Valuing Life over the Life Cycle

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Abstract

The (unobserved) economic valuation of a human life pays a central role in safety, public health, legal, as well as quality vs quantity of life debates. Its shadow value must be inferred from the explicit or implicit willingness to pay (WTP) or to accept compensation (WTA) for beneficial and detrimental changes in longevity. Both evolve over the life cycle (LC) through (i) accumulation and decumulation phases in financial, human and life capital, (ii) age-increasing morbidity and mortality risks, and (iii) differing mixes between market (e.g. consumption) and non-market (e.g. leisure) activities. The objective of this paper is to characterize these age-dependent sources of variation in life values. I solve and calibrate a LC model of consumption, leisure and health investment choices, featuring generalized recursive preferences and age-dependent wages, exposure to death and sickness risks. A calibration exercise reproduces observed labour, wealth and health patterns and yields plausible WTP/WTA. It reveals how the willingness to pay/accept compensation for beneficial and detrimental changes are altered by ageing, and what are the implications for life valuations.

Keywords Value of Human Life; Value of Statistical Life; Gunpoint Value; Hicksian Compensating and Equivalent Variations; Willingness to Pay; Willingness to Accept Compensation; Mortality; Longevity; Non-Expected Utility.

JEL classification J17, D15, G11

1 Introduction

1.1 Motivation and overview

The COVID pandemic has highlighted the relevance of the economic value of a human life. Indeed, the consequential macroeconomic costs of sanitary measures such as lockdown had to be contrasted with both the costs of illness and the value of the lives saved. Moreover, the allocation of scarce medical resources (e.g. access to intensive care) raised the specter of uncomfortable tradeoffs between saving one person against another. These arbitrage were complicated by the fact that both the pandemic incidence, and consequences varied greatly across age and underlying medical conditions.

Computing the (non-marketed) value of a human life relies on identifying a shadow price corresponding to a marginal rate of substitution (MRS) between additional life/mortality and wealth. The two associated life valuations can be statistical, i.e. a collective willingness to pay to save *someone* in the group, such as in the Value of a Statistical Life (VSL), or it can be for an *identified* person, such as in the Human Capital (HK), i.e. the market value of foregone net income, and Gunpoint (GPV) values, i.e. the willingness to pay and accept compensation for immediate and certain death. For both statistical and identified perspectives, preferences towards life can be either explicitly (i.e. stated preferences), or implicitly (i.e. revealed preferences) recovered. Identifying the effects of age implies understanding how the life cycle (LC) of the MRS is affected by factors as changing death and sickness risk exposures, or in disposable resources, e.g. wealth, wages.

This paper adopts a revealed-preference perspective to solve the closed-form expressions for the life cycles of both statistical and identified life values. I resort to a flexible LC model where the agent optimally chooses investment in her health, labor/leisure, and consumption/savings in an environment where wages peak at mid-life and decrease afterwards, whereas mortality and morbidity risks exposures monotonously increase in age. I append separate intra-temporal substitution between labor and consumption, as well as bequest motives to a generalized recursive preferences framework (Epstein and Zin, 1989, 1991; Weil, 1990, EZW) that disentangles inter-temporal substitution from risk aversion.

The identification of the life cycle of life values is calculated through the continuation (indirect) utility, i.e. the forward-looking welfare of continued living along the optimal path. More precisely, I calculate the Hicksian Equivalent (EV), and Compensating (CV) Variations for both beneficial, and detrimental changes in death risk exposure using the continuation utility from optimal investment, leisure, and consumption choices. In addition to the MRS, the two Hicksian variations directly identify four key life valuations: the Willingness to Pay (WTP) to attain beneficial, and to prevent detrimental changes, and the Willingness to Accept (WTA) compensation to forego beneficial and to accept detrimental changes in mortality. In addition to providing closed-form characterization on how ageing alters how we value increases and decreases in longevity, the WTP/WTA allows for the calculation of the life cycles of both Statistical and Gunpoint values of life. Importantly, all measures directly address quality of life concerns by incorporating the shadow value of the human (i.e. health) capital in the substitution between life and resources.

A revealed preference perspective is relied upon to compute numerical life value estimates. The model is calibrated to reproduce observed LC's for both exogenous (mortality, morbidity and wages) and endogenous variables (health, wealth, leisure, labor income). Since no unique data set regroups all these variables, I combine Panel Study of Income Dynamics (PSID), American Time Use Survey (ATUS), and U.S. Life Tables under a common time frame, and under the assumption that the datasets are representative of a common set of agents. The parametrization is selected to realistic values and adequately replicates the observed life cycles in both exogenous and endogenous variables.

The four key takeaways from this paper are the following. First, consistent with preference for life over death, the change in the continuation utility induced by a change in mortality risk exposure is positive for reductions and negative for increases in death risk. Second, the effects are asymmetric, with welfare losses from detrimental changes being much lower than welfare gains from beneficial changes of similar magnitude. These nonlinearities translate into larger WTP/WTA to attain/forego for longevity gains than for to prevent/accept longevity losses. They also reveal endowment effects where selling prices (WTA) are much larger than buying prices (WTP).

Third, ageing is invariably associated with flatter Hicksian variations and WTP/WTA measures. Equivalently, older agents are both willing to spend less and are less reactive to changes in longevity. Several reasons explain why this is the case. First, the combination of optimally falling net total wealth and marginal values imply falling continuation utility. Since this welfare is the measure against which changes in death risk are valued, elders have both less resources, and willingness to pledge resources for longevity. Related to this is the effect of finite maximal longevity; any gain or loss in death risk exposure is operational on a shorter remaining horizon compared to younger agents.

Fourth, the structural life values are often higher than other estimates. First, unlike reduced-form VSL calculations, I explicitly compute the value of the health capital stock, and of the time endowment of agents. Since both are much larger than financial wealth, net total wealth is also much larger. Second, accounting for leisure is also important. The quality of life made possible by leisure implies that agents are willing to reduce consumption more to prolong life.

This paper is primarily related to integrated, life cycle model-based approaches to life valuations initiated by Conley (1976); Shepard and Zeckhauser (1984); Rosen (1988).[1] Córdoba and Ripoll (2017) also consider a LC framework of consumption and leisure choices featuring EZW preferences to analyze the value of changes in mortality exposure. They emphasize the importance of non-linearities in death probabilities made possible by recursive preferences

[1] See Hugonnier et al. (2021a) for a more thorough review of the life valuation literature.

to generate rich predictions regarding WTP's. However, they do not integrate endogenous human capital choices, nor morbidity shocks in their analysis of life valuations. Hugonnier et al. (2013, 2021b) do incorporate human capital considerations in a continuous-time model with similar recursive preferences. However, they abstract from leisure choices, as well as from the role of bequests in life values. Bommier et al. (2019) also resort to a LC model of consumption and financial decisions, and analyze its implications for life values However, they abstract from both leisure and human capital considerations and consider non-EZW (risk sensitive) preferences that are less general than the ones I rely upon. Importantly, none of these papers provide full characterization of EV and CV measures for beneficial and detrimental changes in longevity, and none explicitly focus on the LC trajectories for life values.

The rest of the paper is organized as follows. I present the theoretical LC model, and its optimal solution in Sections 2 and 3. Section 4 presents the theoretical implications for life valuation measures. The empirical calibration strategy is detailed in Section 5. The numerical life value estimates are presented in Section 6, and are discussed in Section 7.

2 Model

2.1 Horizon, savings, labour choices and human capital

Horizon Let $t \in [0, T]$ denote discrete time, where T is the maximal biological longevity, $T^m \in (0, T]$ be the stochastic timing of death following a Poisson process with age-increasing intensity λ_t^m. The age-t one-period s_t and $k-$period $S_{t,t+k}$ ahead survivals, as well as expected longevity L_t are:

$$s_t = \exp(-\lambda_t^m), \tag{1a}$$

$$S_{t,t+k} = \prod_{\tau=0}^{k-1} s_{t+\tau} = \exp\left(-\sum_{\tau=0}^{k-1} \lambda_{t+\tau}^m\right), \tag{1b}$$

$$L_t = \sum_{k=1}^{T-t} S_{t,t+k}. \tag{1c}$$

Financial and health capital dynamics Let W_t denote the agent's financial wealth, c_t her consumption and $\ell_t \in [0,1]$ her leisure. The agent's faces the following financial constraints:

$$W_{t+1} = [W_t + Y_t - M_t - c_t]\, R, \tag{2a}$$

$$Y_t = y + w_t(1-\ell_t), \tag{2b}$$

$$M_t = i + I_t - BH_t. \tag{2c}$$

The budget constraint (2a) assumes a constant risk-free rate $R = \exp(r)$. Income Y_t in (2b) is net wages w_t over work time $n_t = (1-\ell_t)$, plus constant revenues y (e.g. social security). Medical expenses M_t in (2c) include a constant level i (e.g. health insurance) plus the time-varying expenses that increase in chosen level of investment I_t, but decrease in the agent's health H_t.

Regarding the latter, I assume the following dynamics for health capital:

$$H_{t+1} = AI_t^\alpha H_t^{1-\alpha} + (1-\delta-\epsilon_{t+1}^h\phi)H_t, \tag{3a}$$

where stochastic morbidity schock

$$\epsilon_{t+1}^h = \begin{cases} 0 & \text{with prob. } \exp(-\lambda_t^h), \\ 1 & \text{with prob. } 1 - \exp(-\lambda_t^h). \end{cases} \tag{3b}$$

The Grossman (1972); Ehrlich and Chuma (1990) demand for health function (3a) is similar to Hugonnier et al. (2013, 2020, 2021b) and assumes Cobb-Douglas technology. Stochastic morbidity ϵ_{t+1}^h is appended to gross investment in (3b) where illness occurs at age-increasing hazard rate λ_t^h and induces additional depreciation $\phi \in (0, 1 - \delta)$ in the health stock.

We close our discussion of financial constraints and health dynamics by assuming perfect financial markets. In particular, health shocks in ϵ_{t+1}^h can be fully insured against at actuarially-fair insurance rate. Moreover, a claim to any net income stream can be sold at no additional costs in exchange for its risk-adjusted capitalized value.

Agent's problem I resort to Epstein and Zin (1989, 1991) non-expected utility to represent the agent's preferences. I append to that framework separate iso-elastic preferences over consumption and leisure, as well as a bequest motive. More precisely the agent's problem can be written as:

$$V_t = V_t(W_t, H_t) = \max_{\{c_t, \ell_t, I_t\}} \left[(1-\beta)u(c_t, \ell_t)^{1-\varepsilon} + \beta \mathrm{CE}_t(V_{t+1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{4a}$$

where the felicity function is:

$$u(c_t, \ell_t) = \left[\theta c_t^{1-\sigma} + (1-\theta)\ell_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{4b}$$

where the certainty-equivalent of the continuation utility is:

$$
\begin{aligned}
\mathrm{CE}_t(V_{t+1}) &= \left[\mathrm{E}_t V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \\
&= \left[s_t \mathrm{E}_t^h V_{t+1}^{1-\gamma} + (1-s_t)\, b\, \mathrm{E}_t^h [W_{t+1} + V_{t+1}^H]^{1-\gamma}\right]^{\frac{1}{1-\gamma}},
\end{aligned}
\tag{4c}
$$

and where the value of human capital V_{t+1}^H at death is:

$$
V_{t+1}^H = \eta_{t+1} H_{t+1}. \tag{4d}
$$

The dynamic problem is subject to financial (2) and health dynamics (3) and where initial wealth and health (W_0, H_0) are taken as given.

First, $\beta \in (0,1)$ is a subjective discount factor, whereas $1/\varepsilon > 0$ measures the elasticity of *inter*-temporal substitution in the agent's problem (4a). Second, the parameter $\theta \in (0,1)$ in the felicity (4b) measures the consumption share whereas $1/\sigma > 0$ is the elasticity of *intra*-temporal substitution between consumption and leisure. Third, $\mathrm{E}_t = \mathrm{E}_t^{m,h}$ in (4c) denotes the joint conditional expectations with respect to mortality T^m, and morbidity ϵ_{t+1}^h, and E_t^h denotes the expectation with respect to the latter in (3b). Fourth, $\gamma > 0$ is the agent's relative risk aversion.

Finally, $b > 0$ captures the warm-glow utility benefit of the financial and health statuses evaluated at death. More precisely, the non-zero agent's utility at death depends on her end-of-life financial wealth W_{t+1} and health V_{t+1}^H statuses. The latter in (4d) captures lower terminal care expenses, as well as better life quality (e.g. less suffering, more sense of fulfillment) for healthier agents at the very end of life. Alternatively, b can be interpreted as the bequest motive, with $b^{1/(1-\gamma)}$ measuring the share of next-period total wealth, i.e. the sum of financial W_{t+1} and human capital $V_{t+1}^H = \eta_{t+1} H_{t+1}$, to be bequeathed. For both interpretations, the age-dependent shadow price (i.e. Tobin's-q) of health $\eta_{t+1} \geq 0$ is determined endogenously and satisfies transversality restriction $\eta_T = 0$.

Time variation Several elements concur to induce an age-dependent allocation $\{c_t, I_t, \ell_t\}$, for $0 < t \leq T^m \leq T$. First, longevity is bounded above by T. Second, both risk exposures to death λ_t^m and illness λ_t^h increase with age. Third, wages w_t are subject to exogenous time variation. As will become apparent shortly, the induced time variation in the associated continuation utility $V_t(W_t, H_t)$ will generate age-dependency in the life valuation measures calculated from the welfare function.

3 Optimal allocation

3.1 Solution method

The solution method relies on four building blocks. I first exploit static optimization between labour and consumption choices to recast the agent's problem in terms of total expenses (i.e. consumption and the cost of leisure). Second, under the perfect markets assumption, I invoke separation properties (e.g. Bodie et al., 1992; Hugonnier et al., 2013; Palacios, 2015; Acemoglu and Autor, 2018) to solve the optimal human capital dynamics independently from financial decisions. Third, the agent's problem is recoded as an equivalent one where the agent maximizes utility over total expenses subject to the dynamics for net total (i.e. financial plus human wealth). Finally, the dynamic optimization for both human capital and total expenses calculates the optimal policies by backward iteration starting at maximal longevity T.

3.1.1 Optimal labour-consumption choices

A standard argument, applicable in our setting, establishes the well-known a-temporal optimization equalizing the marginal rate of substitution between leisure and consumption

$U_{\ell,t}/U_{c,t}$ to the wages w_t to obtain:

$$\ell_t = \left[\left(\frac{1-\theta}{\theta}\right)\frac{1}{w_t}\right]^{\frac{1}{\sigma}} c_t.$$

It follows that total spending on consumption and leisure $\tilde{c}_t$, as well as felicity u_t at the static optimum can be recast as:

$$\tilde{c}_t \equiv c_t + w_t\ell_t = c_t\mu(w_t) \tag{5}$$
$$u(c_t, \ell_t) = \tilde{c}_t\nu(w_t),$$

where the wage- (and therefore age-) dependent loadings are:

$$\mu(w_t) \equiv 1 + \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\sigma}} w_t^{1-\frac{1}{\sigma}} \geq 0 \tag{6a}$$
$$\nu(w_t) \equiv [\theta\mu(w_t)^{\sigma}]^{\frac{1}{1-\sigma}} \geq 0. \tag{6b}$$

Hence, $1 - 1/\mu(w_t)$ represent the leisure share $w_t\ell_t$ in total expenses $\tilde{c}_t$. At high elasticity of intra-temporal substitution $1/\sigma > 1$, substitution effects outweigh income effects; an increase in wages lowers $\mu(w_t)$, thereby lowering the leisure share. Low elasticity $1/\sigma < 1$ induce opposite patterns, whereas $1/\sigma = 1$ leads to exact cancellation of income and substitution effects, resulting in constant (μ, ν) and consumption and leisure shares.

3.1.2 Separation between human capital and financial choices

Next, under perfect markets assumption, two claims can be sold on financial markets:

1. a claim to the expected net present value of exogenous lifetime wages, net of fixed medical expenses, independent of mortality risk, and over horizon $t = 0, \ldots, T$:

$$V_0^w = \mathrm{E}_0 \sum_{t=0}^{T} \exp(-rt)(w_t - m); \tag{7}$$

where $m = i - y$ is the excess medical expenses above fixed income. Observe that V_t^w encompasses the expected net present value of the unit of time endowment (a share ℓ_t of which is spent on leisure and is accounted for in total expenses $\tilde{c}_t$).

2. a claim to health benefits net of investment, independent of mortality risk, and over finite horizon $t = 0, \ldots, T$, and whose value $V_t^H = V_t^H(H_t)$ satisfies:

$$V_t^H = \max_{I_t} (BH_t - I_t) + \exp(-r)\mathrm{E}_t V_{t+1}^H, \tag{8}$$

subject to health dynamics (3), which is solved by backward induction, independently from the other allocation for (c_t, ℓ_t). V_t^H corresponds to the shadow value of the human capital, i.e. the expected net present value of its dividend stream $BH_t - I_t$.

At the initial period $t = 0$, the agent then cashes-in those two claims that are added to financial wealth W_t to obtain (non-stochastic) net total wealth N_t, and the corresponding dynamics for the latter are adjusted accordingly. Regrouping our static optimization and separation results imply that the original problem can be equivalently recast with $V_t = V_t(N_t)$ as follows:

$$V_t = \max_{\tilde{c}_t} \left\{ (1-\beta)(\nu_t \tilde{c}_t)^{1-\varepsilon} + \beta \left[s_t V_{t+1}^{1-\gamma} + (1-s_t)\, b\, N_{t+1}^{1-\gamma} \right]^{\frac{1-\varepsilon}{1-\gamma}} \right\}^{\frac{1}{1-\varepsilon}} \tag{9a}$$

where s_t is the on-period ahead survival probability in (1a), and subject to

$$
\begin{aligned}
N_0 &\equiv W_0 + V_0^w + V_0^H(H_0), \\
N_{t+1} &= [N_t - \tilde{c}_t]\, R,
\end{aligned} \tag{9b}
$$

Appendix C proves the separable health investment and total expenses solution coincides with the direct solution method where perfect financial markets are abstracted from and the optimal rules for $I_t, \tilde{c}_t$ are solved simultaneously under the assumption of actuarially-fair insurance against health shock ϵ_{t+1}^h.

3.2 Optimal rules

Timing convention We will henceforth recode timing t in terms of maximal remaining survival time before T, and refer to $s = 1, 2, \ldots$ as the number of remaining periods before maximal longevity is reached, such that current period is $t = T - s$. To alleviate notation we omit time-subscript for contemporary variables, use prime ($'$) for next-period variables, and use s to emphasize feedback rules calculated s-periods from T.

3.2.1 Health investment

I first characterize the optimal solution for the separate health investment decision I, and the associated value of the health stock V^H.

Theorem 1 *The optimal investment and corresponding value of human capital solving* (8) *are:*

$$
\begin{aligned}
I_s(H) &= \kappa_s H, \\
V_s^H(H) &= \eta_s H,
\end{aligned}
$$

where the loadings $\{\kappa_s, \eta_s\}_{s=1}^{T}$ *satisfy the following recursion:*

$$\kappa_s = \left[\eta_{s-1} R^{-1} \alpha A\right]^{\frac{1}{1-\alpha}}$$
$$\eta_s = B - \kappa_s + \eta_{s-1} R^{-1} \left\{A \kappa_s^{\alpha} + (1-\delta) - [1 - \exp(-\lambda^h)]\phi\right\}$$

with initial values $(\kappa_1, \eta_1) = (0, B)$.

The age-dependent feedback rule κ_s crucially determines optimal health decumulation (see eq. (20) below), whereas η_s corresponds to a shadow price, i.e. marginal and average Tobin's-q of the health capital H. Both κ_s, η_s explicitly account for benefits B, accumulation technology A, α, δ as well as age-increasing sickness risk exposures λ^h, and consequences ϕ, yet are independent of mortality λ^m.

3.2.2 Total expenses

Next, the health capital V^H is added to financial and wage-related resources and the optimal total expenses $\tilde{c}$ and value function are calculated as functions of the net total wealth N as follows.

Theorem 2 *The optimal total expenditures and corresponding value function solving* (9) *are:*

$$\tilde{c}_s(\lambda^m, N) = \omega_s(\lambda^m) N \tag{10}$$
$$V_s(\lambda^m, N) = \psi_s(\lambda^m) N \tag{11}$$

where the loadings $\{\omega_s, \psi_s\}_{s=1}^{T}$ *satisfy the following recursion:*

$$\beta_s(\lambda^m) = \beta \left\{ s(\lambda^m)\psi_{s-1}(\lambda^m)^{1-\gamma} + [1 - s(\lambda^m)]\, b \right\}^{\frac{1-\varepsilon}{1-\gamma}}, \tag{12a}$$

$$\omega_s(\lambda^m) = \frac{(1-\beta)^{\frac{1}{\varepsilon}} \nu_s^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} \nu_s^{\frac{1-\varepsilon}{\varepsilon}} + \beta_s(\lambda^m)^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}} \tag{12b}$$

$$\psi_s(\lambda^m) = \left\{ (1-\beta)\left[\nu_s \omega_s(\lambda^m)\right]^{1-\varepsilon} + \beta_s(\lambda^m)\left[(1 - \omega_s(\lambda^m))\, R\right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}} \tag{12c}$$

with initial value $\beta_1(\lambda^m) = \beta b^{\frac{1-\varepsilon}{1-\gamma}}$*, and where* (μ_s, ν_s) *are given in* (6).

The time-varying β_s in (12a) corrects the subjective discount applied on future wealth N' for the time variation in death risk exposure $s(\lambda^m)$, and for the importance of bequest motives b. The age-dependent feedback rule $\omega_s \in [0,1]$ in (10) is the marginal (and average) propensity to spend on total expenditures $\tilde{c}$ out of net total wealth N, whereas ψ_s is the marginal (and average) value of the latter. All three depend nonlinearly on death risk exposure λ^m. As will become clear shortly, the linearity of the continuation utility (11) will nonetheless simplify the interpretation of the life valuation measures associated with changes in λ^m.

4 Implications for life valuations

4.1 Hicksian Equivalent and Compensating Variations

The implications of the life cycle model for the valuations of human life are next derived. In particular, I rely on the Hicksian Equivalent and Compensating Variations to calculate the maximal willingness to pay and to accept compensation to prevent/accept permanent and constant detrimental modifications and to attain/forego beneficial changes in death risk exposure.[2]

Definition 1 (EV and CV along optimal path) *Consider a permanent, constant change of magnitude $\Delta \geq -\lambda^m$ in base exposure to death risk λ^m. Then, given net total wealth N and remaining time s to maximal longevity T, the Equivalent (v_s^e) and Compensating (v_s^c) Variations along the optimal path are the implicit solutions to the indifference conditions:*

$$V_s(\lambda^m, N - v_s^e) = V_s(\lambda^m + \Delta, N) \tag{13a}$$

$$V_s(\lambda^m + \Delta, N - v_s^c) = V_s(\lambda^m, N) \tag{13b}$$

where V_s is the continuation utility solved in (11).

Definition 1 adapts standard Hicksian variational analysis (e.g. Hanemann, 1991; Varian, 1984, p. 264) to our life valuation setting and evaluates the variations along the optimal life cycle path generated by the indirect utility V_s. A number of characteristics can be derived. First, the link between the two variations follows directly from standard principles:

$$v_s^c(\Delta, \lambda^m, N) = v_s^e(-\Delta, \lambda^m + \Delta, N)$$

i.e. the CV is equal to the EV evaluated at $\lambda^m + \Delta$ for change equal to $-\Delta$. Second, the links between the two Hicksian variation measures and the WTP's, and WTA's can be deducted from (13). The Equivalent Variation (13a) takes current exposure λ^m as status-quo to calculate maximal $WTP_s = v_s^e$ to prevent detrimental change $\Delta > 0$ or minimal $WTA_s = -v_s^e$ to forego beneficial changes $\Delta < 0$. The Compensating Variation (13b) instead takes altered exposure $\lambda^m + \Delta$ as status-quo and computes the $WTP_s = v_s^c > 0$ to

[2] See Murphy and Topel (2006) for a similar constant change perspective, and Jones-Lee et al. (2015) for alternative one-shot, and proportional changes in death exposure.

attain beneficial change $\Delta < 0$ and $WTA_s = -v_s^c$ to accept detrimental changes $\Delta > 0$:

$$WTP_s(\Delta, \lambda^m, N) = \begin{cases} v_s^c, (\Delta, \lambda^m, N), & \text{for } \Delta < 0 \text{ (attain benef. chg.)} \\ v_s^e(\Delta, \lambda^m, N), & \text{for } \Delta > 0 \text{ (prevent detrim. chg.)} \end{cases} \tag{14a}$$

$$WTA_s(\Delta, \lambda^m, N) = \begin{cases} -v_s^e(\Delta, \lambda^m, N), & \text{for } \Delta < 0 \text{ (forego benef. chg.)} \\ -v_s^c(\Delta, \lambda^m, N), & \text{for } \Delta > 0 \text{ (accept detrim. chg.)} \end{cases} \tag{14b}$$

It follows directly that both EV and CV in (13), and therefore that the WTP (14a) and WTA (14b) are zero at base risk $\Delta = 0$.

Third, we can substitute $v_s^e(\Delta, \lambda^m, N)$ in (13a) and $v_s^c(\Delta, \lambda^m, N)$ in (13b), take derivatives with respect to change Δ and re-arrange to obtain the marginal rate of substitution between life and net total wealth as:

$$MRS(\lambda^m, N) \equiv \frac{\frac{-\partial V_s(\lambda^m, N)}{\partial \lambda^m}}{\frac{\partial V_s(\lambda^m, N)}{\partial N}} = \left.\frac{\partial v_s^e(\Delta, \lambda^m, N)}{\partial \Delta}\right|_{\Delta=0} = \left.\frac{-\partial v_s^c(\Delta, \lambda^m, N)}{\partial \Delta}\right|_{\Delta=0}. \tag{15}$$

Hence, the shadow relative price of life, i.e. the required marginal change in net total wealth to leave an agent indifferent to a marginal change in longevity is simply the slope of the tangent of the EV (and negative of tangent slope for the CV) evaluated at base risk $\Delta = 0$. Importantly, observe that all variational/willingness measures, as well as MRS account for quality-of-life considerations. Indeed, the net total wealth $N_s = N(W_s, H_s)$ in (9b) explicitly incorporates the shadow value of the human capital stock H_s, as as such accounts for health-related variations in V_s^H in (8) caused by ageing and/or illness.

The indirect utility (11) is proportional to net total wealth, where all utilitarian effects of mortality risk exposure are captured through the marginal utility of wealth $\psi_s(\lambda^m)$. The calculation of the EV and CV are therefore simplified and can be derived as:

Proposition 1 (Equivalent and Compensating Variations) *The Hicksian Equivalent and Compensating variations along the optimal path are given as:*

$$v_s^e(\Delta, \lambda^m, N) = \left[1 - \frac{\psi_s(\lambda^m + \Delta)}{\psi_s(\lambda^m)}\right] N$$

and

$$v_s^c(\Delta, \lambda^m, N) = \left[1 - \frac{\psi_s(\lambda^m)}{\psi_s(\lambda^m + \Delta)}\right] N$$

where ψ_s is given in (12)*.*

The proof follows directly by substituting continuation utility (11) in Hicksian variations (13) and is therefore omitted.

Proposition 1 shows that both EV and CV are proportional to net total wealth. It further reveals that the key determinant for life valuations is how continuation utility is affected by Δ. Indeed, rearranging (16) reveals that:

$$\begin{aligned} v_s^e(\Delta, \lambda^m, N) &= -\Psi(\Delta, \lambda^m) N, \\ v_s^c(\Delta, \lambda^m, N) &= \left(\frac{\Psi_s(\Delta, \lambda^m)}{1 + \Psi(\Delta, \lambda^m)}\right) N, \end{aligned}$$

where

$$\Psi_s(\Delta, \lambda^m) \equiv \frac{V_s(\lambda^m + \Delta, N) - V_s(\lambda^m, N)}{V_s(\lambda^m, N)} \tag{17}$$

is the elasticity of the continuation utility V_s induced by infra-marginal change Δ in base mortality risk exposure λ^m. Clearly, preference for life implies that the elasticity is positive for beneficial changes ($\Delta < 0$) and negative for detrimental ones ($\Delta > 0$).

4.2 Additional life valuations

The two Hicksian variational measures in Proposition 1 can also be relied upon to calculate closed-form expressions for additional life valuation measures.

4.2.1 Value of Statistical Life

The theoretical VSL is the marginal rate of substitution given by (15) and is therefore independent from the EV or CV perspective. The empirical VSL commonly resorted to in the literature is an *infra*-MRS approximation equal to the WTP or WTA value divided by Δ:

$$\begin{aligned} VSL_s^p(\lambda^m, N) &= \lim_{\Delta \to 0} \frac{WTP_s(\Delta, \lambda^m, N)}{\Delta} \approx \left.\frac{WTP_s(\Delta, \lambda^m, N)}{\Delta}\right|_{\Delta \text{ small}}, \\ VSL_s^a(\lambda^m, N) &= \lim_{\Delta \to 0} \frac{WTA_s(\Delta, \lambda^m, N)}{\Delta} \approx \left.\frac{WTA_s(\Delta, \lambda^m, N)}{\Delta}\right|_{\Delta \text{ small}}. \end{aligned} \tag{18}$$

Typical practices set $\Delta = 1/n$ where n is the size of the population under study; the VSL can then be conveniently interpreted as the collective WTP to save one unidentified (i.e. statistical) life in the group.

4.2.2 Gunpoint Value

The Gunpoint value is the maximal willingness to pay to prevent (GPV_s^p), or willingness to accept compensation (GPV_s^a) for instantaneous, certain death corresponding to $\Delta = \infty$. Substituting in Proposition 1 reveals that:

$$\begin{aligned} GPV_s^p &= WTP_s(\infty, \lambda^m, N) \\ GPV_s^a &= WTA_s(\infty, \lambda^m, N) \end{aligned} \tag{19}$$

Both Gunpoint measures are useful in ex-ante instances where death is a certain outcome under a specific action or inaction, such as in terminal care decisions, or in ex-post instances where death has occurred, such as in wrongful death litigation.[3]

4.2.3 Graphical links EV, CV, MRS, VSL and GPV

The previous discussion allows for a graphical representation of the theoretical links between the various life valuations in Figure 1. First, for monotone increasing and concave EV (in green), the MRS in (15) is the slope of the blue tangent evaluated at $\Delta = 0$, and is also equal to the negative of the slope of the tangent at the origin of the decreasing convex CV (in brown). Equation (18) shows that both are equal to the theoretical VSL. The empirical counterpart of the VSL is the slope of a green arc from the origin to a point on the EV at infra-marginal change Δ_0 for the WTP-based VSL^p, whereas the negative of the slope of a brown arc from origin to the CV is the WTA-based VSL^a. From (19), the GPV^p is maximal WTP to avoid instantaneous and certain death, i.e. the upper bound of the EV, whereas the GPV^a is the minimal WTA to accept imminent death is the negative of the lower bound of the CV.

5 Empirical strategy

The empirical strategy relies on a calibration of the model's parameters to generate predicted life cycles of a subset of key variables for which observable counterparts exist. Unfortunately, no unique data set can be found for the variables to be matched. I therefore combine several well-known databases over a common period under the assumption that they are representative of US agents. The theoretical optimal rules and associated optimal dynamics provide predictions for the life cycles of variables such as wealth, leisure, income, and health.

[3] See Jones-Lee (1974); Cook and Graham (1977); Weinstein et al. (1980); Eeckhoudt and Hammitt (2004); Hugonnier et al. (2021b) for GPV-related definitions, applications and discussion.

Since these depend on age only, so must their observable counterparts. Consequently, observed life cycles must be computed taking into account the statics and the dynamics in socio-economic variables to recover pure age-dependent effects.

5.1 Data

5.1.1 Health, sickness and mortality variables

Health and sickness intensities I rely on Panel Study on Income Dynamics (PSID) to recover health-related data. More specifically, I use a panel ordered probit, with random effects over the unbalanced household data for the period 2003-2019 to regress the polytomous self-reported health status on socio-demographics,[4] as well as on a fractional polynomial on age. The associated score function,[5] evaluated by age, is taken to represent the health variable H_t, whereas the imputed marginal probability of being in the worst health state, by age, is used as proxy for the sickness intensity λ_t^h.

Death intensity The Life Tables of the United States (Arias and Xu, 2020, Tab. 1) report age-specific one-year survival probabilities s_t, from which the intensity λ_t^m is directly recovered as:

$$\lambda_t^m = -\ln(s_t).$$

Figure 2, panel a displays a continuous decline in health which accelerates after 70. Unsurprisingly, the exposure to sickness risk (panel b) is larger in absolute terms, yet does not increase as rapidly with age compared to death risk (panel c) which displays the familiar exponential growth associated with ageing (Gompertz).

[4] More specifically, I use sex, gender, race, education and year dummies, as well as financial wealth as regressors.

[5] The predicted score is scaled upwards to guarantee positive observed values for H_t, consistent with the growth process in (3a).

5.1.2 Financial and income variables

Wealth The PSID data is also resorted to for wealth. In particular, the latter is taken to be the net financial and residential wealth of agents. Again, wealth is regressed on socio-economic variables as well as a fractional polynomial in age, and accounting for random effects.[6] The fitted variables by age is our wealth variable W_t. Panel a in Figure 3 shows the accumulation of wealth up to mid-70's and slow de-cumulation afterwards. Summary statistics for the PSID variables are provided in Table 1, panel a.

Hours, wages and income Labor market variables are taken from the American Time Use Survey (ATUS) for the 2003-2019 period. Controlling for random effects, sex, occupation and industrial sector, as well as year dummies, wages, hours and income are again regressed on a fractional polynomial from which the fitted values by age are recovered. Average hourly wages in Figure 3 panel b increase up to mid-life and slowly decline afterwards, whereas panel c shows relatively constant hours up to mid-life and rapid decline after 55. Finally, the salaried income in panel d shows similar inverted-U patterns. Summary statistics for the ATUS variables are provided in Table 1, panel b.

5.2 Calibration strategy

Calculating the optimal dynamics The model's deep parameters are calibrated so as to match the joint lifetime dynamics of health, wealth and labor market variables along the optimal path with their observable counterparts. This strategy involves four steps.

1. Given a set of parameters, the optimal recursions are solved for $\{\kappa_s, \eta_s\}_{s=1}^{T}$ (Theorem 1) as well as for $\{\beta_s, \omega_s, \psi_s\}_{s=1}^{T}$ (Theorem 2).

[6] I use year, sex, gender, race, and education dummies, as well as self-reported health as regressors.

2. Relying on separability between financial- and health-related decisions, as well as complete markets, the value of net wages $\{V_t^w\}_{t_0}^T$ is exogenous and calculated from (7), while the human wealth $\left\{V_t^H\right\}_{t_0}^T$ is endogenous and calculated from Theorem 1.

3. For given initial financial wealth and health (W_0, H_0) set equal to their empirical counterparts, we solve initial total wealth:

$$N_0 = W_0 + V_0^w + V_0^H(H_0).$$

   The predicted optimal paths for health and total wealth $\{H_t, N_t\}_{t=0}^T$ are solved forward for each $t = T - s$ as:

$$\mathrm{E}[H'(H)] = H\left\{A\kappa_s^\alpha + (1-\delta) - [1 - \exp(-\lambda^h)]\phi\right\}, \tag{20}$$

$$N' = N(1-\omega_s)R. \tag{21}$$

4. The associated optimal paths for total expenses, and continuation utility $\{\tilde{c}_t, V_t\}$ are derived from Theorem 2. The corresponding expressions for the leisure share and income can be calculated from the definitions of total expenses $\tilde{c}_t$ in (5) and $\mu(w_t)$ in (6a) as:

$$\ell_t = \frac{\tilde{c}_t}{w_t}\left(\frac{\mu(w_t) - 1}{\mu(w_t)}\right),$$

$$Y_t = w_t(1-\ell_t).$$

5. Given net total N_t, and the human capital components V_t^w, V_t^H, the financial wealth W_t along the optimal path can be recovered as:

$$W_t = N_t - V_t^w - V_t^H.$$

Calibrated parameters The calibrated parameters reported in Table 2 are chosen to minimize the distance between observed and predicted life cycles for health H_t, hours $n_t = 1 - \ell_t$, labor income Y_t, and financial wealth W_t.

First, by exploiting the separation properties between health-related and financial decisions, the health production parameters in panel a are selected to reproduce the observed life cycle dynamics of health (Figure 2, panel a, scaled upwards for positive values) via its predicted optimal path (20). The parameters are indicative of significant diminishing returns to investment ($\alpha = 0.75$), non-negligible depreciation ($\delta = 1.75\%$), and consequential additional depreciation through illness ($\phi = 3.50\%$).

Second, in panel b, all nominal variables expressed in dollars are scaled by a factor of 10^{-3}. The risk-free discount rate is set at 5%. The parameters $m = i - y$, and B are obtained by regressing income net of medical expenses (both from PSID data) on a constant and the health level H.

Third, the preference parameters in panel c are obtained by minimizing a weighted sum of squares of residuals between observed and predicted LC's for wealth, hours and income. The consumption share $\theta = 0.45$, is set at a realistic value, and $\sigma = 0.8164$ is indicative of strong intra-temporal substitutability $1/\sigma$ between consumption and leisure. Similarly, the parameter $\varepsilon = 0.5009$ shows high elasticity of inter-temporal substitution $1/\varepsilon$ between current consumption and the certainty equivalent of future utility. The risk aversion $\gamma = 3.5382$ and discount rate $\rho = 0.0438$ are both set at realistic values. Finally, the bequest motive b is indicative of low bequests motives,[7]

Fit adequacy Contrasting the observed and predicted life cycles in Figure 4 reveals that the model performs reasonably well in reproducing the data. First, both the level, and rate of decline in health are precisely matched and remain within confidence bounds in panel a.

[7]More precisely, the agent expects to bequest on average $b^{1/(1-\gamma)}\mathrm{E}(N_t) = 3,279\$$.

Similarly, the level, accumulation and de-cumulation phases of financial wealth are precisely predicted in panel b. The model tends to under-predict observed hours, and labor income for younger, and older agents in panels c, and d yet both levels and timing of declines are correctly predicted. The excessive adjustments in hours worked is likely related to the absence of labor market frictions in the model, such as statutory number of hours.

Predicted net total wealth and continuation utility Figure 5 plots the predicted net total wealth N_t calculated from the recursion (21). First, the level is considerably higher than financial wealth, confirming the importance of human assets. Second, the decline in the value of human capital V_t^w and V_t^H dominates the accumulation of financial wealth W_t, resulting in a decline in total resources N_t. Finally, the continuous drop in net total wealth N_t, combined with declining marginal utility ψ_t jointly lead to a monotonous in the continuation utility $V_t = \psi_t N_t$ in Figure 6 . Put differently, the metric against which changes in longevity are valued is falling throughout the life cycle under the combined influences of diminishing resources and falling marginal value of total wealth.

6 Empirical estimates life valuations

6.1 Methodology

The Hicksian EV and CV measures as well as their WTP, WTA implications are calculated from the LC model for each change Δ in the death risk intensity λ^m, and along the optimal path for each age t. To isolate the former, it will also be useful to consider survival-weighted population averages that are computed for any variable X_t as follows

$$\mathrm{E}(X_t) = \sum_{k=1}^{T-t} f_{t,t+k} X_{t+k}, \quad \text{where } f_{t,t+k} = \left(\frac{S_{t,t+k}}{L_t} \right)$$

where survival and longevity are given in (1). Equivalently, the density $f_{t,t+k}$ is the share of total population of initial size L_t that is surviving at each age $t+k$.

Second, the valuations can be calculated for permanent changes Δ or in terms of associated changes in longevity. For the latter, I fix detrimental and beneficial changes in expected remaining longevity L_t in (1c) and then compute the associated change Δ in death intensity λ_t^m. Because the death intensity is age-dependent, and in light of the non-linearities involved, this procedure must be repeated at all ages in our sample, while the detrimental and beneficial changes must be calculated separately from one another. More specifically, for base sequence $\lambda^m = \{\lambda_t^m\}_{t=0}^{T-1}$, I use (1a), (1b) and (1c) to compute the detrimental $\lambda_+^m = \lambda^m + \Delta$ and beneficial $\lambda_-^m = \lambda^m - \Delta$ sequences such that,

$$L_t\left(\lambda^m\right) - L_t\left(\lambda_+^m\right) = L_t\left(\lambda_-^m\right) - L_t\left(\lambda^m\right) \in \{6, 12, \ldots, 36\} \text{ months}$$

The associated EV's, CV's, WTP's and WTA's are then calculated at each age, for each decrease/increase in remaining longevity.

6.2 Effects of changes in mortality Δ

The figures plotting the effects of changes Δ are regrouped in Appendix B.3.

Mortality elasticity Figure 7 plots elasticity of the continuation utility $\Psi_t(\Delta, \lambda^m, N)$ with respect to mortality given by (17) in function of change Δ, at ages $t = 25$ (in blue), and $t = 65$ (in red), as well as the survival-weighted population average (in black). This elasticity is decreasing and convex in change Δ. Consistent with preference for life, it is positive for beneficial changes $\Delta < 0$, whereas it is negative for detrimental ones $\Delta > 0$. Convexity implies that beneficial gains will have a comparatively more potent marginal effect than detrimental ones of similar magnitude. Equivalently, the agent will benefit more from beneficial changes than suffer from detrimental changes in longevity. The combination of a fixed terminal horizon T with age-increasing death risks λ^m implies that the remaining effects on continuation utility are weaker, translating into a flatter profile at older age.

Hicksian variations and willingness measures Figure 8 plots the Hicksian EV $v_t^e(\Delta, \lambda^m)$ (solid lines) and the CV $v_t^c(\Delta, \lambda^m)$ (dots) calculated from (16), at ages $t = 25$ (in blue), and $t = 65$ (in red), as well as the survival-weighted population average (in black) for beneficial ($\Delta < 0$) and detrimental ($\Delta > 0$) changes. From equations (14), the North quadrant corresponds to a WTP to attain beneficial changes; the West quadrant corresponds to a WTA to forego beneficial changes; the East quadrant measures the WTP to prevent detrimental and the South quadrant measures the WTA to accept detrimental changes.

The results reveal that the EV is increasing and concave, whereas the CV is decreasing and convex in Δ. The implications of functional differences between EV and CV are twofold. Focusing first on horizontal differences highlights disparities between beneficial and detrimental changes in mortality. They reveal that both WTP and WTA measures are much larger in the gains ($\Delta < 0$) than in the loss ($\Delta > 0$) domain, confirming previous results for the mortality elasticity Ψ_t in Figure 7. Second, vertical differences highlights differences between buying (WTP) and selling (WTA) prices. They confirm standard variational results linked to endowment effects that the latter outweigh the former. Indeed, the WTA to forego is larger than the WTP to attain beneficial changes, while the WTA to accept is larger than the WTP to prevent detrimental changes in longevity of similar magnitude. Next, again consistent with Figure 7, ageing attenuates both levels and differences between the two Hicksian measures by flattening the EV's and CV's, and therefore lowering both the WTP's and WTA's for older agents.

The previous results are confirmed in Figure 9 which converts the Hicksian EV's and CV's into WTP and WTA measures using (14). The willingness to pay and to accept compensation for beneficial changes $\Delta < 0$ is increasing and convex in the size of $|\Delta|$. For detrimental changes, $\Delta > 0$ the WTP and WTA are increasing and concave in Δ. Equivalently, the two willingness measures are much higher for beneficial, than for detrimental changes in death risk exposure. Second, the WTA-WTP gaps are positive and increasing in levels Δ. Ageing lowers both the levels of and the differences between WTP's and WTA's, and between detrimental and beneficial changes.

VSL Figure 10 plots the survival-weighted population average of the VSL calculated from (18) for both WTP and WTA perspectives, in function of changes Δ, i.e. the slope of arc from origin to the WTP or WTA. The results first confirm that the *collective* selling price $E(VSL^a)$ (dots) is always larger than the buying price $E(VSL^p)$ (line), except at $\Delta = 0$ where the two coincide.[8] The latter corresponds to the theoretical Value of Statistical Life, i.e. the MRS between life and wealth in (15), and is equal to 58.5 M$, which is in the upper range of the estimates reported in the empirical VSL literature (e.g. Viscusi and Masterman, 2017). Second, both VSL measures are decreasing and convex in Δ confirming that the collective WTP/WTA are much larger in the gains domain than in the loss domain.

6.3 Life cycles of life values

The figures related to the effects of age t are regrouped in Appendix B.4.

Hicksian willingness measures Figure 11 plots the life cycles of both the WTP's (in blue) and WTA's (in red) for changes in expected longevity corresponding to $\pm 12, 24$ months. First, the two willingnesses are unsurprisingly increasing in the changes in longevity. Second, all values are decreasing in age. Third, consistent with previous results, the WTA measures (selling price) are always larger than the WTP's, (buying price) and both are larger for

[8] See also Guria et al. (2005) for much larger WTA-based empirical VSL estimates than their WTP-based counterparts.

beneficial gains, than for detrimental losses in longevity, yet the differences are attenuated with age. Put differently, attaining one year additional longevity is valued more than losing one year for most of the life cycle, except at older age when the two values tend to converge.

VSL Figure 12 plots the life cycle of the two VSL measures VSL_t^p, VSL_t^a calculated from (18). Choosing a small $\Delta = 1.0e - 05$ ensures that both the WTP-based (line) and WTA-based (dots) VSL's coincide with one another and correspond to the theoretical VSL estimate equal to the MRS in (15). First, the VSL are falling rapidly in age, except for elders where the rate of decline is reduced. The ranges of values are consequently very large.

GPV Figure 13 plots the life cycle of the two Gunpoint measures GPV_t^p, GPV_t^a computed from (19). First, both the willingness to accept compensation for immediate and certain death (dots) and to pay (line) to avoid death are falling in age. These patterns are consistent with the declining continuation utility V_t which was identified in Figure 6. As the welfare from remaining alive falls through the combined influences of falling total wealth N_t (Figure 5) and falling marginal utility of wealth ψ_t, so does the WTP to prevent and WTA to accept imminent death. Second, concavity of both WTP and WTA in the loss domain guarantees that the WTA is finite and much larger than the WTP, consistent with some degree of substitutability between wealth and own life.[9] Again the difference between the two is attenuated with ageing.

[9] Hanemann (1991, Prop. 2) shows that there exists no finite acceptable compensation in the absence of substitutability between a given good and others, implying infinite WTA for immediate death. In our setting, utility for bequests guarantees finite tradeoff between more bequeathed wealth and loss of life.

7 Discussion

7.1 Comparative statics

Table 3 presents survival-weighted population averages for our baseline model (column 1), and for alternative parametric choices (columns 2-5). The comparative statics are calculated by re-computing the optimal life cycles of (N_t, ψ_t) at the alternative parametric set, and then recalculating the implied life valuations (v_t^e, v_t^c) and (WTP_t, WTA_t) for beneficial increases, and detrimental decreases in expected longevity of 6 to 36 months, as well as for the Gunpoint and Statistical life values. Consistent with comparative statics principles, only one parameter is modified at a time with others remaining at base values.

Bequest motive In column (2), I analyze the effects of bequests by increasing the intended bequest equal to $b^{1/(1-\gamma)}$ via a 50% decrease in b. This results in two opposing forces with respect to life valuations. On the one hand, agents wish to increase bequeathed resources, resulting in an increase in net total wealth N_t from 2.74 M\$ to 3.14 M\$. On the other hand, the utilitarian cost of dying is attenuated by leaving bequests, thereby reducing the propensity to pay or to accept compensation for changes in longevity. The results confirm that the latter is more important, leading to an overall decline in WTP's, WTA's, as well as GPV and VSL.

Leisure motive In column (3), I analyze the effects of the utility for leisure by removing its benefit in setting $\theta = \sigma = 1$ in (4b). Consequently, the agent inelastically supplies her full time endowment for work ($n_t = 1$) and the absence of spending on leisure ($w_t \ell_t = 0$) implies a decrease in total expenditures and an increase in total wealth from 2.74 M\$ to 3.37 M\$. However, the fall in felicity from leisure activities implies a lower continuation utility and a reduction in the WTP/WTA for life, as well as in VSL and GPV values.

Impatience The effects of impatience are reported in column (4) where the subjective discount rate ρ is increased by 50% from its calibrated value. As the agent discounts the future more heavily, savings drops, resulting in a large decrease in net total wealth from 2.74 M\$ to 1.38 M\$. Heavier discounting also implies a much lower net present value of the gains and costs associated with changes in longevity. Both effects of a higher ρ concur to lower all life valuations.

Market imperfections The theoretical results depend crucially on a complete, perfect markets assumption where any revenue stream can be sold at its expected net present value. I relax this assumption in column (5) by assuming that only 50% of the human capital wealth (V_t^w, V_t^H) can be collected by the agent because of market imperfections such as taxes, information asymmetries, or markups. Unsurprisingly, net total (i.e. financial and human) wealth falls sharply and so do life valuation measures.

7.2 Comparison with other estimates

Several reasons explain why the value of a statistical life estimates are larger than other values found in the VSL literature. First, all WTP and WTA measures are based on the net total wealth N_t, instead of financial resources W_t. As evidenced in Figure 5 this value is much larger than financial wealth W_t. Indeed, it incorporates explicit corrections for the shadow values of the time endowment V_t^w of of the health capital stock V_t^H. Moreover, N_t is calculated under a perfect market assumption and therefore abstracts from borrowing constraints or incomplete markets to calculate an agent's net human and financial worth. Market imperfections were previously shown to greatly reduce both the marginal and total WTP and WTA measures. Third, the life valuations are recovered as shadow values of life under a revealed preference perspective applied to life cycle patterns of health, wealth and work/leisure choices. This perspective is unrelated to traditional VSL elicitation approaches such as the wages/fatality nexus which capture both the employer's WTP and the worker's WTA for compensation against death risk exposure. There is no ex-ante reason why the two measures should coincide.

The value of life estimates are more comparable with other theoretical models of life valuations. Córdoba and Ripoll (2017) and Bommier et al. (2019) also resort to non-expected utility LC settings and find lower estimates for the VSL. However, they abstract from the shadow value of the agent's health capital, thereby imposing that total wealth is equal to financial wealth only. Both Gunpoint and Statistical life values found in Hugonnier et al. (2021b) incorporate human wealth, yet their life values are lower than our estimates. Their modelling approach is however different as it abstracts from ageing, bequests ($b = 0$), and leisure ($\theta = 1$). As seen earlier the omission of leisure lowers the utilitarian benefits from living, resulting in lower life valuations. Moreover, Hugonnier et al. (2013, 2021b) allow for subsistence consumption which cannot be pledged in life valuations and must be deducted from N_t. Finally, they abstract from a finite upper bound T on longevity and focus exclusively on WTP measures in the loss domain $\Delta > 0$ where they identify strongly diminishing returns in the WTP function. The latter are likely related to the possibly infinite life horizon which reduces the utility loss of (and therefore WTP to prevent) marginal increases in death risk exposure compared to our finite lives setting.

7.3 Interpretation of life valuations

Longevity gains versus losses All our valuation results convey the same message. Consistent with preference for life, the mortality gradient, i.e. the change in continuation utility induced by Δ is positive for beneficial changes and negative for detrimental ones. Crucially, the welfare elasticity is decreasing and convex in mortality risk change. These properties imply that the continuation utility benefit of more life always outweighs the utility cost of more mortality for any given Δ (Figure 7). This asymmetry explains functional differences between the EV and CV and rationalizes why the WTP/WTA are much larger for beneficial than for detrimental changes of similar magnitude. It also induces well-known endowment effects where the selling prices (WTA) are much larger than the buying prices (WTP). Whereas the asymmetry between the life values suggest no diminishing returns in the gains versus loss domain, they do not rule out diminishing marginal utility in the additional mortality dimension. Indeed both WTP's and WTA's are concave in the loss domain. Moreover, the limiting WTP's to prevent and WTA's to accept certain death are both finite.

Ageing and life values All our results are consistent with ageing being associated with (a) flatter Hicksian measures in the Δ domain, and (b) continuous declines in life valuation measures in the age domain. Both results entail much lower, and less reactive life values for elders than for younger agents. Three reasons explain why this is the case. First, net total wealth N_t is optimally falling throughout the life cycle (see Figure 5). Financial wealth-poor, but health capital-rich young agents have much more human wealth (and therefore net total capital N_t) to pledge than elders in order to attain/forego more life or to prevent/accept shorter longevity. Second, the marginal utility of wealth is also falling with age (see Figure 5). Combining falling wealth with falling marginal utility leads to falling continuation utility of living (see Figure 6). Since this forward-looking welfare is the metric against which changes in longevity are contrasted, the willingness to pay or accept compensation for both beneficial and detrimental changes is lowered by ageing. This is especially apparent in the Gunpoint value (see Figure 13) where falling continuation utility of living implies falling WTP to prevent and WTA to accept certain and immediate death. Third, finite lives T mechanically imply that permanent changes in death exposure are effective over a shorter maximal horizon for elders than for young agents. The three effects of falling net total wealth declining continuation utility, weakening effects of permanent changes in mortality concord to generate much higher WTP and WTA for younger adults who have both more net total wealth (consisting primarily from human wealth) and more willingness to engage resources for permanent changes that will have a longer-lasting impact, on a higher valued continuation utility compared to elders. The results are consistent with ageing being associated with accumulating stock of “lived life”; the willingness to protect the remaining accumulation or to accept the end of accumulation decreases in age.

A Tables

Table 1: Descriptive statistics

Variable	Mean	Median	Std	Min	Max
			a.PSID		
Age	46.53	45.00	16.34	21.00	100.00
Wealth (K$)	243.47	185.73	164.79	-3.26	544.01
Health	-0.06	-0.01	0.52	-2.62	0.59
Sick	0.04	0.03	0.04	0.01	0.43
			b.ATUS		
Age	49.72	48.00	16.40	21.00	85.00
Hours	36.09	38.93	5.53	21.56	40.33
Income($)	692.95	743.76	126.83	399.52	822.16
Wages ($)	17.87	18.58	2.02	10.82	19.84

Notes: a. PSID. Wealth: net financial and residential. Health: Score function from panel multinomial probit on self-reported polytomous health status. Sickness: marginal probability of reporting worst health outcome from panel multinomial probit on self-reported polytomous health status. b. ATUS. Hours: spent working, per week. Income: salaried income per week. Wages: per hour.

Table 2: Calibrated parameters

Parameter	Value	Parameter	Value
	a. Health		
A	0.0080	α	0.7500
δ	0.0175	ϕ	0.0350
	b. Scaling, income and wealth		
scale	0.001	r	0.0500
B	13.3762	$m = i - y$	-34.7817
	c. Preferences		
θ	0.4500	σ	0.8164
γ	3.5382	ε	0.5009
b	$2.6e+07$	ρ	0.0438

Table 3: Survival-weighted population averages and comparative statics (in K$)

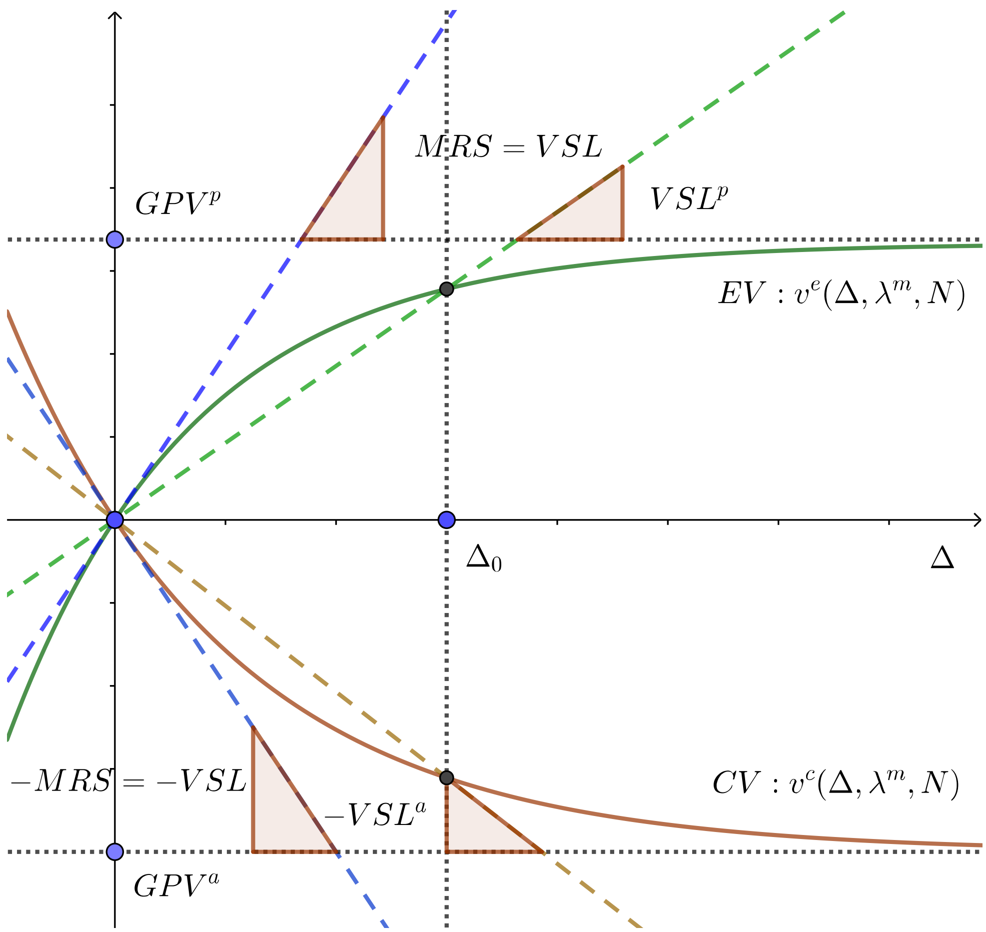
Δ longev. (months)	(1) Base WTP	(1) Base WTA	(2) High bequest WTP	(2) High bequest WTA	(3) No leisure WTP	(3) No leisure WTA	(4) Impatience WTP	(4) Impatience WTA	(5) Imperfect mkts. WTP	(5) Imperfect mkts. WTA
36	387	522	233	264	161	174	195	263	287	518
30	303	373	191	211	134	142	153	188	198	264
24	231	268	151	163	107	112	117	135	142	172
18	167	185	112	118	80	83	84	93	99	112
12	108	115	74	76	53	54	54	58	62	67
6	53	54	37	37	27	27	27	27	29	30
0	0	0	0	0	0	0	0	0	0	0
-6	49	51	36	36	26	27	25	26	27	27
-12	95	100	70	72	52	54	48	50	50	53
-18	138	148	103	108	78	81	69	75	72	78
-24	178	195	135	144	104	108	90	98	91	102
-30	217	242	167	180	129	136	109	122	110	125
-36	254	289	198	217	155	165	128	146	127	148
GPV	1'321	2'686	1'245	2'151	1'120	1'720	666	1'354	574	1'357
VSL	58'467	58'485	41'467	41'474	30'668	30'671	29'468	29'477	31'823	31'834
N_t	2,736		3,138		3,374		1,379		1,006	

Notes: Weighted average of age-dependent life valuations where weights given by survival rates at each age. (1) Baseline calibration; (2) High Bequest: High intended bequest $b^{1/(1-\gamma)}\uparrow$; (3) No leisure: $\theta = \sigma = 1$; (4) Impatience: $\rho\uparrow$; (5) Imperfect markets: $V_t^w, V_t^H\downarrow$.

B Figures

B.1 Theoretical life valuations

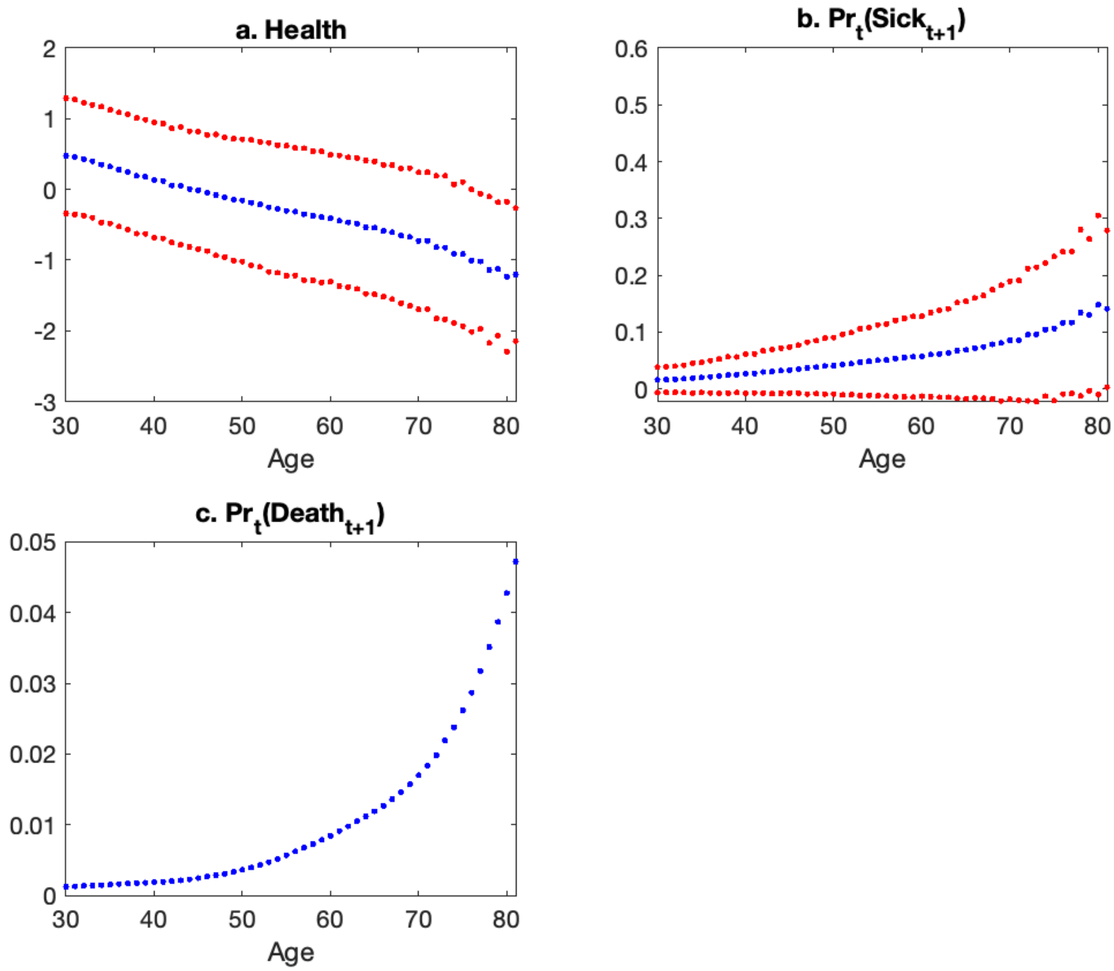
Figure 1: EV, CV, MRS, VSL and GPV



Notes: EV (resp. CV) is Hicksian Equivalent (resp. Compensating) Variation for change Δ in death risk intensity λ^m. MRS is slope (resp. negative slope) of tangent of EV (resp. CV) evaluated at origin; Both are equal to theoretical VSL. Empirical WTP- (resp. WTA-) VSL^p (resp. VSL^a) is WTP- (resp. WTA-)based is slope (resp. negative slope) of arc from origin to EV (resp. CV) evaluated at infra-marginal change Δ_0. GPV^p: Maximum WTP to prevent instantaneous, certain death is upper bound on EV; GPV^a: Minimum WTA to accept instantaneous, certain death is minus lower bound on CV.

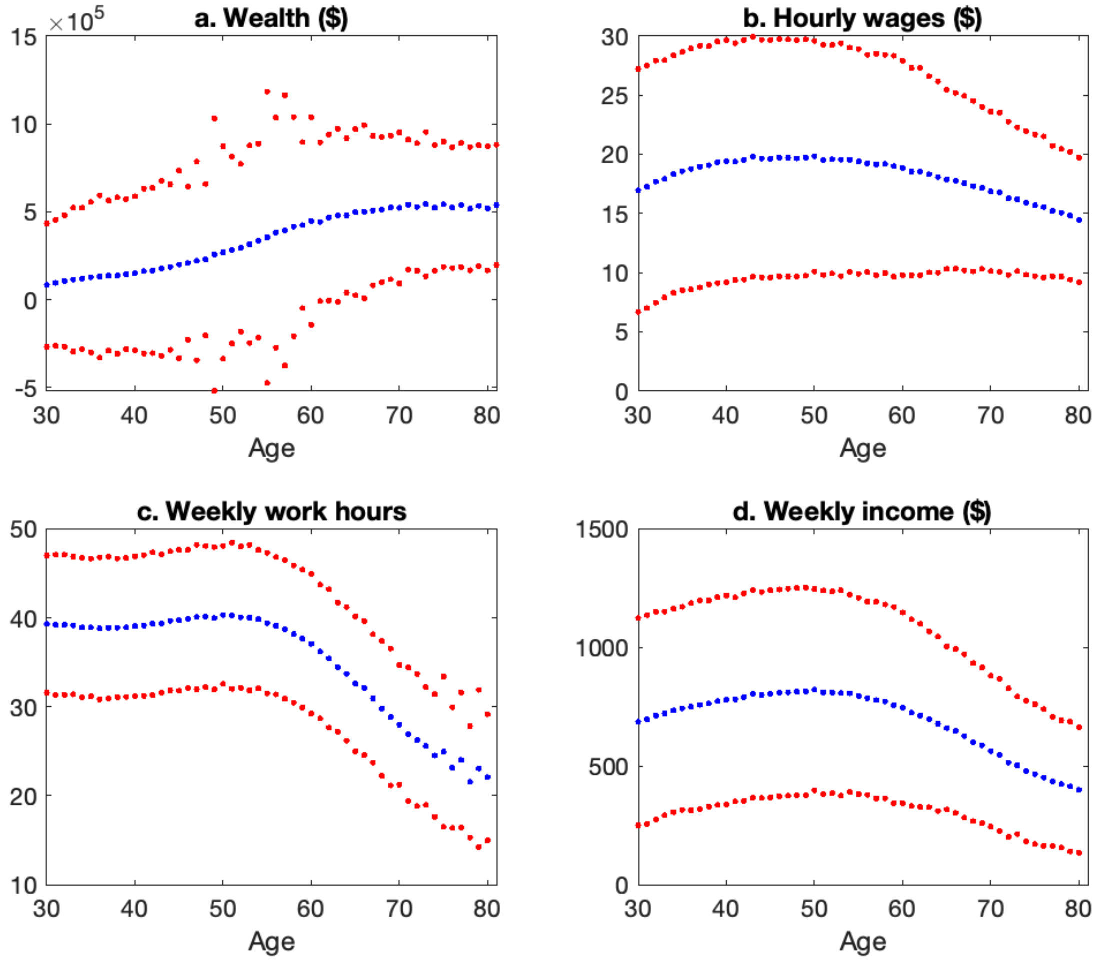
B.2 Data and fitted variables

Figure 2: Health, sickness, and mortality



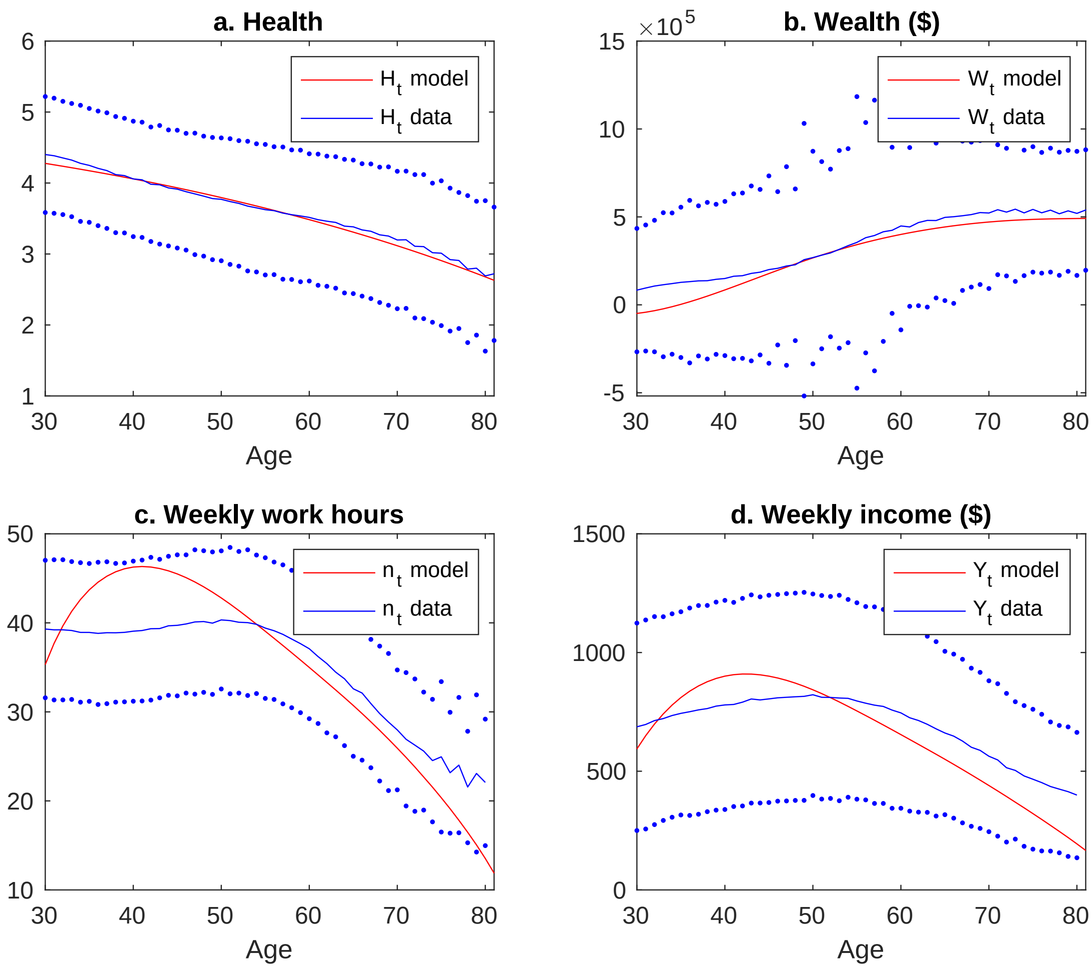
Notes: Sources: Panels a, b: PSID and author's calculations. Confidence bands corresponding to $\pm 2\sigma$, where the standard error is taken from the longitudinal Probit estimation. Panel c: Arias and Xu (2020) and author's calculations.

Figure 3: Wealth, hours, wages and income



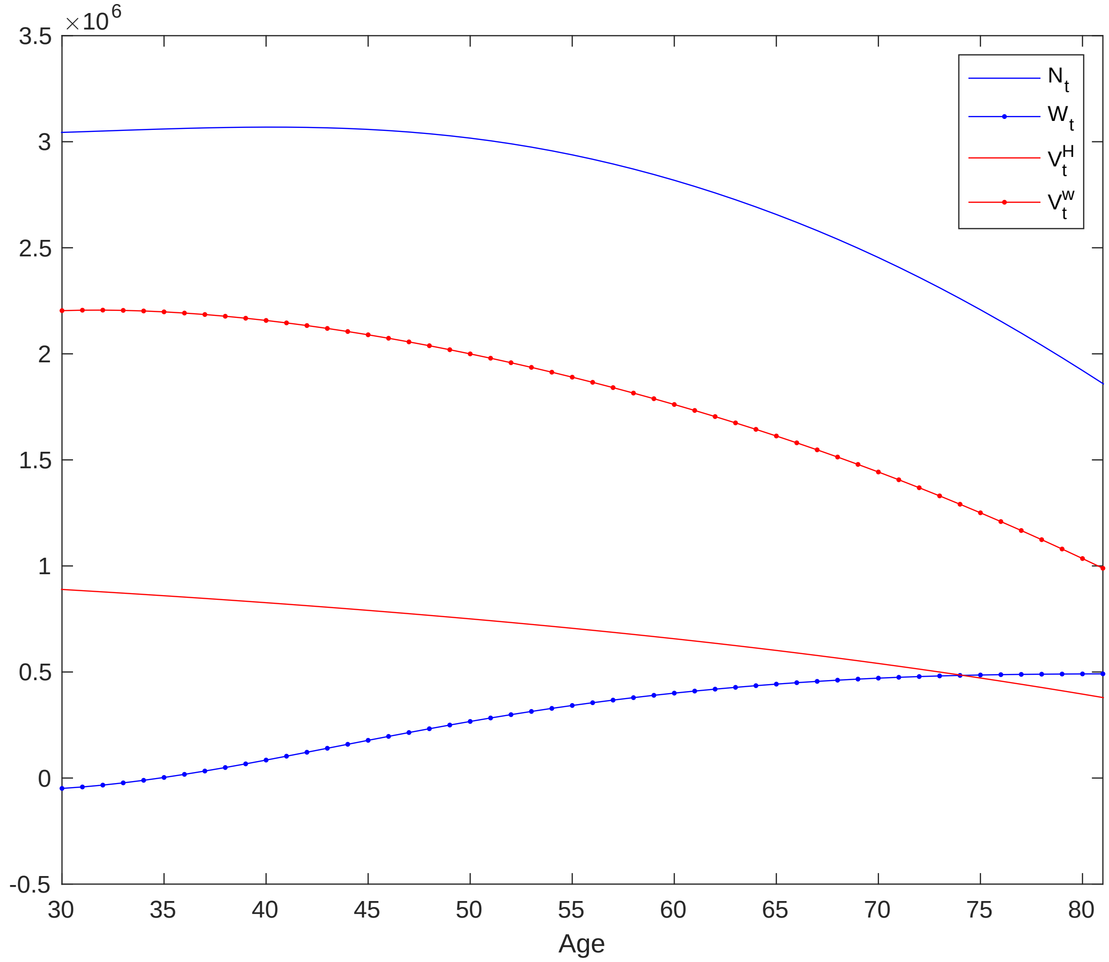
Notes: Sources: Panel a: PSID and author's calculations. Panels b, c and d: ATUS and author's calculations. Confidence bands corresponding to $\pm 2\sigma$, where the standard error is taken from the longitudinal OLS estimation.

Figure 4: Observed and predicted life cycles



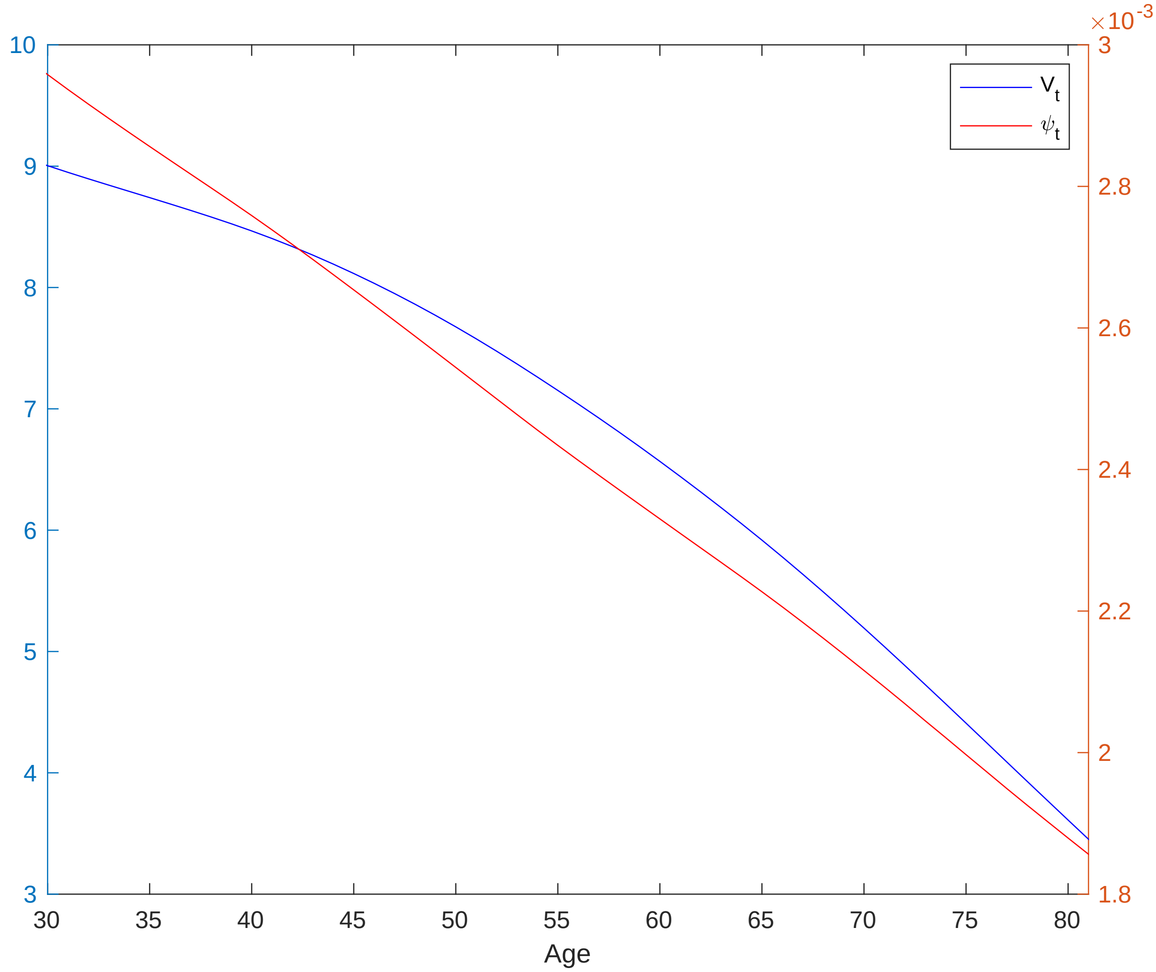
Notes: Calculated along optimal path at calibrated parameters.

Figure 5: Financial, human and total wealth by age (in \$)



Notes: Financial (W_t), Human (V_t^w, V_t^H) and Net total wealth (N_t). Calculated along optimal path at calibrated parameters.

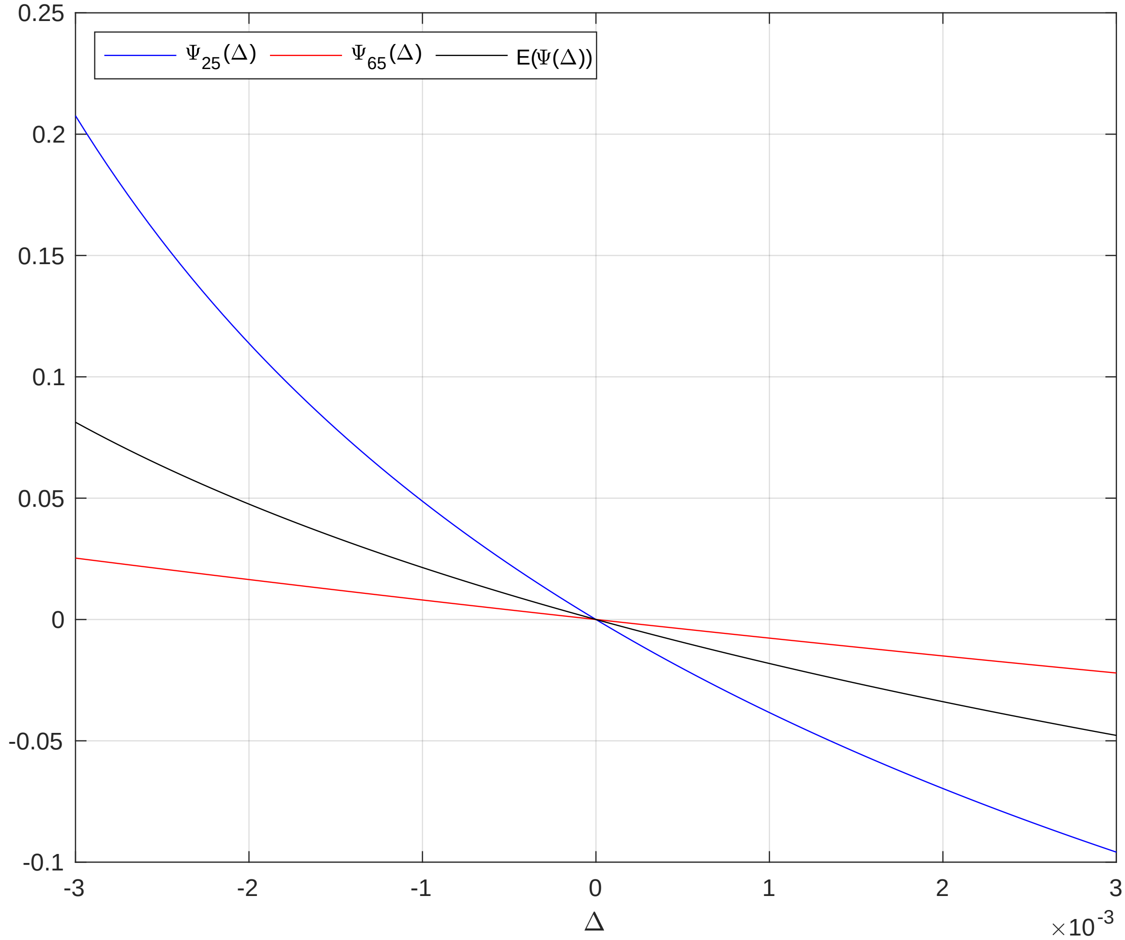
Figure 6: Continuation and marginal continuation utility by age



Notes: Continuation utility (V_t) in blue, left-hand scale and marginal utility of net total wealth (ψ_t) in red, right-hand scale. Calculated along optimal path at calibrated parameters.

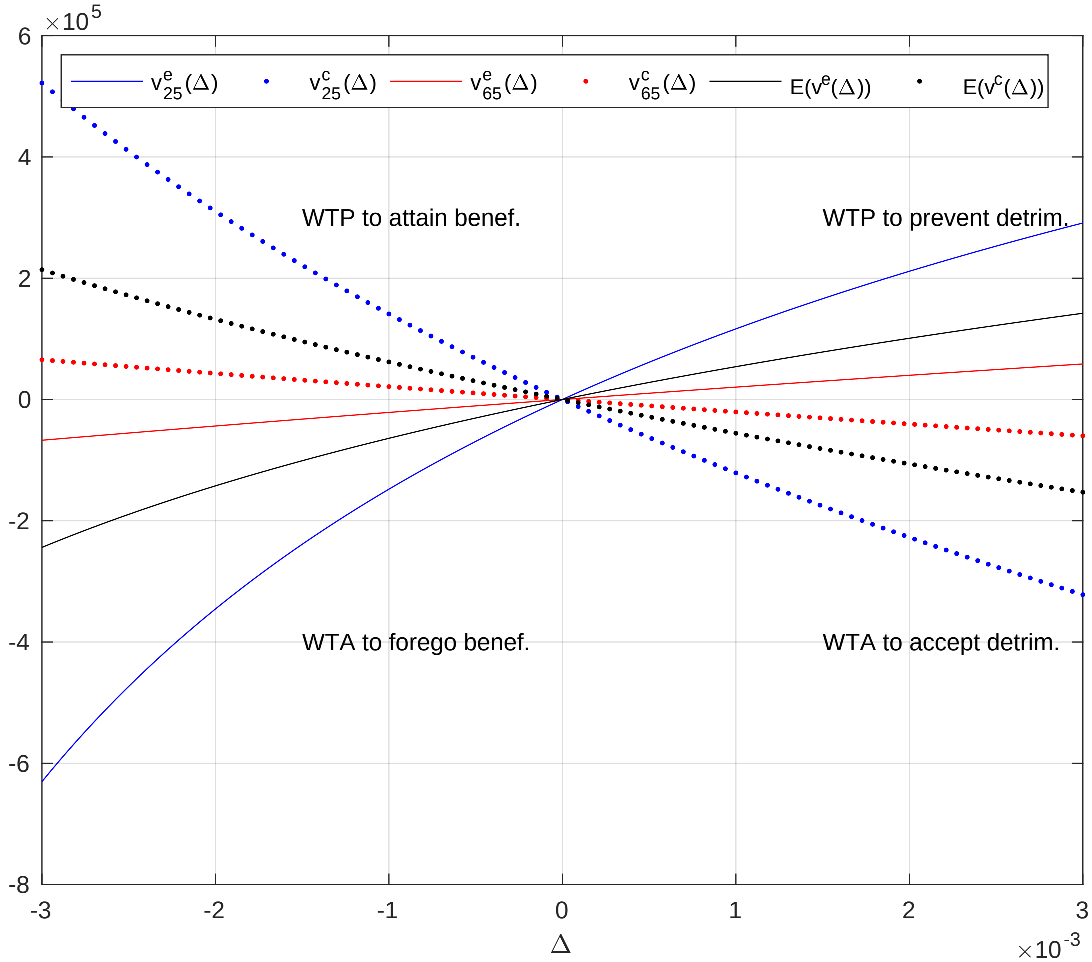
B.3 Effects of changes Δ

Figure 7: Mortality elasticity of welfare by change in mortality risk



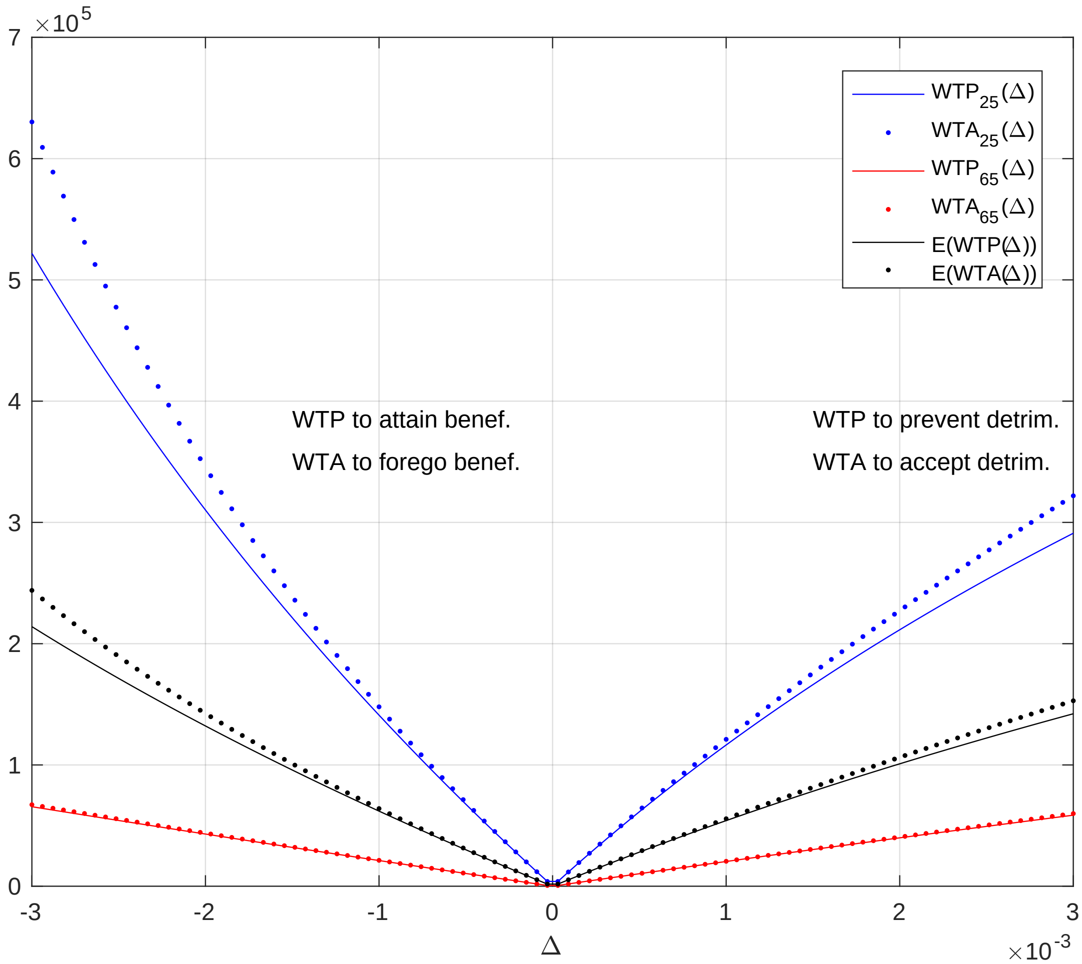
Notes: Mortality elasticity of continuation utility $\Psi(\Delta, \lambda^m, N)$ induced by change Δ in mortality exposure, at age t, and survival-weighted mean $\mathrm{E}(\Psi_t)$ for beneficial ($\Delta < 0$) and detrimental (Δ) changes in base exposure λ^m. Calculated from (17) at calibrated parameters.

Figure 8: Hicksian EV and CV by change in mortality risk



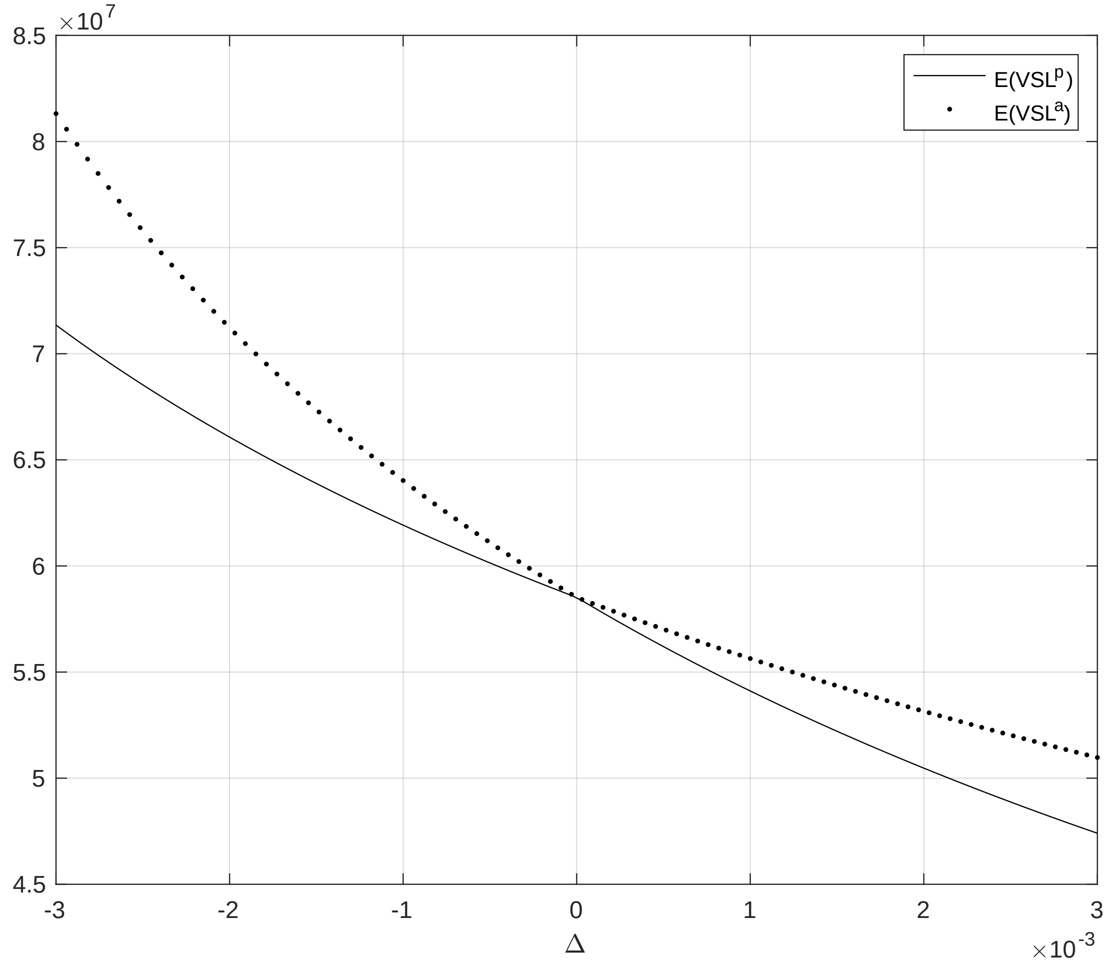
Notes: Hicksian EV: $v^e_t(\Delta, \lambda^m, N)$ (solid line) and CV: $v^c_t(\Delta, \lambda^m)$ (dots) at age t and survival-weighted mean $\mathrm{E}(v^e_t), \mathrm{E}(v^c_t)$ for beneficial ($\Delta < 0$) and detrimental (Δ) changes in base exposure λ^m. Calculated from (16) at calibrated parameters.

Figure 9: WTP and WTA by change in mortality risk (in \$)



Notes: Willingness to pay: $WTP_t(\Delta, \lambda^m, N)$ (solid line) and to accept compensation: $WTA_t(\Delta, \lambda^m, N)$ (dots) at age t, and survival-weighted mean $\mathrm{E}(WTP_t), \mathrm{E}(WTA_t)$ for beneficial ($\Delta < 0$) and detrimental (Δ) changes in base exposure λ^m at age t. Calculated from (14) at calibrated parameters.

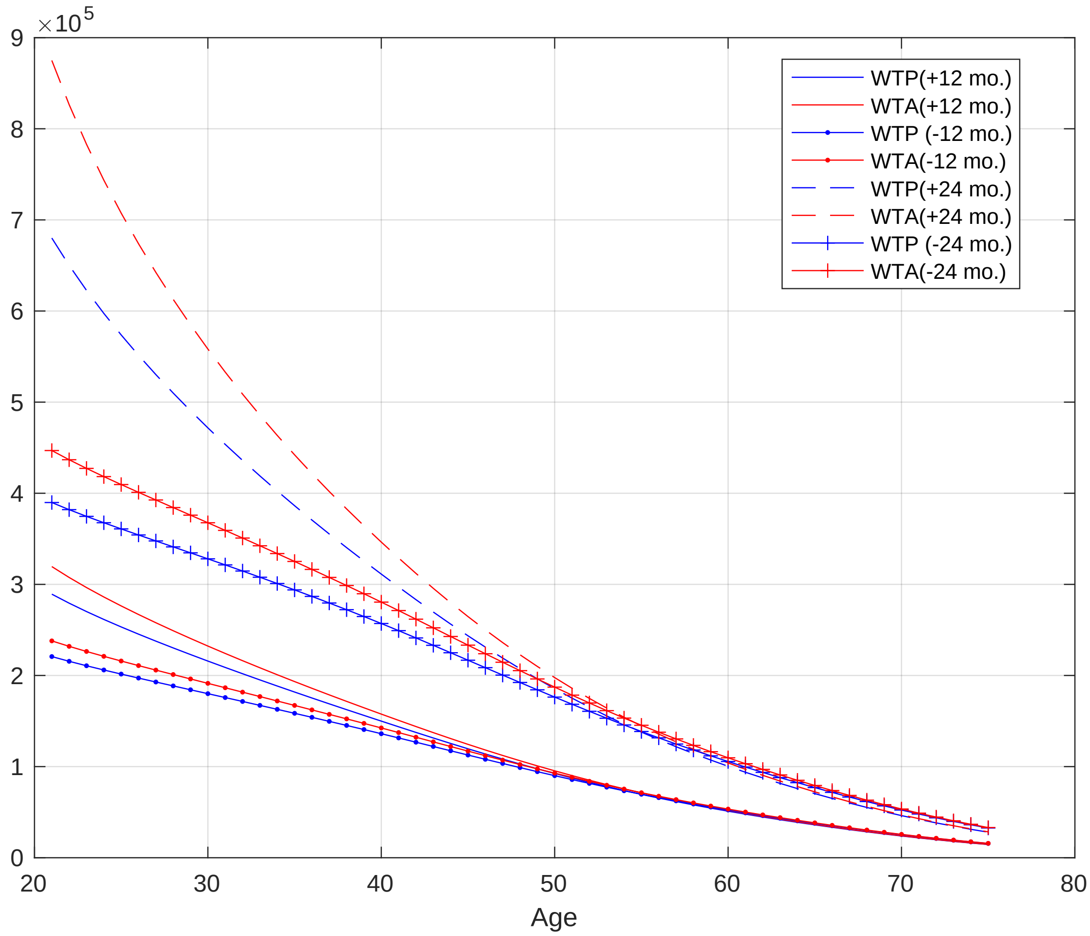
Figure 10: VSL by change in mortality risk (in \$)



Notes: Survival-weighted mean Value of Statistical Life, from WTP: $\mathrm{E}(VSL_t^p(\Delta, \lambda^m, N))$, (solid line) and from WTA compensation $\mathrm{E}(VSL_t^a(\Delta, \lambda^m, N))$ (dots) for beneficial ($\Delta < 0$) and detrimental (Δ) changes in base exposure λ^m at age t. Calculated from (18) at calibrated parameters.

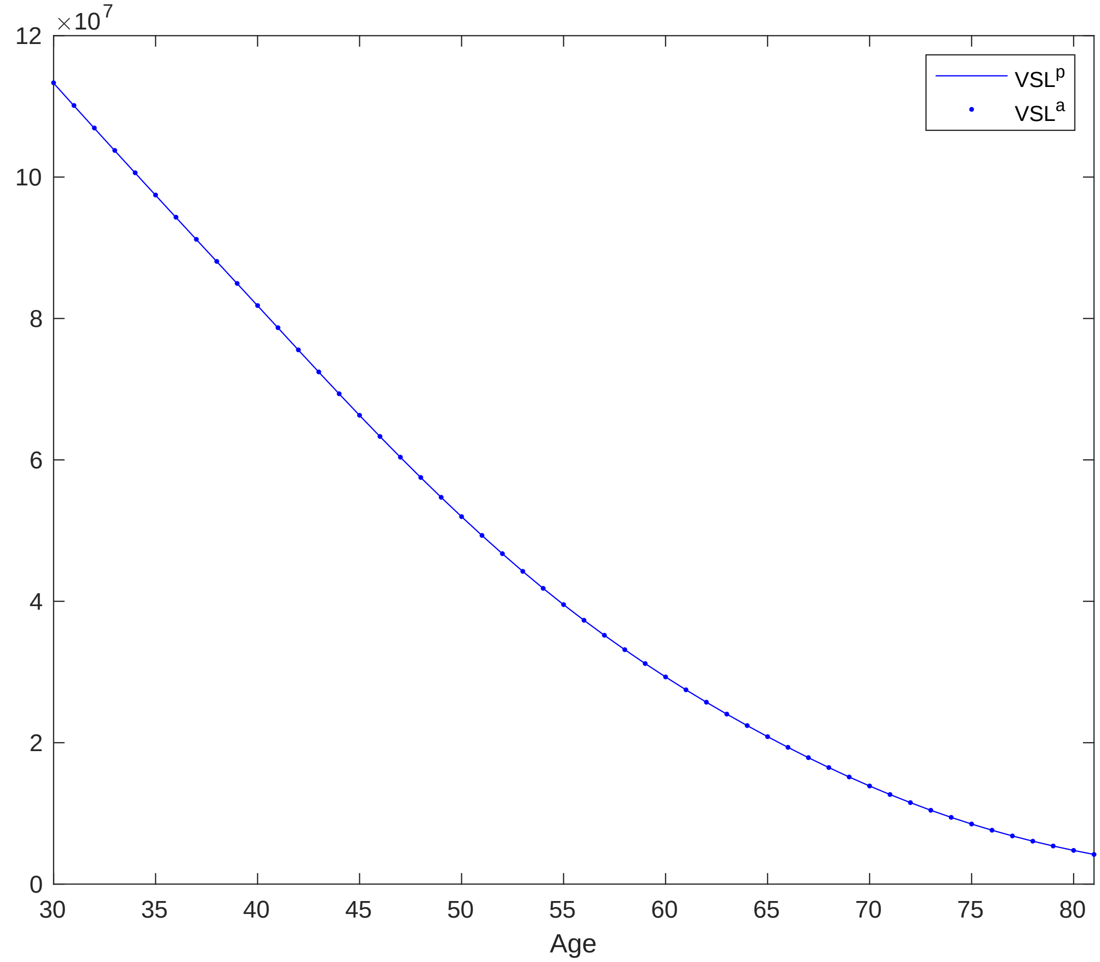
B.4 Effects of age

Figure 11: WTP and WTA by change in longevity and by age (in \$)



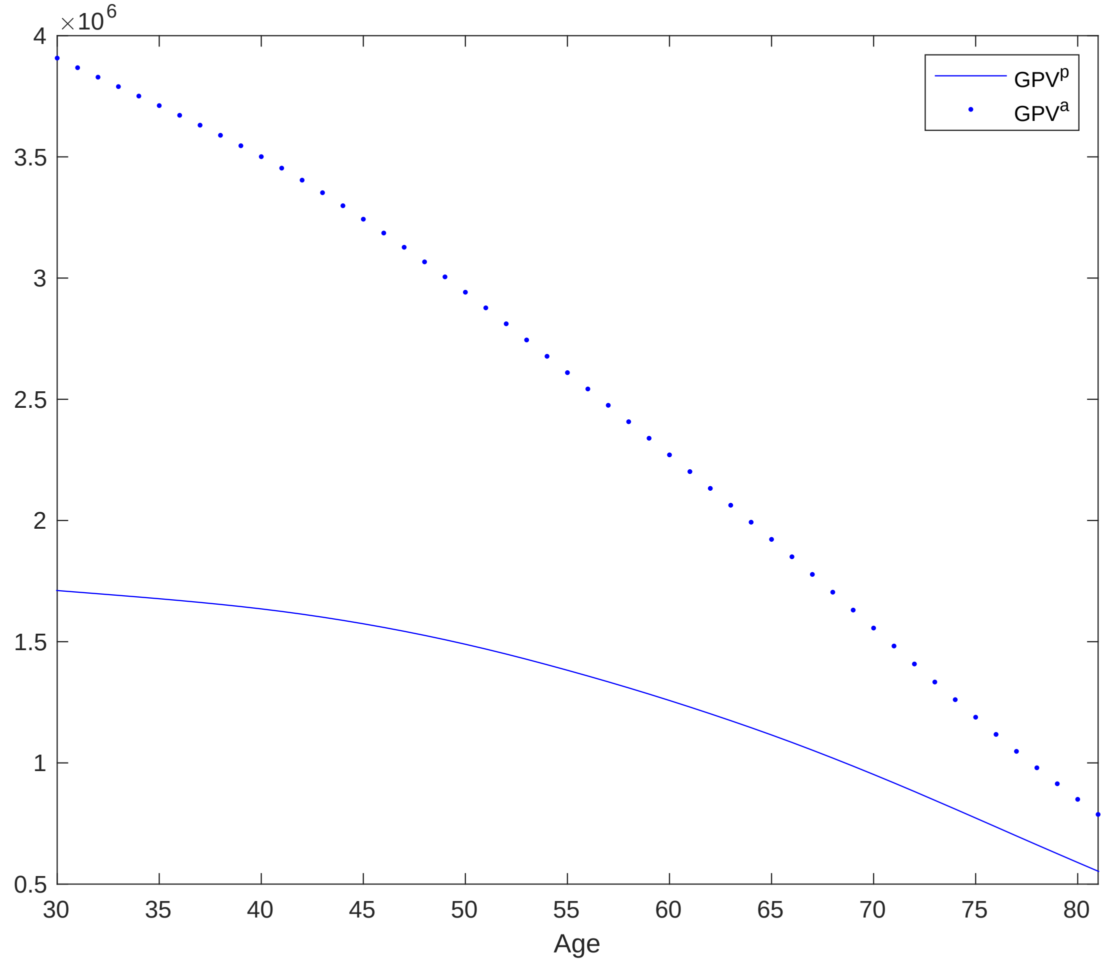
Notes: Willingness to pay $WTP_t(\Delta, \lambda^m, N)$ and to accept compensation $WTA_t(\Delta, \lambda^m, N)$ for beneficial $(\Delta < 0)$ and detrimental (Δ) changes in base exposure λ^m at age t corresponding to changes in longevity $\Delta L_t = 12, 24$ months. Calculated from (14) at calibrated parameters.

Figure 12: Value of Statistical Life by age (in \$)



Notes: Value of Statistical Life, from WTP $VSL_t^p(\Delta, \lambda^m, N)$, and from WTA compensation $VSL_t^a(\Delta, \lambda^m, N)$ for marginal change (Δ) in base exposure λ^m at age t. Calculated from (18) at calibrated parameters along optimal path for N_t.

Figure 13: Gunpoint Value by age (in $)



Notes: Gunpoint value, from WTP $GPV_t^p(\lambda^m, N)$, and from WTA compensation $GPV_t^a(\lambda^m, N)$ for infinite detrimental change ($\Delta = \infty$) in base exposure λ^m at age t. Calculated from (19) at calibrated parameters along optimal path for N_t.

C Proofs

The following proofs are based on backward induction. As mentioned in the text, we refer to $s = 1, 2, \ldots$ as the number of remaining periods before maximal longevity is reached, such that current period is $t = T - s$. Time-subscript are omitted for contemporary variables, with prime ($'$) used for next-period variables. We rely on s subscripts to emphasize feedback rules s-periods from T.

In what follows, I make use of Contraction Mapping results applicable to homogeneous problems to calculate optimal policies $\left\{I_s(H), V_s^H(H)\right\}_{s=1}^T$ and $\{\tilde{c}_s(N), V_s(N)\}_{s=1}^T$ where the feedback rules to the respective state variables are solved backward in closed-form recursions.

Theorem 1. The optimal investment problem (8) can be rewritten as:

$$V_s^H(H) = \max_I BH - I + \exp(-r)\left\{\exp(-\lambda^h)V_{s-1}^H(H'_+) + \left[1 - \exp(-\lambda^h)\right]V_{s-1}^H(H'_-)\right\}$$

subject to

$$H'_+ = AI^\alpha H^{1-\alpha} + (1-\delta)H$$
$$H'_- = AI^\alpha H^{1-\alpha} + (1-\delta-\phi)H$$

for $t = T - s$ periods away from maximal longevity. The candidate solutions are:

$$I_s(H) = \kappa_s H$$
$$V_s^H(H) = \eta_s H$$

$s = 1$ Longevity being bounded leads trivially to $I_s = 0$ and $V_s^H = BH$; the initial loadings are $(\kappa_1, \eta_1) = (0, B)$.

$s = 2$ Substituting the previous solution for $s = 1$ reveals that

$$V_s^H = \max_I BH - I + \exp(-r)\eta_{s-1}\left\{AI^{\alpha}H^{1-\alpha} + (1-\delta)H - [1-\exp(-\lambda^h)]\phi H\right\}$$

with the solution to the FOC:

$$I_s = \underbrace{[\eta_{s-1}\exp(-r)\alpha A]^{\frac{1}{1-\alpha}}}_{\kappa_s} H$$

substituting back in the objective function simplifies to:

$$V_s^H = \underbrace{\left[B - \kappa_s + \eta_{s-1}\exp(-r)\left\{A\kappa_s^{\alpha} + (1-\delta) - [1-\exp(-\lambda^h)]\phi\right\}\right]}_{\eta_s} H$$

$s \geq 3$ It is readily verifiable that the solutions converge to the same form for the other periods.

Regrouping terms shows that the sequence for the loadings $\{\kappa_s, \eta_s\}_{s=1}^{T}$ are solved recursively as stated and completes the proof. ■

Theorem 2. The candidate solutions to the optimal expenses problem (9) are:

$$\tilde{c}_s(N) = \omega_s N$$

$$V_s(N) = \psi_s N$$

$s = 1$ Since longevity is bounded, $s^m = \exp(-\lambda^m) = 0$, leading to the following problem:

$$
\begin{aligned}
V(N) &= \max_{\tilde{c}} \left\{ (1-\beta)(\nu \tilde{c}^{1-\varepsilon}) + \beta [b(N')^{1-\gamma}]^{\frac{1-\varepsilon}{1-\gamma}} \right\}^{\frac{1}{1-\varepsilon}} \\
&= \max_{\tilde{c}} \left\{ (1-\beta)(\nu \tilde{c}^{1-\varepsilon}) + (N')^{1-\varepsilon} \underbrace{\beta b^{\frac{1-\varepsilon}{1-\gamma}}}_{\beta_1} \right\}^{\frac{1}{1-\varepsilon}}
\end{aligned}
$$

subject to $N' = (N - \tilde{C})R$. The solution to the FOC is:

$$
\tilde{c}_1(N) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}} + \beta_1^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}} \right]}_{\omega_1} N
$$

Substituting back into the objective function implies that:

$$
V_1(N) = \underbrace{\left\{ (1-\beta)(\nu \omega_1)^{1-\varepsilon} + \beta_1 \left[(1-\omega_1) R \right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}}_{\psi_1} N
$$

$s = 2$ Noting that $s^m = \exp(-\lambda^m) \neq 0$ and using our previous solution simplifies the agent's problem to:

$$
\begin{aligned}
V &= \max_{\tilde{c}} \left\{ (1-\beta)(\nu \tilde{c})^{1-\varepsilon} + \beta \left[s^m (\psi_1 N')^{1-\gamma} + (1-s^m)\, b\, (N')^{1-\gamma} \right]^{\frac{1-\varepsilon}{1-\gamma}} \right\}^{\frac{1}{1-\varepsilon}}, \\
&= \max_{\tilde{c}} \left\{ (1-\beta)(\nu \tilde{c})^{1-\varepsilon} + (N')^{1-\varepsilon} \underbrace{\beta \left[s^m \psi_1^{1-\gamma} + (1-s^m)\, b\, \right]^{\frac{1-\varepsilon}{1-\gamma}}}_{\beta_2} \right\}^{\frac{1}{1-\varepsilon}},
\end{aligned}
$$

subject to $N' = (N - \tilde{C})R$. The solution to the FOC is:

$$
\tilde{c}_2(N) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}} + \beta_2^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}} \right]}_{\omega_2} N
$$

Substituting back into the objective function implies that:

$$V_2(N) = \underbrace{\left\{(1-\beta)(\nu\omega_2)^{1-\varepsilon} + \beta_2\left[(1-\omega_2)R\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}}_{\psi_2} N$$

$s \geq 3$ It is readily verifiable that the solutions converge to the same form for the other periods.

Regrouping terms shows that the sequence for the loadings $\{\omega_s, \psi_s\}_{s=1}^{T}$ are solved recursively as stated and completes the proof. ■

Separability. I now formally show that health-related and financial decisions are separable, i.e. that a joint optimization problem yields the same solutions as the ones obtained under separability. First, the risk-averse agent will fully insure against health shocks ϵ_{t+1}^{h} at actuarially-fair prices. Consequently, the problem can be recast as a deterministic one with respect to morbidity, i.e. by setting $\epsilon_{t+1}^{h} = 0, \forall t$, with insurance premium calculated endogenously and deducted below from health capital value. Second, recast financial wealth as $W = W + V^w$ to include the value of the time endowment. The agent's problem can then be written as:

$$V(W,H) = \max_{\tilde{c},I}\left\{(1-\beta)\left(\nu\tilde{c}_s\right)^{1-\varepsilon} + \beta\left[s^m V(W',H')^{1-\gamma} + (1-s^m)b(N')^{1-\gamma}\right]^{\frac{1-\varepsilon}{1-\gamma}}\right\}^{\frac{1}{1-\varepsilon}}$$

subject to:

$$\begin{aligned} W' &= \left[W + BH - I - \tilde{c}\right]R \\ H' &= AI^{\alpha}H^{1-\alpha} + (1-\delta)H \\ N' &= W' + \eta' H'. \end{aligned}$$

The candidate solutions are the following:

$$
\begin{aligned}
V_s(W,H) &= V_s(N) \\
&= \psi_s N = \psi_s(W+\eta_s H) \\
I_s &= \kappa_s H \\
\tilde{c}_s &= \omega_s N
\end{aligned}
$$

where the age-dependent loadings $\{\psi_s, \eta_s, \kappa_s, \omega_s\}$ are determined recursively.

$s=1$ Observing that $s^m = 0$ and by transversality $\eta' = \eta_0 = 0$ directly implies zero investment, i.e. $\kappa_1 = I_1 = 0$. The agent's problem simplifies to:

$$
V(W,H) = \max_{\tilde{c}} \left\{ (1-\beta)\left(\nu \tilde{c}_s\right)^{1-\varepsilon} + \underbrace{\beta b^{\frac{1-\varepsilon}{1-\gamma}}}_{\beta_1} \left([\underbrace{W+BH}_{N_1} - \tilde{c}]R \right)^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}
$$

The optimal consumption and continuation utility are characterized by:

$$
\tilde{c}_1(N_1) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}} + \beta_1^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}} \right]}_{\omega_1} N_1
$$

and

$$
V_1(W,H) = V_1(N_1) = \underbrace{\left\{ (1-\beta)(\nu\omega_1)^{1-\varepsilon} + \beta_1 \left[(1-\omega_1)R\right]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}}_{\psi_1} N_1
$$

which is the same as under the separable problem, and establishes that the Tobin's-q in $V^H = \eta_1 H = BH$.

$s = 2$ The problem is:

$$V = \max_{\tilde{c},I} \left\{ (1-\beta)(\nu\tilde{c})^{1-\varepsilon} + \beta \left[s^m(\psi_1 N')^{1-\gamma} + (1-s^m)\, b\, (N')^{1-\gamma} \right]^{\frac{1-\varepsilon}{1-\gamma}} \right\}^{\frac{1}{1-\varepsilon}},$$

$$= \max_{\tilde{c},I} \left\{ (1-\beta)(\nu\tilde{c})^{1-\varepsilon} + (N')^{1-\varepsilon} \underbrace{\beta \left[s^m \psi_1^{1-\gamma} + (1-s^m)\, b\, \right]^{\frac{1-\varepsilon}{1-\gamma}}}_{\beta_2} \right\}^{\frac{1}{1-\varepsilon}},$$

subject to:

$$N' = \left\{ [W + BH - I - \tilde{c}] + R^{-1}\eta_1 \left[AI^{\alpha}H^{1-\alpha} + (1-\delta)H \right] \right\} R$$

Solving for optimal investment reveals that

$$I_2 = \underbrace{\left(R^{-1}\eta_1 \alpha A \right)^{\frac{1}{1-\alpha}}}_{\kappa_2} H$$

$$V_2^H = \underbrace{\left[B - \kappa_2 + R^{-1}\eta_1 \left(A\kappa_2^{\alpha} + (1-\delta) \right) \right]}_{\eta_2} H$$

$$N' = \left[\underbrace{W + V_2^H}_{N_2} - \tilde{c} \right] R$$

i.e. the optimal investment is independent of mortality risk. The optimal expenditures choices solves:

$$V = \max_{\tilde{c}} \left\{ (1-\beta)(\nu\tilde{c})^{1-\varepsilon} + \beta_2 (N')^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}},$$

The solution to the FOC is:

$$\tilde{c}_2(N) = \underbrace{\left[\frac{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}}}{(1-\beta)^{\frac{1}{\varepsilon}} \nu^{\frac{1-\varepsilon}{\varepsilon}} + \beta_2^{\frac{1}{\varepsilon}} R^{\frac{1-\varepsilon}{\varepsilon}}} \right]}_{\omega_2} N$$

Substituting back into the objective function implies that:

$$V_2(N) = \underbrace{\left\{(1-\beta)(\nu\omega_2)^{1-\varepsilon} + \beta_2\left[(1-\omega_2)R\right]^{1-\varepsilon}\right\}^{\frac{1}{1-\varepsilon}}}_{\psi_2} N$$

$s \geq 3$ It is readily verifiable that the solutions converge to the same form for the other periods.

Health insurance The risk-averse agent purchases full insurance against health shocks ϵ^h_{s-1} when sold at actuarially-fair prices. The insurance premium is the expected loss in human capital value induced by morbidity:

$$\begin{aligned}
\pi_s &= \left[1-\exp(-\lambda^h)\right]\nabla V^H_s, \quad \text{where} \\
\nabla V^H_s &= V^H_s(\epsilon^h_s = 1) - V^H_s(\epsilon^h_s = 0) \\
&= R^{-1}\eta_{s-1}\phi H
\end{aligned}$$

subtracting the insurance premium π_s from the shadow value V^H_s and regrouping terms establishes that the Tobin's-q is:

$$\eta_s = B - \kappa_s + \eta_{s-1}R^{-1}\left\{A\kappa^\alpha_s + (1-\delta) - [1-\exp(-\lambda^h)]\phi\right\}$$

which completes the proof that the separable and joint allocations coincide.

■

References

Acemoglu, Daron, and David Autor (2018) 'Lectures in labor economics.' Lecture notes, MIT

Arias, Elizabeth, and Jiaquan Xu (2020) 'United States life tables, 2019.' *National Vital Statistics Report* 68(7), 1–65

Bodie, Zvi, Robert C Merton, and William F Samuelson (1992) 'Labor supply flexibility and portfolio choice in a life cycle model.' *Journal of Economic Dynamics and Control* 16(3-4), 427–449

Bommier, Antoine, Danier Harenberg, and François Le Grand (2019) 'Recursive preferences, the value of life, and household finance.' Working paper, Available at SSRN https://ssrn.com/abstract=2867570, May

Conley, Bryan C. (1976) 'The value of human life in the demand for safety.' *American Economic Review* 66(1), 45 – 55

Cook, Philip J., and Daniel A. Graham (1977) 'The demand for insurance and protection: The case of irreplaceable commodities.' *Quarterly Journal of Economics* 91(1), 143 – 156

Córdoba, Juan Carlos, and Marla Ripoll (2017) 'Risk aversion and the value of life.' *Review of Economic Studies* 84(4), 1472–1509

Eeckhoudt, Louis R., and James K. Hammitt (2004) 'Does risk aversion increase the value of mortality risk?.' *Journal of Environmental Economics and Management* 47(1), 13 – 29

Ehrlich, Isaac, and Hiroyuki Chuma (1990) 'A model of the demand for longevity and the value of life extension.' *Journal of Political Economy* 98(4), 761–782

Epstein, Larry G., and Stanley E. Zin (1989) 'Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework.' *Econometrica* 57(4), 937–969

— (1991) 'Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical analysis.' *Journal of Political Economy* 99(2), 263–268

Grossman, Michael (1972) 'On the concept of health capital and the demand for health.' *Journal of Political Economy* 80(2), 223–255

Guria, Jagadish, Joanne Leung, Michael Jones-Lee, and Graham Loomes (2005) 'The willingness to accept value of statistical life relative to the willingness to pay value: Evidence and policy implications.' *Environmental and Resource Economics* 32(1), 113 – 127

Hanemann, W. Michael (1991) 'Willingness to pay and willingness to accept: How much can they differ?' *The American Economic Review* 81(3), 635–647

Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour (2013) 'Health and (other) asset holdings.' *The Review of Economic Studies* 80(2), 663–710

— (2020) 'Closing down the shop: Optimal health and wealth dynamics near the end of life.' *Health Economics* 29(2), 138–153

— (2021a) 'Technical appendix valuing life as an asset, as a statistic and at gunpoint: Endogenous mortality and morbidity, source-dependent risk aversion.' https://people.unil.ch/pascalst-amour/files/2021/02/TechApxHPS13-1.pdf

— (2021b) 'Valuing life as an asset, as a statistic and at gunpoint.' *The Economic Journal.* (forthcoming)

Jones-Lee, Michael (1974) 'The value of changes in the probability of death or injury.' *Journal of Political Economy* 82(4), 835–849

Jones-Lee, Michael, Susan Chilton, Hugh Metcalf, and Jytte Seested Nielsen (2015) 'Valuing gains in life expectancy: Clarifying some ambiguities.' *Journal of Risk and Uncertainty* 51(1), 1 – 21

Murphy, Kevin M., and Robert H. Topel (2006) 'The value of health and longevity.' *Journal of Political Economy* 114(5), 871–904

Palacios, Miguel (2015) 'Human capital as an asset class implications from a general equilibrium model.' *The Review of Financial Studies* 28(4), 978–1023

Rosen, Sherwin (1988) 'The value of changes in life expectancy.' *Journal of Risk and Uncertainty* 1(3), 285–304

Shepard, Donald S., and Richard J. Zeckhauser (1984) 'Survival versus consumption.' *Management Science* 30(4), 423–439

Varian, Hal R. (1984) *Microeconomic Analysis,* 2nd ed. (New York and London: W. W. Norton & Company)

Viscusi, W. Kip, and Clayton Masterman (2017) 'Anchoring biases in international estimates of the value of a statistical life.' *Journal of Risk and Uncertainty* 54(2), 103 – 128

Weil, Philippe (1990) 'Non-expected utility in macroeconomics.' *Quarterly Journal of Economics* 105(1), 29–42

Weinstein, Milton C., Donald S. Shepard, and Joseph S. Pliskin (1980) 'The economic value of changing mortality probabilities: A decision-theoretic approach.' *The Quarterly Journal of Economics* 94(2), 373–396