International Diversification, Reallocation, and the Labor Share *

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Abstract

When firms choose inputs under uncertainty, the price and quantity of risk affects the allocation of resources across firms and labor's share of income. We develop a model of input allocation and international risk sharing. Increasing international diversification lowers the price of local risk and leads to a reallocation of labor towards riskier firms and a rise in the median (or within-firm) labor's share of income, matching key micro-level facts. Under plausible assumptions, the reallocation effects dominates and labor's share of national income declines. We use firm-level and cross-country data to document a number of empirical patterns consistent with our model: (1) riskier firms have lower labor share, (2) international diversification is associated with reallocation towards riskier firms and declines in aggregate labor's share of income, (3) industries with greater heterogeneity have greater sensitivity of their labor share to international diversification.

*The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

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1 Introduction

The last 40 years have witnessed a global decline in labor's share of income (Karabarbounis and Neiman (2014)) concurrent with a rapid deepening in international financial integration (Lane and Milesi-Ferretti (2018)). This paper explores theoretically and empirically the links between the international diversification of risk, risk-sharing between investors and workers, and the resulting labor share. The paper stresses the dual effect of deeper international diversification on the aggregate labor share: a within-firm effect that increases labor share, and a reallocation of production towards firms with a lower labor share, which decreases it. We derive and test empirically conditions for which the latter effect dominates, explaining the negative relationship between international financial integration and the aggregate labor share observed in the data.

The paper proposes a simple yet novel framework to assess how the exposure of firms to market risk affects their choice of inputs and equilibrium wages. Firms insure workers against market risk, but the price of such implicit insurance depends on firms' exposure to that risk and to their ability to diversify it away. International diversification, by reducing risk exposure, reduces the premium that workers must implicitly pay to get obtain insurance. Under general conditions, such risk-diversification leads to an increase in the labor share within a given firm. At the same time, however, international risk-diversification opportunities lead firms more exposed to market risk to expand. The resulting reallocation effect can generate a reduction in the aggregate labor share even if the median labor share increase. Such a result is consistent with the evidence on labor share declines at the industry or aggregate level despite an increase in the median firm labor share (Hartman-glaser et al. (2019),Kehrig and Vincent (2021)).

Empirically, the within firm insurance effect implies that firms that are more exposed to market risk display a lower labor share. This paper is the first to establish this robust fact using 47 years of data on US firms and controlling for a rich array of fixed effects and firm characteristics. We also provide empirical evidence of the reallocation effect by showing that riskier firms (with lower labor share) expands their share of output as international diversification increases. Furthermore, using a panel of 25 countries over 38 years, we establish a robust negative link between the aggregate labor share and international diversification, suggesting that the reallocation effect dominates. These results indicate that international diversification can explain part of the decline in the labor share in a way that is consistent both with the within firm/between firm empirical decomposition of such decline and with a standard model of the labor share under market uncertainty. The paper also provides empirical evidence using ORBIS cross-country firm-level data to show that industries in which there is greater dispersion in firm labor shares of income see their labor shares fall by more in response to an increase in international diversification. A version of our model with heterogeneous industries and normally distributed risk suggests that this fact should arise because there is more scope for reallocation from safer, high labor share firms to riskier, lower labor share firms.

Firms are subject to market risk and firm-specific risk when they choose their inputs. Our model captures the influence of these two sources of risk by introducing a standard multiple firm production model with uncertainty. Market risk is introduced through a standard stochastic discount factor that is used to value firm cash flows while firm-specific risk is introduced as a shock to the firm's productivity. Factor payments are decided before the shocks are realized. In this framework, the firm-specific labor share depends both on the share of labor in the production function and on the covariance between market risk and firm-specific risk. As long as the two sources of risk are uncorrelated, the equilibrium labor share will differ from the share of labor in production due to an insurance effect. In particular, if firm productivity is procyclical while market risk is countercyclical, as standard theory and empirics suggest, then this covariance term is negative and reduces labor's share of expected income. Workers suffer from a wage discount in terms of reduced compensation in order to be insured. Furthermore, firms more exposed to aggregate risk, i.e. for which the covariance between firm-specific and market risk is more negative, display a lower labor share.

The model enables us to assess the consequences of a change in the price of risk and, more specifically, of diversification opportunities that reduce the correlation between firm-specific risk and market risk. Such a change induces a dual effect: (i) the decrease in the effective price of risk faced by firms leads to a decline in the wage discount and to an increase in the labor share within each individual firm. This is the within-firm insurance effect. On the other hand, (ii) diversification opportunities disproportionately affect firms with higher risk exposure and lower labor share. This leads to a reallocation of production between firms.

The aggregate consequences of risk-diversification depend on whether the within-firm insurance effect or between firm reallocation effect dominates. We demonstrate that the effect of risk diversification on the aggregate labor share is U-shaped: there is a unique threshold in the price of risk below (beyond) which a decrease in the price of risk leads to a decrease (increase) in labor share. it is only in the extreme case in which market risk can been fully diversified away, that the equilibrium labor share is fully determined by the relative importance of labor inputs for production.

We show further that this key novel result carries through in a stochastic general equilibrium model with multiple countries and sectoral production. In such a world, investors diversify country-specific risk by exchanging shares with investors in other countries. Limits to diversification are captured by a cost of trading in international assets. We show analytically that as barriers to international diversification are reduced, the labor share increases within firms (insurance effect) while production and labor are reallocated towards more risky firms (reallocation affect). Here again, the relationship between international financial diversification and the labor share follows a U-shaped pattern.

This paper builds a bridge between two literatures: the literature on international financial integration and the literature on the global dynamics of labor share. That international integration favors risk-taking and growth has been demonstrated theoretically by Obstfeld (1994). Empirically, Thesmar and Thoenig (2011) show using French firm-level data that diversification in ownership leads to more risk-taking at the firm level. Levchenko et al. (2009) find sector-level volatility increases permanently following international financial liberalization suggesting an underlying risk-taking channel.

Karabarbounis and Neiman (2014) document a global decline in the labor share, which is mostly a within-industry phenomenon. Hartman-glaser et al. (2019) and Kehrig and Vincent (2021) decompose further this decline, in the case of the US, as the resulting outcome of two opposite effects: a within-firm increase in labor share and a reallocation towards firms with lower labor share within each industry. The observed decline in the industry labor share results from the reallocation effect dominating the within firm effect. Our theoretical mechanism and our empirical findings also stress these dual effects, relating each of them to the decline in the price of risk.

There is a vast literature on the cause of the global decline in labor shares summarized in the recent survey by Grossman and Oberfield (2021). Proposed causes range from technical change and the relative price of capital goods (Karabarbounis and Neiman (2014)) to the rise of superstar firms (Lashkari et al. (2018), Autor et al. (2020)), to robotization (Humlum (2019), Autor and Salomons (2018)), to increased trade globalization (Elsby et al. (2013), to a decline in the market power of workers Benmelech et al. (2020). By contrast with these approaches, this paper focuses on the role played by financial globalization and its implications for the labor share due to risk-diversification, risk-taking, and wage-based insurance within the firm.

The consequences of market integration for risk-sharing goes back to Newbery and Stiglitz (1984), who show that openness to free-trade can be Pareto inferior to Autarky in absence of insurance markets. In our context, without risk-sharing within the firm, financial integration leading to increased risk-taking would necessarily make wages more volatile and potentially reduce welfare. Fernandez (1992) shows that the result by Newbery and Stiglitz (1984) can be overturned via efficient risk-sharing within the firm through wage contracts, but entailing a reduction of wages to compensate firms for the additional insurance provided. In our context with firm heterogeneity in risk-exposure, increased risk also reduces the wage bill but does so through the reallocation of production towards more risk-exposed firms. For a given firm, financial integration reduces rather than increases risk exposure and thus reduces the implicit cost of insurance offered to workers.

Regarding international financial integration Levchenko (2005) looks at its effect on consumption volatility in an environment in which access to a perfect international insurance market is unevenly distributed and domestic risk-sharing is limited to self-enforcing contracts. In such a context, the benefits to financial integration might no longer be passed on the workers, as perfect international insurance contracts crowd out domestic self-enforcing contracts. A counterfactual implication would be that the labor share decreases within the firm. In contrast to Levchenko (2005), our framework enables endogenous risk-taking and reallocation of production towards riskier firms and yields a decline in labor share due to reallocation, consistently with the micro-level evidence, even as within-firm labor shares rise.

Our paper also contributes to the nascent literature on risk-adjusted input allocation. David et al. (2021) develop a theory of risk-adjusted capital allocation and show that firms more exposed to aggregate risk have higher marginal products of capital. David and Zeke (2022) show that risk-adjusted capital allocation affect the dynamics of aggregate tfp and modify the effects of monetary policy (and thus change optimal policy).

Finally, there is significant evidence that workers are insured within the firm and especially so against temporary shocks (e.g. Guiso et al. (2005)) supporting the main assumption of our model framework. Hartman-glaser et al. (2019) suggests that such a mechanism can explain the divergence between the declining average capital share and the increasing aggregate labor share observed in the US, UK, and continental Europe. Their paper is the closest to ours it emphasizes the within-firms risk-sharing between capitalists and workers as driving the dynamics of the capital/labor shares. The emphasis and the mechanism are however very different. They look at the effect of increasing idiosyncratic volatility on the ex-post distribution of capital shares while we look at the exposure to aggregate risk on the ex-ante distribution of expected labor share and the implication of international diversification of aggregate risk. Because of such differences, we see our empirical results and model-based explanation as complementary to theirs.

The rest of the paper is organized as follows. Section 2 demonstrates how labor's share of income, both at the micro and macro level, are influenced by changes in the price of risk in a standard production environment with labor chosen under uncertainty. Section 3 develops a general equilibrium model of international diversification with heterogeneous firms and demonstrates how changes in the extent of international diversification can affect labor's share of income. Section 4 documents that several key implications of our model are supported by both cross-country and firm-level panel data. Section 5 concludes.

2 Labor's share of income, risk, and input allocation

In this section we develop analytical characterizations of how labor's share of income is affected by the heterogeneous risks of production technologies when inputs are made under uncertainty. Here we assume that there is a common, exogenous stochastic discount (SDF) factor used to value cash flows from these technologies in different states of the world; the SDF is endogenized in section 3. For ease of notation we model this as a static problem and omit time subscripts, but the results extend to dynamic versions of the model as well.

There are multiple production technologies that produce a homogeneous output as follows:

$$\begin{aligned} Y_i &= A_i K_i^{\alpha_1} L_i^{\alpha_2} \\ Y &= \sum_i Y_i, \end{aligned} \tag{1}$$

where $\alpha_1 + \alpha_2 < 1$ and the level of the productivity of technology i, A_i, is not realized until after input choices are made. We use Cobb-Douglas production for tractability, but the insights in this section extend to a broader class of production functions - in appendix A we discuss the application to other production functions and derive analogues of the results in this section for the CES production functions.

We assume that payments to factors of production (capital and labor) cannot be state-contingent, and that inputs are chosen to maximize the SDF-weighted value of cash flows, yielding the optimization problem:

$$\max_{L_i, K_i} \mathbb{E}\left[\Lambda\left(A_i K_i^{\alpha_1} L_i^{\alpha_2} - W L_i - P_k K_i\right)\right], \tag{2}$$

where Λ is a stochastic discount factor used to price all cash flows. This can be interpreted as each technology/firm chooses inputs to maximize their market values, where investors price states of the world using Λ.[1] The first-order conditions resulting from the maximization problem above yield an expression for the share of a firm's expected sales that are paid as wages, which we refer to as *labor's share of expected income*:

$$\frac{W L_i}{\mathbb{E}\left[Y_i\right]} = \alpha_2 \left(1 + Cov\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{A_i}{\mathbb{E}\left[A_i\right]}\right)\right) \tag{3}$$

If the SDF Λ is risk neutral, we get the standard result that labor's share of expected income is equal to the production function coefficient in labor: α_2. However, if Λ is affected by the

[1] The exact timing of when wages and the rental price of capital are paid are not crucial for our results; what is important is that the payment for labor and capital that are productive at time t are not contingent on shocks at time t.

realization of shocks which also affect firm revenue productivity, A_i, then labor's share of income is modified by a covariance term. If firm productivity is procyclical while the SDF is countercyclical, as standard theory and empirics suggest, then this covariance term is negative and reduces labor's share of expected income. The intuition for this result is as follows: since the quantity and wage of workers is chosen in advance, firms are insuring workers against aggregate shocks and bearing all of the cyclical cash flow risk themselves. If firm profits are procyclical and the SDF countercyclical, then this risk is costly and leads to firms wanting to hire fewer workers at a given wage rate, reducing the (micro) labor's share of income.

The solution to (2) also yields expressions for the relative allocation of labor and capital:

$$\frac{K_i}{K} = \frac{L_i}{L} = \frac{\left(\mathbb{E}\left[A_i\right]\left(1 + Cov\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{A_i}{\mathbb{E}[A_i]}\right)\right)\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\sum_h \left(\mathbb{E}\left[A_h\right]\left(1 + Cov\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{A_h}{\mathbb{E}[A_h]}\right)\right)\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}. \tag{4}$$

We see that this covariance term also affects the relative allocation of inputs: if this covariance term is negative for a firm, this firm will have a lower labor's share and a lower relative allocation of labor as compared to an acyclical firm with the same expected productivity. Note that the production technology, implies that the aggregate labor share of expected income can be written as an output-share weighted average of firm-level labor shares:

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} \frac{WL_i}{\mathbb{E}\left[Y_i\right]} \tag{5}$$

We see that this aggregate labor's share depends on firm-level labor shares and relative output shares of firms; note that both of these depend on the covariance of firm productivity with the SDF. For convenience we have worked with labor's share of expected income; realized labor share depends on this measure and the realization of productivity shocks, $\frac{\mathbb{E}[Y]}{Y}$, which are identical to TFP shocks:[2]

$$\frac{WL}{Y} = \frac{\mathbb{E}\left[Y\right]}{Y} \frac{WL}{\mathbb{E}\left[Y\right]} \tag{6}$$

[2] $\frac{\mathbb{E}[Y]}{Y}$ depends on the realizations of firm-level productivity shocks and the output shares of firms; we provide an analytical characterization in appendix A.

Plugging in (3) and (4) yields an expression as a function of technology and covariances:[3]

$$\frac{WL}{E\left[Y\right]}=\alpha_{2}\frac{\sum_{i}\overline{A_{i}}\left(1+Cov\left(\frac{\Lambda}{\mathbb{E}[\Lambda]},\frac{A_{i}}{\mathbb{E}[A_{i}]}\right)\right)^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}}{\sum_{i}\overline{A_{i}}\left(1+Cov\left(\frac{\Lambda}{\mathbb{E}[\Lambda]},\frac{A_{i}}{\mathbb{E}[A_{i}]}\right)\right)^{\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}}},\tag{7}$$

where $\overline{A_{i}}=\mathbb{E}\left[A_{i}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}$ is the expected productivity of a technology taken to a power to account for convexity in production. Notice that again if the price of risk, Λ, is uncorrelated with firm-level shocks that aggregate labor's share will be equal to α_2. More generally, however, aggregate labor shares depends on the covariances of the price of risk with realized productivity shocks, as they both affect firm-level labor shares and their share of aggregate output.

2.1 Changes to the price of risk

Now consider a change to how risk is priced. We can represent this in general by considering some change to the dynamics of SDF. Formally, we can define a function,χ, which maps the exogenous state variables A_i to a value of the SDF: $\chi:\{A_i\}\rightarrow\Lambda$. A change is the pricing of risk is some change in the function χ; therefore from (5) we can express the derivative of some change to the nature of the SDF function as:

$$\frac{\partial\frac{WL}{\mathbb{E}[Y]}}{\partial\chi}=\sum_{i}\frac{\partial\frac{\mathbb{E}[Y_{i}]}{\mathbb{E}[Y]}}{\partial\chi}\frac{WL_{i}}{\mathbb{E}\left[Y_{i}\right]}+\sum_{i}\frac{\mathbb{E}\left[Y_{i}\right]}{\mathbb{E}\left[Y\right]}\frac{\partial\frac{WL_{i}}{\mathbb{E}[Y_{i}]}}{\partial\chi}.\tag{8}$$

The first term in (8) is the *reallocation effect*; it captures how the changing pricing of risk affects the allocation of inputs across technologies with different labor's shares. The second term is the *within effect premium effect* - it captures the effect of the changing pricing of risk on the labor's shares of individual technologies.

2.1.1 Parametric examples

For greater insight into how the pricing of risk affects labor share, we need to put more structure on the environment. We first consider the parameterization in which there are two production technologies: one risky and one safe. We then consider a parameterization where both the Λ and A_i are log-linear functions of an aggregate shock, where the loading of A_i on shocks are normally distributed. In both of these cases we show that following a reduction in the price of risk, the *reallocation effect* decreases labor's share while the *within effect* increases labor's

[3] Output shares depend on firm productivity and input shares, we characterize them in appendix A.

share. Furthermore, we derive the conditions under which the reallocation effect may dominate and imply that a reduction in the price of risk leads to a decline in labor's share of income.

One safe, one risky firm Assume that there are only two types of technologies: $i \in \{s, r\}$, where s indexes the "safe" technology which is always equal to its expectation, and r indexes the risky technology. In such a world the price (and quantity) of productivity risk is captured by a single statistic χ_r:[4]

$$\chi_r = Cov\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{A_r}{\mathbb{E}[A_r]}\right) \tag{9}$$

Proposition 1 formalizes the effects of changes on the price of risk on aggregate labor's share of income, while panel A of Figure 1 shows how the equilibrium changes as the price of risk ($-\chi_r$) changes. At the firm level, the risky firm's labor share is monotonically decreasing in the price of risk, as the cost of insuring the workers wages rises. The risky firm's share of output and the risky firm's share of inputs (labor/capital) are both monotonically decreasing in the price of risk, as they choose fewer inputs as this risk premia rises. As the price of risk falls (χ_r gets bigger) we have two competing forces on labor's share of expected aggregate income: the labor share of the risky firm rises (the *within effect*) while the share of output produced by the risky firm, which has a lower labor share, rises (the *reallocation effect*). If the price of risk is high enough ($\chi_r < \overline{\chi}$), then the reallocation effect is larger than the within effect and a fall in the price of risk leads to a decline in the aggregate labor share of income; otherwise ($\chi_r > \overline{\chi}$) the within effect dominates and a decline in the price of risk increases labor's share of income. In other words, labor's share of expected aggregate income is falling in the price of risk for low prices of risk ($\chi_r > \overline{\chi}$), and increasing for higher ones ($\chi_r < \overline{\chi}$).

Proposition 1. *If $\chi_r < 0$ (SDF is negatively correlated with risky technology shock), a decrease in the price of risk (increase in χ_r) implies:*

1. *The within effect increases labor share:* $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \chi_r} = \alpha_2 \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} > 0$
2. *The reallocation effect decreases labor share:* $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \chi_r} \frac{WL_i}{\mathbb{E}[Y_i]} = \alpha_2 \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\chi_r}{(1+\chi_r)} < 0$
3. *If χ_r is negative enough, a decrease in the price of risk decreases labor share because the reallocation effect dominates the within effect: There exists a threshold $\overline{\chi} < 0$ such that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r} < 0$ iff $\chi < \overline{\chi}$ and $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r} > 0$ iff $\chi > \overline{\chi}$*

[4]Note that we can also write χ_r as the product of two terms: The quantity of risk, $\sigma^2_{log(A_r)}$, and the price of this risk $\frac{Cov\left(\frac{\Lambda}{\mathbb{E}[A_r]}, \frac{A_r}{\mathbb{E}[\Lambda]}\right)}{\sigma^2_{log(A_r)}}$. The price of risk can be understood as the elasticity of the SDF with respect to the technology shock.

Proof. See appendix A. □

Gaussian shocks and firm risk exposures We now show that this same intuition applies to a specification with richer heterogeneity in the risk of technologies. Assume that there are a continuum of technologies such that $log(A_i) = \overline{a_i} + \beta_i X$ where X is a normally distributed shock and $\overline{a_i}, \beta_i$ are known when input choices are made. The SDF is also log-linear in the shock: $log(\Lambda) = \lambda_0 + \lambda_x X$ where λ_0 is uncorrelated with X and λ_x is constant known when input choices are made. Finally, assume that the logarithm of expected productivity and β_i are normally distributed across firms and independent; we let $\mu_\beta, \sigma_\beta > 0$ denote the mean and standard deviation of betas across firms. Note in this specification that the price of risk is $-\lambda_x$ - so an increase in λ_x is a decrease in the price of risk.

Proposition 2 formalizes the effects of changes on the price of risk on aggregate labor's share of income. Panel B of Figure 1 shows how the equilibrium changes as the price of risk $(-\lambda_r)$ rises. As before, Labor's share of expected aggregate income is falling in the price of risk for low prices of risk $(\lambda_x > \overline{\lambda}_x)$, and rising in the price of risk for higher prices of risk $(\lambda_x < \overline{\lambda}_x)$.[5] The higher a firm's beta, the more negative is its elasticity of firm labor share to the price of risk. Finally, subplot (f) plots the relative share of inputs used and outputs produced by a firm with $\beta = 1$ as compared to a riskless firm with $\beta = 0$.[6].

Proposition 2. *If $\lambda_x < 0$ (SDF is negatively correlated with technology shock), a decrease in the price of risk (increase in λ_x) implies:*

1. *If the input-weighted risk exposure is positive, the within effect increases labor share: If $\left(\sum_i (\beta_i) \frac{L_i}{L}\right) > 0$, then $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda_x} > 0$*

2. *The reallocation effect decreases labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda_x} \frac{WL_i}{\mathbb{E}[Y_i]} < 0$*

3. *If the price of risk is high enough, a decrease in the price of risk decreases labor share because the reallocation effect dominates the within effect:*
   $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \lambda_r} < 0$ *iff* $\mu_\beta < \left(\frac{1-(\alpha_1+\alpha_2)^2}{(1-\alpha_1-\alpha_2)^2}\right)(-\lambda_x)\,\sigma_x^2\sigma_\beta^2$

Proof. See appendix A. □

[5] Since there are firms with negative β, if the price of risk were to be high enough, negative beta firms can have a larger input share than positive beta firms and therefore labor's share of income could theoretically be higher than α_2. For this reason the macro labor share does not asymptote to α_2 as the price of risk diverges, as it does in the two firm case (where the riskless firm produces all output in that limit).

[6] The relative share of firms with $\beta \neq 1$ to the riskless firm is equal to these relative shares to the power of β.

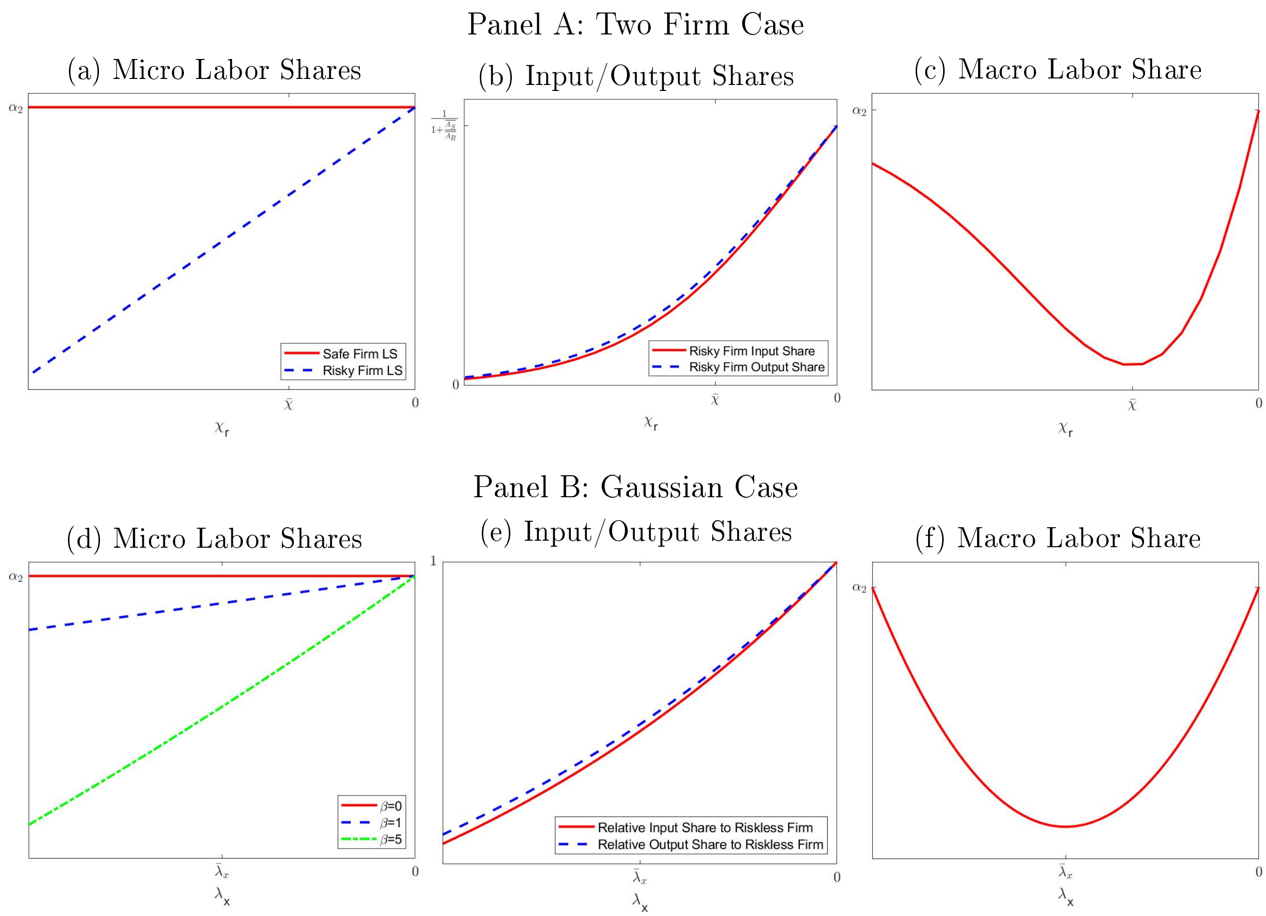

Figure 1: Labor's Share of Income and Resource Allocations as a function of the price of risk

3 International Diversification

We now embed the dynamics of production under uncertainty explored in section 2 in a two-period general equilibrium model of limited international diversification. We demonstrate how a change in the extent of international diversification, by changing the equilibrium price of risk, can lead to changes labor's share of income, both at the macro and micro level. We show that an increase in international diversification will lower the price of risk, as the owners of firms are better able to diversify across countries and reduce the volatility of their consumption. This leads to a decrease in the price of risk, which through its effects on the labor shares of given technologies and the allocation of resources across production technologies influences the aggregate labor share. If the price of risk is high enough, greater international diversification can lead to a decline in the aggregate labor's share of income but an increase in the micro (technology-level) labor share, as the reallocation effect dominates the within effect.

3.1 Model Setup

There are a continuum of islands, indexed by j. In our model consumption goods are homogeneous and fully mobile across islands (no frictions on trade), labor is immobile, and financial assets are imperfectly mobile.

On each island there are two representative production technologies indexed by $i \in \{s, r\}$, each representing a continuum of identical technologies, which produce a homogeneous output (both within and across islands). One of these two technologies will be riskless ($i = s$ for 'safe'), while the other will depend on the realization of an island-specific shock ($i = r$ for 'risky'); these island specific shocks are uncorrelated.

There are two types of households on each island: workers and capitalists. Workers provide labor for the production process and consume some of the final good; they are not mobile and cannot trade financial assets. Capitalists consume the final good and can trade a limited set of financial instruments: shares in firms, possibly both on their own island (*domestic*) or on other islands (*foreign*), and a risk-free bond. Timing in the model works as follows: in the first period, firms decide the quantity of capital and labor to employ and capitalists receive an endowment that can be either consumed then or used for capital investment. Capitalists also make asset allocation decisions - how much of their endowment to sell, and which financial assets to purchase. In the second period production occurs, workers are paid their pages and capitalists their profits, and both workers and capitalists consume the final good.

Capitalist's Problem Each island has a representative capitalist, born with an endowment of capital $K_{0,j}$. In the first period, they can either directly consume units of these endowment or sell it to firms who use it as capital. Capitalists also can purchase shares in firms. Purchasing a share of a foreign firm in country h comes with a cost equal to τ_h of the amount invested abroad. This can be interpreted either as a literal tax on foreign investment or a reduced form representation of informational or administrative costs of foreign investment. In the second period, capitalists receive the operating profits firms pay out to their shareholders and use those funds to purchase consumption goods. In addition, there is an additional charge if a firm in a particular continuum (of a risk type in a certain island) is more foreign-owned than other firms in that continuum, which we denote $\tau^*_{i,h}(S_{i,h})$; in the equilibrium we study all firms of a given risk type in an island will have the same ownership composition and thus this cost will be equal to zero.[7]

The problem of a capitalist/investor in country j consists of choosing consumption and holdings of financial assets, both domestic ($S_{i,j}$ indexes firm i in island j) or foreign ($S^*_{i,j,h}$ indexes capital from island j holding firm i in island $h's$ equity), as well as the risk free asset S_{rf}.

$$
\begin{aligned}
& \max_{S_{i,j},S^*_{i,j,c},S_{rf},K_{1,j}} U_1(C_{E,j,0}) + \rho U_2(C_{E,j}) \\
s.t. \quad & : C_{E,j,0} = K_{0,j} - K_{1,j} - \sum_i S_{i,j}P_{i,j} - \sum_{h\neq j}\left(1+\tau_h+\tau^*_{i,h}(S_{i,h})\right) S^*_{i,j,h}P_{i,h} - P_{rf}S_{rf} + T_{0,j} \\
& C_{E,j} = S_{i,j}V_{i,j} + \sum_{h\neq j}\sum_i S^*_{i,j,h}V_{i,h} + S_{rf} + P_{k,j}K_{1,j} + T_j \\
& S_{i,j}, S^*_{i,j,h}, C_{E,j,0}, C_{E,j} \geq 0
\end{aligned}
$$

where $V_{i,j} = \left(A_{i,j}K^{\alpha_1}_{i,j}L^{\alpha_2}_{i,j} - K_{i,j}P_{k,j} - W_jL_{i,j}\right)$ is the value of company i in island j after shocks are realized and $P_{i,j}$ its market value in period 1, before shocks are realized. $T_{0,j}, T_j$ are lump sum transfers of the purchase prices of firms that are rebated to capitalists to ensure consumption markets clear. Capitalists have CRRA preferences over consumption in both periods.[8] We define the stochastic discount factor of capitalists is island j as $\Lambda_j = \frac{U'_2(C_{E,j})}{U'_1(C_{E,j,0})}$.

[7] This eliminates incentives for one firm in the continuum to deviate and allows for us to model these firms as a representative firm. We describe the details of this cost and the incentive to deviate in appendix B.

[8] The CRRA coefficients need not be the same for the first and second period utility functions; further the majority of our results do not depend on the assumption of CRRA preferences. Assuming CRRA preferences yields a simple proof that the fraction of the risky firms shares held by domestic capitalists is increasing in the tax on foreign investors. For our other results it is enough to assume that capitalists' utility function is a continuous, increasing, and concave function of their consumption.

Workers, firms, and clearing conditions Workers have utility over consumption and leisure, which they maximize subject to their budget constraint:

$$C_{W,j} = W_j \sum_i L_{i,j}$$

The firm's problem is identical to the problem set up in section 2. In equilibrium the domestic capitalist will always be a marginal investor (for $\tau_j > 0$), and therefore we can use Λ_j as the SDF for the input choices of firms on island j.[9]

An equilibrium is an allocation and prices which satisfy firms', workers', and capitalists' problems and satisfies clearing conditions in asset, good, and input markets:

$$S_{i,j} + \sum_{h \neq j} S^*_{i,h,j} = 1 \qquad \sum_j C_{W,j} + C_{E,j} = \sum_j Y_j$$

$$\sum_j S_{rf,j} = 0 \qquad \sum_j C_{E,j,0} = \sum_j \left(K_{j,0} - \sum_i K_{j,i} \right)$$

3.2 Analytical Results

It is straightforward to verify that there exists a threshold, $\tau_j^{f.autarky}$, at which the shares of all island j technologies are held by island j capitalists in equilibrium - we call this 'financial quasi-autarky'.[10] In that case the allocations only depend on local preferences, technology, and the (common) risk-free rate. However, if $\tau_j < \tau_j^{f.autarky}$, then in equilibrium the risky technology in island j is held in positive quantities both by domestic and foreign capitalists; the safe securities are never held by foreign capitalists with $\tau_j > 0$. Note that the solution to the capitalists problems lead to the following first order conditions for the share price of the risky technology in island j:

$$\begin{aligned} P_{r,j} &= \mathbb{E}\left[\Lambda_j \left(A_{r,j} F\left(L_{r,j}, K_{r,j}\right) - L_{r,j} W_j - K_{r,j} P_{k,j}\right)\right] \ \ if\ S_{r,j} > 0 && (10) \\ P_{i,j}\left(1+\tau_j\right) &= \mathbb{E}\left[\Lambda_h \left(A_{i,j} F\left(L_{i,j}, K_{i,j}\right) - L_{i,j} W_j - K_{i,j} P_{k,j}\right)\right] \ \ if\ S^*_{i,h,j} > 0 && (11) \end{aligned}$$

where (10) corresponds to the first order condition of a domestic capitalist, and (11) corresponds to the first order condition of a foreign capitalist. If $\tau_j < \tau_j^{f.autarky}$ and both are held in positive quantities, then both (10) and (11) must bind. If both foreign and domestic investors own shares of the risky technology in equilibrium, their valuations of it must be equal. Note

[9] See appendix B for the details of the firm problem and the derivations of why Λ_j is the SDF used to price assets in country j in equilibrium.

[10] We call this quasi-autarky because it is possible that the risk-free bond may still be traded by the island or that domestic capitalists hold foreign assets.

that the valuation of the domestic capitalist can be expressed as the risk-free discounted (by the common risk-free interest rate) value of the firm, less a risk premia due to the covariance of the domestic SDF with firm productivity. Foreign investors valuation, on the other hand, will be equal to the risk-free discounted value less the cost of foreign investment; they have no risk premium because in equilibrium their SDF is independent of foreign country j productivity. Combining these yields an equilibrium condition in which the technology-specific risk premia term that influences labor shares and allocations are pinned down by the cost to foreign investors:

$$Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right) = -\frac{\tau_j\left(1-\alpha_1-\alpha_2\right)}{1+\tau_j\left(1-\alpha_1-\alpha_2\right)} \tag{12}$$

When both foreign and domestic investors own shares of the risky technology, (12) shows that the equilibrium risk premia term is pinned down by the cost for foreign investors. Intuitively, if the cost for foreign investors is lower, there will be greater foreign demand for the risky firm shares; therefore in equilibrium domestic capitalists will hold less of the domestic risky firm shares as a fraction of their portfolio and thus their SDF will be less sensitive to the productivity of the domestic risky firm.

We can use (12) and the results from section 2 to derive expressions for labor's share of expected income for each technology and the relative allocation of resources in this environment if $\tau_j \in [0, \tau_j^{f.autarky})$. First, note that we can write the micro level labor shares for the safe and risky firms as follows:

$$\frac{L_{s,j}W_j}{E\left[Y_{s,j}\right]} = \alpha_2$$
$$\frac{L_{r,j}W_j}{E\left[Y_{r,j}\right]} = \alpha_2\frac{1}{1+\tau_j\left(1-\alpha_1-\alpha_2\right)}$$

As before, the safe technology's labor share is just equal to the production function coefficient on labor. The risky technology's labor share, on the other hand, can be expressed as a function of the cost for foreign investors. As the cost for foreign investors, τ_j, falls the risk premia term for the risky technology on island j must decrease (in magnitude) in equilibrium due to (12), leading to an increase in the labor share for the risky technology. As the allocation of resources also depends on this risk premia term, we can express that as a function of the cost for foreign investors:

$$\frac{K_{r,j}}{K_j} = \frac{L_{r,j}}{L_j} = \frac{\overline{A_{r,j}} \left(\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\overline{A_{s,j}} + \overline{A_{r,j}} \left(\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}},$$

where $\overline{A_{i,j}} = \mathbb{E}[A_{r,j}]^{\frac{1}{1-\alpha_1-\alpha_2}}$ is the expected productivity of a technology taken to a power to account for convexity in production. Again, a decrease in the cost for foreign investors, τ_j, reduces the magnitude of risk premia term because the risky technology is a smaller share of the domestic investor's portfolio. This leads to an increase in the share of labor/capital allocated to the risky technology in equilibrium.

We can express the aggregate labor's share and decompose the changes on the aggregate labor's share of income resulting from a change in the cost for foreign investors:

$$\frac{W_j L_j}{\mathbb{E}[Y_j]} = \alpha_2 \frac{1 + \frac{\overline{A_{r,j}}}{\overline{A_{s,j}}} \left(\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{1 + \frac{\overline{A_{r,j}}}{\overline{A_{s,j}}} \left(\frac{1}{1+\tau_j(1-\alpha_1-\alpha_2)}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \tag{13}$$

$$\frac{\partial \frac{W_j L_j}{\mathbb{E}[Y_j]}}{\partial \tau_j} = \sum_i \frac{\partial \frac{\mathbb{E}[Y_{i,j}]}{\mathbb{E}[Y_j]}}{\partial \tau_j} \frac{W_j L_{i,j}}{\mathbb{E}[Y_{i,j}]} + \sum_i \frac{\mathbb{E}[Y_{i,j}]}{\mathbb{E}[Y_j]} \frac{\partial \frac{W_j L_{i,j}}{\mathbb{E}[Y_{i,j}]}}{\partial \tau_j}.$$

Analogously to (8), a decrease in the cost of investment for foreign capitalists leads to an increase in the risky technology's labor share, and a reallocation of inputs towards the risky firm, which decreases labor share. Proposition 3 formalizes the effects of these tax changes for foreign investors, τ_j, for the aggregate labor's share of income in this economy.

Proposition 3. *If $\tau_j < \tau_j^{f.autarky}$, then a fall in the cost for foreign investors, τ_j, implies:*

1. *The equilibrium price of risk decreases*
2. *The fraction of shares in the risky firm, $S_{r,j}$ held by domestic investors falls if the real interest rate is non-negative.*
3. *The within effect increases labor share: $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \tau_j} < 0$*
4. *The reallocation effect decreases labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \tau_j} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$*
5. *There exists a threshold $\widehat{\tau_j}$ such that*[11]

[11] In some parameterizations (ie there is relatively little risk or capitalists are close to risk neutral) it is possible that $\widehat{\tau_j} > \tau_j^{f.autarky}$.

(a) Labor share is falling (reallocation effect>within effect): $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \tau_j} > 0$ *iff* $\tau_j > \widehat{\tau}_j$*:*

(b) Labor share is rising (reallocation effect<within effect): $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \tau_j} < 0$ *iff* $\tau_j < \widehat{\tau}_j$

Proof. See Appendix B. □

4 Empirical evidence

In this section we test the key predictions of the model developed in sections 2 and 3. We first use cross-country data to show that the process of greater financial liberalization/risk-sharing is associated with falling labor shares in cross-country data, and that there has been a significant increase in cross-country risk-sharing over the past several decades. We then use U.S. firm-level data to show two facts consistent with the predictions of our model: first firm with greater exposure to systematic risk have lower labor shares of firm income as compared to otherwise similar firms within their industries. Second, increased financial integration (foreign portfolio investment) is associated with reallocation of industry input and output towards riskier firms. Finally, we use cross-country and industry data to verify that labor share falls by more in response to increased financial integration in industries with greater scope for reallocation (greater heterogeneity in labor shares).

4.1 From model to data

The model we developed in sections 2 and 3 has several testable empirical implications. We lay them out below.

Prediction 1: International diversification is associated with lower aggregate labor shares of income The general equilibrium model in section 3 shows that countries with lower costs for foreign investors (τ_j) should have lower labor's share of expected income. In particular, a first-order Taylor expansion of the equation for aggregate labor share yields the following expression for labor's share of income:

$$log(\frac{W_j L_j}{Y_j}) = \alpha_j + \gamma_\tau \tau_j + \gamma_{tfp} \left(tfp_j - \mathbb{E}\left[tfp_j\right]\right)$$

Our theory implies that if the *reallocation effect* dominates the *within effect*, then $\gamma_\tau > 0$; further, preset wages imply that the coefficient $\gamma_{tfp} < 0$. In equilibrium, τ_j is monotonically decreasing in foreign investor's holdings of firm equity, suggesting the following regression:

$$log(\frac{W_{j,t}L_{j,t}}{Y_{j,t}}) = \alpha_j + \gamma_{FEQ} FEQ_{j,t} + \gamma_{tfp} \left(tfp_{j,t} - \mathbb{E}_{t-1}\left[tfp_{j,t}\right]\right) \tag{14}$$

where FEQ_j is a measure of foreign investors holding of country j equity. Again, if the *reallocation effect* dominates the *within effect*, our model implies that $\gamma_{FEQ} < 0$ - labor share should be falling in international diversification.[12]

Prediction 2: Risky firms have lower labor shares of income Our model also has implications for micro-level observables. In particular, (3) outlines the fact that, for firms with identical production functions, riskier firms should have lower labor shares of income.[13] Therefore, in firm-level data, we should see that within narrowly defined industries (within which heterogeneity in production technologies is likely to be low), riskier firms have lower labor shares of income. This suggests a regression of the form

$$log(LS_{i,t+1}) = \gamma_\beta \beta_i + controls + \epsilon_{i,t}. \tag{15}$$

Our theory predicts that $\gamma_\beta < 0$.

Prediction 3: International Diversification is associate with reallocation towards firms that are riskier and have lower labor shares Our model has predictions for how the international diversification affects the allocation of resources. In particular, (4) and (20) characterize the allocation of inputs and outputs, and show that the covariance of a technologies' productivity and the stochastic discount factor affects this allocation.

These equations suggest regressions to test whether international diversification is associated with reallocation towards riskier firms:

$$log(\frac{Z_{i,t}}{Z_{ind,t}}) = \gamma_{\beta,FEQ} \beta_i FEQ_t + controls + \epsilon_{i,t} \tag{16}$$

where $Z = Y, L, K$ indexes inputs or outputs, and FEQ_t is the measure of foreign equity liabilities. β_i is a measure of risk exposure. Similarly, the theory suggests an analogue of (16)

[12] If we were to extend our model to one with a CES aggregator, as in appendix A, (14) would change only in that we would have to add controls for the determinants of the aggregate K/L - for example the relative price of investment goods.

[13] This insight does not require that production be Cobb-Douglas as in our baseline model; Appendix B develops an analogue of (3) with CES production.

to test whether international diversification is associated with reallocation towards lower labor share firms:

$$log(\frac{Z_{i,t}}{Z_{ind,t}}) = \gamma_{LS,FEQ} LS_{i,t} FEQ_t + controls + \epsilon_{i,t}. \tag{17}$$

Our theory predicts that increases in international diversification leads to reallocation towards riskier firms with lower labor share, and thus $\gamma_{\beta,FEQ} > 0$ and $\gamma_{LS,FEQ} < 0$.

Prediction 4: Industries with greater dispersion in labor shares should see larger declines in industry labor share In section 2, we consider a special parametric case in which firms' risk exposures are normally distributed.[14] If we assume that there are several industries, indexed by s, then we can express the elasticity the labor share of income of that industry with respect to the price of risk as a function of the mean and dispersion of labor shares of income (within that industry):[15].

$$\frac{\partial log\left(\frac{W_{s,j,t}L_{s,j,t}}{E[Y_{s,j,t}]}\right)}{\partial \lambda_{j,t}} = \frac{1}{\lambda_j}\left(\left(\mu_{LS,j,s,t} - log\left(\alpha_{2,s}\right)\right) + \sigma^2_{LS,j,s,t}\frac{1-\left(\alpha_1+\alpha_2\right)^2}{\left(1-\alpha_1-\alpha_2\right)}\right)$$

where $\mu_{LS,s,j}, \sigma_{\beta,s,j}$ are the average and standard deviation of risk exposures of industry s in country j. The intuition for this is as follows: in industries with greater dispersion in risk exposures (and therefore greater dispersion in labor shares), the *reallocation effect* should be larger (decreasing industry labor share of income if the price of risk falls); in industries with greater average risk-exposures (and therefore lower average labor shares), the *within effect* should be smaller (increasing industry labor share of income if the price of risk falls). This suggests a regression to test whether the interaction between industry average and dispersion:

$$\log\left(LS_{s,j,t}\right) = \gamma_\mu \mu_{LS,s,j,t-1} FEQ_{j,t} + \gamma_\sigma \sigma^2_{LS,s,j,t-1} Div_{j,t} + controls + \varepsilon_{i,t} \tag{18}$$

This regression tests whether the mean and dispersion of firm labor shares of income, interacted with measures of international diversification (which lower the price of risk, according to our theory) are associated with changes in industry aggregate labor shares. Our theory predicts that $\gamma_\mu.\gamma_\sigma < 0$ - an increase in diversification should lower labor share by more in industries

[14] For tractability we focus on the case with only two firms in our general equilibrium model in section 3.
[15] The formal setup for these results are in Appendix C.

(a) GDP-Weighted Averages (b) United States

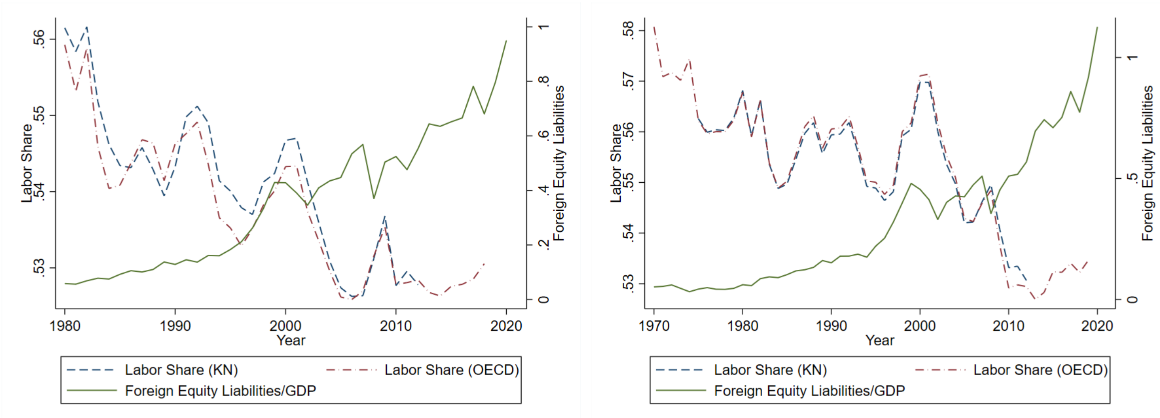

Figure 2: Trends in Labor Share and Equity Share of Foreign Investors

Displays the level of the aggregate labor share (left axis) and foreign investor's stock of equity, divided by GDP (right axis). Panel (a) displays an average of both of these measures across countries, while panel (b) displays these statistics for the United States.

with greater dispersion in labor shares because the *reallocation effect* is larger. Similarly, in industries with high labor shares on average, the *within effect* will be smaller (as our models suggests these firms have smaller risk exposures). Once nice feature of (18) is that is requires only measures of firm-level labor shares.

4.2 Cross-Country evidence

We use a panel dataset of country labor shares to document facts relating to international diversification and labor shares and evaluate prediction 1. We measure labor's share of income at the country level following Karabarbounis and Neiman (2014); for robustness we also use OECD's measure which extends beyond their sample. Our measure of international diversification is the domestic equity holdings of foreign investor, which we normalize by dividing by GDP. We take measure equity holdings from the External Wealth of Nations dataset of Lane and Milesi-Ferretti (2018), and include both foreign equity holdings via FDI and portfolio investment. We drop countries which are tax havens or have a disproportionate share of financial activity. Figure 2 shows that both in the U.S. and worldwide, measures of labor's share of income have been falling while our measure of international diversification, foreign holdings of a country's equity, have been rising.

Figure 3 displays a binned scatter plots of country labor share of income against our measure of international diversification, controlling for country-fixed effects.[16] We see that when a

[16]Binned scatter plots have been widely used in applied microeconomics to visualize relationships between variables in large datasets since Chetty and Szeidl (2005) and Chetty et al. (2009). With controls, this procedure

(a) Karabarbounis and Neiman (2014) (b) OECD

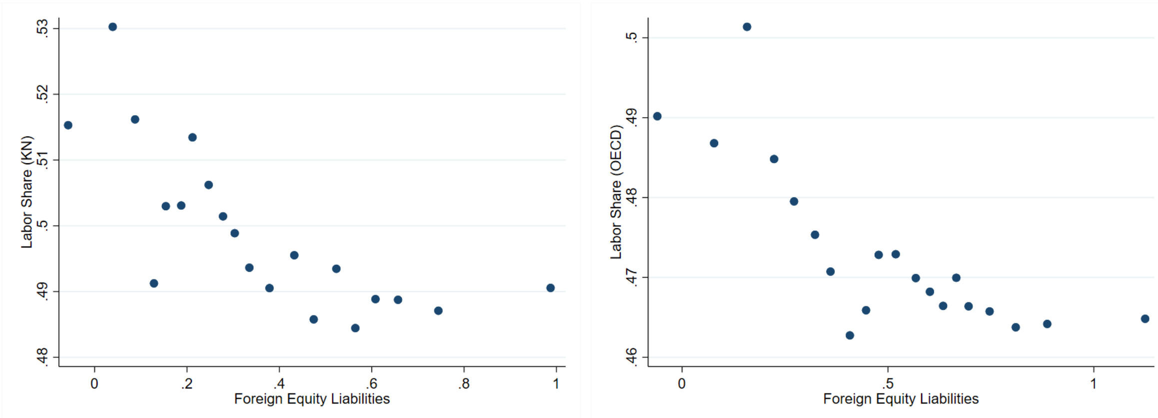

Figure 3: Labor Share vs Equity of Foreign Investors

These binned scatter plots display the relationship between labor's share of income and foreign equity holdings, controlling for country fixed effects. Each observations is a bin of country-year observations.

country has more equity in the hands of foreign investors as a share of GDP (as compared to its average level of foreign investor holdings), on average it has lower labor's share of income.

Table 1 formally tests prediction 1 by running the cross-country regression (14). The results confirm prediction 1 - foreign investors owning more of a country's equity is associated with statistically significant reduction in labor's share of income within that country. Our results are robust to adding fixed effects for country and year and controlling for variables related to the ratio of capital to labor (In Appendix A we show how our risk adjustment extends to the case with CES, in which labor share is affected by both risk and this ratio), including the relative price of investment. We find that the relative price of investment is associated with higher labor shares, consistent with Karabarbounis and Neiman (2014). The relationship between foreign investment in equity and labor's share of income is also economically significant: given the increase in foreign investors' equity share seen in Figure 2, the coefficient estimate in column 4 of Table 1 implies a reduction of U.S. labor share of between 2.5-3% over that same horizon, and a similar value for a global average.

first residualizes both the x and y variables on the fixed effects and then adds back the unconditional means. Then it plots dots, which represents the average of the x and y variable for percentile bins of the x axis variable. Foreign equity liabilities are always positive, by construction - but after the residualization procedure to control for fixed effects negative values are possible.

	(1)	(2)	(3)	(4)
Foreign Equity Liabilities	-0.0685**	-0.0491**	-0.0283**	-0.0387***
	(-2.51)	(-2.62)	(-2.30)	(-3.88)
TFP shock			-0.237	-0.397**
			(-1.17)	(-2.18)
Average hours				-0.127
				(-0.90)
Relative price of investment				0.102***
				(3.38)
Country fixed effects	yes	yes	yes	yes
Year fixed effects	no	yes	yes	yes
R^2	0.853	0.878	0.921	0.937
Observations	661	661	423	402

Notes: This table presents a regression of country labor share on foreign investors holding of domestic equity for advanced economics, excluding tax havens and small financial centers. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by country and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.

Table 1: Labor's share of income on international diversification

4.3 Firm-Level evidence

4.3.1 Data and Measurement

We use U.S. firm-level data from Compustat on publicly traded firms to test our second and third predictions. This dataset lets us compute measures of firm aggregate risk exposure from financial markets using a high quality dataset with relatively good coverage. We measure firm CAPM betas by running a regression of daily stock returns on the aggregate daily market return for trading days within a calendar year. Note that, by definition, a weighted average of betas (by market-cap) is always equal to one. In other words, if there is reallocation towards riskier firms, that makes the market portfolio riskier itself and lowers measured betas. This poses a challenge for our predictions relating to reallocation. To this end, we construct measure of firm *relative beta*, $\widehat{\beta}_i$, by residualizing firm CAPM betas on industry-year fixed effects and computing the average of these residuals for each firm.

Measuring labor share in Compustat also presents challenges - while the number of employees is reported for most firms in Compustat, only a subset have data on their levels of labor compensation, which is a challenge for computing measures of labor's share of income. Our

(a) Labor Intensity

(b) Extended Labor Share

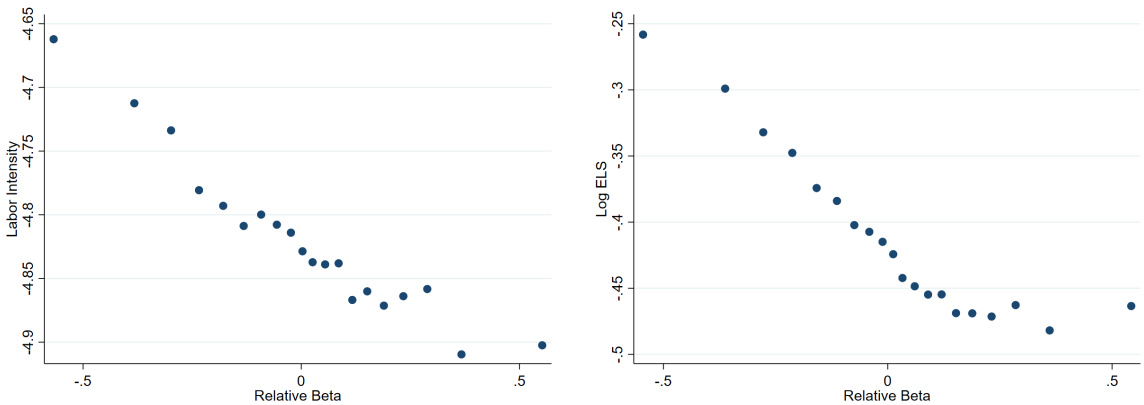

Figure 4: Exposure to Aggregate Risk and Labor's Share of Income

These binned scatter plots display the relationship between labor's share of income and firm relative capm beta, controlling for industry by year fixed effects. Each observations is a bin of firm-year observations.

analysis focuses on differences in labor share in firms within the same industries, as labor's share of income can vary markedly across industries due to heterogeneity in production technologies. We use three measures of firm labor share. The first is labor intensity, $(log(\frac{emp}{sale}))$. Note that since our analyses all include industry fixed effects, this is equal to labor share if firms within an industry have the same average compensation per employee. The two other measures we use are from Donangelo et al. (2018) - 'Labor Share' (LS) using data on firm labor compensation and 'Extended Labor Share' (ELS) which infers labor compensation per employee from industry averages for firms with missing labor compensation.

4.3.2 Prediction 2

We document that firms which are more exposed to aggregate market risk (have higher market betas) have, on average, lower labor shares of income than other firms in their industries. Figure 4 presents binned scatter plots of firm market betas against measures of the labor share, controlling for industry by time fixed effects. [17] Panel (a) plots the results for labor intensity $(log(\frac{emp}{sale}))$, while panel (b) plots the Extended Labor Share measure.[18]

To confirm that this relationship is not only economically, but statistically significant and robust to standard controls, we run panel regressions of the form:

[17] Our measure of market beta is the estimated regression coefficient of daily firm equity returns on the aggregate market return in the previous year.

[18] The other labor share measure from Donangelo et al. (2018), 'Labor Share', also has a negative relationship with firm market betas; however it is more volatile due to the small number of firms that report the measure within a typical industry.

$$log(LS_{i,t+1}) = \gamma_{j,t} + \gamma_\beta \widehat{\beta}_i + \gamma_{FEQ} FEQ_{i,t} + \epsilon_{i,t} \tag{19}$$

where i indexes firms, j indexes industries, t denotes the year, and the γ terms are the estimated coefficients. $\widehat{\beta}_i$ is the measure of firm market betas, and $FEQ_{i,t}$ is a vector of controls. $\gamma_{j,t}$ indexes the industry-by-year fixed effects, and we cluster standard errors by firm and by year. Table 2 plots the results of regression (19) for three measures of firm labor share. It shows that firms that are more exposed to aggregate shocks (have higher CAPM betas) have lower labor shares on average than firms with lower betas within their industries, and that this effect is economically and statistically significant. These regressions imply that a firm with beta of 1 has, on average, a labor share of income that is more than 15% lower than a firm in the same industry with a beta of 0.

	(1) $\log\left(\frac{L}{Y}\right)$	(2) $\log(ELS)$	(3) $\log(LS)$	(4) $\log\left(\frac{L}{Y}\right)$	(5) $\log(ELS)$	(6) $\log(LS)$
$\widehat{\beta}$	-0.238***	-0.241***	-0.105***	-0.336***	-0.176***	-0.166***
	(-12.57)	(-16.24)	(-3.16)	(-16.35)	(-14.62)	(-5.77)
Profitability				-1.019***	-1.514***	-1.470***
				(-24.10)	(-33.93)	(-20.92)
Age				-0.0275***	-0.00781***	-0.00738
				(-5.85)	(-2.73)	(-1.14)
Size				0.0609***	-0.00607***	0.0253***
				(14.69)	(-2.74)	(4.36)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
R^2	0.677	0.405	0.718	0.716	0.510	0.797
Observations	153676	126730	11536	142760	118455	10039

Notes: This table presents the results of regressions of log firm labor share on measures of firms risk exposure and controls. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * $p < .10$ ** $p < 0.05$, *** $p < 0.01$.

Table 2: Firm Labor Share and Market Betas

4.3.3 Prediction 3

We now turn to our third prediction, that increasing international diversification should lead to reallocation towards technologies that are riskier and have lower labor shares. Figure 5 displays trends in employment growth along different firm characteristics by plotting firm employment growth, relative to their industry, against select firm characteristics. The figures clearly show that riskier firms have had employment reallocated towards them, as have firms with lower labor share.

(a) Relative β^{CAPM} (b) Labor Intensity (c) Extended Labor Share

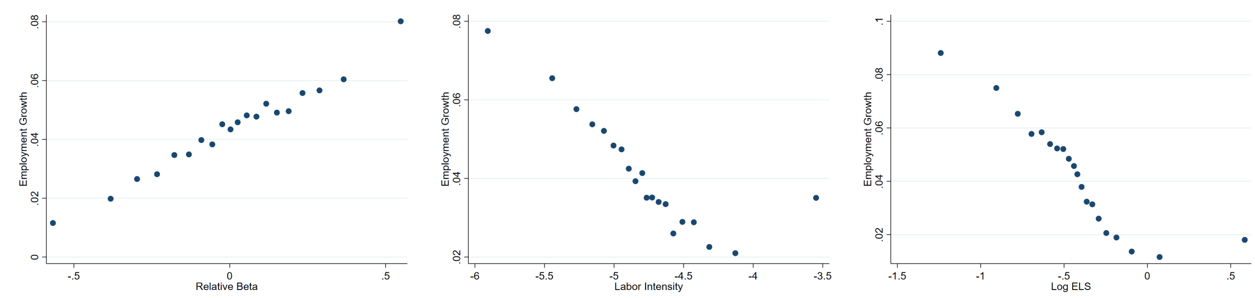

Figure 5: Trends in the Reallocation of Employment within Industry

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Table 3 displays the result of regressions of firm shares of industry output/inputs on the interaction of international diversification with a firm's relative Capm beta (following (16)); we include firm and industry-year fixed effects, control variables, and cluster standard errors two ways by firm and year.[19] We see that higher levels of international diversification are associated with riskier firms making up a higher fraction of industry output/inputs. The coefficient estimates imply that, on average, in response to the annual average increase in international diversification, a firm with a beta of 1 increased its share of industry inputs/outputs by around 2.5% more than a firm with a beta of 0 in the same industry.

Table (4) displays the results of an analogous regression of firm share of industry output/inputs on the interaction between firm labor share and international diversification. We see that higher levels of international diversification are associated with high labor share firms making up a lower fraction of industry output and inputs.[20]

4.4 Industry-Level evidence

We use firm panel data from several countries from ORBIS, merged with our measure of country financial diversification from Lane and Milesi-Ferretti (2018), to test our fourth prediction. We construct measures of both the aggregate labor share for every industry-country-year, as well as the mean and standard deviation of log labor shares within each industry-country-year.

[19]The individual components of the interaction do not have to be controlled for as they are absorbed by the fixed effects.

[20]The results are qualitatively similar for the other two measures of the labor share we consider, we report them for the other measures in Appendix D.

	(1) $\log(\frac{Y_i}{Y_{ind}})$	(2) $\log(\frac{L_i}{L_{ind}})$	(3) $\log(\frac{K_i}{K_{ind}})$	(4) $\log(\frac{Y_i}{Y_{ind}})$	(5) $\log(\frac{L_i}{L_{ind}})$	(6) $\log(\frac{K_i}{K_{ind}})$
$\widehat{\beta}$ × FEQ	2.318***	1.804***	2.372***	2.324***	1.829***	2.392***
	(11.29)	(9.11)	(9.44)	(10.59)	(8.82)	(9.02)
Profitability				1.479***	0.908***	1.058***
				(34.38)	(25.86)	(26.56)
Age				0.278***	0.247***	0.233***
				(21.46)	(18.69)	(14.33)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.943	0.941	0.938	0.950	0.945	0.942
Observations	151652	150763	152913	146330	145021	147208

Notes: This table presents the results of a regression of a firm's share of industry outputs & inputs on the interaction of firm relative beta and foreign investors holdings of U.S. equity, scaled by GDP. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.

Table 3: Reallocation, International Diversification, and Firm Relative Betas

	(1) $\log(\frac{Y_i}{Y_{ind}})$	(2) $\log(\frac{L_i}{L_{ind}})$	(3) $\log(\frac{K_i}{K_{ind}})$	(4) $\log(\frac{Y_i}{Y_{ind}})$	(5) $\log(\frac{L_i}{L_{ind}})$	(6) $\log(\frac{K_i}{K_{ind}})$
$\log(ELS)$ × FEQ	-0.328***	-0.256***	-0.406***	-0.398***	-0.311***	-0.447***
	(-5.65)	(-4.44)	(-6.08)	(-6.97)	(-5.69)	(-6.65)
Profitability				0.984***	0.948***	0.550***
				(19.19)	(18.56)	(8.74)
Age				0.237***	0.238***	0.251***
				(19.37)	(20.14)	(16.06)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.954	0.942	0.947	0.957	0.945	0.949
Observations	119414	118338	118994	116579	115551	116180

Notes: This table presents the results of a regression of a firm's share of industry outputs & inputs on the interaction of firm labor share and foreign investors holdings of U.S. equity, scaled by GDP. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.

Table 4: Reallocation, International Diversification, and Labor Share

We measure industries as 3 digit SIC codes and again drop countries that are tax havens or financial centers, and include only observations (industry-country-years) which are computed from at least 50 firms. Table 5 presents the results of the panel regression implied by (18): We regress the labor share of an industry-country-year on the interaction between foreign ownership of domestic equity and the mean/standard deviation of log firm labor shares within that industry (lagged to account for possible simultaneity bias). We include rich fixed effects in this regression, featuring industry-year, country-year, and industry-country fixed effects (the unit of observation is industry-country-year), and include the variables in interactions separately as control variables. Corresponding to our theory, country-industries with greater dispersion in firm labor shares see their labor share fall my more in response to an increase in foreign holdings of domestic equity, as do country-industries with greater average labor shares. We show that this result is robust to controlling for the interaction of the mean and standard deviation of firm sales - so it is not just more pronounced in industries that just happen to have greater heterogeneity in size.

	(1)	(2)	(3)
Foreign Equity Liabilities × mean log(LS)	-0.127*	-0.0869**	-0.0940**
	(-1.86)	(-2.74)	(-2.62)
Foreign Equity Liabilities × stdev log(LS)	-0.0983*	-0.0513**	-0.0592**
	(-1.95)	(-2.40)	(-2.26)
Foreign Equity Liabilities × stdev log(sales)			-0.0099
			(-1.22)
Foreign Equity Liabilities × mean log(sales)			-0.0047
			(-1.07)
Fixed effects	no	yes	yes
R^2	0.485	0.791	0.804
Observations	71346	69431	57325

Notes: This table presents a regression of the aggregate labor share of an industry (in a given country in a given year) on the mean and standard deviation of log labor shares interacted with our measure of international diversification, foreign ownership of equity divided by GDP. Our sample uses Orbis data merged with the *External Wealth of Nations* dataset of Lane and Milesi-Ferretti (2018), and we exclude tax havens and small financial centers. All specifications include the terms in the interactions individually as controls; fixed effects, when included, consist of three fixed effects terms: industry-year fixed effects, country-year fixed effects, and industry-country fixed effects. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by industry-country and year.

Table 5: Industry labor share dispersion and labor share dynamics

5 Conclusion

We develop a model of the allocation of inputs and labor's share of income in an environment with heterogeneous firms and where input decisions are made under uncertainty. In equilibrium, riskier firms have lower labor's share of income and allocated a smaller share of the inputs/outputs relative to their productivity, and the magnitude of this risk adjustment depends on the price of risk. In a model of international capital where the price of risk is endogenous to the degree of international risk sharing, we show that an increase in international diversification leads to lower prices of risk and leads to two competing effects on country labor shares of income. First, a fall in the price of risk leads to an increase in labor share for a given technology. Second, a fall in the price of risk leads to reallocation of labor and capital to riskier firms, who have lower labor shares. If the second effect dominates the first one, then an increase in international diversification can lead to a fall in aggregate labor's share of income, while within-technology labor's share of income rises. We document that the empirical predictions of the model are supported by both U.S. firm-level data, cross-country aggregate data, and cross-country firm-level data. Our empirical results imply that a sizable part of the observed decline in labor's share of income over the past several decades, both in the U.S. and internationally, could be explained by the concurrent increase in international diversification.

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Appendix

A Derivations and proofs for section 2

A.1 Derivations and additional results

Output shares Note that the output share of a given firm can be written as:

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]}=\frac{\mathbb{E}\left[A_i\right]K_i^{\alpha_1}L_i^{\alpha_2}}{\sum_h \mathbb{E}\left[A_h\right]K_h^{\alpha_1}L_h^{\alpha_2}}$$

Plugging in for input shares from (4) yields a characterization of output shares as a function of technology and covariance terms:

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]}=\frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1+\frac{Cov(\Lambda,A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_h \mathbb{E}\left[A_h\right]^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1+\frac{Cov(\Lambda,A_h)}{\mathbb{E}[A_h]\mathbb{E}[\Lambda]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}. \tag{20}$$

Realized labor share of income To derive the realized aggregate labor share, note that we can write:

$$\frac{WL}{Y}=\frac{\mathbb{E}\left[Y\right]}{Y}\frac{WL}{\mathbb{E}\left[Y\right]}$$

We can solve for the ratio of expected to realized output:

$$\begin{aligned}
\frac{\mathbb{E}\left[Y\right]}{Y} &= \frac{\sum_i \mathbb{E}\left[A_i\right]\left(\frac{K_i}{K}\right)^{\alpha_1}\left(\frac{L_i}{L}\right)^{\alpha_2}}{\sum_i A_i\left(\frac{K_i}{K}\right)^{\alpha_1}\left(\frac{L_i}{L}\right)^{\alpha_2}}\\
\Rightarrow \frac{\mathbb{E}\left[Y\right]}{Y} &= \frac{\sum_i \mathbb{E}\left[A_i\right]\left(\mathbb{E}\left[A_i\right]\left(1+\frac{Cov(\Lambda,A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_i A_i\left(\mathbb{E}\left[A_i\right]\left(1+\frac{Cov(\Lambda,A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}\\
\Rightarrow \frac{\mathbb{E}\left[Y\right]}{Y} &= \frac{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1+\frac{Cov(\Lambda,A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_i \frac{A_i}{\mathbb{E}[A_i]}\mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}}\left(1+\frac{Cov(\Lambda,A_i)}{\mathbb{E}[\Lambda]\mathbb{E}[A_i]}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \qquad (21)
\end{aligned}$$

Note that this only depends on firm productivities and their covariance with the SDF. If TFP is properly measured ($TFP_t=\frac{Y_t}{K_t^{\alpha_1}L_t^{\alpha_2}}$) then

$$\frac{\mathbb{E}\left[Y\right]}{Y}=\frac{\mathbb{E}\left[TFP\right]}{TFP} \tag{22}$$

A.2 Proofs

Proposition 1

1. Within effect derivative Taking the derivative of the labor share of each technology yields:

$$\sum_{i}\frac{\mathbb{E}\left[Y_{i}\right]}{\mathbb{E}\left[Y\right]}\frac{\partial\frac{WL_{i}}{\mathbb{E}\left[Y_{i}\right]}}{\partial\chi_{r}}=\alpha_{2}\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}>0 \tag{23}$$

2. Reallocation effect derivative Note plugging in the assumptions of the two firms into (20) yields:

$$\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}=\frac{\mathbb{E}\left[A_{r}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}\left(1+\chi_{r}\right)^{\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}}}{\mathbb{E}\left[A_{s}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}+\mathbb{E}\left[A_{r}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}\left(1+\chi_{r}\right)^{\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}}}. \tag{24}$$

Taking the derivative w.r.t χ_r yields:

$$\frac{\partial\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}}{\partial\chi_{r}}=\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}\frac{\left(1+\chi_{r}\right)^{\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}-1}\mathbb{E}\left[A_{r}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}\mathbb{E}\left[A_{s}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}}{\left(\mathbb{E}\left[A_{s}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}+\mathbb{E}\left[A_{r}\right]^{\frac{1}{1-\alpha_{1}-\alpha_{2}}}\left(1+\chi_{r}\right)^{\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}}\right)^{2}} \tag{25}$$

Which can be simplified as:

$$\frac{\partial\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}}{\partial\chi_{r}}=\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}\frac{\mathbb{E}\left[Y_{s}\right]}{\mathbb{E}\left[Y\right]}\frac{1}{1+\chi_{r}} \tag{26}$$

Plugging in for labor's share of income implies that

$$\sum_{i}\frac{\partial\frac{\mathbb{E}\left[Y_{i}\right]}{\mathbb{E}\left[Y\right]}}{\partial\chi_{r}}\frac{WL_{i}}{\mathbb{E}\left[Y_{i}\right]}=\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}\frac{\mathbb{E}\left[Y_{s}\right]}{\mathbb{E}\left[Y\right]}\frac{1}{1+\chi_{r}}\left(\alpha_{2}\left(1+\chi_{r}\right)-\alpha_{2}\right)$$

$$\Rightarrow\sum_{i}\frac{\partial\frac{\mathbb{E}\left[Y_{i}\right]}{\mathbb{E}\left[Y\right]}}{\partial\chi_{r}}\frac{WL_{i}}{\mathbb{E}\left[Y_{i}\right]}=\alpha_{2}\frac{\alpha_{1}+\alpha_{2}}{1-\alpha_{1}-\alpha_{2}}\frac{\mathbb{E}\left[Y_{r}\right]}{\mathbb{E}\left[Y\right]}\frac{\mathbb{E}\left[Y_{s}\right]}{\mathbb{E}\left[Y\right]}\frac{\chi_{r}}{1+\chi_{r}} \tag{27}$$

If $\chi_r < 0$ then clearly $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \chi_r} \frac{WL_i}{\mathbb{E}[Y_i]} < 0$.[21]

3. Threshold $\overline{\chi_r}$ Combining the results above yields:

$$\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r} = \alpha_2 \frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} + \alpha_2 \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{\chi_r}{1 + \chi_r} \tag{28}$$

Define $\overline{\chi_r}$ such that:

$$1 + \frac{\mathbb{E}\left[A_r\right]^{\frac{1}{1-\alpha_1-\alpha_2}}}{\mathbb{E}\left[A_s\right]^{\frac{1}{1-\alpha_1-\alpha_2}}} \left(1 + \overline{\chi_r}\right)^{\frac{1}{1-\alpha_1-\alpha_2}} + \frac{1}{1 - \alpha_1 - \alpha_2} \overline{\chi_r} = 0 \tag{29}$$

It is easy to verify that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r} = 0$ at $\chi_r = \overline{\chi_r}$.

Note that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r}$ has the same sign as

$$1 + \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{\chi_r}{1 + \chi_r} < 0 \tag{30}$$

note that the derivative of this w.r.t. χ_r is

$$\frac{\partial 1 + \frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\chi_r}{1+\chi_r}}{\partial \chi_r} = \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{1}{\left(1 + \chi_r\right)^2} + \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \chi_r} \frac{\chi_r}{1 + \chi_r} > 0 \tag{31}$$

Both of these terms are positive if $\chi_r \in (-1, 0)$, since $\frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \chi_r} = -\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \chi_r}$ and we derived $\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \chi_r}$ above.

It immediately follows that if $\chi_r > \overline{\chi_r}$, $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r} > 0$. Similarly, if $\chi_r < \overline{\chi_r}$, $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \chi_r} < 0$.

Proposition 2

1. Within effect derivative Note that solving for output shares and firm labor shares yields:

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} = \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{\lambda_x \beta_i \sigma_x^2}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{\lambda_x \beta_i \sigma_x^2}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \tag{32}$$

[21] Note that if $\chi < -1$ would imply negative expected labor's share of income, which would be impossible. In practice, that would mean that the extent of risk aversion is so high that the firm would choose no labor in equilibrium, and therefore we would not have an interior solution.

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \alpha_2 e^{\lambda_x \beta_i \sigma_x^2} \tag{33}$$

Solving for the within risk premium derivative yields the expression:

$$\sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda_x} = \left(\sum_i \left(\beta_i \sigma_x^2\right) \frac{L_i}{L} \right) \frac{WL}{E\left[Y\right]} \frac{1}{\alpha_2} \tag{34}$$

This is clearly positive if the input-share weighted risk exposure, $\left(\sum_i \left(\beta_i\right) \frac{L_i}{L}\right)$, is positive.

2. Reallocation effect derivative Using similar algebra, we derive an expression the reallocation component as:

$$\sum_i \frac{WL_i}{\mathbb{E}\left[Y_i\right]} \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda_x} = \frac{\alpha_1 + \alpha_2}{1 - \alpha_1 - \alpha_2} \sigma_x^2 \left(\lambda_x \sigma_x^2 \sigma_\beta^2\right) e^{\lambda_x \sigma_x^2 \mu_\beta + \frac{1}{2} \sigma_\beta^2 \left(\lambda_x \sigma_x^2\right)^1 \left(\left(\frac{1}{1-\alpha_1-\alpha_2}\right)^2 - \left(\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}\right)^2 \right)} \tag{35}$$

Which is clearly negative if $\lambda_x < 0$.

3. Threshold $\overline{\chi_r}$ Note that we can write:

$$\frac{WL}{E\left[Y\right]} = \alpha_2 \frac{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{\lambda_x \beta_i \sigma_x^2}\right)^{\frac{1}{1-\alpha_1-\alpha_2}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\alpha_1-\alpha_2}} \left(e^{\lambda_x \beta_i \sigma_x^2}\right)^{\frac{\alpha_1+\alpha_2}{1-\alpha_1-\alpha_2}}} \tag{36}$$

Evaluating the integrals, and given the assumed independence of β_i and $\mathbb{E}\left[A_i\right]$ yields:

$$log\left(\frac{WL}{E\left[Y\right]}\right) = log\left(\alpha_2\right) + \mu_\beta \lambda_x \sigma_x^2 + \frac{1}{2} \sigma_\beta^2 \left(\lambda_x \sigma_x^2\right)^2 \frac{1 - \left(\alpha_1 + \alpha_2\right)^2}{\left(1 - \alpha_1 - \alpha_2\right)} \tag{37}$$

Which has derivative w.r.t. λ_x:

$$\frac{\partial log\left(\frac{WL}{E[Y]}\right)}{\partial \lambda_x} = \sigma_x^2 \left(\mu_\beta + \lambda_x \sigma_\beta^2 \sigma_x^2 \frac{1 - \left(\alpha_1 + \alpha_2\right)^2}{\left(1 - \alpha_1 - \alpha_2\right)} \right) \tag{38}$$

From which the condition for the sign of $\frac{\partial \frac{WL}{E[Y]}}{\partial \lambda_x}$ immediately follows.

A.3 Alternate production functions

While we consider Cobb-Douglas production functions in our baseline model, the insights of our model on how risk apply more broadly. First, consider a generic production function that produces output using labor, capital, and possibly other inputs. If we assume that labor is chosen in advance, the optimization problem of the owner of this technology would yield:

$$\mathbb{E}\left[\Lambda\left(MRPL_i\right)\right]=\mathbb{E}\left[\Lambda\right]W \tag{39}$$

We can rearrange (39) to yield:

$$\mathbb{E}\left[MRPL_i\right]\left(1+\frac{Cov\left(\Lambda, MRPL_i\right)}{\mathbb{E}\left[\Lambda\right]\mathbb{E}\left[MRPL_i\right]}\right)=W \tag{40}$$

This tells us that firms do not equalize the realized (or expected) Marginal revenue product of labor equal to their wage rates, but rather that this is adjusted by a covariance term which depends on the co-movement of their marginal revenue product of labor with the stochastic discount factor. Note that we can write labor's share of expected income as follows:

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]}=\frac{\mathbb{E}\left[MRPL_i\right]L_i}{\mathbb{E}\left[Y_i\right]}\left(1+\frac{Cov\left(\Lambda, MRPL_i\right)}{\mathbb{E}\left[\Lambda\right]\mathbb{E}\left[MRPL_i\right]}\right) \tag{41}$$

With the Cobb-Douglas production function, $\frac{\mathbb{E}[MRPL_i]L_i}{\mathbb{E}[Y_i]}$ is equal to α_2.

A.3.1 CES production function

Here we derive analogues of the main expressions in section 2, the production function were instead to be a CES production function of capital and labor:

There are multiple production technologies that produce a homogeneous output as follows:

$$Y_i \quad = \quad A_i\left(K_i^{\rho}\left(1-\theta\right)+\theta L_i^{\rho}\right)^{\frac{\nu}{\rho}} \tag{42}$$

This yields an analogue of (3)

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]}=\frac{\nu\theta}{\left(\frac{K}{L}\right)^{\rho}\left(1-\theta\right)+\theta}\left(1+\frac{Cov\left(\Lambda, A_i\right)}{\mathbb{E}\left[A_i\right]\mathbb{E}\left[\Lambda\right]}\right), \tag{43}$$

where $\frac{K}{L} = \frac{\sum_i K_i}{\sum_i L_i}$ is the aggregate ratio of capital to labor. We can therefore derive input shares:

$$\frac{L_i}{L} = \frac{K_i}{K} = \frac{\left(\mathbb{E}\left[A_i\right]\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)\right)^{\frac{1}{1-\nu}}}{\sum_h \left(\mathbb{E}\left[A_h\right]\left(1 + \frac{Cov(\Lambda, A_h)}{\mathbb{E}[A_h]\mathbb{E}[\Lambda]}\right)\right)^{\frac{1}{1-\nu}}} \tag{44}$$

Note that this is identical to (4), except that returns to scale with CES is denoted by ν instead of $\alpha_1 + \alpha_2$.

We can write aggregate expected output shares as:

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} = \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\nu}{1-\nu}}}{\sum_h \mathbb{E}\left[A_h\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\nu}{1-\nu}}} \tag{45}$$

and plug these into (5) (which is an identity and continues to hold) to yield an expression for the aggregate labor's share of expected income:

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \frac{\nu\theta}{\left(\frac{K}{L}\right)^{\rho}(1-\theta) + \theta} \sum_i \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{1}{1-\nu}}}{\sum_h \mathbb{E}\left[A_h\right]^{\frac{1}{1-\nu}}\left(1 + \frac{Cov(\Lambda, A_i)}{\mathbb{E}[A_i]\mathbb{E}[\Lambda]}\right)^{\frac{\nu}{1-\nu}}} \tag{46}$$

Note that (6) and (8) are identities and also still hold. Given that our expressions for labor shares and output shares are closely related to the Cobb-Douglas case, it can easily be verified that versions of Propositions 1 and 2 also hold, though the exact threshold at which labor share is rising/falling in the price of risk differ slightly.

B Derivations and proofs for section 3

B.1 Firm's problem and the marginal investor

A firm wants to maximize the market value of their shares. This optimization is complicated by the fact that there are multiple possible investors.

If there is no penalty for firms being "more foreign" than other firms of the same risk level in the same island, then we end up with no equilibrium with representative firms, in which each risky firm has an incentive to deviate.[22]

[22] More precisely, there may be an equilibrium here in which these risky firms are either wholly owned by domestic investors or foreign investors, which each firm making input choices using the corresponding SDF.

We can formally set up the firm's problem as:

$$\max_{L_{i,j},K_{i,j}} \max \left\{ \mathbb{E}_j \left[\Lambda_j \left(A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2} - W_j L_{i,j} - P_{k,j} K_{i,j} \right) \right], \max_{h \neq j} \frac{\mathbb{E}_h \left[\Lambda_h \left(A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2} - W_j L_{i,j} - P_{k,j} K_{i,j} \right) \right]}{1 + \tau_j + \tau_{i,j}^*(S_{i,j})} \right\}$$

In the absence of the cost $\tau_{i,j}^*(S_{i,j})$, consider an equilibrium in which (1) all firms of a certain risk profile in island j make the same choice, and (2) are jointly owned by domestic and (some) foreign investors. If input decisions are made according to any SDF, firms have incentives to deviate. If they are made using the domestic SDF, then they can increase their valuation to foreign investors by changing their choices to be more risk-neutral. If they are made using the foreign (risk-neutral) SDF, then they can increase their valuation by domestic investors by changing their choices to be more risk-averse. If they are not following either, then deviations towards the direction suggested by either SDF can increase their value. It is easy to verify that there is no such equilibrium.

However, if we add the cost $\tau_{i,j}^*(S_{i,j})$ and assume that the cost if the firm is all foreign-owned (when other firms in its continuum are not) is large enough, then the incentive to deviate is eliminated, and firms make input choices using the domestic SDF.

B.2 Definition of an Equilibrium

An equilibrium consists of

- Physical allocations $L_{i,j}, K_{i,j}, Y_{i,j}, C_{E,j}, C_{W,j}, C_{E,j,0}, Y_j$
- Prices $W_j, P_k, P_{i,j}, \Lambda_j, P_{rf}, V_{i,j}$
- Asset holdings $S_{i,j}, S_{i,h,j}^*, S_{rf,j}, K_{1,j}$

Such that

$$
\begin{aligned}
&\Lambda_j = \frac{U_2'(C_{E,j})}{U_1'(C_{E,j,0})} \\
&\mathbb{E}[\Lambda_j] = P_{rf} \\
&(P_{i,j} - \mathbb{E}[\Lambda_j(A_{i,j}F(L_{i,j}, K_{i,j}) - L_{i,j}W_j - K_{i,j}P_{k,j})])\, S_{i,j} = 0 \\
&(P_{i,j}(1+\tau_j) - \mathbb{E}[\Lambda_h(A_{i,j}F(L_{i,j}, K_{i,j}) - L_{i,j}W_j - K_{i,j}P_{k,j})])\, S_{i,h,j} = 0 \\
&\alpha_2 \mathbb{E}\left[\Lambda_j A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2 - 1}\right] = \mathbb{E}[\Lambda_j]\, W_j \\
&\alpha_1 \mathbb{E}\left[\Lambda_j A_{i,j} K_{i,j}^{\alpha_1 - 1} L_{i,j}^{\alpha_2}\right] = \mathbb{E}[\Lambda_j]\, P_{k,j} \\
&Y_{i,j} = A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2} \\
&V_{i,j} = \left(A_{i,j} K_{i,j}^{\alpha_1} L_{i,j}^{\alpha_2} - K_{i,j}P_{k,j} - W_j L_{i,j}\right) \\
&Y_j = \sum_i Y_{i,j} \\
&C_{W,j} = W_j \sum_i L_{i,j} \\
&C_{E,j,0} = K_{0,j} - K_{1,j} - \sum_i S_{i,j}P_{i,j} - \sum_{h \neq j} (1+\tau_h)\, S^*_{i,j,h} P_{i,h} - P_{rf} S_{rf,j} + T_{0,j} \\
&C_{E,j} = S_{i,j}V_{i,j} + \sum_{h \neq j} \sum_i S^*_{i,j,h} V_{i,h} + S_{rf,j} + P_{k,j} K_{1,j} + T_j \\
&S_{i,j} + \sum_{h \neq j} S^*_{i,h,j} = 1 \\
&\sum_j S_{rf,j} = 0 \\
&\frac{\partial U_j^W}{\partial L_j} + W_j \mathbb{E}\left[\frac{\partial U_j^W}{\partial C_j}\right] = 0 \\
&1 = \mathbb{E}[\Lambda_j]\, P_{k,j} \\
&\sum_j K_{1,j} = \sum_j \sum_i K_{i,j}
\end{aligned}
$$

Where $T_{0,j}, T_j$ are chosen to make the clearing conditions hold and govern how the proceeds of purchasing the shares are distributed. For instance, we could set:

$$
\begin{aligned}
&T_{0,j} = \sum_i S_{i,j}P_{i,j} + \sum_{h \neq j} (1+\tau_j)\, S^*_{i,h,j} P_{i,j} \\
&T_j = 0
\end{aligned}
$$

B.3 Proofs to Proposition 3

Points 3-5 follow directly from proposition 1, since the 'risk premia' term χ_r can be written as a decreasing function of τ_j: $\chi_r = -\frac{\tau_j(1-\alpha_1-\alpha_2)}{1+\tau_j(1-\alpha_1-\alpha_2)}$. The proofs for (1) and (2) are below:

1. The equilibrium price of risk decreases (12) implies that $Cov\left(\frac{\Lambda_j}{\mathbb{E}[\Lambda_j]}, \frac{A_{r,j}}{\mathbb{E}[A_{r,j}]}\right)$, which is a function of the quantity of technology r, j risk and price of risk, decreases in magnitude as τ_j falls. We can see this clearly if we decompose the risk adjustment, $Cov\left(\frac{\Lambda_j}{\mathbb{E}[\Lambda_j]}, \frac{A_{r,j}}{\mathbb{E}[A_{r,j}]}\right)$, into the quantity and price of risk:

$$Cov\left(\frac{\Lambda}{\mathbb{E}\left[A_r\right]}, \frac{A_r}{\mathbb{E}\left[\Lambda\right]}\right) = \sigma^2_{log(A_{r,j})}\frac{Cov\left(\frac{\Lambda_j}{\mathbb{E}[\Lambda_j]}, \frac{A_{r,j}}{\mathbb{E}[A_{r,j}]}\right)}{\sigma^2_{log(A_{r,j})}} \tag{47}$$

where $-\frac{Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right)}{\sigma^2_{log\left(A_{r,j}\right)}}$ is the price of risk: the (negative) elasticity of the island j capitalist's SDF with respect the the productivity of the risky technology in island j. Since the quantity of risk is exogenous, as τ_j gets smaller the price of risk is falling.

2. The fraction of shares in the risky firm, $S_{r,j}$, held by domestic investors falls Note that we can express the covariance term as:

$$\chi_{r,j} \equiv -\frac{\tau_j\left(1-\alpha_1-\alpha_2\right)}{1+\tau_j\left(1-\alpha_1-\alpha_2\right)} = \frac{Cov\left(U_2'\left(C_{E,j}\right), A_{r,j}\right)}{\mathbb{E}\left[A_{r,j}\right]\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]} \tag{48}$$

For analytic tractability, we will therefore show that $\frac{\partial S_{r,j}}{\partial \chi_{r,j}} < 0$, which from the above will show that $\frac{\partial S_{r,j}}{\partial \tau_j} > 0$

Expression for consumption in the second period Taking the equilibrium conditions for consumption

$$\begin{aligned} C_{E,j} &= \sum_i S_{i,j}V_{i,j} + \sum_{h\neq j}\sum_i S^*_{i,j,h}V_{i,h} + S_{rf,j} + P_{k,j}K_{1,j} + T_j \\ C_{E,j,0} &= K_{0,j} - K_{1,j} - \sum_i S_{i,j}P_{i,j} - \sum_{h\neq j}\left(1+\tau_h+\tau^*_{i,h}(S_{i,h})\right)S^*_{i,j,h}P_{i,h} - P_{rf}S_{rf,j} + T_{0,j} \end{aligned}$$

Note that we know the value of firm shares in the second period conditional on shocks:

$$V_{r,j} = K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2} \left(A_{r,j} - \mathbb{E}\left[A_{r,j}\right](\alpha_1+\alpha_2) \left(1 + Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right)\right)\right)$$

$$V_{s,j} = K_{s,j}^{\alpha_1} L_{s,j}^{\alpha_2} A_{s,j}\left(1-(\alpha_1+\alpha_2)\right)$$

and that in equilibrium domestic investors hold only infinitesimal shares of each foreign security and are thus fully diversified:

$$\sum_{h \neq j} \sum_i S_{i,j,h}^* V_{i,h} = \sum_{h \neq j} S_{r,j,h}^* K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2} \mathbb{E}\left[A_{r,h}\right] \left(1 - (\alpha_1+\alpha_2)\left(1 + Cov\left(\frac{\Lambda_h}{\mathbb{E}\left[\Lambda_h\right]}, \frac{A_{r,h}}{\mathbb{E}\left[A_{r,h}\right]}\right)\right)\right)$$

Further the prices of these firm shares in the first period can be expressed as:

$$\begin{aligned} P_{s,j} &= \mathbb{E}\left[\Lambda_j\right] K_{s,j}^{\alpha_1} L_{s,j}^{\alpha_2} A_{s,j}\left(1-(\alpha_1+\alpha_2)\right) \\ P_{r,j} &= K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2} \left(\mathbb{E}\left[\Lambda_j A_{r,j}\right] - \mathbb{E}\left[\Lambda_j\right]\mathbb{E}\left[A_{r,j}\right](\alpha_1+\alpha_2)\left(1 + \frac{Cov\left(\Lambda_j, A_{r,j}\right)}{\mathbb{E}\left[A_{r,j}\right]\mathbb{E}\left[\Lambda_j\right]}\right)\right) \end{aligned}$$

plugging these into $C_{E,j}, C_{E,j,0}$ are simplifying yields:

$$C_{E,j} = S_{r,j} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2} \left(A_{r,j} - \mathbb{E}\left[A_{r,j}\right]\left(1 + Cov\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{r,j}}{\mathbb{E}\left[A_{r,j}\right]}\right)\right)\right) + \left(T_j + \frac{T_{0,j}}{\mathbb{E}\left[\Lambda\right]} + \frac{K_{0,j}}{\mathbb{E}\left[\Lambda\right]}\right) - \frac{C_{E,j,0}}{\mathbb{E}\left[\Lambda_j\right]} \tag{49}$$

Note further that $C_{E,j,0}$ has a relation with $C_{E,j,0}$ via the expectation of the SDF; This SDF is equated across capitalists as they can freely trade a risk-free bond.

$$\mathbb{E}\left[\Lambda\right] = \frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{U_1'\left(C_{E,j,0}\right)} \tag{50}$$

Derivative of $C_{E,j,0}$ Note that (50) yields:

$$C_{E,j,0} = U_1^{\prime(-1)}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}\left[\Lambda\right]}\right) \tag{51}$$

and therefore

$$\frac{\partial \frac{C_{E,j,0}}{\mathbb{E}[\Lambda]}}{\partial \chi_{r,j}} = \frac{U_1^{\prime(-1)\prime}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}[\Lambda]}\right)}{\mathbb{E}\left[\Lambda\right]^2} \frac{\partial \mathbb{E}\left[U_2''\left(C_{E,j}\right)\right]}{\partial \chi_{r,j}} \tag{52}$$

Note that (49) implies:

$$\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]=\mathbb{E}\left[U_2'\left(S_{r,j}K_{r,j}^{\alpha_1}L_{r,j}^{\alpha_2}\left(A_{r,j}-\mathbb{E}\left[A_{r,j}\right]\left(1+\chi_{r,j}\right)\right)+\left(T_j+\frac{T_{0,j}}{\mathbb{E}\left[\Lambda\right]}+\frac{K_{0,j}}{\mathbb{E}\left[\Lambda\right]}\right)-\frac{U_1'^{(-1)}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}[\Lambda]}\right)}{\mathbb{E}\left[\Lambda\right]}\right)\right]$$

and thus

$$\frac{\partial\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\partial\chi_{r,j}}=\frac{K_{r,j}^{\alpha_1}L_{r,j}^{\alpha_2}\mathbb{E}\left[U_2''\left(C_{E,j}\right)\left(\frac{\partial S_{r,j}}{\partial\chi_{r,j}}\left(A_{r,j}-\mathbb{E}\left[A_{r,j}\right]\left(1+\chi_{r,j}\right)\right)-\mathbb{E}\left[A_{r,j}\right]S_{r,j}\right)\right]}{1+\mathbb{E}\left[U_2''\left(C_{E,j}\right)\frac{U_1'^{(-1)\prime}\left(\frac{\mathbb{E}\left[U_2'\left(C_{E,j}\right)\right]}{\mathbb{E}[\Lambda]}\right)}{\mathbb{E}[\Lambda]^2}\right]} \tag{53}$$

Plugging in CRRA preferences yields:

$$\frac{\partial\frac{C_{E,j,0}}{\mathbb{E}[\Lambda]}}{\partial\chi_{r,j}}=\frac{\frac{\gamma}{\gamma_0}\frac{\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma_0}-1}}{\mathbb{E}[\Lambda]^2}\left(\frac{\partial S_{r,j}}{\partial\chi_{r,j}}\frac{1}{S_{r,j}}\left(\mathbb{E}\left[C_{E,j}^{-\gamma}\right]-\left(\overline{C}+C_0\right)\mathbb{E}\left[C_{E,j}^{-\gamma-1}\right]\right)-\mathbb{E}\left[A_{r,j}\right]S_{r,j}K_{r,j}^{\alpha_1}L_{r,j}^{\alpha_2}\mathbb{E}\left[C_{E,j}^{-\gamma-1}\right]\right)}{1+\frac{\gamma}{\gamma_0}\mathbb{E}\left[C_{E,j}^{-\gamma-1}\frac{\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma_0}-1}}{\mathbb{E}[\Lambda]^2}\right]} \tag{54}$$

Showing $E\left[x^{-\gamma}\right]^2-E\left[x^{-\gamma-1}\right]E\left[x^{-\gamma+1}\right]<0$ Note that we can write

$$\begin{aligned}E\left[x^{-\gamma}\right]^2-E\left[x^{-\gamma-1}\right]E\left[x^{-\gamma+1}\right] &= \int_x\int_y (xy)^{-\gamma}\left(1-\frac{y}{x}\right)f(x)f(y)\,dydx\\ &= \int_x\int_{y>x}(xy)^{-\gamma}\left(2-\left(\frac{y}{x}+\frac{x}{y}\right)\right)f(x)f(y)\,dydx\end{aligned} \tag{55}$$

note that for $x,y\geq 0$, $\left(\frac{y}{x}+\frac{x}{y}\right)\geq 2$. Thus $E\left[x^{-\gamma}\right]^2-E\left[x^{-\gamma-1}\right]E\left[x^{-\gamma+1}\right]<0$

Finishing proof Note that plugging in (49) into (48) and taking the derivative w.r.t $\chi_{r,j}$ yields:

$$\frac{\partial S_{r,j}}{\partial\chi_{r,j}}=\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2\mathbb{E}\left[A_{r,j}\right]S_{r,j}K}{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2-\mathbb{E}\left[C_{E,j}^{-\gamma-1}\right]\mathbb{E}\left[C_{E,j}^{-\gamma+1}\right]}\frac{1}{\gamma}-\mathbb{E}\left[A_{r,j}\right]S_{r,j}K+\frac{\partial C_0}{\partial\chi_{r,j}}\right)\frac{S_{r,j}}{\overline{C}+C_0} \tag{56}$$

Plugging in (54) and simplifying yields:

$$\frac{\partial S_{r,j}}{\partial \chi_{r,j}} = \frac{\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2 \mathbb{E}[A_{r,j}] S_{r,j} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2}}{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]^2 - \mathbb{E}\left[C_{E,j}^{-\gamma-1}\right] \mathbb{E}\left[C_{E,j}^{-\gamma+1}\right]} \frac{1}{\gamma} \left(1 + \mathbb{E}\left[C_{E,j}^{-\gamma-1}\right] \frac{\left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma}-1}}{\mathbb{E}[\Lambda]^2}\right) - \mathbb{E}\left[A_{r,j}\right] S_{r,j} K_{r,j}^{\alpha_1} L_{r,j}^{\alpha_2}}{\left(T_j + \frac{T_{0,j}}{\mathbb{E}[\Lambda]} + \frac{K_{0,j}}{\mathbb{E}[\Lambda]}\right) + \left(\frac{1}{\mathbb{E}[\Lambda]} - 1\right) \left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}[\Lambda]}\right)^{-\frac{1}{\gamma}}}$$

We have shown above that $E\left[x^{-\gamma}\right]^2 - E\left[x^{-\gamma-1}\right] E\left[x^{-\gamma+1}\right] < 0$, therefore the numerator is negative. So $\frac{\partial S_{r,j}}{\partial \chi_{r,j}} < 0$ (and therefore $\frac{\partial S_{r,j}}{\partial \tau_j} > 0$) iff

$$\left(T_j + \frac{T_{0,j}}{\mathbb{E}\left[\Lambda\right]} + \frac{K_{0,j}}{\mathbb{E}\left[\Lambda\right]}\right) + \left(\frac{1}{\mathbb{E}\left[\Lambda\right]} - 1\right) \left(\frac{\mathbb{E}\left[C_{E,j}^{-\gamma}\right]}{\mathbb{E}\left[\Lambda\right]}\right)^{-\frac{1}{\gamma}} > 0$$

The first term in parenthesis is the present value of transfers and the endowment, which are positive by assumption. The second term is nonnegative if $\mathbb{E}\left[\Lambda\right] \leq 1$, which is equivalent to saying that the risk free rate is non-negative.

C Empirical Appendix

C.1 Implications for cross-industry heterogeneity

Let us extend our model to one in which there are several industries s, and within each industry risk exposures are normally distributed with mean $\mu_{\beta_{s,j}}$ and variance $\sigma_{s,j}^2$ for industry s in country j. Then (38) implies that we can write the change in industry labor share as:

$$\frac{\partial log\left(\frac{W_{s,j} L_{s,j}}{E[Y_{s,j}]}\right)}{\partial \lambda_j} = \sigma_{s,j}^2 \left(\mu_{\beta,s,j} + \lambda_j \sigma_{\beta,s,j}^2 \sigma_{s,j}^2 \frac{1 - (\alpha_1 + \alpha_2)^2}{(1 - \alpha_1 - \alpha_2)}\right) \tag{57}$$

This suggests a regression of the form:

$$\Delta log\left(LS_{s,j,t}\right) = \alpha_t + \gamma_\mu \mu_{\beta,s,j} \Delta Div_{j,t} + \gamma_\sigma \sigma_{\beta,s,j}^2 \Delta Div_{j,t} + controls + \varepsilon_{i,t} \tag{58}$$

where each observation is an industry-country-year.

Note that if there is risk aversion (ie $\lambda_j < 0$), then our model implies $\gamma_\mu > 0$ and $\gamma_\sigma < 0$.

Note that in many datasets we cannot measure $\mu_{\beta,s,j}$ and $\sigma_{\beta,s,j}^2$, however firm-level labor shares are more readily observable:

$$log\left(\mathbb{E}\left[LS_{j,s,i,t}\right]\right) = log\left(\alpha_{2,s}\right) + \lambda_j \beta_i \sigma_{j,s}^2 \tag{59}$$

plugging this in yields:

$$\frac{\partial log\left(\frac{W_{s,j}L_{s,j}}{\mathbb{E}[Y_{s,j}]}\right)}{\partial \lambda_j} = \frac{1}{\lambda_j}\left(\left(\overline{log(\mathbb{E}\left[LS_{j,s,t}\right])} - log\left(\alpha_{2,s}\right)\right) + \sigma^2_{\mathbb{E}LS,j,s,t}\frac{1-\left(\alpha_1+\alpha_2\right)^2}{\left(1-\alpha_1-\alpha_2\right)}\right)$$

where $\overline{log(\mathbb{E}\left[LS_{j,s,t}\right])}$ is the mean of log labor share for industry s in country j at time t and $\sigma_{\mathbb{E}LS,j,s,t}$ is the cross-sectional standard deviation of labor shares in industry s in country j at time t. If firm betas are This suggests regression of the form:

$$\Delta log\left(LS_{s,j,t}\right) = \alpha_t + \gamma_\mu \mu_{LS,s,j,t}\Delta Div_{j,t} + \gamma_\sigma \sigma^2_{LS,s,j,t}\Delta Div_{j,t} + controls + \varepsilon_{i,t} \tag{60}$$

Where $\mu_{LS,s,j,t}$ and $\sigma^2_{LS,s,j,t}$ are the mean and variance of firm labor shares within the industry s in country j at time t.

D Additional Tables and Figures

D.1 Cross-Country Regressions with OECD Data

	(1)	(2)	(3)	(4)
Foreign Equity Liabilities/GDP	-0.0551**	-0.0627**	-0.0394**	-0.0467**
	(-2.72)	(-2.57)	(-2.14)	(-2.12)
TFP shock			-0.365*	-0.413**
			(-1.96)	(-2.09)
Average hours				0.00130
				(0.01)
Relative price of investment				0.0222
				(0.75)
Country fixed effects	yes	yes	yes	yes
Year fixed effects	no	yes	yes	yes
R^2	0.849	0.905	0.920	0.927
Observations	755	754	567	529

Notes: This table presents a regression of country labor share on foreign investors holding of domestic equity for advanced economics, excluding tax havens and small financial centers. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by country and year. Significance levels denoted by: * $p < .10$ ** $p < 0.05$, *** $p < 0.01$.

Table 6: Labor's share of income on international diversification

D.2 Reallocation in Changes

	(1)	(2)	(3)	(4)
	$\Delta\log(\frac{Y_i}{Y_{ind}})$	$\Delta\log(\frac{L_i}{L_{ind}})$	$\Delta\log(\frac{Y_i}{Y_{ind}})$	$\Delta\log(\frac{L_i}{L_{ind}})$
$\widehat{\beta}\times\ \Delta$ FEQ	0.508***	0.292***	0.521***	0.270***
	(4.01)	(2.94)	(4.03)	(2.72)
$\widehat{\beta}$	0.0574***	0.0492***	0.0748***	0.0825***
	(10.67)	(11.33)	(10.34)	(14.23)
Profitability			0.0293**	0.241***
			(2.23)	(17.80)
Age			-0.0361***	-0.0259***
			(-13.73)	(-12.69)
Size			-0.00451***	-0.0142***
			(-5.04)	(-15.31)
Industry-year F.E.	yes	yes	yes	yes
R^2	0.204	0.169	0.228	0.211
Observations	137742	136188	127373	127141

Notes:

Table 7: Reallocation, International Diversification, and Labor Share

D.3 Reallocation Interaction with Alternate Labor Share Measures

	(1)	(2)	(3)	(4)	(5)	(6)
	$\log(\frac{Y_i}{Y_{ind}})$	$\log(\frac{L_i}{L_{ind}})$	$\log(\frac{K_i}{K_{ind}})$	$\log(\frac{Y_i}{Y_{ind}})$	$\log(\frac{L_i}{L_{ind}})$	$\log(\frac{K_i}{K_{ind}})$
$\log(LS) \times$ FEQ	-0.882*	-1.179***	-1.343**	-1.123***	-1.075***	-1.711***
	(-1.90)	(-3.41)	(-2.47)	(-3.34)	(-3.50)	(-4.41)
Profitability				0.300	0.609**	-0.360
				(1.38)	(2.65)	(-1.23)
Age				0.261***	0.220***	0.240***
				(6.74)	(5.12)	(4.60)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.979	0.971	0.974	0.981	0.972	0.975
Observations	10060	9849	9641	9858	9660	9435

Notes: This table presents the results of a regression of a firm's share of industry outputs & inputs on the interaction of firm labor share and foreign investors holdings of U.S. equity, scaled by GDP. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.

Table 8: Reallocation, International Diversification, and Labor Share

	(1)	(2)	(3)	(4)	(5)	(6)
	$\log(\frac{Y_i}{Y_{ind}})$	$\log(\frac{L_i}{L_{ind}})$	$\log(\frac{K_i}{K_{ind}})$	$\log(\frac{Y_i}{Y_{ind}})$	$\log(\frac{L_i}{L_{ind}})$	$\log(\frac{K_i}{K_{ind}})$
$\log\left(\frac{L}{Y}\right) \times$ FEQ	-0.190***	-0.219***	-0.290***	-0.167***	-0.187***	-0.244***
	(-3.17)	(-4.17)	(-4.64)	(-2.79)	(-3.70)	(-3.85)
Profitability				1.241***	1.300***	1.154***
				(29.87)	(31.48)	(26.49)
Age				0.244***	0.249***	0.231***
				(18.40)	(19.04)	(13.14)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes	yes	yes
R^2	0.948	0.942	0.940	0.952	0.948	0.943
Observations	144566	143370	143770	140047	138809	139300

Notes: This table presents the results of a regression of a firm's share of industry outputs & inputs on the interaction of firm labor share and foreign investors holdings of U.S. equity, scaled by GDP. T statistics are reported in parenthesis below the coefficient estimates, computed from standard errors clustered two ways by firm and year. Significance levels denoted by: * p < .10 ** p < 0.05, *** p < 0.01.

Table 9: Reallocation, International Diversification, and Labor Share