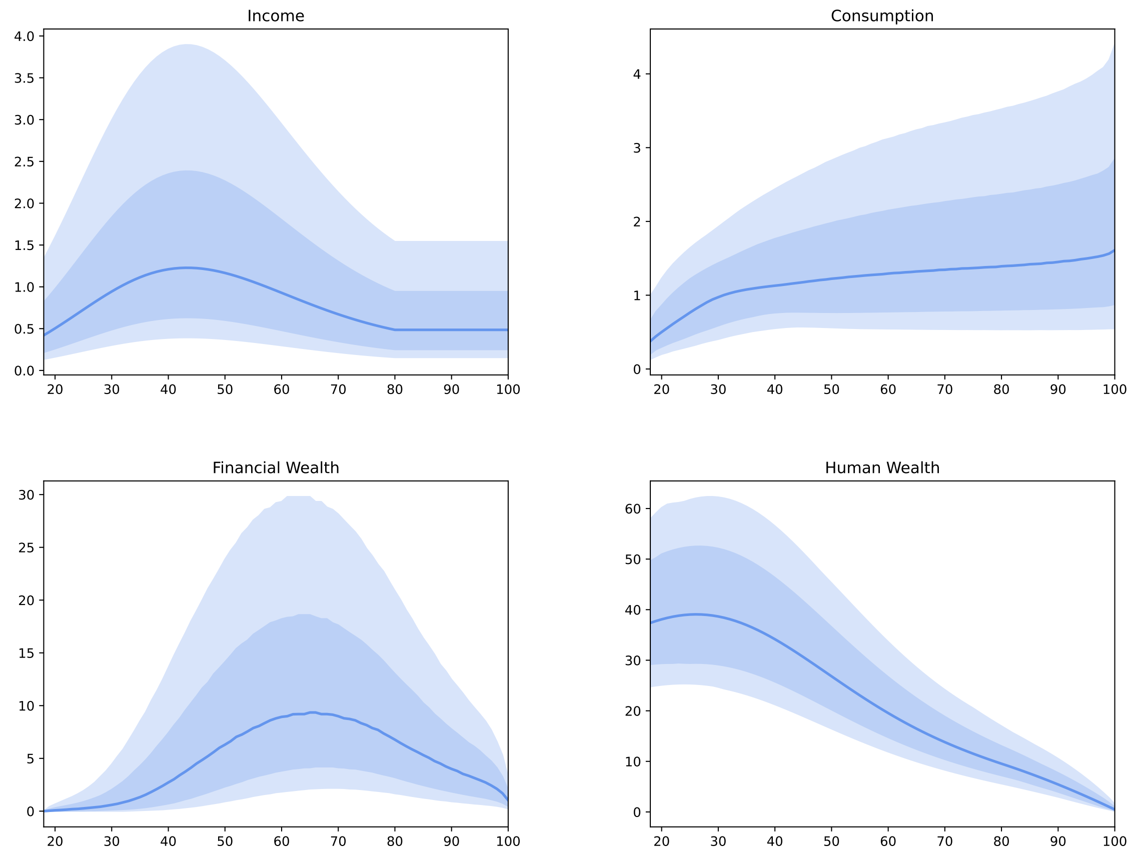
We begin by describing the properties of the model in its stationary distribution under the high interest rate regime. Figure 6 displays the life cycle profiles of income, consumption, financial wealth, and human wealth. The axes are normalized such that 1 represents the typical income during working life. Income inequality is increasing over the life cycle because of the accumulation of income shocks and because of the increase in average income over the life cycle profile. The income inequality drops after retirement but is still non-negligible since agents have heterogeneous retirement income and still face some income risk.

Turning to wealth, financial wealth in the bottom left panel increases in preparation for retirement, and is subsequently run down during retirement. Financial wealth inequality rises and falls over the life cycle. Human wealth in the bottom right panel is decreasing in age. There are two effects at play. Human wealth rises as the households' highest-earning periods are brought closer to the present. Human wealth falls due to the overall decrease in the remaining periods of work. The latter effect dominates. Total wealth consists almost exclusively of human wealth when young. As households age and accumulate financial wealth, a larger share of total wealth becomes financial wealth. However, human wealth remains a large component of total wealth throughout the life-cycle.

Figure 7 displays the Lorenz curves for consumption and wealth for all households (in all groups), and reports the Gini coefficients. The model generates a Gini coefficient for (after-transfer) household income of 0.544. Consumption inequality (not plotted) closely tracks income inequality and has a Gini coefficient of 0.496. Financial wealth is much more unequally distributed than human wealth or total wealth. The Gini coefficients of human and total wealth are 0.419 and 0.429,

[14] Our results are not sensitive to this assumption. Future extensions could add an operative bequest motive. They would make the life-cycle model closer to the infinite-horizon model of the previous section.

Figure 6: Life Cycle Profiles

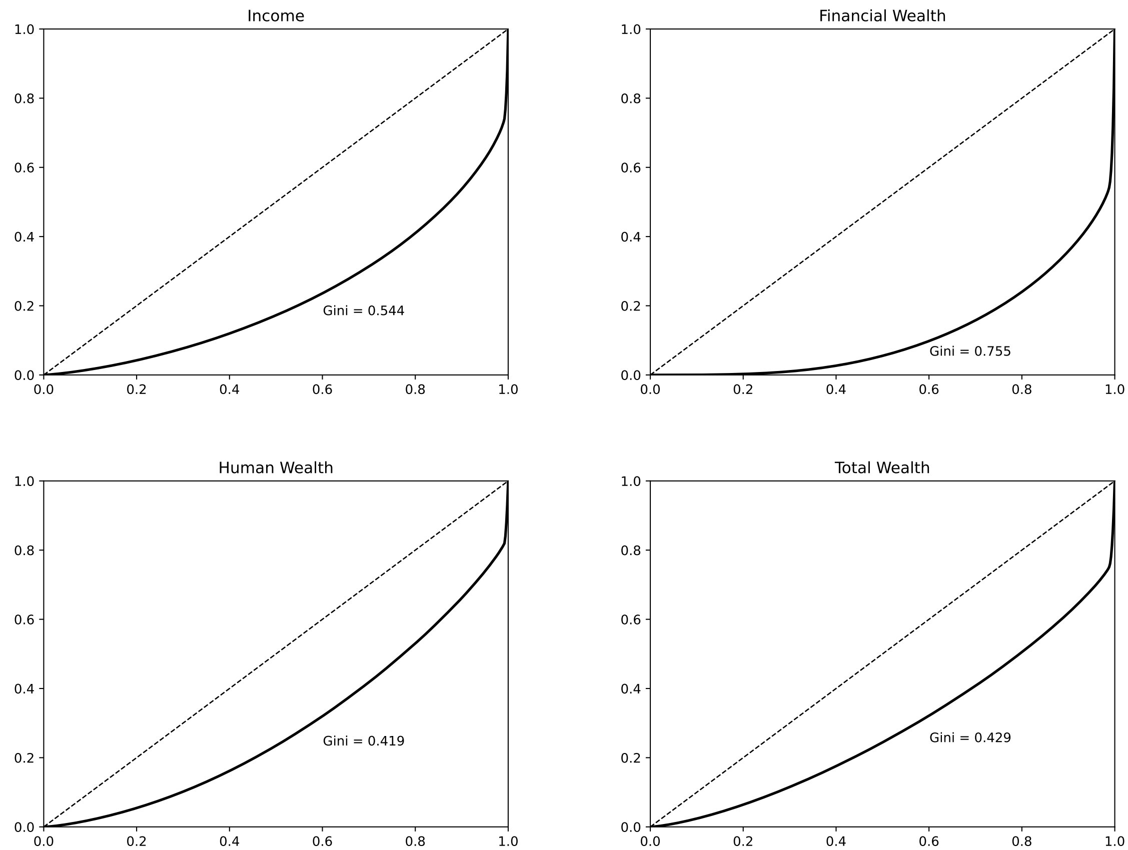


Note: This figure plots the life cycle profiles by age for the all agents of all groups combined. The axes are normalized so that the average income across all agents of all ages is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands. Although agents in the model have a maximum age of 100, we truncate the plot at age 90 due the relatively small sample of agents surviving past this age.

compared to the Gini of financial wealth of 0.755. The low total wealth inequality arises from (i) the importance of human wealth in total wealth, and (ii) the negative cross-sectional correlation between financial and human wealth.

Figure 8 displays the duration of human and total wealth by age. Human wealth represents a claim on lifetime income whereas total wealth represents a claim on lifetime consumption. Both of these durations are similar because of the importance of human wealth in total wealth. These durations are high when young, around 30, and drop rapidly as age increases, since there are fewer years of life remaining to earn labor/pension income.

Figure 7: Lorenz Curves



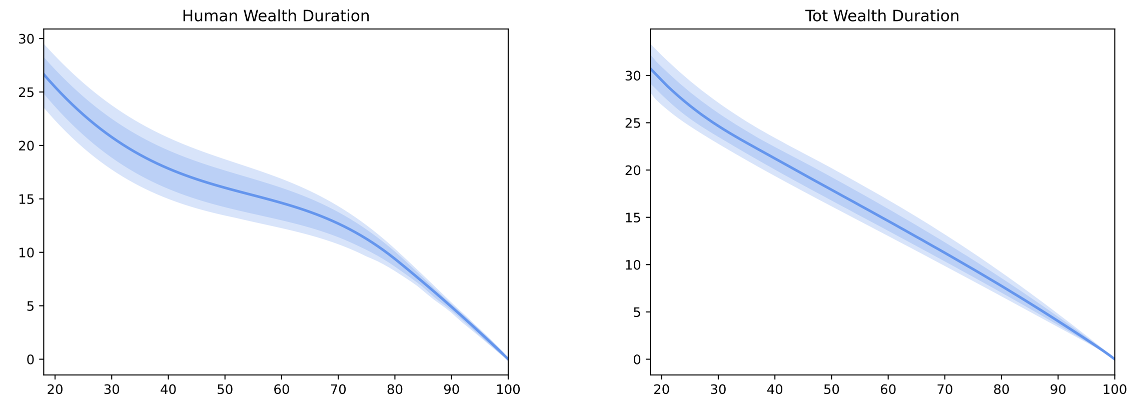
Note: This figure plots the Lorenz curve for each variable, obtained from a long simulation of the model.

6.3 Change to Low Interest Rates

In this section we apply the main experiment of an unanticipated, permanent decline in the real interest rate from 4.82% to 0.34% in the growing economy, corresponding to a decline from 2.83% to -1.57% in the stationary economy. Before turning to the response of households' actual wealth portfolios, we first note that agents' prior consumption plans may no longer be budget feasible. Thus, even if financial wealth were unchanged, the change in interest rates could have large effects on lifetime consumption and welfare.

To study the impact of the change in interest rates, we first simulate the model to generate an initial draw from the model's stationary distribution. We then change the interest rate, re-solve the model at the new interest rate, and simulate forward 50 periods (years). To isolate the effect of the rate change, we subtract out the results of the simulation with the same idiosyncratic shock realizations under the old interest rate. We do not clear the bond market in this exercise. As a result, when interest rates decline, the economy produces excess savings. We rebate those savings

Figure 8: Wealth Durations



Note: This figure plots the durations of labor income (human) wealth (left panel) and consumption (right panel). The plots display durations computed for many agents simulated from the stationary equilibrium of the model. The economy is normalized so that the average income is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands.

to households so as to keep the total resources of the economy unchanged before and after the interest rate change. Appendix E explains the details.

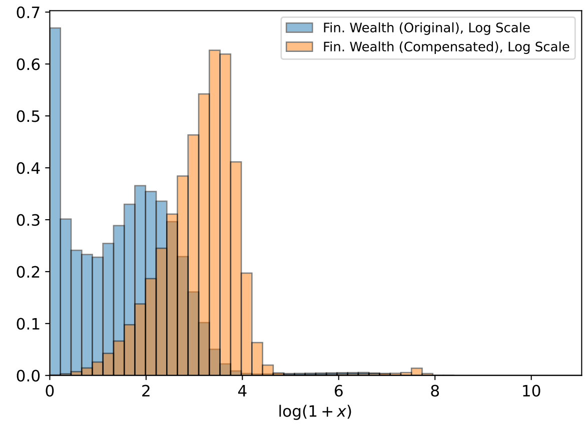
6.3.1 The Compensated Wealth Distribution

To establish an intuitive baseline that is consistent with the theoretical analysis in Proposition 5.4 of Section 5.2, we compute the change in financial wealth that would be required to maintain the prior consumption allocation in the high interest-rate economy. We refer to the counterfactual wealth allocation in which "fully hedged" households receive this financial wealth as the *compensated* financial wealth distribution (defined as $\widetilde{\theta}$ in the theory above).

The resulting distribution of financial wealth, alongside the original (pre-shock) distribution, is displayed in Figure 9. To ensure that the full distribution is visible, we display transformed variables $\log(1 + x)$ on the x-axis.[15] This comparison shows two major differences between the pre-shock and compensated distribution. First, the compensated distribution is shifted substantially to the right. Households in this economy mostly save ($c_t < y_t$) earlier in life before dissaving ($c_t > y_t$) in old age. When rates are much lower, households lose much of the effect of compound interest on their retirement savings. As a result, the aggregate amount of financial wealth in the compensated distribution exceeds the pre-shock total by 170.8%. As can be seen from the plot, this rightward shift extends up to the very top, implying that even the wealthiest individuals must be compensated with additional financial assets to attain their old consumption plans. Indeed, more

[15]Because many agents have zero financial wealth, a standard log transform would be inappropriate in this context.

Figure 9: Histogram, Compensated vs. Original Financial Wealth Distribution



Note: This plot displays the distribution of financial wealth under the stationary distribution and under the compensated distribution drawn from the stationary distribution of the economy. The x-axis displays a transformation $\log(1+x)$ of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

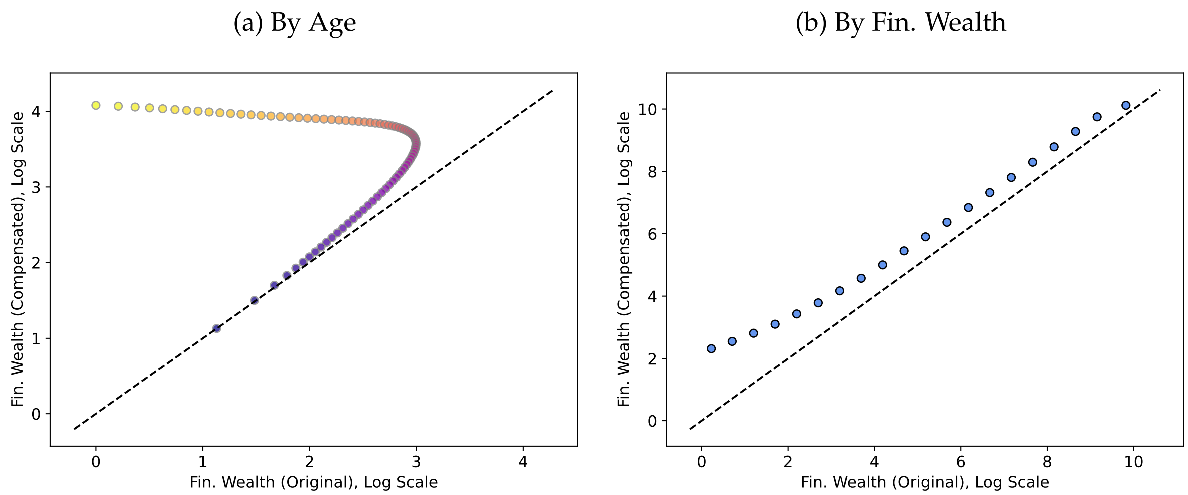
than one third (32.1%) of new financial wealth accrues to top-1% financial wealth holders under the compensated distribution.

Second, although the wealthiest gain under this compensated distribution, the financial wealth Gini falls substantially in the compensated distribution, as the less wealthy gain proportionally more. Visually, while the original high interest-rate distribution of financial wealth is heavily right-skewed, the compensated distribution is actually left-skewed. Quantitatively, the share of financial wealth held by the top-1% decreases from 44.1% in the baseline economy to 35.4% in the compensated economy.

To see why inequality falls in the compensated distribution, we can turn to Figure 10. Panel (a) compares the original (horizontal axis) and compensated financial wealth distributions (vertical axis) by age. The youngest agents (light/yellow) in the top left have close to zero financial wealth in the original distribution, but require the most financial wealth in the compensated distribution. As households age, their actual wealth initially increases, but their compensated wealth falls. Finally, late in life, both actual and compensated wealth fall rapidly toward zero, with the actual and compensated distributions close to coinciding for these older households.

This result is perhaps surprising, since the young have virtually their entire asset portfolio invested in human wealth. Because human wealth has a very long duration (left panel of Figure 8), it is well-hedged against interest rate changes. The key challenge the young face in a low interest rate environment, however, is not from their current portfolio, but their future portfolios. Due to the life cycle profile of income, the young plan to save during middle age, then dissave during retirement. Under a low interest rate, the young will be unable to accumulate enough

Figure 10: Scatterplots, Compensated vs. Original Financial Wealth Distribution



Note: Panel (a) plots the distribution of original financial wealth against the distribution of compensated financial wealth by age. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Both variables are plotted using the transform $\log(1+x)$. The dashed line represents equality between the original and compensated distributions. Panel (b) plots the same distribution by bins of original financial wealth in place of age.

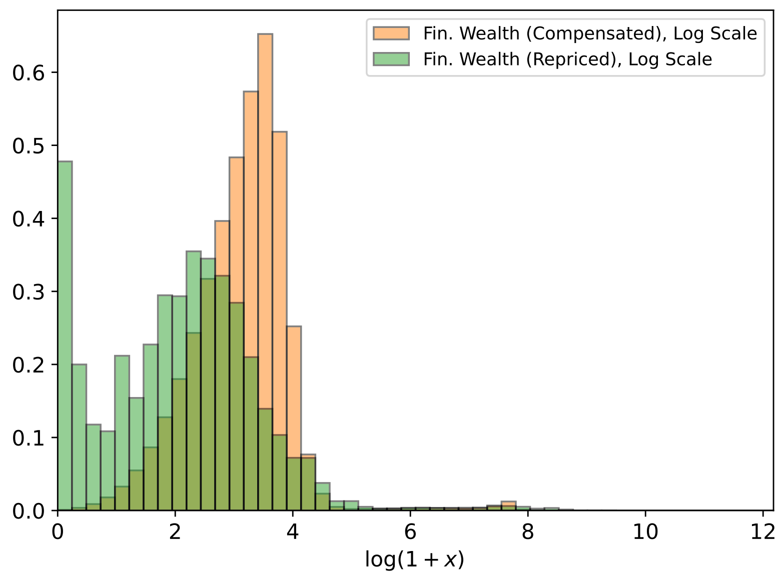
interest on their future savings, making their original consumption plans unattainable without large infusions of financial wealth today. In contrast, older agents have already benefited from the higher rate of return in accumulating their retirement assets, while the oldest are dissaving, consuming principal rather than interest. These households are less affected by the loss of high-return investment opportunities, and require little compensation.

Panel (b) aggregates over ages to present the total compensation required for various levels of pre-shock financial wealth. The lowest levels of financial wealth mix young agents who have not begun saving with old agents who are spending down assets late in life. As a result, this group mixes over agents requiring the largest and smallest amounts of compensation. Quantitatively, the young make up a disproportionate share of this group and dominate the aggregate result, so that the least wealthy agents in this economy require the most compensation, measured as the vertical distance from the dot to the dashed 45-degree line. As wealth increases, we move toward the middle-aged individuals in the economy, who require a non-zero level of compensation, but less than those at the bottom of the wealth distribution. Finally, the wealthiest agents in the top bin, whose wealth is more driven by their income realizations than by demographics, also require a strictly positive level of compensation, but less than that of the least wealthy.

6.3.2 The Repriced Wealth Distribution

Having computed the compensated financial wealth distribution required to keep consumption plans constant, we can compare it to the financial wealth distributions that actually results under low interest rates. We discussed this repriced distribution earlier in Section 3.4. Figure 11 plots both repriced and compensated distributions in one graph. Lower interest rates increase aggregate financial wealth by 116.2%, less than the increase in aggregate wealth required under the compensated distribution (170.8%). The compensated and repriced distributions display strikingly different shapes, with the repriced distribution leaving many more agents at low wealth levels.

Figure 11: Histograms, Repriced Financial Wealth Distribution

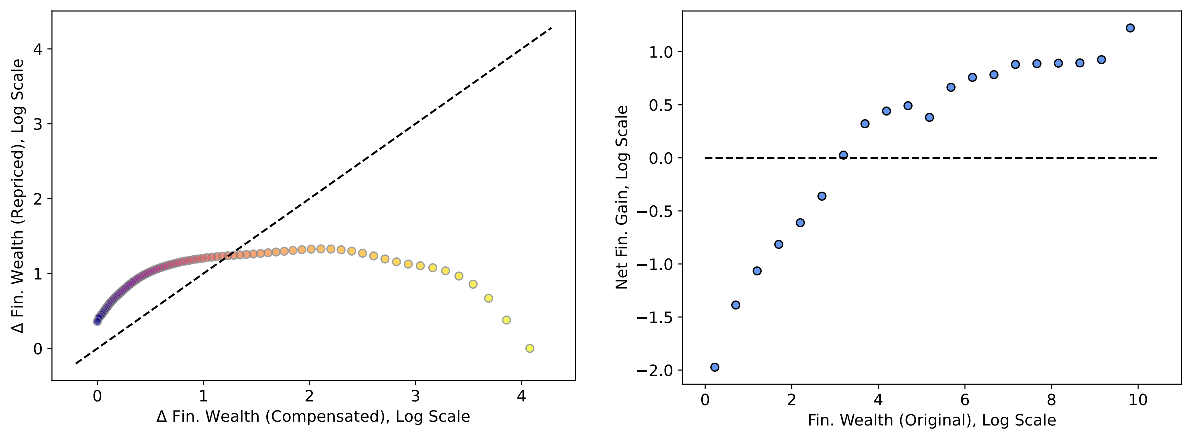


Note: This plot displays the distribution of financial wealth under the repriced distribution, compared to the original distribution and compensated distribution. All distributions are drawn from the stationary distribution of the economy. The x-axis displays a transformation $\log(1 + x)$ of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

Figure 12 compares changes in the repriced vs. compensated distributions by age in Panel (a) and by wealth in Panel (b). Panel (a) shows that repricing delivers virtually no additional financial wealth to the young, despite their large need for compensating transfers. In contrast, the old are, if anything, slightly over-hedged, receiving more wealth under repricing than needed to afford their former consumption plan. These are the points above the 45-degree line. Panel (b) displays the net gain from repricing, defined as the change in repriced wealth net of the change in compensated wealth. The figure reinforces this finding, showing that only the wealthiest agents gain on net from repricing, while the least wealthy experience a large net loss from the interest rate change, as

Figure 12: Scatterplots, Repriced Financial Wealth Distribution

(a) Compensated vs. Repriced | (b) Scatter: Repriced Gain



Note: This plot displays the distribution of financial wealth under the repriced distribution, compared to the compensated distribution. Panel (a) displays the change in financial wealth relative to the original distribution for the compensated (x-axis) and repriced (y-axis) distributions. Both axes display a transformation $\log(1 + x)$ of the original data. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Panel (b) displays original financial wealth on the x-axis and the net financial gain (repriced minus compensated wealth) on the y-axis. The x-axis displays the transform $\log(1 + x)$, while the y-axis displays the difference in transformed values. Each dot represents one bin from the original wealth distribution. All distributions are drawn from the stationary distribution of the economy.

repricing fails to appropriately compensate these households.

6.3.3 Financial and Total Wealth Inequality

Our model's combined implications for inequality following a fall in interest rates are summarized in Table 3.

As discussed before, the repriced distribution produces a realistic level of financial wealth inequality both for the pre-shock and the post-shock periods, generating a 13.2pp rise in the top-10% financial wealth share, which is even larger than the observed increase of 8.5pp. The repriced distribution also generates a large increase in the financial wealth gini of 0.063 and a large increase in the top-1% financial wealth share of 15.2pp. In short, the combination of lower expected returns on financial assets and heterogeneity in financial durations is quantitatively strong enough to explain (more than) all of the rise in financial wealth inequality in the data.

The last column of Table 3 shows that the compensated distribution, which allows households to afford their prior consumption plans, features a major *decrease* in inequality. The top-10% financial wealth share falls by 14.1pp compared to pre-shock levels, the top-1% by 8.7pp and the Gini coefficient by . This suggests that the actual allocations in the data failed to fully compensate younger and less wealthy individuals, leaving them less well off than they were prior to the rate shock.

Turning to the center panel of Table 3, we observe that all three human wealth inequality indicators are much lower than their financial wealth inequality counterparts in the initial distribution. Lower interest rates modestly increase the human wealth Gini from 0.419 to 0.452. Younger households own most of the human wealth, and have a high duration of human wealth. The interest rate decline generates the largest increase for the highest-human wealth households, explaining the rise in human wealth inequality. However, the 0.033 increase in the human wealth Gini is much smaller than the size of the 0.082 increase in the financial wealth Gini implied by the model. The model predicts a substantial decline in the top-1% human wealth share, which spills over to a modest decline in the top-10% share. The top percentile of human wealth contains many households who currently are in the superstar income state. Since that state arrives at random times in the life-cycle, ends with 2.5% probability each period, the human wealth duration of the superstars is lower than that of typical young households.[16] Hence, a decline in interest rates lowers the top-1% human wealth share.

The bottom panel of Table 3 reports on total wealth inequality, where total wealth is the sum of financial and human wealth. Since human wealth is by far the largest component of total wealth for most households, the total wealth Gini (0.429) is close to the human wealth Gini (0.419) and much lower than the financial wealth Gini (0.755). When interest rates decline, the total wealth Gini rises by 0.055 , a magnitude much lower than the rise in the financial wealth Gini. At the top of the wealth distribution, the changes in inequality are even smaller. The top-10% total wealth share of the repriced distribution rises by 1.4pp, far less than the 13.2pp rise in the corresponding financial wealth share. The top-1% total wealth share rises by a similar 1.8pp, far below the 15.2pp increase in the top-1% financial wealth share.

The behavior of the top total wealth percentile in response to an interest rate decline can be thought of as the composition of the responses of two types of households in the top 1%. The first group consists of older households who hold most of their wealth in financial wealth. These households have typically saved for a long time, and likely entered the superstar state sometime in the past, but have since transitioned out of it. The second cluster are households who currently are in the superstar state. They are younger on average and have much lower ratios of financial to total wealth. The wealth dynamics of the former cluster are governed by the dynamics of the top-1% financial wealth share, which increases sharply, while the wealth dynamics of the second cluster are governed by the dynamics of the top-1% human wealth share, which falls sharply. The effect of the first cluster dominates, and on net, there is a modest increase in the total wealth share of the top-1%. The main take-away is that top total wealth inequality does not rise nearly as

[16]The average human wealth duration of households in the superstar state is 12.0 compared to 17.5 for those not in the superstar state. Intuitively, the exit rate acts as an additional discount rate which lowers the duration. Moreover, when younger agents enter the superstar state, it pulls forward their income profile, again lowering its duration.

much as top financial wealth inequality when rates decline. Since consumption is ultimately what matters to the households in the model, and total wealth is the present value of consumption, the most relevant measure of wealth inequality has changed less than inequality in the more easily measured financial wealth data.

Finally, we note that the repriced distribution for total wealth features more inequality than the compensated distribution. Abstracting from incentive effects—which may well be very important—progressive (total) wealth taxation would help move the economy under the repriced distribution closer to that under the compensated distribution.

7 Conclusion

A persistent decline in real interest rates, like the one experienced in much of the world between the 1980s and the 2010s, naturally leads to a rise in financial wealth inequality. Households whose wealth is predominantly made up of financial rather than human wealth, and particularly those with short-maturity assets, must increase savings to be able to afford the same consumption plan. We show how a standard incomplete markets Bewley model predicts that a decline in rates increases financial wealth inequality. We establish that households display large heterogeneity in the duration of their financial wealth portfolio. Once the observed positive correlation between financial wealth and financial wealth duration is taken into account, the model that feeds in the observed decline in interest rates explains all of the rise in financial wealth inequality. Human wealth inequality is much lower than financial wealth inequality, and increases by much less when rates decline. Since human wealth represents a majority of total wealth, the effect of lower rates on top total wealth shares is modest. While most households have been made worse off by the decline in interest rates, due to imperfectly hedged portfolios of human and financial wealth, the costs have fallen disproportionately on young and low-wealth households.

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A Affine Asset Pricing Model

This appendix develops a reduced-form asset pricing model. The asset pricing model is used for three main purposes. First, to compute long-term real bonds yields, the cost of a 30-year real annuity, and expected returns on stocks and housing wealth. Second, to compute the McCauley duration of the aggregate stock market, small stocks, and real estate wealth in a manner that is consistent with the history of bond and stock prices. Third, the model delivers the price and duration of a claim to aggregate consumption and to aggregate labor income.

The asset pricing model in the class of exponentially-affine SDF models. A virtue of the reduced-form model is that it can accommodate a substantial number of aggregate risk factors. We argue that it is important to go beyond the aggregate stock and bond markets to capture the risk embedded in households' financial asset portfolios as well as the aggregate risk in consumption and labor income claims. Similar models are estimated in Lustig et al. (2013); Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019); Gupta and Van Nieuwerburgh (2021).

A.1 Setup

A.1.1 State Variable Dynamics

Time is denoted in quarters. We assume that the $N \times 1$ vector of state variables follows a Gaussian first-order VAR:

$$\boldsymbol{z}_t = \boldsymbol{\Psi} \boldsymbol{z}_{t-1} + \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\varepsilon}_t, \tag{13}$$

with shocks $\boldsymbol{\varepsilon}_t \sim i.i.d. \mathcal{N}(0, I)$ whose variance is the identity matrix. The companion matrix Ψ is a $N \times N$ matrix. The vector z is demeaned. The covariance matrix of the innovations to the state variables is Σ; the model is homoscedastic. We use a Cholesky decomposition of the covariance matrix, $\Sigma = \Sigma^{\frac{1}{2}} \Sigma^{\frac{1}{2}\prime}$, which has non-zero elements only on and below the diagonal. The Cholesky decomposition of the residual covariance matrix allows us to interpret the shock to each state variable as the shock that is orthogonal to the shocks of all state variables that precede it in the VAR. We discuss the elements of the state vector and their ordering below. The (demeaned) one-quarter bond nominal yield is one of the elements of the state vector: $y_{t,1}^{\$} = y_{0,1}^{\$} + e_{yn}' z_t$, where $y_{0,1}^{\$}$ is the unconditional average 1-quarter nominal bond yield and e_{yn} is a vector that selects the element of the state vector corresponding to the one-quarter yield. Similarly, the (demeaned) inflation rate is part of the state vector: $\pi_t = \pi_0 + e_\pi' z_t$ is the (log) inflation rate between $t-1$ and t. Lowercase letters denote logs.

A.1.2 Stochastic Discount Factor

The nominal SDF $M^{\$}_{t+1} = \exp(m^{\$}_{t+1})$ is conditionally log-normal:

$$m^{\$}_{t+1} = -y^{\$}_{t,1} - \frac{1}{2}\mathbf{\Lambda}'_t\mathbf{\Lambda}_t - \mathbf{\Lambda}'_t\varepsilon_{t+1}. \tag{14}$$

Note that $y^{\$}_{t,1} = -\mathbb{E}_t[m^{\$}_{t+1}] - 0.5\text{Var}_t[m^{\$}_{t+1}]$. The real log SDF $m_{t+1} = m^{\$}_{t+1} + \pi_{t+1}$ is also conditionally Gaussian. The innovations in the vector ε_{t+1} are associated with a $N \times 1$ market price of risk vector $\mathbf{\Lambda}_t$ of the affine form:

$$\mathbf{\Lambda}_t = \mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 z_t. \tag{15}$$

The $N \times 1$ vector $\mathbf{\Lambda}_0$ collects the average prices of risk while the $N \times N$ matrix $\mathbf{\Lambda}_1$ governs the time variation in risk premia. Asset pricing amounts to estimating the market prices of risk $(\mathbf{\Lambda}_0, \mathbf{\Lambda}_1)$. We specify the moment conditions used to identify the market prices of risk below.

A.1.3 State Vector Elements

The state vector contains the following $N = 22$ variables, in order of appearance: (1) real GDP growth, (2) GDP price inflation, (3) the nominal short rate (3-month nominal Treasury bill rate), (4) the spread between the yield on a five-year Treasury note and a three-month Treasury bill, (5) the log price-dividend ratio on the CRSP value-weighted stock market, (6) the log real dividend growth rate on the CRSP stock market. Elements 7, 9, 11, and 13 are the log price-dividend ratios on the first size quintile of stocks (small), the first book-to-market quintile of stocks (growth), the fifth book-to-market quintile of stocks (value), and a listed infrastructure index (infra). Elements 8, 10, 12, and 14 are the corresponding log real dividend growth rates. Element 15 is the log price-dividend ratio on housing wealth, element 16 is log real dividend growth on housing wealth. Finally, the state vector contains the log change in the consumption/GDP ratio Δcx in 17th, the log change in the log labor income/GDP ratio Δlx in 18th, the log level of the consumption/GDP ratio cx in 19th, and the log level of the labor income/GDP ratio lx in 20th position.

$$\begin{aligned} z_t = \quad & \Big[\pi_t, x_t, y^{\$}_{t,1}, y^{\$}_{t,20} - y^{\$}_{t,1}, pd^m_t, \Delta d^m_t, pd^{small}_t, \Delta d^{small}_t, \\ & pd^{growth}_t, \Delta d^{growth}_t, pd^{value}_t, \Delta d^{value}_t, pd^{infra}_t, \Delta d^{infra}_t \\ & pd^{hw}_t, \Delta d^{hw}_t, \Delta cx_{t+1}, \Delta lx_{t+1}, cx_{t+1}, lx_{t+1}\Big]'. \end{aligned} \tag{16}$$

This state vector is observed at quarterly frequency from 1947.Q1 until 2019.Q4 (292 observations). This is the longest available time series for which all variables are available. Inflation is the log change in the GDP price deflator. For the yields, we use the average of daily Constant

Maturity Treasury yields within the quarter. All dividend series are deseasonalized by summing dividends across the current month and past 11 months. Small stocks are the bottom 20% of the market capitalization distribution, growth stocks the bottom 20% of the book-to-market distribution, and value stocks the top 20% of the book-to-market distribution. The infrastructure stock index is measured as the value-weighted average of the eight relevant Fama-French industries (Aero, Ships, Mines, Coal, Oil, Util, Telcm, Trans). We subtract inflation from all nominal dividend growth rates to obtain real dividend growth rates.

Dividend growth on housing wealth is measured as housing services consumption growth from the Bureau of Economic analysis Table 2.3.5. The price-dividend ratio is the ratio of owner-occupied housing wealth from the Financial Accounts of the United States Table B.101.h divided by housing services consumption. The resulting price-dividend ratio on housing wealth averages 16.1 (for annualized dividends) between 1947 and 2019. We subtract inflation from dividend growth on housing wealth and we also subtract 0.6% per quarter to reflect the fact that the size of the housing stock is growing and we are only interested in the rental price change, not the change in the quantity of housing. The resulting real rental growth rate is 1.82% per year, which is in line with (and still on the higher end of the numbers reported in) the literature.

Aggregate consumption is measured as non-durables plus services plus durable services consumption. Durable services consumption is constructed as the depreciation rate (20%) multiplied by the stock of durables. The stock of durables itself is computed using the perpetual inventory method. This series is divided by nominal GDP and logs are taken.

Aggregate labor income is measured as wages and salaries plus business income (proprietors' income with inventory valuation and capital consumption adjustments) plus transfer income (personal current transfer receipts) minus taxes (Personal current taxes and Contributions for government social insurance, domestic). This series is divided by nominal GDP and logs are taken. Real consumption growth can then be written as the sum of real GDP growth plus the change in the consumption/GDP ratio:

$$\Delta c^a_{t+1} = x_{t+1} + \Delta cx_{t+1}$$

and similar for labor income growth.

All state variables are demeaned with the observed full-sample mean. The first 18 equations of the VAR are estimated by OLS equation by equation. We recursively zero out all elements of the companion matrix Ψ whose t-statistic is below 2.2. The resulting point estimates for Ψ and $\Sigma^{\frac{1}{2}}$ are reported below.

The dynamics of cx are pinned down by the dynamics of Δcx:

$$cx_{t+1} = cx_t + \Delta cx_{t+1} = \left(e_{cx} + e_{cxgr}\Psi\right)' z_t + e_{cxgr}\gamma^{\frac{1}{2}}\varepsilon_{t+1}$$

Therefore the 19st row of Ψ is identical to the 17th row, except that $\Psi(19,19) = \Psi(17,19) + 1$.

Similarly, the 20th row of Ψ is identical to the 18th row, except that $\Psi(20,20) = \Psi(18,20) + 1$. The innovations to the 19th and 20th row are not independent innovations but determined by the innovations that precede it. The level variables cx and lx are only added to the VAR to enforce cointegration between consumption and GDP and between labor income and GDP. As a result of this cointegration, the aggregate consumption and labor income claims will have the same aggregate risk as the GDP claim.

A.2 Estimation

A.2.1 Bond Pricing

In this setting, nominal bond yields of maturity τ are affine in the state variables:

$$y^{\$}_{t,\tau} = -\frac{1}{\tau}A^{\$}_{\tau} - \frac{1}{\tau}\left(B^{\$}_{\tau}\right)' z_t.$$

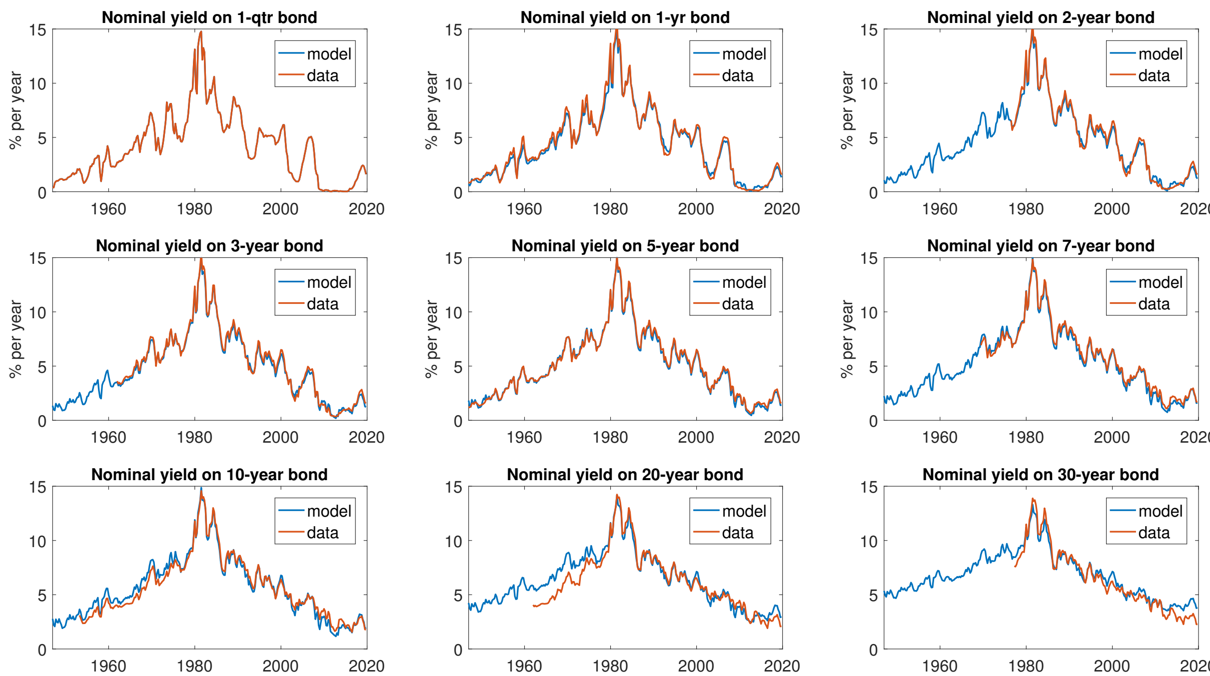
The scalar $A^{\$}(\tau)$ and the vector $B^{\$}_{\tau}$ follow ordinary difference equations (ODE) that depend on the properties of the state vector and on the market prices of risk. Real bond yield are also exponentially affine with coefficients that follow their own ODEs. We will price the cross-section of nominal and real bond yields (price levels), putting more weight on matching the time series of one- and twenty-quarter nominal bond yields since those yields are part of the state vector $\boldsymbol{z}_t$. We also fit the dynamics of 20-quarter nominal bond risk premia (price changes).

Figure A1 plots the nominal bond yields on bonds of maturities 1 quarter, 1-, 2-, 3-, 5-, 7-, 10-, 20-, and 30-years. These are all available bond yields in the data. The 20-, and 3-year bond yields are not available in parts of the sample, but the estimation minimizes the distance between observed and model-implied yields for every period where data is available. The model matches the time series of bond yields in the data closely. It matches nearly perfectly the 1-quarter and 5-year bond yield which are part of the state space.

Figure A2 shows that the model also does a good job matching real bond yields. These yields are available over a much shorter sample in the data, and we only plot the relevant subsample for the model-implied yields as well.

The top panels of Figure A3 show the model's implications for the average nominal (left panel) and real (right panel) yield curves at longer maturities. These long-term yields are well behaved. The bottom left panel shows that the model matches the dynamics of the nominal bond risk premium, defined as the expected excess return on five-year nominal bonds. The compensation for interest rate risk varies substantially over time, both in data and in the model. The bottom right panel shows a decomposition of the yield on a five-year nominal bond into the five-year real bond yield, annual expected inflation over the next five years, and the five-year inflation risk premium. The importance of these components fluctuates over time. This graph shows the secular rise and fall of real bond yields, with a peak in the early 1980s.

Figure A1: Dynamics of the Nominal Term Structure of Interest Rates



Note: The figure plots the observed and model-implied nominal bond yields. Data are from FRED: constant-maturity Treasury yields, daily averages within the quarter.

A.2.2 Equity Factors and Housing Wealth Pricing

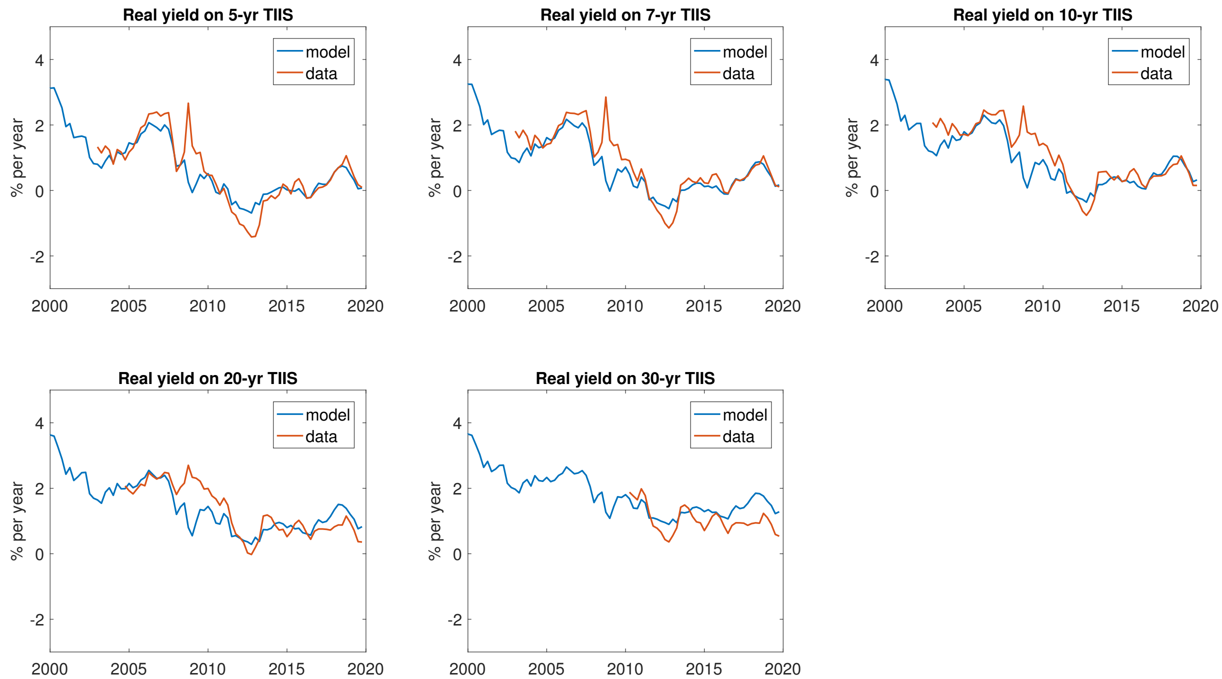
The VAR contains both the log price-dividend ratio and log dividend growth for five equity risk factors (the aggregate stock market, small stocks, growth stocks, value stocks, and infrastructure stocks), and residential real estate wealth. Together these two time-series imply a time-series for log returns through the definition of a log stock return. Hence, the VAR implies linear dynamics for the expected excess stock return, or equity risk premium, for each of these seven assets. We estimate market prices of risk to match the VAR-implied risk premium levels and dynamics.

The price of a stock equals the present-discounted value of its future cash-flows. By value-additivity, the price of the aggregate stock index, P_t^m, is the sum of the prices to each of its future cash-flows D_t^m. These future cash-flow claims are the so-called market dividend strips or zero-coupon equity (Wachter, 2005). Dividing by the current dividend D_t^m:

$$\frac{P_t^m}{D_t^m} = \sum_{\tau=1}^{\infty} P_{t,\tau}^d \tag{17}$$

$$\exp\left(\overline{pd} + e'_{pd^m} z_t\right) = \sum_{\tau=0}^{\infty} \exp\left(A_\tau^m + B_\tau^{m\prime} z_t\right), \tag{18}$$

Figure A2: Dynamics of the Real Term Structure of Interest Rates

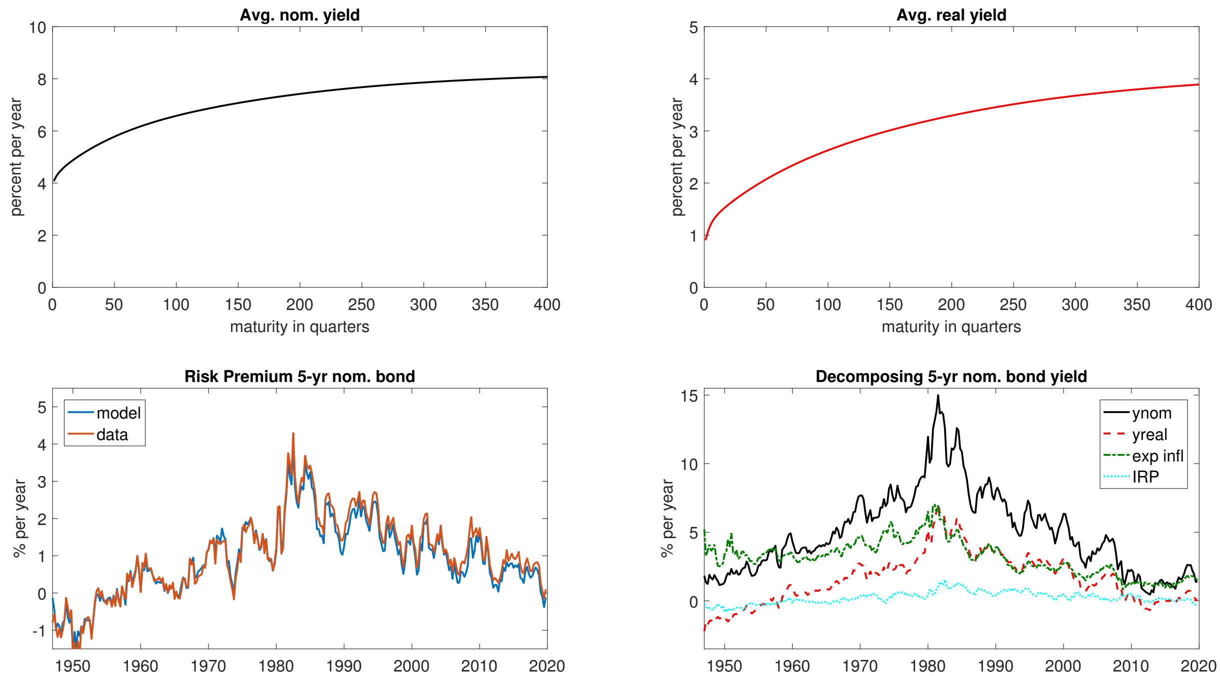


Note: The figure plots the observed and model-implied real bond yields. Data are from FRED: constant-maturity Treasury inflation-indexed bond yields, daily averages within the quarter.

where $P^d_{t,\tau}$ denotes the price of a τ-period dividend strip divided by the current dividend. The log price-dividend ratio on each dividend strip, $p^d_{t,\tau} = \log\left(P^d_{t,\tau}\right)$, is affine in the state vector and the coefficients $\left(A^m_\tau, \boldsymbol{B}^m_\tau\right)$ follow an ODE. Since the log price-dividend ratio on the stock market is an element of the state vector, it is affine in the state vector by assumption. Equation (18) restates the present-value relationship from equation (17). It articulates a non-linear restriction on the coefficients $\{(A^m_\tau, B^m_\tau)\}_{\tau=1}^{\infty}$ at each date (for each state z_t), which we impose in the estimation. Analogous present value restrictions are imposed for each of the other four equity factors, and for housing wealth.

If dividend growth were unpredictable and its innovations carried a zero risk price, then dividend strips would be priced like real zero-coupon bonds. The strips' dividend-price ratios would equal yields on real bonds with the coupon adjusted for deterministic dividend growth. All variation in the price-dividend ratio would reflect variation in the real yield curve. In reality, the dynamics of real bond yields only account for a small fraction of the variation in the price-dividend ratio, implying large prices of risk associated with shocks to dividend growth that are orthogonal to shocks to bond yields. Hence, matching price-dividend ratios (price levels) and expected returns (price changes) allow us to pin down the market prices of risk associated with orthogonal dividend growth shocks (shocks to the state variables in rows 6, 8, 10, 12, 14, 16, and 18 of the

Figure A3: Long-term Yields and Bond Risk Premia

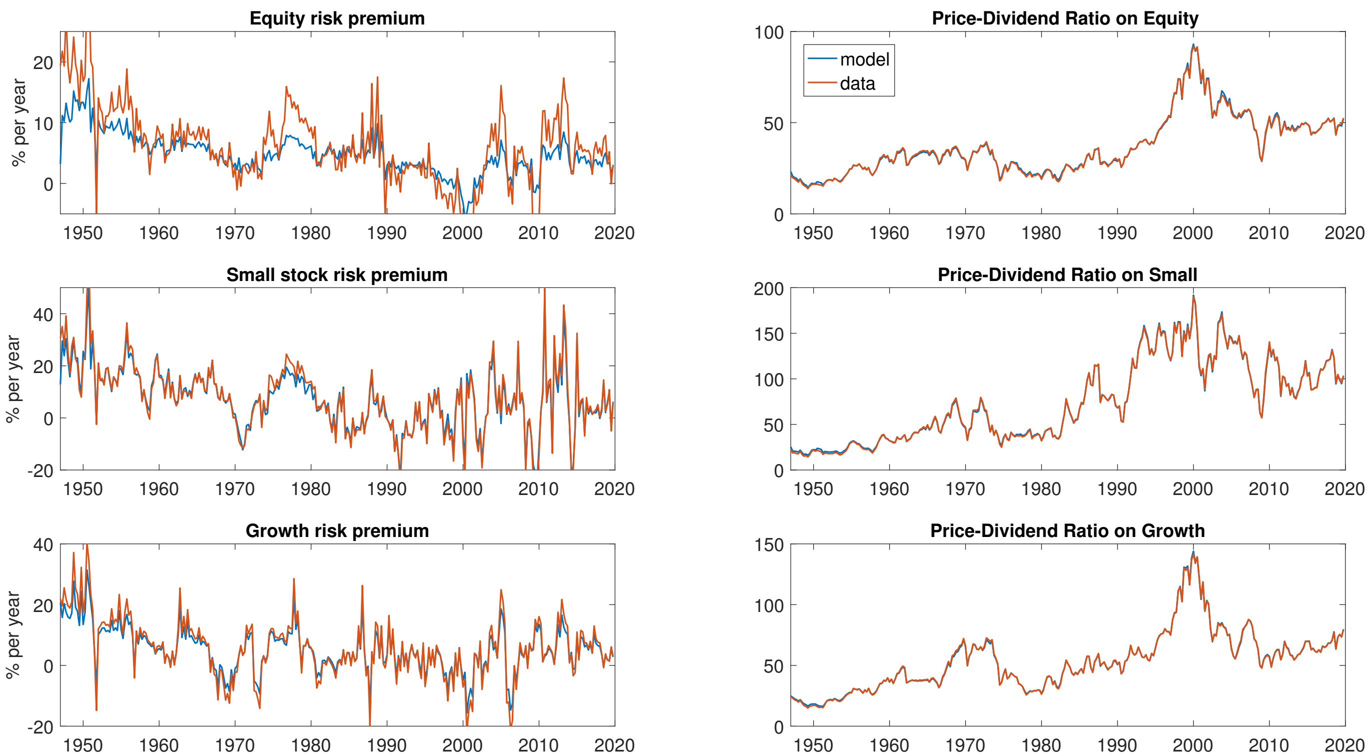


Note: The top panels plot the average bond yield on nominal (left panel) and real (right panel) bonds for maturities ranging from 1 quarter to 400 quarters. The bottom left panel plots the nominal bond risk premium in model and data. The bottom right panel decomposes the model's five-year nominal bond yield into the five-year real bond yield, the five-year inflation risk premium and the five-year real risk premium.

VAR).

Figures A4 and A5 show the equity risk premium, the expected excess return, in the left panels and the price-dividend ratio in the right panels. The various rows cover the five equity indices and the housing wealth series we price. The dynamics of the risk premia in the data are dictated by the VAR. The model chooses the market prices of risk to fit these risk premium dynamics as closely as possible alongside with the price-dividend ratio levels. The price-dividend ratios in the model are formed from the price-dividend ratios on the strips of maturities ranging from 1 to 3600 quarters, as explained above. The figure shows an excellent fit for price-dividend levels and a good fit for risk premium dynamics. Some of the VAR-implied risk premia have outliers which the model does not fully capture. This is in part because the good deal bounds restrict the SDF from becoming too volatile and extreme. We note large level differences in valuation ratios across the various stock factors, as well as big differences in the dynamics of both risk premia and price levels, which the model is able to capture well.

Figure A4: Equity Risk Premia and Price-Dividend Ratios (1/2)



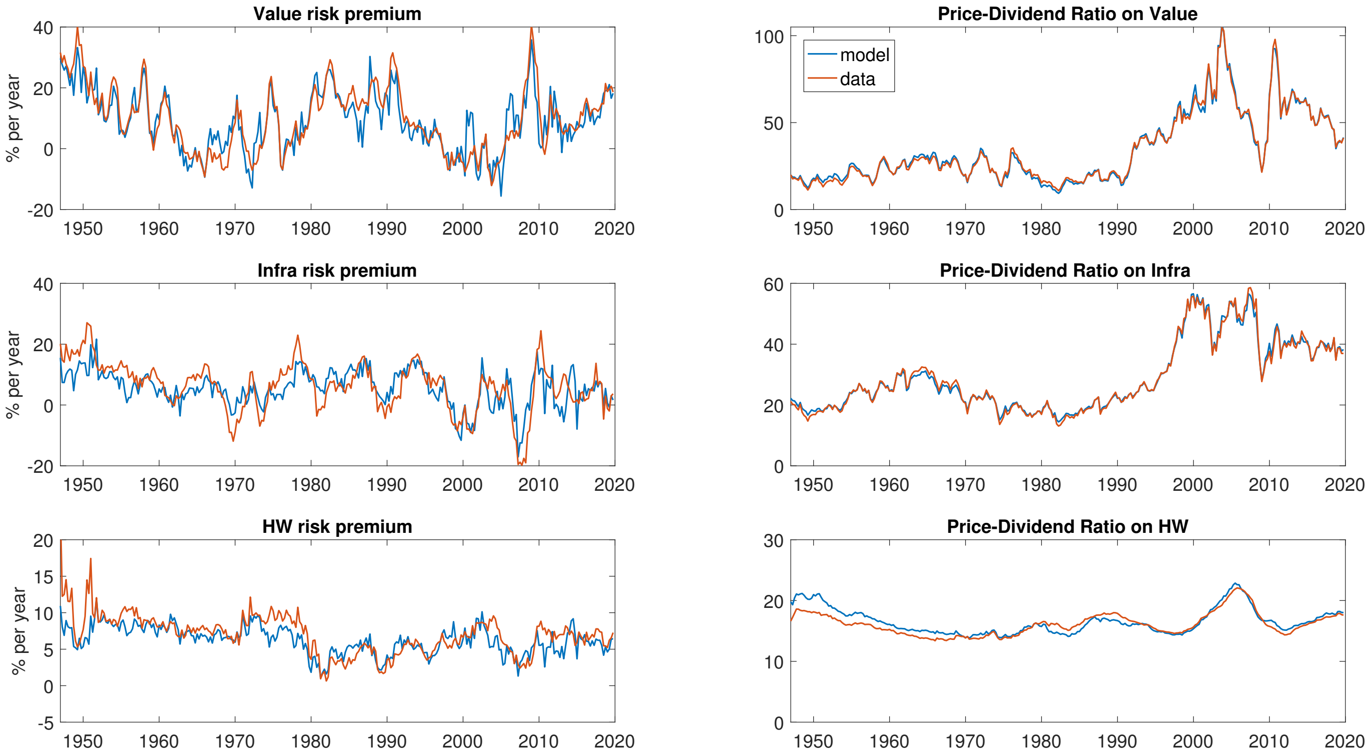
Note: The figure plots the observed and model-implied equity risk premium on the overall stock market, small stocks, and growth stocks in the left panels, as well as the corresponding price-dividend ratio in the right panels. The model is the blue line, the data are the red line.

A.2.3 Pricing Claims to Aggregate Consumption and Labor Income

Shocks to the growth rate in consumption/GDP (labor income/GDP) ratio are priced only to the extent that they are correlated with other priced sources of risk. The innovation to the change in the consumption/GDP (labor income/GDP) ratio that is orthogonal to all prior shocks is not priced. Since consumption/GDP growth and labor income/GDP growth appear last in the VAR and the model includes many sources of priced aggregate risk, those innovations are as small as possible.

Figure A6 plots the annual price-dividend ratios on the claims to GDP, aggregate consumption, and aggregate labor income. It contrasts these valuation ratios to those for the aggregate stock market, and housing wealth. The valuation ratios of GDP, aggregate consumption, and aggregate labor income claims are all highly correlated. They are high at the start of the sample, low in the early 1980s, and high at the end of the sample. Since total wealth is a claim to aggregate consumption, this suggests that expected returns on total wealth were highest in the early 1980s and have been falling ever since.

Figure A5: Equity Risk Premia and Price-Dividend Ratios (2/2)



Note: The figure plots the observed and model-implied equity risk premium on value stocks, infrastructure stocks, and housing wealth in the left panels, as well as the corresponding price-dividend ratio in the right panels. The model is the blue line, the data are the red line.

A.2.4 Cash-flow Duration

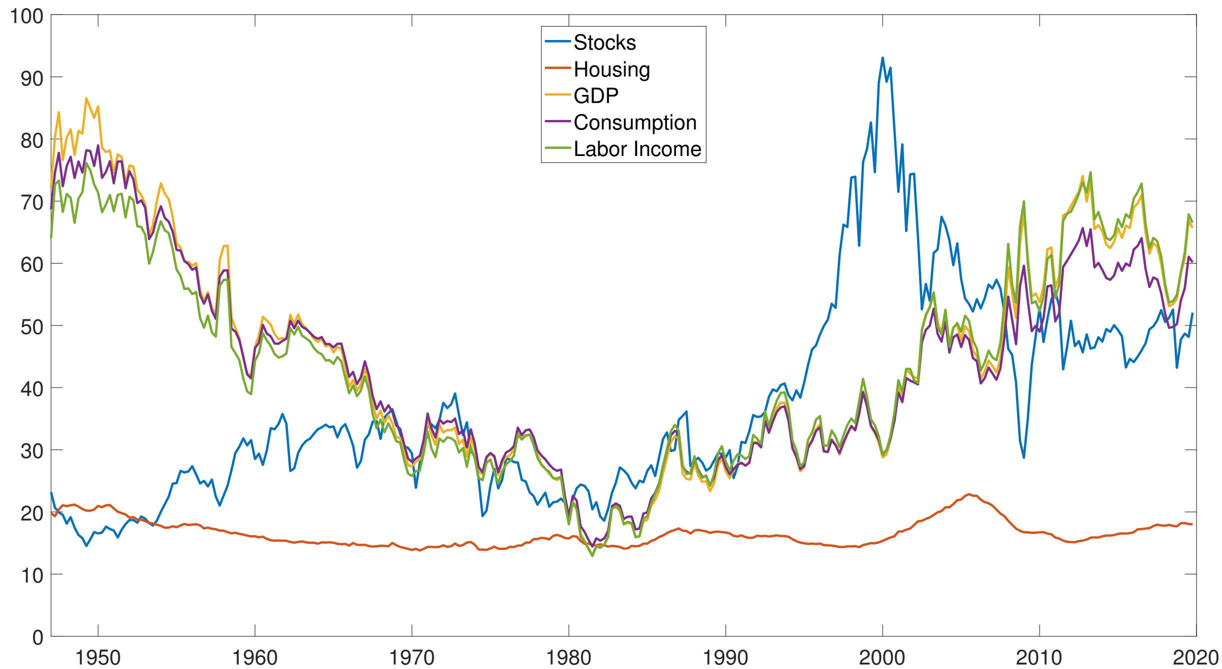
The (McCauley) duration is the weighted average time for an investor to receive cash flows. For the aggregate stock market, this measure is computed as follows:

$$D_t^{CF,m} = \sum_{\tau=1}^{\infty} w_{t,\tau}\tau, \qquad w_{t,h} = \frac{P^d_{t,\tau}}{\frac{P^m_t}{D^m_t}} = \frac{\exp\left(A^m_\tau + B^{m\prime}_\tau z_t\right)}{\exp\left(\overline{pd} + e'_{pd^m} z_t\right)}$$

where $P^d_{t,\tau}$ is the price-dividend ratio of a τ-period dividend strip. Since durations are usually expressed in years while time runs in quarters in our model, we divide by 4. Duration is defined analogously for the other four equity indices, housing wealth, and for the GDP, consumption, and labor income claims. Note that for a nominal or real zero-coupon bond of maturity τ, $D_t^{CF} = \tau$.

Figure A7 The figure plots the model-implied time series of cash-flow durations on the overall stock market, small stocks, growth stocks, value stocks, infrastructure stocks, housing wealth, the GDP claim, the aggregate consumption claim, and the aggregate labor income claim. Durations tend to be positively correlated with the price-dividend ratios: high at the start of the sample,

Figure A6: Valuation Ratios



Note: The figure plots the annual price-dividend ratios on the aggregate stock market, housing wealth, and on claims to GDP, aggregate consumption, and aggregate labor income.

lowest in the early 1980s, and high at the end of the sample. The duration of housing wealth is highest during the housing boom in 2003–2007 when the valuation ratio of housing peaks. It then falls sharply in the housing bust before rising again in the housing boom that starts in 2013.

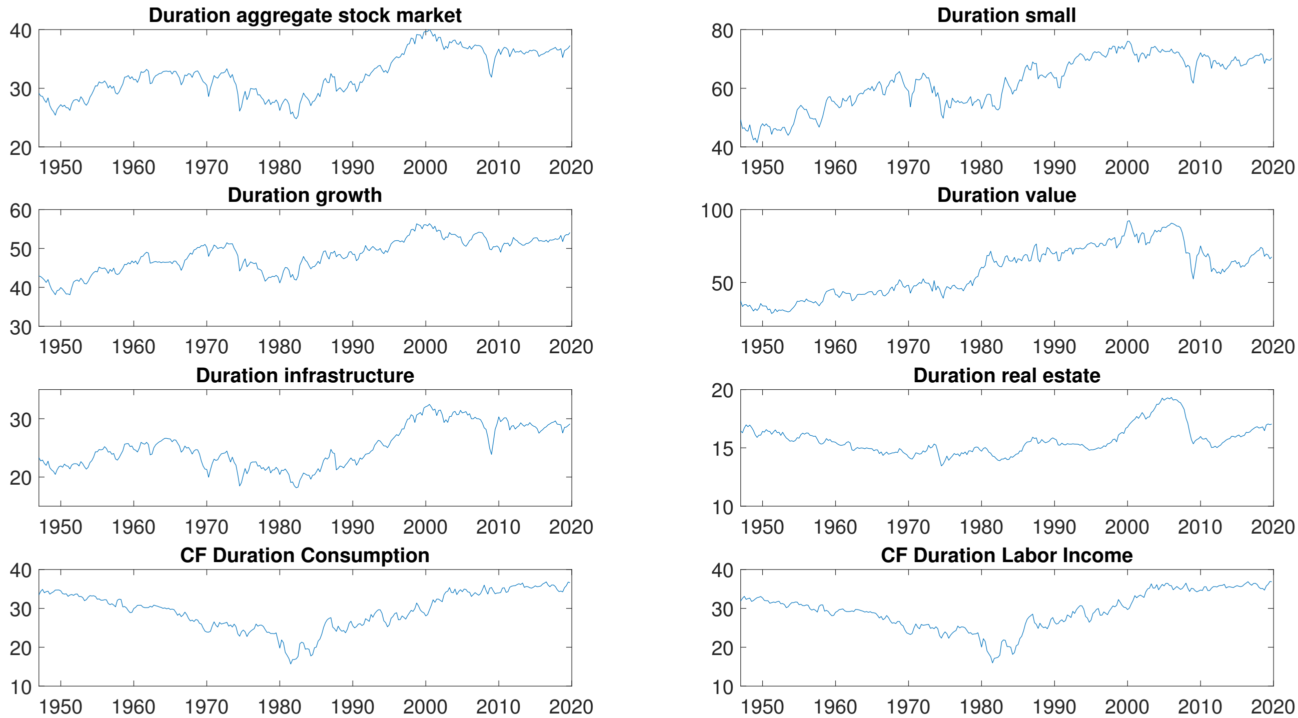
A.2.5 Market Price of Risk Estimates

The market prices of risk are pinned down by the moments discussed in the main text. Here we report and discuss the point estimates. Note that the prices of risk are associated with the orthogonal VAR innovations $\varepsilon \sim \mathcal{N}(0, I)$. Therefore, their magnitudes can be interpreted as (quarterly) Sharpe ratios. The constant in the market price of risk estimate $\widehat{\Lambda_0}$ is:

0.11	0.00	-0.36	0.06	0.00	0.43	0.00	-0.01	0.00	0.12	0.00	0.25	0.00	0.26	0.00	2.76	0.00	0.00	0.00	0.00

The matrix that governs the time variation in the market price of risk is estimated to be $\widehat{\Lambda_1} =$:

Figure A7: Cash-Flow Duration



Note: The figure plots the model-implied time series of cash-flow durations on the overall stock market, small stocks, growth stocks, value stocks, infrastructure stocks, housing wealth, the GDP claim, the aggregate consumption claim, and the aggregate labor income claim. The duration is expressed in years.

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19.3	16.5	-31.7	-250.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37.5	15.1	0.0	3.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.2	19.5	0.9	0.9
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23.3	4.0	-29.7	-160.0	-0.9	12.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	3.6	0.1	1.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-5.0	-1.6	1.5	-21.1	0.6	2.0	-0.5	8.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.9	-4.0	0.2	0.7
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.1	34.3	-23.7	14.0	0.9	-11.2	-0.1	1.8	-1.1	16.8	0.0	0.0	0.0	0.0	0.0	0.0	0.8	1.2	-0.1	-0.7
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	34.7	-11.1	34.1	0.8	-15.6	0.1	-1.1	-0.3	0.5	-2.0	0.6	0.0	0.0	0.0	0.0	-1.5	1.3	17.6	0.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10.1	46.0	-40.9	-20.8	7.6	-10.9	1.0	-2.4	-5.9	-2.3	0.5	1.6	-4.4	0.1	0.0	0.0	3.3	3.7	-5.2	-2.3
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8.6	27.2	-113.3	66.6	5.7	1.3	-2.2	-5.7	-0.4	-3.0	0.8	0.2	-3.3	0.0	1.3	0.4	0.1	-0.2	-15.0	-6.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

B Model-Free Repricing

Now that we have measured the exposure of household portfolios to changes in real rates, we can now feed in the actual path of interest rates, reprice these portfolios according to our duration measure, and compute the resulting impact on financial wealth inequality.

To begin, we use our auxiliary asset pricing model to compute implied 10-year real rates for the three years prior to each SCF wave, denoted R_t, where t belongs to the set of SCF wave years. For each household in SCF wave t, we now compute the effect on that household's net wealth of moving from interest rate R_t to rate R_{t+1}. Recall that financial wealth duration is the negative of the semielasticity of financial wealth with respect to interest rates:

$$\frac{\partial \log \theta^{i,t}}{\partial R_t} = -D^{fin}_{i,t}$$

where $\theta_{i,t}$ is financial wealth for household i at time t, and $D^{fin}_{i,t}$ is its duration. Turning this into a first-order approximation and rearranging yields

$$\theta^{repriced}_{i,t+1} \simeq \theta_{i,t} \exp\left\{-\Delta R_{t+1} \times D^{fin}_{i,t}\right\} \tag{19}$$

where $\theta^{repriced}_{i,t+1}$ is the implied value of the household's net wealth following revaluation due exclusively to the change in real interest rates, ignoring other factors such as savings that will influence the actual evolution of wealth between time t and $t+1$.

Using (19), we compute this repriced value of wealth $\theta^{repriced}_{i,t+1}$ for each household i at each date t. We next compute the impact of this change on financial wealth inequality according to

$$dS^{q,repriced}_{i,t} = S^{q,repriced}_{i,t+1} - S^{q}_{i,t}$$

where $S^q_{i,t}$ is the share of financial wealth $\theta_{i,t}$ held by the top q% of the population, and $S^{q,repriced}_{i,t}$ is the equivalent statistic using the repriced distribution $\theta^{repriced}_{i,t}$.

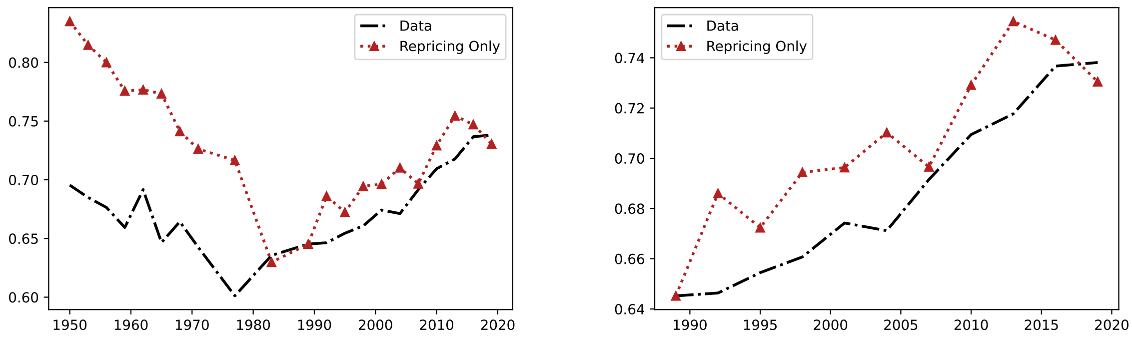
This statistic $dS^{q,repriced}_{i,t}$ reflects the impact of repricing on inequality between a single pair of SCF waves. To compute the combined impact of changes along our entire real interest rate series, we cumulate these changes as

$$\hat{S}^{q,repriced}_{i,t} = \text{const} + \sum_{\tau \le t} dS^{q,repriced}_{i,\tau}$$

and choose the constant term so that $\hat{S}^{q,repriced}_{i,t}$ is exactly equal to the actual top wealth share $S^q_{i,t}$ in the 1989 wave of the SCF.

To measure the contribution of interest rates to financial wealth inequality, Figure A8 compares

Figure A8: Top 10% Financial Wealth Shares, Actual vs. Repriced



(a) Full Sample

(b) Since 1989

our implied top-10% wealth share $\hat{S}_{i,t}^{10,repriced}$ that incorporates changes due to repricing only (red dots), against the true series $S_{i,t}^{10}$ (black line). The figure shows that repricing has been a powerful force, implying a rise of 10.2pp in the top-10% financial wealth share from 1989 to the end of the sample, which accounts for more than half of the 9.1pp rise observed in the data.

C Data Appendix

C.1 Inequality Data

The top wealth shares presented in 2 are from the World Inequality Database. The data for the U.S. are available until 2019, for the U.K. until 2012, and for France until 2014.

Our primary source of data for the top wealth shares presented in 2 is the World Inequality Database maintained by WID team. As the WID time series for the UK and France have a limited window of observation, we augment the WID estimates with additional measures of top wealth shares obtained from survey data and the Credit Suisse Global Wealth Report (Shorrocks, Davies, and Lluberas, 2020), where available and necessary. This serves to increase the size of the observation window and provides additional robustness to our results.

For the United States the WID time series provides complete coverage. In addition, we report survey estimates of top wealth shares from the Survey of Consumer Finances (SCF) and the SCF+, the database developed by Kuhn et al. (2020), from 1950 to 1983. We slightly modify the definition of total financial net-wealth by subtracting vehicles (for both the SCF and SCF+ data).[17]

For the United Kingdom the time series of top wealth shares from the WID ends in 2012. From 2012 onwards, we rely on top wealth share estimates from the Credit Suisse Global Wealth Report (Shorrocks et al., 2020). In addition, we construct estimates of the top 1% wealth share by augmenting survey microdata from the U.K. Wealth and Assets Survey (WAS) with observations from the Sunday Times Rich List to estimate the top 1% wealth share implied by fitting a Pareto tail to the wealth distribution, following Vermeulen (2018). We choose this method of estimating the top wealth share because in the periods of overlap between the WAS and WID the estimates of top wealth inequality in the raw survey data do not align well with the estimates from the WID which are based on the work of Alvaredo, Atkinson, and Morelli (2018a) using administrative estate tax records. The most likely cause of this misalignment is undersampling of the rich in the WAS, which can be remediated by the Pareto-tail fitting exercise using rich list observations proposed by Vermeulen (2018).

For France the time series of top wealth shares from the WID ends in 2014. As for the U.K., we rely on top wealth share estimates from the Credit Suisse Global Wealth Report (Shorrocks et al., 2020) for the time period from 2014 onwards.

We construct the price of a real 30 year annuity by estimating the historical real yield curve for each country. Letting $y^r_{t,m}$ denote the real yield at maturity m at time t the cost of the annuity is calculated as

$$a_t = \sum_{m=1}^{30} \frac{1}{(1 + y^r_{t,m})^m}$$

[17]Note that the SCF+ database uses a definition of total financial net-wealth that is consistent with the SCF.

Due to varying availability of data we use three different approaches to estimate the real yield curve that lead to broadly consistent estimates. Firstly, for the UK post 1985 we use historical time series of real yields from to fit a spline through these points and construct the real yield curve directly. Secondly, for the U.S. and France we use the time series of historical nominal yields and inflation provided by Global Financial Data, augmented with data from the Macrohistory database constructed by Jordà, Schularick, and Taylor (2017), to annually estimate real yields at different maturities and then fit a spline through the estimated real yields to construct the real yield curve. We construct real yields for each year by estimating an AR(1) process for inflation on a sample of 50 years prior and then subtracting forecasted inflation from nominal yields at all available maturities (3-month treasury yields and 10-year government bond yields for all periods, as well as 30-year government bond yields for later periods). Thirdly, for the U.K. and U.S. we also use model estimates of the real yield curve from .

We construct the price of a real 30 year annuity by estimating the historical real yield curve for each country. Letting $y_t^r(h)$ denote the real yield at maturity h at time t the cost of the annuity is calculated as:

$$\sum_{h=1}^{30} \frac{1}{(1 + y_t^r(h))^h}$$

Due to varying availability of data and for robustness, we use three different approaches to estimate the real yield curve that lead to broadly consistent estimates.

First, for the UK post 1985 we use historical time series of real yields of various maturities available from the Bank of England. We fit a spline through these points and construct the real yield curve directly.

Second, for the U.S. and France we use the time series of historical nominal yields and inflation provided by Global Financial Data, augmented with data from the Macrohistory database constructed by Jordà et al. (2017), to estimate real yields at different maturities and then fit a spline through the estimated real yields to construct the real yield curve. We construct real yields for each year by estimating an AR(1) process for inflation on a rolling sample of 50 years of past data, and then subtracting forecasted inflation from nominal yields at all available maturities. Those are 3-month treasury yields and 10-year government bond yields for all periods, as well as 30-year government bond yields for later years.

Third, for the U.K. and U.S. we also use model estimates of the real yield curve. The U.S. estimates are from the model in Section A. The U.K. estimates are from a similar model estimated for the U.K. in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2021).

C.2 Income Data

C.2.1 Data Source: PSID

The Panel Study of Income Dynamics (PSID) is a household panel survey that began in 1968. The PSID was originally designed to study the dynamics of income and poverty. Thus, the original 1968 PSID sample was drawn from two independent samples: an over-sample of 1,872 low income families from the Survey of Economic Opportunity (the "SEO sample") and a nationally representative sample of 2,930 families designed by the Survey Research Center at the University of Michigan (the "SRC sample"). A total of approximately 500 post-1968 immigrant families were added in 1997/1999 to update the PSID by adding a representative sample of recent immigrants to the United States: this sample is called the 1997 PSID Immigrant Refresher Sample. A total of 615 post-1997 immigrant families were added in 2017 to update the PSID by adding a representative sample of recent immigrants to the United States: this sample is called the 2017 PSID Immigrant Refresher Sample.

C.2.2 PSID Income variables

We now describe the construction of the relevant income variables used in the paper. We construct the following variables: *labinc2f* is labor income excluding transfers but including the labor part of business and farm income for both head and eventual spouse; *transf* which are total households transfer (including Social Security Income and other transfers); *labinc3f*, which is our measure of total household income for both head and eventual spouse,is the sum of *labinc2f* and *transf*.

We provide further details on how we build these three variables. As the variables included in the PSID are subject to change, the variable construction vary with different sample period. For this reason, below we provide details on the variables used in different time periods. Moreover, the ticker for each variable changed in each survey. We therefore define the ticker used in a specific year as (YYYY:Ticker).[18]

labinc2f In the 1970 - 1993 sample, this variable is defined as the sum of Total labor income of head, including wages and salaries, labor part of business income and farm income (1993:V23323), and Spouse's total labor income, including labor part of business income and farm income (1993:V23324). In the 1993 - 2017 sample, this variable is defined as the sum of Reference Person's total labor (including wages and other labor) excluding Farm and Unincorporated Business Income, (2017:ER71293), Labor Part of Business Income from Unincorporated Businesses (2017:ER71274), Reference Person's and Spouse's/Partner's Income from Farming (2017:ER71272), Wife's Labor Income, Excluding Farm and Unincorporated Business Income (2017:ER71321), Wife's Labor Part

[18]The PSID website provides information on how to harmonize tickers across different surveys.

of Business Income from Unincorporated Businesses (2017:ER71302). Note that farm's income includes both labor and asset portions of income.

transf In the 1970-1993 sample, this variable is defined as Total Transfer Income of Head and Wife/"Wife" (1993:V22366) and Total Transfer Income of Others (1993:V22397) In the 1994-2003 sample, this variable is defined as Head's and Wife's Total Transfer Income, Except Social Security (2017:ER71391), Other Total Transfer Income, Except Social Security (2017:ER71419), Total Family Income from Social Security (1994:ER4152). In the 2004-2017 sample, this variable is defined as: Head's and Wife's Total Transfer Income, Except Social Security (2017:ER71391), Other Total Transfer Income, Except Social Security (2017:ER71419), Reference Person's Income from Social Security (2017:ER71420), Spouse's/Partner's Income from Social Security (2017:ER71422), Others Income from Social Security (2017:ER71424).

labinc3f We then construct *labinc3f* by summing *labinc2f* and total family transfers *transf*.

C.2.3 Aggregation: NIPA vs PSID

We compare the PSID aggregates to the NIPA table aggregates from NIPA Table 2.1. We use NIPA *Wages and salaries* and compare to *labinc2f*. We then use the Census data on US number of households to compute *Wages and salaries* per households (note that our PSID measures are at the household level).

C.2.4 Group Definitions

Our groups are defined based on gender, race and education. Here we detail the variables used from the PSID. **Sex.** We use the sex of the head of the household (2017:ER66018).

Race. We use the variable race (2017:ER70882). We only have an indicator function if the head is white and zero for all other races.

Education. We use a measure of years completed of education (2017:ER34548).[19] We define an individual to be college educated if they have 16 years of schooling or more. This definition is consistent with Heathcote, Perri, and Violante (2010). Before 1975, we use the variable (1975:V4198).

C.2.5 Estimating Income Process

We estimate the income profile for different groups following Meghir and Pistaferri (2004). The income process for household i in group g of age a at time t is given by (10)-(12). The estimation

[19] The question in the survey is: "What is the highest grade or year of school that (you/he/she) has completed?". We make the following assumption: Education is based on highest level of educational achievement with perfect foresight. So, income of an 18 year old who goes to college later should be part of the college income profile.

proceeds in two steps. In the first step, we estimate the year-fixed effects and the coefficients on the deterministic income profile χ from (10). In the second stage, we estimate the risk parameters using the residuals z_{it} from the first step. This estimation is done by GMM as detailed below.

Figure A9 plots the deterministic income profile of the different groups, evaluated at the 2016 year-fixed effects. The graph plots the expected income profile for the average person in each group who is 18 years old in 2016, expressed in thousands of 2016 dollars.

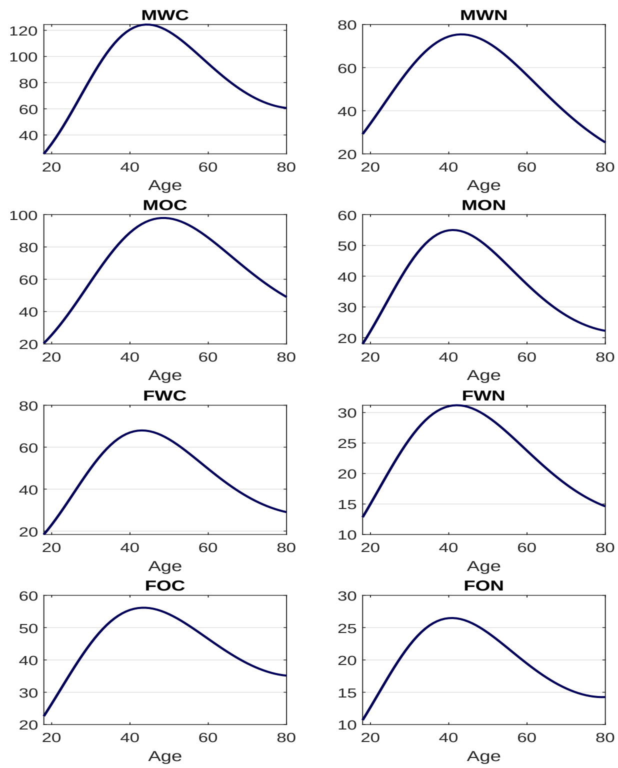


Figure A9: Income Profile by Group

Note: This figure displays the life cycle income profile of households within different groups. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college. We use the 2016 year fixed effects. The figure is in thousands of 2016 dollars. The model is estimated according to Equation (10)-(12) on PSID data from 1970 to 2017.

Using Equation (10)-(12), and define j as equal to the age of the households minus the minimum age (18), we find that:

$$
\begin{aligned}
E[\eta_j^i, \eta_{j+h}^i] &= \sigma_\alpha^2 + E[{\varepsilon_j^i}^2] + \sigma_\nu^2 \qquad \text{if } h = 0, \\
E[\eta_j^i, \eta_{j+h}^i] &= \sigma_\alpha^2 + \rho^h E[{\varepsilon_j^i}^2] \qquad \text{if } h > 0, \\
E[{\varepsilon_j^i}^2] &= \rho^{2j}\sigma_{\varepsilon_0}^2 + \sum_{k=1}^{j} \rho^{2(j-k)}\sigma_u^2.
\end{aligned}
$$

We then use a GMM estimation to estimate $\theta = (\rho, \sigma_\nu, \sigma_\eta, \sigma_\alpha, \sigma_{\varepsilon_0})$. We use a Minimum Distance Estimator, where the weighting matrix is the identity matrix. We only include sample moments estimated on more than 10 observations.

Sample Selection. We use PSID data from 1970 to 2017. We only include households whose head is 18 to 80 years old. We only include households which were in the survey for three or more periods. We exclude households with zero or negative income. In each year, we trim the top 2.5% of households by their income.

We pool all households together, after removing group-specific year-fixed effects and cubic age-profiles, and estimate the idiosyncratic risk parameters θ. The point estimates are displayed in Table A1. These are the parameters used in the main text.

Table A1: Idiosyncratic Risk Parameter Estimates

	ρ	σ_u^2	σ_v^2	σ_α^2	$\sigma_{\varepsilon_0}^2$	N. Obs.
Pooled	0.952	0.021	0.201	0.073	0.163	10638

Note: $\rho, \sigma_u^2, \sigma_v^2, \sigma_\alpha^2, \sigma_{\varepsilon_0}^2$ are estimated using Equation (10)-(12). Data are based on PSID and runs from 1970 to 2017.

C.3 Portfolio Shares

C.3.1 Data Source: SCF

The Survey of Consumer Finances (SCF) is a statistical survey of the balance sheet, pension, income and other demographic characteristics of families in the United States. We use data from the Summary Extract Data – that is, the extract data set of summary variables used in the Federal Reserve Bulletin. It includes data from the triennial surveys beginning in 1989.[20] We collect the following variables.

Total Financial Assets. This includes: All types of transaction account (liquid assets), Certificates of deposit, Directly held pooled investment funds (exc. money mkt funds), Savings bonds, Directly held stocks, Directly held bonds (excl. bond funds savings bonds), Cash value of whole life insurance, Other managed assets, Quasi-liquid retirement accounts, Other misc. financial assets.

Cash & Deposits This includes all types of transaction account (liquid assets) and certificated of deposits. The list of variables are: Money market accounts; Checking accounts (excl. money market); Savings accounts; Call accounts; Prepaid cards; Certificates of deposit.

Equities (direct & indirect). Total value of financial assets held by household that are invested in stock. That includes: directly-held stock, Stock mutual funds: full value if described as stock mutual fund, 1/2 value of combination mutual funds; RAs/Keoghs invested in stock: full value if mostly invested in stock, 1/2 value if split between stocks/bonds or stocks/money market,

[20]The SCF Flow Chart provides information on how variables are constructed https://www.federalreserve.gov/econres/files/networth%20flowchart.pdf. The code on how different variables in the Summary Extract Data are constructed can be found here: https://www.federalreserve.gov/econres/files/bulletin.macro.txt

1/3 value if split between stocks/bonds/money market; Other managed assets with equity interest (annuities, trusts, MIAs): full value if mostly invested in stock, 1/2 value if split between stocks/MFs & bonds/CDs, or ”mixed/diversified”, 1/3 value if ”other”; Thrift-type retirement accounts invested in stock: full value if mostly invested in stock,1/2 value if split between stocks and interest earning assets.

The allocation rules for mixed investments in 3), 4), and 5) do not apply to 2004 since new questions in 2004 directly ask the share of stock in those assets.

Real Estate. The real estate variable includes: Primary residence; Residential property excluding primary residence (e.g., vacation homes);

Private Business Wealth. Businesses (with either an active or nonactive interest). Businesses include both actively and nonactively-managed business(es). Value of active business(es) calculated as net equity if business(es) were sold today, plus loans from the household to the business(es), minus loans from the business(es) to the household not previously reported, plus value of personal assets used as collateral for business(es) loans that were reported earlier. Value of nonactive business(es) is calculated as the market value of the business(es).

Fixed Income. Fixed income is calculated as the residual of Total financial assets minus Cash & Deposits and Equity (direct & indirect).

Mortgage Debt. This includes: Debt secured by prim. resid. (mortgages, home equity loans, HELOCs); Debt secured by other residential property.

Student Debt. Total value of education loans held by household. This includes education loans that are currently in deferment and loans in scheduled repayment period. We exclude installment loans: these are mostly student loans (which we accounts for separately), vehicle loans (which we do not account as debt as vehicles are part of consumption).

Consumer and Other Debt. This includes: Other lines of credit (not secured by resid. real estate); Credit card balances after last payment; Other installments other than vehicles debt and student debt

Net Wealth. We calculate net wealth for each household as the difference between total assets (Cash & Deposits, Equities (direct & indirect), Real Estate, Private Business Wealth and Fixed Income) and total liabilities (Mortgage Debt, Student Debt and Consumer and Other Debt).

C.3.2 Data Source: SCF+

We use the SCF+ database developed by Kuhn et al. (2020) in order to extend our sample backward. Here we detail how we construct income and financial variables to be consistent with the data in the SCF described above.

Income: tinc - total household income, excluding capital gains

Real Estate: house - asset value of house; oest - other real estate (net position); hoestdebt -

other real estate debt (note: we add back the debt to the other real estate net position).

Cash and Deposits: liqcer - liquid assets and certificates of deposit.

Equities: ffaequ - equity and other managed assets; Indirect holdings through mutual funds and pension funds.

Fixed Income: ffafin - financial assets; Equities.

Private Business Wealth: ffabus - business wealth.

Indirect Equity holdings

As in the SCF+ the indirect holdings of equity are not available, we follow the methodology of Leombroni, Piazzesi, Schneider, and Rogers (2020) to compute indirect holdings exploiting aggregate information from the US financial account. In order to look-through the mutual funds and pension holdings we use data from the US financial account. We compute the mutual funds equity holdings using Corporate Equities (LM653064100)[21]. We compute the shares dividing equity holdings by total mutual funds assets (LM654090000).

We compute the DC pension total equity holdings as the sum of Corporate equities (LM573064133) and indirect holdings of equity through mutual funds. We compute the pension fund indirect holdings as mutual fund shares (LM573064255) $\times$ mutual funds equity shares (as computed above). We then divide the total equity holdings by total DC pension assets (FL574090055) to estimate the pension equity shares. For each household we calculate the indirect equity holdings by multiplying the holdings (in dollar) of mutual funds and pension by the calculated indirect portfolio shares.

Mortgage Debt: hdebt - housing debt on owner-occupied real estate; oestdebt - other real estate debt. **Student Debt**: Student debt is not available in the SCF+. We assume it is 0. **Other Debt**: pdebt - personal debt.

C.3.3 Groups

From the SCF and SCF+ data, we extract the sex of the reference person[22], the education attainment and the race. Using the education attainment we divide the sample into households with college degree and households without college degree. We only include households older than 25 years old. Table A2 provides information on the different groups.

C.3.4 Holdings

We compute the holdings for each of the assets and liabilities for each household. Table A3 shows summary statistics for the distribution of asset holdings. Note that Private Business Wealth is

[21] The code refers to the ones used in the US financial account.

[22] In the SCF+ this information is not available.

Table A2: Summary Statistics by Group

Groups	Population Share (%)	Median Age	Median NW	Average NW	Std NW	Negative NW	Zero NW
MWC	16.82	42	298.02	918.26	3731.29	4.88	0.00
MWN	41.20	46	117.34	336.79	1777.63	7.44	2.12
MOC	2.61	41	84.11	517.01	1960.47	10.84	1.02
MON	12.26	42	28.43	102.57	338.46	13.12	10.46
FWC	3.62	51	168.08	335.90	657.92	8.83	0.00
FWN	14.12	63	54.95	151.99	459.71	9.65	3.99
FOC	0.79	40	20.56	105.85	224.83	38.73	5.89
FON	8.57	49	0.39	36.43	112.86	16.88	27.93
All	100.00	46	91.86	356.90	1970.48	9.21	5.19

Note: Groups Information. SCF 1989. Column 1 is in percentage. Column 3, 4 and 5 in 2016 thousands dollars, Column 6 and 7 are in percentage. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

a measure net of loans from the business to the households and hence may also be a negative number.

Table A3: Portfolio Holdings

	Mean	std	Min	25%	50%	75%	90%	95%	Max
Cash and Deposits	37.55	241.49	0.00	0.75	4.11	22.11	82.99	158.87	67856
Equities	37.58	327.56	0.00	0.00	0.00	5.01	52.33	142.98	103966
Real Estate	190.97	820.86	-33929.77	0.00	91.58	214.94	420.54	631.74	201293
Private Business Wealth	79.53	1227.62	-1336.37	0.00	0.00	0.00	9.35	186.90	258433
Fixed Income	57.49	459.77	-0.00	0.00	3.74	27.66	103.17	223.72	171369
Mortgage Debt	39.26	96.52	0.00	0.00	0.00	50.46	123.36	192.51	30035.59
Student Debt	0.87	5.58	0.00	0.00	0.00	0.00	0.00	3.74	166.35
Other Debt	6.09	49.90	-0.00	0.00	0.26	3.74	11.03	20.56	4205.36
Net Wealth	356.90	1970.48	-39721.95	9.35	91.86	278.49	682.20	1322.91	290573.32

Note: Data are based on SCF 1989 and are reported in 2016 thousands dollars. Note that Private Business Wealth is a measure net of loans from the business to the households. For this reason some observations are negative.

C.3.5 Financial Duration

For the purpose of our duration calculation, we exclude households with zero net-wealth but positive assets. We then compute household's portfolio share in each asset by dividing the dollar holdings in the asset by the households net wealth. Using the portfolio shares, we compute the durations of the household's financial portfolio by multiplying the asset duration of an asset (assets durations are reported in the first column of Table 2) by the portfolio share of that asset, and summing over all assets in the portfolio. We trim household financial durations by excluding the top and bottom 2.5% of observations. In the last row of Table 2, we report the average duration, by averaging over all households (using the SCF sampling weights). Similarly, we compute average durations by group by averaging durations among the households in a group (using the SCF sampling weights).

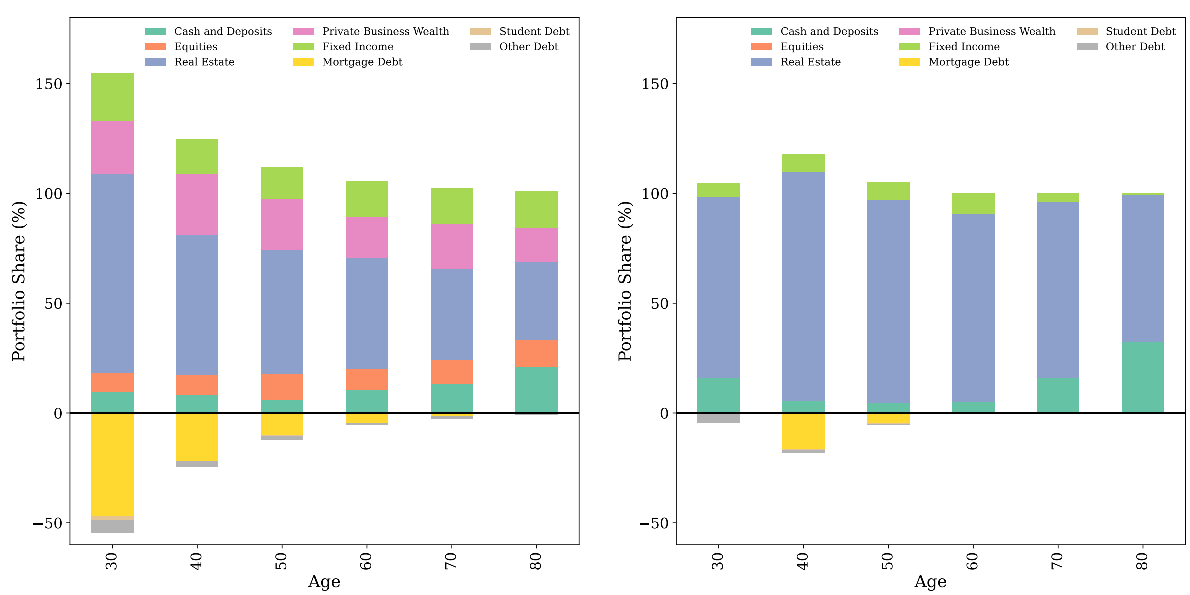
Table 2 also reports value-weighted portfolio shares for each asset. They are obtained by summing dollar holdings of an asset among all households (households in a group) by the total dollar holdings of all assets among all households (households in a group). Aggregate durations are then obtained by multiplying the value-weighted portfolio weights for each asset by the duration of that asset, and summing over assets. They are reported in the last but one row of Table 2.

Figure A10 shows portfolio shares by age in the 1989 SCF. We bundle households into different cohort groups: 25-35, 35-45, 45-55, 55-65, 65-75, 75-85. Figure A10a uses the value-weighted portfolio shares. Figure A10b plots the median portoflio share in each asset category, and then rescales the resulting shares so that they sum to 100%.

Figure A10: Portfolio Shares by Cohorts

(a) Wealth-Weighted Portfolio Shares

(b) Median Portfolio Shares



Note: Portfolio shares by age in the 1989 SCF. We bundle households into different cohort groups: 25-35, 35-45, 45-55, 55-65, 65-75, 75-85. The top panel uses the value-weighted portfolio shares. The bottom panel uses the median portfolio share in each asset category, and then rescales the resulting shares so that they sum to 100%. We exclude households with zero net wealth as the portfolio shares are undefined.

Figure A11 provides further information on the distribution of durations across households. Figure A11a plots the average duration by cohort. We bundle households into cohort groups and estimate the average duration. Figure A11b bundles households in wealth-weighted percentile and estimate the average duration of households in each bin.[23] Figure A11c and A11d rank

[23]Households are ranked according to their net-wealth and allocated to different bins. Each bin is designed such that the share of total wealth held by the households in each bin is the same across different bins.

households according to their wealth and income percentile, respectively, and estimate the average duration of each group. Figure A11e and Figure A11f also rank households according to their wealth and income. Then plot the average duration of each group against the average net-wealth or income.

We also evaluate more formally the correlation between financial duration and some covariates of interest. First, we regress household financial duration on household position in the Lorenz Curve. To calculate households' positions, we rank households by their net-wealth, then calculate the cumulative sum of net-wealth and divide by the aggregate net-wealth. We then add a dummy for each group, a quadratic function of age and the log of household income. We exclude households with zero net-wealth and positive assets (as the duration is indeterminate) and trim the bottom/top 2.5% of households ranked by their duration. The regression estimate take into account survey weights. Table A4 reports the estimation results.

C.3.6 Financial Duration Over Time

Figure A12 uses information from each SCF survey from 1989 till 2019. Figure A12a compute the aggregate (wealth-weighted) while A12b computes the average average (equally-weighted) duration over time. We use two different specifications for the duration of assets. Full sample computes the duration of the asset using the information over the whole sample; the duration of each asset is kept constant over time. The time varying specification computes time varying duration measures for equity, private business wealth and real estate. We then use these time varying measures to compute the portfolio duration.

C.3.7 Financial Duration - Robustness

To make sure our results are robust we replicate Figure 3 using different samples and different duration estimates for business wealth. Figure A13a uses a measure of business duration equal to 50, Figure A13b uses as duration of business wealth, the same duration used for equity (28.7), A13c plots the median duration instead of the average duration, Figure A13d uses a different trimming: trim the bottom/top 1%.

Table A5 reproduces the last two rows of Table 2 but using a duration of business wealth equal to 50.

We also provide a robustness for the figure on the distribution of duration, by plotting both the median duration as well as the average duration for the different percentiles. In Figure A14 we plot both average duration (Figure A14a) as well as the median duration (Figure A14b).

Table A4: Determinants of Household-level Financial Duration

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Age	0.034	-0.36***	-0.33***				-0.18***	-0.20***
	(0.51)	(-5.80)	(-5.12)				(-2.76)	(-3.11)
Age Squard	-0.0023***	0.00100*	0.00097*				-0.00044	-0.00013
	(-3.96)	(1.84)	(1.76)				(-0.78)	(-0.23)
Net-Wealth	0.0000010***							
	(8.18)							
Income	0.0000015***							
	(3.06)							
Log-Net-Wealth		2.43***						
		(27.40)						
Log-Income		0.065						0.60***
		(0.42)						(3.33)
Net-Wealth Pctl			0.11***					
			(16.04)					
Income Pctl			0.038***					
			(5.88)					
Lorenz				0.17***		0.15***	0.20***	0.19***
				(18.97)		(14.48)	(20.40)	(16.74)
MWC					7.87***	4.63***	1.67**	0.99
					(12.09)	(6.65)	(2.29)	(1.34)
MWN					5.39***	3.89***	2.46***	2.03***
					(9.04)	(6.52)	(3.92)	(3.18)
MOC					6.74***	4.97***	2.19*	1.73
					(5.07)	(3.81)	(1.68)	(1.34)
MON					4.56***	4.16***	2.50***	2.15***
					(6.11)	(5.63)	(3.29)	(2.81)
FWC					0.19	-1.52	-2.44**	-2.86***
					(0.19)	(-1.62)	(-2.50)	(-2.93)
FWN					-0.76	-1.47**	-0.040	-0.25
					(-1.19)	(-2.33)	(-0.06)	(-0.39)
FOC					3.06***	2.13*	1.15	0.69
					(2.61)	(1.84)	(1.11)	(0.68)
Constant	21.8***	4.56**	23.8***	15.1***	12.9***	12.6***	23.6***	17.9***
	(11.85)	(2.10)	(13.46)	(77.03)	(23.23)	(22.72)	(12.34)	(6.76)
Observations	13145	13145	13145	13145	13145	13145	13145	13145
R^2	0.098	0.201	0.159	0.053	0.051	0.084	0.157	0.159
Adjusted R^2								

Note: Data based on SCF 1989. T-stats in parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$)

Table A5: Robust Aggregate and Average Duration

	All	MWC	MWN	MOC	MON	FWC	FWN	FOC	FON
Aggregate Duration	22.21	23.35	22.49	27.01	20.15	15.93	14.91	18.05	16.68
Average Duration	14.85	19.55	16.50	16.33	13.71	12.30	10.27	11.89	7.68

Note: Data are based on SCF 1989. This table reproduces the last two rows of Table 2 but using a duration of business wealth equal to 50. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

C.3.8 Wealth Shares, Income Shares, and Gini Coefficients

We estimate the net-wealth shares held by the top-10% and top-1%. We also estimate gini coefficients. We use the SCF+ database developed by Kuhn et al. (2020) in order to have a longer time series of wealth and income. We slightly modify their definition of total financial net-wealth by subtracting vehicles and other non-financial wealth.

Figure A15 plots the top shares and the gini coefficient for financial (net) wealth. Table A6 computes averages for these moments, computed over all surveys in the 1980s and all surveys in the 2010s, for both financial wealth and income. The income moments in this table are from the SCF. We define household income as SCF total household income minus capital income.

Table A6: Summary Statistics Wealth and Income Inequality in SCF

	SCF		WID					
	1980s	2010s	1980s	2010s				
Wealth: Top 1 Share (%)	26.9	36.4	25.3	35.1				
Wealth: Top 10 Share (%)	64.9	76.2	63.2	71.8				
Wealth: gini (×100)	77.0	85.8	77.8	83.6				
	SCF		WID		PSID		PSID (ex transf.)	
	1980s	2010s	1980s	2010s	1980s	2010s	1980s	2010s
Income: Top 1 Share (%)	11.5	18.3	12.2	18.6	6.4	9.5	8.1	11.8
Income: Top 10 Share (%)	36.3	45.5	36.3	45.1	29.2	34.3	35.4	41.7
Income: gini (×100)	48.2	56.1	48.7	57.9	42.8	47.8	56.9	62.7

Note: Shares and Gini coefficients estimated using the SCF+ developed by Kuhn et al. (2020), the WID database and the PSID. We use our income variable *labinc3f* from the PSID as well the income variable excluding transfers (*labinc2f*). From the SCF+ we use the total income variable excluding capital gain. SCF+ 1980s average over the surveys in 1977, 1983 and 1989.

C.3.9 Inheritance

The survey of consumer finances also includes data on inheritance. We include the total of gift, intheritance or assets received through trusts or in some other forms.[24] We calculate the total amount of inheritances received: the SCF provides some details on three main inheritance and then aggregate all the other inheritances in an additional variable. We first compute the sum

[24] Note that this also includes life insurance or other settlements.

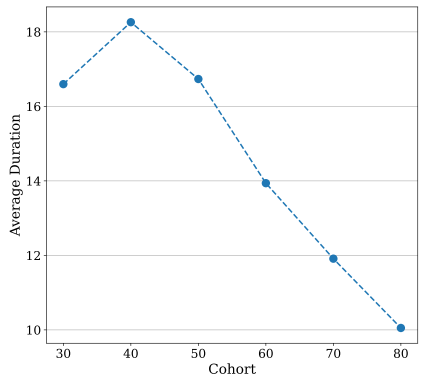
of SCF variables: X5804 + X5809 + X5814, which report the approximate value of the three main inheritances at the time they were received. We only include transfers received from grandparents and parents (variables X5806, X5811, X5816 have to be equal to 1 or 2).[25] We also use the variables that detail the year in which the inheritances were received and deflate the inheritance values to be in 2016 dollars (variables X5805, X5810 and X5815). We also calculate the age at which the inheritance was received using the current age and subtracting the number of years to the time the main inheritance was received (X5805).[26]

[25] We do not therefore include transfer from Child, Aunt/Uncle, Sibling, Friend, Government settlement; compensation , Family, n.e.c., Divorced former spouse

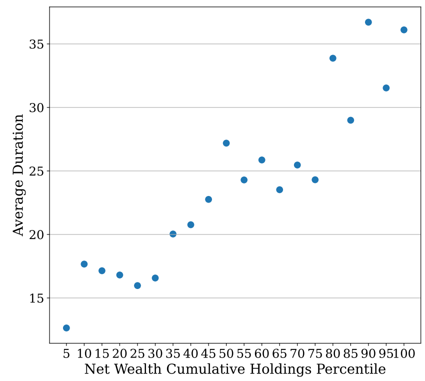
[26] We assume it was received at birth if the year it was received is before the household was born.

Figure A11: Distribution of Durations

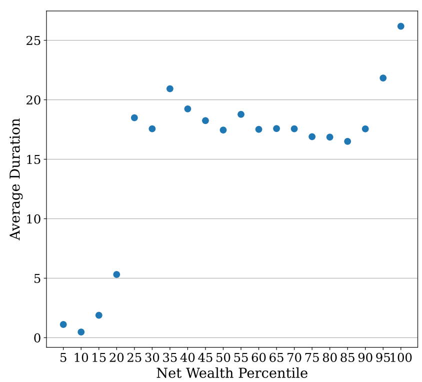
(a) Average Duration by Cohort



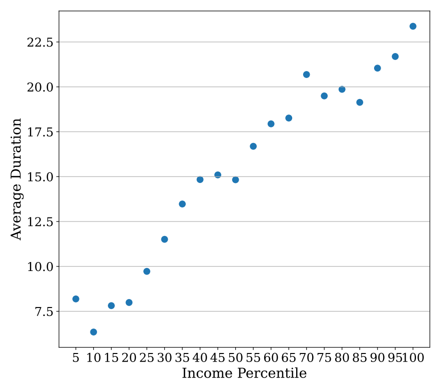
(b) Average Duration by Wealth-Weighted Percentiles



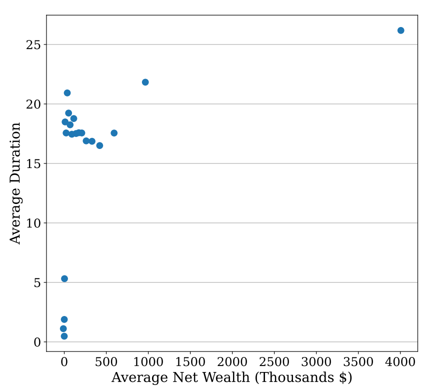
(c) Average Duration by Net-Wealth Percentile



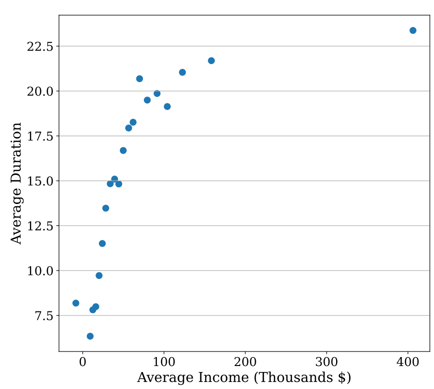
(d) Average Duration by Income Percentile



(e) Duration vs Net-Wealth



(f) Duration vs Income

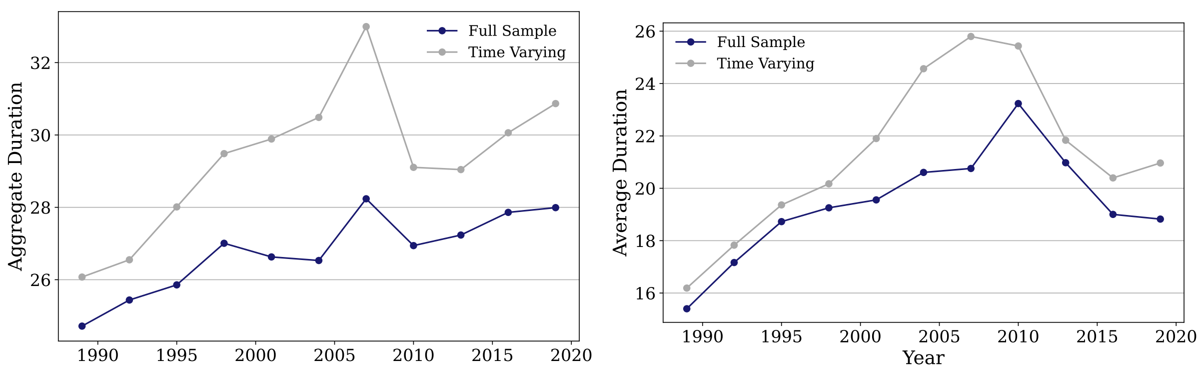


Note: Data are based on SCF 1989. We exclude households with zero net wealth and positive assets (as their portfolio shares would be indeterminate) and we trim the data based on households' overall duration: we exclude the top/bottom 2.5%. Panel (a) plots the average duration by cohort. We bundle households into cohort groups and estimate the average duration. Panel (b) bundles households in wealth-weighted percentile and estimate the average duration of households in each bin. Panel (c) and Panel (d) rank households according to their wealth and income percentile, respectively, and estimate the average duration of each group. Then plot the average duration of each group against the average net-wealth (Panel (e)) or income (Panel (f)).

Figure A12: Financial Duration Over Time

(a) Wealth-Weighted Duration

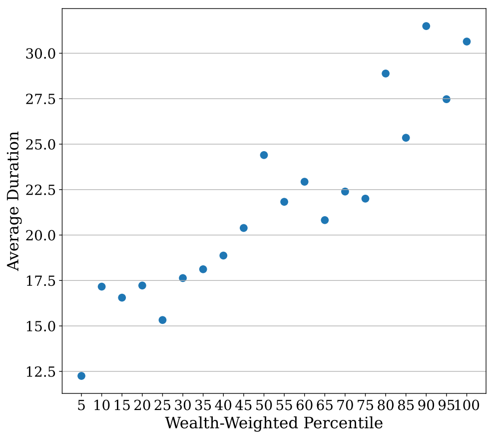
(b) Equally-Weighted Duration



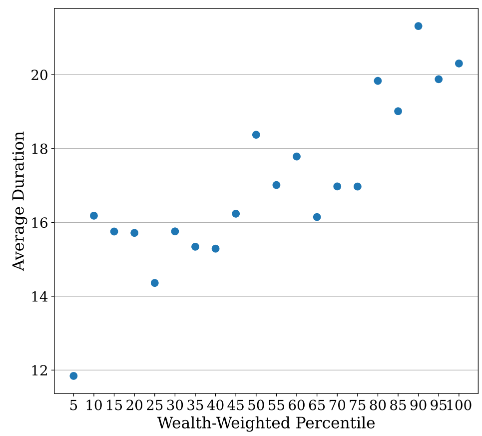
Note: A12 uses information from each SCF survey from 1989 till 2019. Figure A12a compute the aggregate (wealth-weighted) while A12b computes the average average (equally-weighted) duration over time. We use two different specifications for the duration of assets. Full sample computes the duration of the asset using the information over the whole sample; the duration of each asset is kept constant over time. The time varying specification computes time varying duration measures for equity, private business wealth and housing.

Figure A13: Duration by Net Worth, Robustness

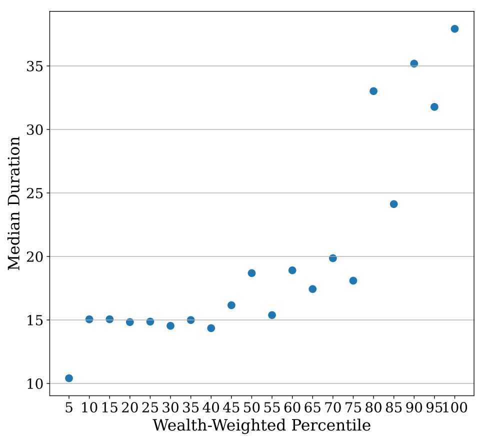
(a) Business Wealth Duration = 50



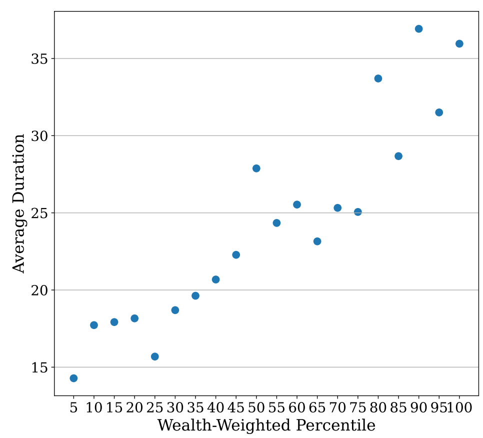
(b) Business Wealth Duration = Equity Duration



(c) Median Duration



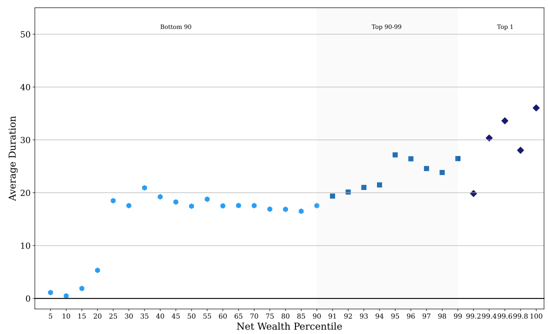
(d) Trim top / bottom 1%



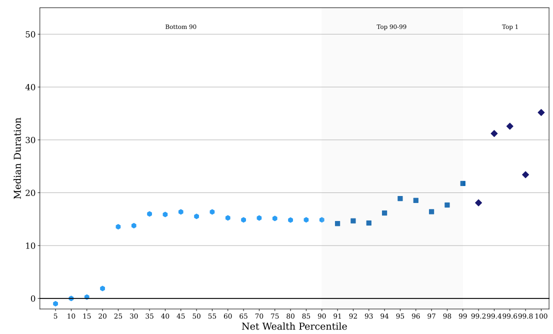
Note: Data are based on SCF 1989. We always exclude households with zero net wealth and positive assets (as their portfolio shares would be indeterminate). Figure A13a uses a measure of business duration equal to 50, Figure A13b uses as duration of business wealth, the same duration used for equity (28.7), A13c plots the median duration instead of the average duration, Figure A13d uses a different trimming: trim the bottom/top 1%.

Figure A14: Financial Duration by Net Worth Population Percentiles, Robustness

(a) Average Duration



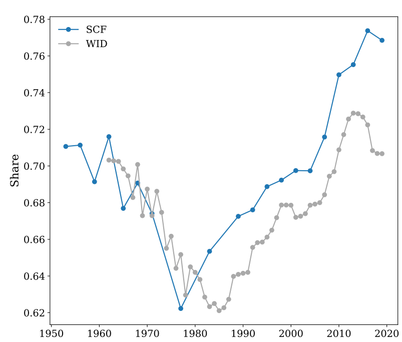
(b) Median Duration



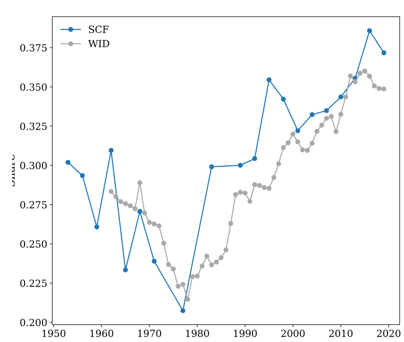
Note: This figure plots both average duration (Figure A14a) as well as the median duration (Figure A14b) estimated for different percentiles of net-wealth. Data are based on SCF 1989. We exclude households with zero wealth and positive assets holdings. We trim the top/bottom 2.5% of households ranked by the duration of their portfolio.

Figure A15: Financial Wealth Inequality in the SCF+

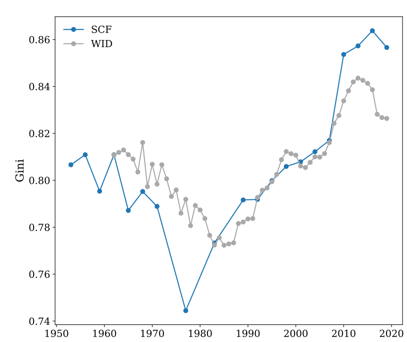
(a) Top 10% Share



(b) Top 1% Share



(c) gini



Note: Data are based on SCF+ database developed by Kuhn et al. (2020) and WID database.

D Proofs

D.1 Proof of proposition 5.1

Proof. The one-period budget constraint:

$$\widehat{c}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t = (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha),$$

can be restated, using equation (6), as:

$$\widehat{c}_t(\eta^t) - (1-\alpha)\widehat{y}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha)}{\widehat{R}_t} = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \tag{20}$$

Rewriting (20) one period later:

$$\widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha)}{\widehat{R}_{t+1}} = \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha).$$

Multiply this equation by $\varphi(\eta_{t+1}|\eta^t)$ and sum across all states η_{t+1} to obtain:

$$\begin{aligned} & \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left(\widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha)}{\widehat{R}_{t+1}} \right) \\ = \; & \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha), \end{aligned}$$

where we used the fact that $\sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) = 1$ on the right-hand side. Next, substitute this expression back into (20) to obtain:

$$\begin{aligned} & \widehat{c}_t(\eta^t) - (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{R}_t^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left(\widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) \right) \\ & + \widehat{R}_{t\to t+1}^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left(\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha) \right) = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \end{aligned}$$

Define financial wealth, scaled by the aggregate endowment, as:

$$\widehat{\theta}_t = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha).$$

Continuing the forward substitution, we end up with the following expression:

$$\widehat{\theta}_t \;=\; \sum_{\tau=t}^{\infty} \widehat{R}_{t\to\tau-1}^{-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau) \right).$$

where $\varphi(\eta^t|\eta^t) = 1$. Financial wealth must equal the cost of the household's excess consumption plan, where excess refers to the part not paid for with labor income. Noting that $e_0 = 1$ so that

$\widehat{\theta}_0 = \theta_0$, writing this expression at time zero:

$$\theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)$$

recovers the statement of the proposition. □

D.2 Proof of Proposition 5.2

Proof. We note that the cross-sectional expectation of the product can be decomposed in the standard way:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau)\right) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{C}ov_0[\psi_\tau, c_\tau] + \mathbb{E}_0\left[\psi_\tau\right] \mathbb{E}_0\left[c_\tau\right].$$

If the orthogonality condition is satisfied, then the following result obtains:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau)\right) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{E}_0[\psi_\tau]\mathbb{E}_0[c_\tau] = \mathbb{E}_0[c_\tau] = 1,$$

because $\mathbb{E}_0[\psi_t] = 1$. □

D.3 Proof of Proposition 5.3

Proof. This inequality $0 \geq Cov(\psi_t, \widehat{c}_t)$ directly implies that the following inequalities obtain:

$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 \leq \int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1},$$
$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau)\widehat{y}_\tau(\eta^\tau) d\Theta_0 \leq \int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{y}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}.$$

As a result, this new measure implies an aggregate value of individual wealth that falls short of total wealth, $\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$. Note that even though this claim to total consumption is itself not traded, the Lucas tree is a claim to α of the same cash flow stream. The market value of the Lucas tree is $\alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$, and hence the value of total wealth has to be $\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$. □

D.4 Proof of proposition 5.4

Proof. An unconstrained household's Euler equation in the high-growth economy is given by:

$$1 = \widehat{\beta}\widehat{R}_t \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \frac{u'(\widehat{c}(\eta_{t+1}, \eta^t))}{u'(\widehat{c}_t(\eta^t))}.$$

This Euler equation is satisfied because the allocations and prices constitute a Bewley equilibrium in the high-growth economy. This household's Euler equation in the new economy with lower interest rates is still satisfied at the old consumption allocation. This can be seen by plugging in the new equilibrium interest rates:

$$\widetilde{R}_t \widetilde{\beta} = \widehat{\beta} \widehat{R}_t,$$

to recover the unconstrained household's Euler equation in the low-growth economy:

$$1 = \widetilde{\beta} \widetilde{R}_t \sum_{\eta_{t+1}} \phi(\eta_{t+1} | \eta_t) \frac{u'(\widehat{c}(\eta^t, \eta_{t+1})}{u'(\widehat{c}_t(\eta^t))}.$$

We allocate the following amount of financial wealth at time 0 to ensure the household can afford the same consumption plan:

$$\widetilde{\theta}_0(\theta_0, \eta_0) = \sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{0 \to \tau-1} \sum_{\eta^\tau} \varphi(\eta^t) \left(\widehat{c}_\tau(\eta^\tau) - (1 - \alpha) \widehat{y}_\tau(\eta^\tau) \right).$$

Aggregating this initial financial wealth across households:

$$\int \widetilde{\theta}_0 d\Theta_0 = \alpha \sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{0 \to \tau} = \tilde{v}_0,$$

where we have used the goods market clearing condition and the definition of labor income shares. The last equation shows that the new allocation of initial financial wealth uses up all aggregate financial wealth in the economy. Finally, note that the natural borrowing constraints are not binding in the high-growth economy. They remain non-binding in the low-growth economy because consumption is nonnegative. Hence, the allocations are feasible, and they satisfy the sufficient conditions for optimality. □

E Life-Cycle Model Details

Each agent in the life-cycle model with age j, portfolio of financial assets $\{a_{k,t}\}$, and idiosyncratic labor income state z solves the Bellman equation:

$$V_j(a_t; z_t) = \max_{a_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta s_j \mathbb{E}_t \left[V_{j+1}(a_{t+1}; z_{t+1}) \right] \tag{21}$$

subject to the budget constraint:

$$c_t \leq y_t + \sum_{k=0}^{K} (q_k + \delta_k) s_j^{-1} a_{k,t} - q_k a_{k,t+1} \tag{22}$$

where y is after-tax income as specified in equations (10) and (11), s_j is the probability of surviving to age $j+1$, q_k and δ_k are the prices and cash flows, respectively, of the set of risk free financial assets available to the household. The term s_j^{-1} in the budget constraint (22) represents that households enter an annuity or tontine system in which surviving households receive the assets of households in their age cohort who died, proportional to their asset holdings. This assumption ensures a sufficiently strong savings motive for older households in the absence of a bequest motive.

We can generalize the problem through some convenient variable substitutions. First, we can simplify the asset structure. In a stationary equilibrium, without aggregate shocks or changes to the interest rate, the specific form of the financial assets is arbitrary, although it will be relevant for repricing assets following an interest rate shock. As a result, we can define x to be the start-of-period value of the entire portfolio, including both its cash flow and continuation value:

$$\theta_t = \sum_{k=0}^{K} (q_k + \delta_k) a_{k,t}.$$

By no arbitrage, we have

$$\frac{q_k + \delta_k}{q_k} = R$$

for all k, which implies

$$\sum_{k=0}^{K} q_k a_{k,t+1} = \sum_{k=0}^{K} (q_k + \delta_k) R^{-1} a_{k,t+1} = R^{-1}\theta_{t+1}.$$

Substituting now yields the simplified the budget constraint

$$c_t \leq y_t + \theta_t - R^{-1}\theta_{t+1}. \tag{23}$$

Under a constant interest rate, the problem can therefore be solved as if the agents held one-period debt with face value θ in each period, allowing us to use a single solution to characterize economies with portfolios over many possible assets.

Compensated Distribution. To compute the compensated distribution under a change from interest rate R to $\tilde{R}$, we first compute total wealth under the original and new interest rates:

$$\Omega_t = \sum_{\tau=0}^{\infty} R^{-\tau} c_{t+\tau}$$
$$\tilde{\Omega}_t = \sum_{\tau=0}^{\infty} \tilde{R}^{-\tau} c_{t+\tau}.$$

We next compute human wealth under the original and new interest rates:

$$
\begin{aligned}
\Upsilon_t &= \sum_{\tau=0}^{\infty} R^{-\tau} y_{t+\tau} \\
\tilde{\Upsilon}_t &= \sum_{\tau=0}^{\infty} \tilde{R}^{-\tau} y_{t+\tau}.
\end{aligned}
$$

The implied amount of financial wealth that makes the original consumption plan affordable is therefore

$$
\begin{aligned}
\theta_t^{comp} &= \tilde{\Omega}_t - \tilde{\Upsilon}_t \\
&= \theta_t + (\tilde{\Omega}_t - \Omega_t) + (\tilde{\Upsilon}_t - \Upsilon_t)
\end{aligned}
$$

where θ_t is pre-shock financial wealth.

Repriced Distribution. To compute the repriced distribution following a change from interest rate R to $\tilde{R}$, we will need to specify the specific asset structure. We assume that agents hold zero coupon bonds with maturity m, which implies $q_m = R^{-m}$. At the moment of the interest rate change, the repriced (post-shock) financial wealth $\theta_t^{repriced}$ is related to pre-shock financial wealth θ_t according to the formula

$$
\theta_t^{repriced} = \left(\frac{\tilde{q}_m}{q_m}\right)\theta_t = \left(\frac{\tilde{R}}{R}\right)^{-m}\theta_t \tag{24}
$$

for a household with bonds of maturity (duration) m. For our computations, we set m equal to financial wealth duration, and apply (24).

F Robustness: Measurement of Private Business Duration

Our benchmark model uses the duration of small stocks—those in the bottom decile of the market capitalization distribution of publicly listed firms—as a proxy for the duration of private business wealth. Since the price-dividend ratio and dividend growth rates of small stocks are included in the state vector of the auxiliary asset pricing model, the latter model fits the post-war quarterly time series of small stock returns and cash flow growth rates exactly and implies a quarterly time series for the duration of small stocks. The resulting private business duration, averaged over the 40 quarters of the 1980s, is 61.25.

This approach may be understating the duration of private business wealth, to the extent that firms in the bottom decile of publicly listed firms already experienced a lot of (cash flow) growth leading up to their inclusion in the publicly-listed universe. Including the cash-flow (growth) leading up to the point of the IPO would result in a higher duration.

The approach may also be overstating duration in that it measures the duration of small public firms, holding fixed inclusion in this group. In reality, firms in the bottom decile of publicly-listed firms may grow further and transition into higher deciles of the market capitalization distribution. Since larger firms may have lower cash flow pay-out ratios, taking into account that small firms do not remain small may lead us to overstate the duration of small public firms. In this appendix, we address this potential overstatement issue, and explain how to arrive at the duration of firms that are *currently* in the bottom decile (or quintile) of the market cap distribution , but may not remain there in the future.

F.1 Measuring Duration with Firm Life-Cycles

The duration of a firm is the weighted average time to its cash flows:

$$Dur = \sum_{t=1}^{\infty} t \frac{PV_t}{\sum_{t=1}^{\infty} PV_t}$$

Let s indicate the current-year size group of a firm, where size is measured by market capitalization. Let there be S groups. We assume that $PV_t = CF_t(1+R)^{-t}$ for some constant discount rate R, calibrated as discussed below. We model the cash flow of the median firm in size group s_t, which came from size group s_{t-1} in the previous period, as the product of the payout-asset ratio of the median firm in that size group and the assets of the median firm in that size group:

$$CF_t(s_t|s_{t-1}) = (CF_t/A_t)\,(s_t|s_{t-1}) \cdot A_t(s_t|s_{t-1}) \tag{25}$$

The state (market capitalization group) transition matrix is denoted by $\mathcal{P}(s_t|s_{t-1})$. Conditional on starting out in the smallest decile at time zero, the cash flow of a typical firm t periods later is:

$$D_t|s_0 = \sum_{s_t=1}^{S} \mathcal{P}^{t-1} \cdot (\mathcal{P} \cdot (D_t/A_t) \cdot A_t) \tag{26}$$

F.2 Implementation

We use CRSP-Compustat data on the universe of publicly-listed firms for the standard sample from 1967–2020. Market capitalization is measured as price per share times shares outstanding, properly adjusted for stock splits. We also make an adjustment for mergers & acquisitions. As is commonly done, we delete stocks whose price is below $1 per share and whose market capitalization is less than $10 million at the first time of observation (and only then).

Cash flow CF is either computed as cash dividends or as cash dividends plus net share repurchases, with the latter bounded from below at zero. Cash flows and assets are deflated by the consumer price index. To compute assets and the cash flow-to-asset ratio in each size group, we first compute book assets and CF/asset ratios for each firm, then winsorize at the 1% level, then compute the median across the firms that are in size group s_t in the current year and were in size group s_{t-1} in the prior year. This delivers a time series for the $S \times S$ matrices $(CF_t/A_t)(s_t|s_{t-1})$ and $A_t(s_t|s_{t-1})$. We then average these objects across years.

Our groups are either market capitalization deciles ($S = 10$) or quintiles ($S = 5$). When computing the size transition probability matrix $\mathcal{P}$, we collapse set all transition probabilities that are more than three notches up (down) to zero and add the empirical weight of those transitions to the state that is exactly three notches up (down). We take the time-series average of the state transition probability matrices in each year.

Finally, we calibrate the discount rate R, needed in the duration calculation, in order to obtain a duration of 28 for the value-weighted market portfolio of all stocks. This is the duration of the aggregate stock market we estimate in the auxiliary asset pricing model. This enables comparability across approaches.

F.3 Results

Size Deciles. Using deciles for size groups, the transition probability matrix is $\mathcal{P}(s'|s) =$

$$\begin{bmatrix}
75.1\% & 19.5\% & 3.6\% & 1.7\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% \\
20.3\% & 49.8\% & 22.2\% & 5.8\% & 2.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% \\
3.7\% & 20.9\% & 43.9\% & 22.8\% & 6.9\% & 1.8\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% \\
0.8\% & 5.3\% & 20.7\% & 42.2\% & 23.8\% & 5.9\% & 1.2\% & 0.0\% & 0.0\% & 0.0\% \\
0.0\% & 1.6\% & 5.2\% & 19.8\% & 43.3\% & 24.5\% & 4.9\% & 0.6\% & 0.0\% & 0.0\% \\
0.0\% & 0.0\% & 1.7\% & 4.2\% & 18.8\% & 47.2\% & 24.3\% & 3.6\% & 0.2\% & 0.0\% \\
0.0\% & 0.0\% & 0.0\% & 1.4\% & 3.5\% & 17.6\% & 52.2\% & 23.7\% & 1.5\% & 0.0\% \\
0.0\% & 0.0\% & 0.0\% & 0.0\% & 1.4\% & 2.5\% & 15.4\% & 61.0\% & 19.5\% & 0.2\% \\
0.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% & 1.2\% & 1.3\% & 12.1\% & 72.9\% & 12.5\% \\
0.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% & 0.0\% & 1.8\% & 0.9\% & 8.6\% & 88.6\%
\end{bmatrix}$$

Table A7 shows, for each of the size groups, the dividend/asset ratio, the payout/asset ratio (which includes net share repurchases in the numerator), log assets, and the duration using either dividends or payouts. For the smallest decile of listed firms, which is our proxy for private businesses, we obtain a duration of 62.5 using cash dividends and 62.3 using the broader payout measure. We conclude that this number is quite similar to the 61.25 number we use in our benchmark results.

Table A7: Duration by Size Decile

Deciles	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Log asset	4.09	4.82	5.18	5.53	6.04	6.43	6.94	7.48	8.32	9.58
CF / asset (div, %)	0.10	0.21	0.30	0.37	0.51	0.56	0.77	1.03	1.36	1.93
CF / asset (payout, %)	0.13	0.34	0.42	0.49	0.68	0.81	1.06	1.36	1.70	2.35
Duration (div)	**62.5**	59.8	56.7	53.4	49.6	45.4	40.8	35.6	29.7	23.2
Duration (payout)	**62.3**	59.6	56.5	53.3	49.5	45.3	40.7	35.6	29.7	23.3

Note: The first row reports the log of book assets of the median firm in each decile of market capitalization. The second and third rows report the ratio of cash flows to book assets for the median firm in each decile of market capitalization, where cash flows are measured as cash dividends (div) in the first instance and dividends plus the max of net share repurchases and zero in the second instance. Assets and CF/assets depend on both the current size decile and the prior year's size decile, but are integrated across the prior year's size deciles for presentation purposes. The last two rows report the durations, using either dividends or dividends plus net share repurchases as the measure of cash flow.

Size Quintiles. As a further robustness check, we also compute durations for quintiles, assuming that private businesses resemble firms in the bottom-20% of the size distribution of listed firms. Using quintiles for size groups, the transition probability matrix is $\mathcal{P}(s'|s) =$

$$\begin{bmatrix} 81.8\% & 17.0\% & 1.2\% & 0.1\% & 0.0\% \\ 15.2\% & 64.9\% & 19.1\% & 0.7\% & 0.0\% \\ 1.0\% & 15.1\% & 66.9\% & 16.8\% & 0.1\% \\ 0.2\% & 1.0\% & 12.1\% & 76.1\% & 10.7\% \\ 0.0\% & 0.5\% & 0.7\% & 7.7\% & 91.0\% \end{bmatrix}$$

Table A8 shows, for each of the size groups, the dividend/asset ratio, the payout/asset ratio (which includes net share repurchases in the numerator), log assets, and the duration using either dividends or payouts. For the smallest decile of listed firms, which is our proxy for private businesses, we obtain a duration of 52.0 using cash dividends and 51.9 using the broader payout measure.

Combining the results for deciles and quintiles suggests that a value around 50 for the duration of private business wealth is conservative. This is particularly true, given the concern of understatement of private business durations mentioned at the beginning of this appendix.

Table A8: Duration by Size Quintile

Quintiles	Q1	Q2	Q3	Q4	Q5
Log asset	4.44	5.34	6.19	7.20	8.88
CF / asset (div, %)	0.12	0.32	0.50	0.89	1.63
CF / asset (payout, %)	0.20	0.42	0.70	1.18	1.99
Duration (div)	**52.0**	47.7	41.8	34.3	25.2
Duration (payout)	**51.9**	47.6	41.7	34.2	25.3

Note: The first row reports the log of book assets of the median firm in each quintile of market capitalization. The second and third rows report the ratio of cash flows to book assets for the median firm in each quintile of market capitalization, where cash flows are measured as cash dividends (div) in the first instance and dividends plus the max of net share repurchases and zero in the second instance. Assets and CF/assets depend on both the current-year and the prior year's size quintiles, but are integrated across the prior-year's size quintiles for presentation purposes. The last two rows report the durations, using either dividends or dividends plus net share repurchases as the measure of cash flow.