Fighting for Lemons: The Encouragement Effect in Dynamic Competition with Private Information*

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Abstract

In a common value environment with multi-stage competition, losing a battle conveys positive news about a rival's estimation of a contested prize, capable of balancing the discouraging effect of falling behind. We show that, due to this *encouragement effect*, aggregate incentives under private information must be higher than under public information and can even exceed the benchmark associated with static competition. By challenging the common wisdom that incentives become undermined by the dynamic nature of competition, our results have implications for our understanding of R&D races, promotion tournaments, or presidential primaries.

Keywords: Dynamic contests, private information, discouragement effect, information design.

JEL: C72, D72, D82.

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1 Introduction

Contests are a frequently employed method of providing incentives, with applications ranging from promotion tournaments and sports competitions to R&D races and political campaigns.[1] There are concerns, however, that due to a contest's *dynamic* nature, incentives may become undermined by the so-called *discouragement effect*. As those contestants who have fallen behind face the extra effort required for catching up, they are discouraged from continuing their fight. This, in turn, allows contestants who are in the lead to reduce their efforts, resulting in the deterioration of incentives on aggregate.[2]

The consequences of the discouragement effect are far reaching and have been noted for a broad variety of settings. In R&D races, an initial breakthrough may mute the investment-incentives of rival firms thereby increasing the time the innovation requires for completion (Fudenberg et al., 1983; Harris and Vickers, 1987; Judd et al., 2012). In promotion tournaments, workers may slack off in response to the achievements of their co-workers, putting under scrutiny the wide-spread use of interim performance evaluations (Klein and Schmutzler, 2017) and feedback policies (Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Goltsman and Mukherjee, 2011). In presidential primaries, overall campaign spending can become reduced and early voting in non-representative districts can become decisive for the overall outcome of the election (Klumpp and Polborn, 2006). Finally, in sports competitions, performance differences accumulated during earlier stages may lead to a deterioration of suspense (Chan et al., 2009).

In this article, we argue that, besides its *direct* effect on a contestant's intermediate standing, a loss during an early stage may have an *indirect*, informational effect, with an opposing, and so far overlooked influence on incentives. Our starting point is the observa-

[1]Further examples and a comprehensive overview can be found in Konrad (2009).

[2]Evidence for the discouragement effect has been reported by Malueg and Yates (2010), Iqbal and Krumer (2019), and Mago et al. (2013) both for tennis and experimental data. Using a vast data set on prediction contests, Lemus and Marshall (2021) find that competitors become more likely to drop out of the competition when they start falling behind in a public leader-board.

tion that, in many of the aforementioned applications, contestants are privately informed about the, arguably, common value of the contest's prize and, while intermediate outcomes are observable, contestants cannot observe each others' efforts. In such situations, an early loss (win) represents good (bad) news about the contest's prize, because the likelihood of a loss (win) is increasing (decreasing) in the opponent's effort which correlates with his information. For example, in an R&D race, an early breakthrough may be the consequence of a large investment by a rival company whose market-research has revealed a profitable future for the contested innovation.[3]

In the presence of private information, it is therefore a priori unclear whether dynamic incentives are subject to a discouragement effect or whether losers of early stages are actually encouraged to exert *larger* efforts than their rivals. Moreover, the implications for aggregate incentives are no longer clear, because the discouragement effect resulting from intermediate performance evaluations might be offset or even overcome by an *encouragement effect* arising from the contestants' ability to learn about their rival's information.

To shed light on these issues, in Section 2, we propose a stylized model of dynamic competition with private information. In our model, two homogeneous contestants compete in a best-of-three contest by exerting efforts with linear costs in three sequential battles. We allow for a generic class of mappings between efforts and battle outcomes, including the frequently employed (generalized) Tullock (1980) success function as a special case. The winner of the overall contest obtains a prize whose (common) value is uncertain, either one or zero.[4] At the start of the contest, contestants receive private, independent, and identically distributed signals, either good or bad, that are informative about the contest's prize. Contestants cannot observe their rival's effort but may learn

[3]Although most of our analysis focuses on the pure common value component of a contest's prize, the possibility of heterogeneity in prize valuations is introduced in Section 7.

[4]An alternative but analogue formulation of our model considers the contest's prize as certain, but assumes that contestants face uncertainty about their (common) marginal costs of effort. For example, R&D expenditures may depend on a common input (e.g. labor) whose price can be uncertain.

from their observation of each battle's outcome.

Our model owes its tractability to the assumption that the underlying information structure is partially conclusive. In particular, we assume that, while conditional on the prize being zero both a good and a bad signal can be observed, conditional on the prize being one, only a good signal can be received. It follows that contestants will conclude from the observation of a bad signal that the contest's prize must be zero and that it is optimal to refrain from exerting effort.[5] The characterization of a Perfect Bayesian equilibrium is thus reduced to the description of the contestants' effort choices, conditional on the receipt of a good signal.

In Section 3 we consider two benchmarks: the *static benchmark*, in which contestants choose efforts for all battles simultaneously rather than sequentially; and the *public information benchmark* where signals are observed publicly rather than privately. We show that in both benchmarks, the expected sum of efforts, aggregated over all contestants and battles, is independent of the signals' *informativeness*, given by the likelihood of receiving a bad signal when the prize is zero. This is reassuring as it implies that, in our dynamic contest with private information, any dependence of aggregate incentives on the signals' informativeness must have its origin in the contestants' learning about their rival's signal. Moreover, a comparison of the two benchmarks allows us to confirm the existence of a discouragement effect and to associate the corresponding loss in aggregate incentives, with the individual battles' *rate of rent dissipation*.

We begin our characterization of Perfect Bayesian equilibrium in the dynamic contest with private information in Section 4, with a focus on the gap between the leader's and the follower's efforts in the intermediate battle. We show that, in accordance with the aforementioned intuition, this gap is reduced relative to the public information benchmark. Moreover, when the battles' rate of rent dissipation is sufficiently low, the follower's effort

[5] This information structure is called the "bad news" model in the literature on strategic experimentation (e.g. Keller and Rady, 2015; Bonatti and Hörner, 2017). As an alternative, we discuss the so called "good news" model, in our Conclusions.

may even exceed the leader's, making it *less* likely that the contest is decided after two rather than three battles. In empirical studies, contests that are decided within a few battles have been interpreted as evidence for the discouragement effect. Our finding, that in the presence of private information, discouragement is countered by encouragement means that long-lasting fights must not necessarily be an indication for the absence of the discouragement effect.[6]

Our two main results are presented in Section 5 and are concerned with the effect of private information on *aggregate* incentives. First, we show that, in the dynamic contest with private signals, expected aggregate effort is strictly larger than in the public information benchmark, *independently* of the signals' informativeness. This finding is surprising as it contrasts with well known results about common value auctions where the linkage principle implies that, expected revenue is maximized when all information is made publicly available (Milgrom and Weber, 1982). Although, in our setting, private information causes a winner's curse familiar from auction theory, the encouragement effect leads to an overall gain in aggregate incentives. Private information raises aggregate incentives as it helps to level the playing field in situations where some contestants have established a lead over others. Naturally, because efforts are costly, a direct implication of this result is that asymmetric information is harmful from the contestants' perspective, i.e. asymmetric information leads to "fighting for lemons".

Second, we show that aggregate incentives in the dynamic contest can be even higher than in the static benchmark. This happens when the encouragement effect is strong relative to the discouragement effect, which is the case when the rate of rent dissipation is low. Our result shows that the common wisdom, that incentives are reduced by the dynamic nature of competition (e.g. Klumpp and Polborn, 2006), must not hold in con-

[6] Ferrall and Smith (1999) argue that in basketball-, hockey-, and baseball-playoffs "a simple model in which players do not give up [...] best explains the outcome of the championship series." Similarly, Zizzo (2002) denotes the lack of evidence for discouragement in experimental patent race data as "a puzzle from the perspective of patent race theory."

tests that are subject to private information. Dynamic competition can improve upon static competition because the encouragement that contestants derive from learning can overcome the discouragement that arises from intermediate performance evaluations.

In Section 6 we relate our work to a nascent literature on information design in contests by characterizing the contest's optimal (partly-conclusive) information structure.[7] We show that aggregate effort is maximized when the players' signal is neither fully informative nor fully uninformative, and that the optimal signal quality is increasing in the likelihood with which the contest's prize has no value. A direct implication of this result is that improving contestants' information may deteriorate dynamic incentives, especially when contestants are "optimistic" about the contest's prospects.

Finally, in Section 7, we extend our model by allowing contestants to have heterogeneous valuations of the contest's prize. A valid concern is that the encouragement effect, although beneficial for aggregate incentives, may have a negative effect on a contest's selective efficiency. In particular, when a low-valuing contestant is more likely to be lagging behind, narrowing the gap between a leader's and a follower's effort may have an adverse effect on the likelihood with which a high-valuing contestant will claim the contest's prize. We argue that this intuition is incomplete and show that, instead, private information can have a *positive* effect not only on aggregate incentives but also on a contest's selective efficiency.

Related literature

The discouragement effect has made its first appearance in the literature on R&D races, where it can take the particularly severe form of ϵ-preemption (Fudenberg et al., 1983): Even the smallest innovation advantage can obstruct the investment of rival firms. The seminal model of Harris and Vickers (1987) takes the format of a best-of-N contest and its battle-components are strategically equivalent to a Tullock contest when investments

[7]The existing literature on information design has mostly restricted attention to static contests. For a detailed discussion of this literature see Section 6.

are lump-sum (Baye and Hoppe, 2003). Our results thus apply and they suggest that, due to the inherently uncertain value of innovation, the dynamic nature of R&D-competition must not be an obstacle but can be a promoter of investment, because firms' become encouraged by the success of their rivals. This finding resonates well with the idea of Choi (1991) that a rival's success may improve a firm's belief in the feasibility of a contested innovation (see also Malueg and Tsutsui (1997) and Bimpikis et al. (2019)). An important difference is that in our setting, information is private rather than public, which means that the observation of progress increases the investment of some firm while decreasing the investment of another.[8]

Our theory combines dynamic competition with private information and it thereby contributes to two, mostly separate branches of the literature. The first branch investigates the role of information under static competition, where a different form of discouragement may arise from potential differences in players' abilities or their individual valuations of the prize. While for private-value environments (Morath and Münster, 2008; Dubey, 2013; Wasser, 2013; Fu et al., 2014; Serena, 2021), asymmetric information is found to have a positive effect on aggregate incentives, in common-value settings, more akin to ours, private information typically has a negative or no effect (Hurley and Shogren, 1998; Wärneryd, 2003; Einy et al., 2017). Our analysis of the static competition benchmark in Section 3 shows that, in our setting, information has an influence on incentives only when competition is dynamic and, in the dynamic contest, the effect of private information turns out to be *positive*.

The second branch of the contest literature typically abstracts from informational issues and investigates incentives in dynamic settings. Konrad and Kovenock (2009) provide the seminal analysis of a best-of-N contest, with individual battles modeled as all-pay auctions, where the rate of rent-dissipation and hence the discouragement effect are ex-

[8]With private information, learning may induce homogeneous investment-behavior from heterogeneous firms (Moscarini and Squintani, 2010) and poorly informed firms may have an advantage due to the possibility of learning from a better informed rival (Awaya and Krishna, 2021).

treme. For more moderate rates of rent dissipation, the characterization of equilibrium in a best-of-N contest has proven rather elusive. Ferrall and Smith (1999) determine a mixed-strategy equilibrium when battles take the form of an additive tournament with normally distributed noise and show, numerically, that the players' likelihood to provide positive effort falls towards zero when the contest reaches an asymmetric state. For standard Tullock-battles, a characterization of equilibrium for a best-of-N contest has been obtained by Klumpp and Polborn (2006). They take the predicted discouragement as an argument in favor of the sequential format of US presidential primaries, where efforts consist of wasteful campaign spending.[9] We contribute to this literature by providing a characterization of equilibrium for generic tournaments with multiplicative noise, including the Tullock specification as a special case.

The few articles that combine dynamic competition and incomplete information belong to a growing literature about the desirability of intermediate performance feedback in labor settings (Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Goltsman and Mukherjee, 2011) or cryptocurrency mining protocols (Ely et al., 2021).[10] Informing players about the outcome of intermediate battles can induce fierce competition when the contest is close, but has a discouraging effect when large performance differences are revealed. As our static competition benchmark is strategically equivalent to a situation where players compete sequentially without knowledge of the individual battles' outcomes, our theory contributes to this literature. In particular, our results imply that, in the presence of private information about the contest's prize, intermediate performance feedback is detri-

[9]By introducing multiplicative biases into a best-of-three version of the Klumpp and Polborn (2006) model, Barbieri and Serena (2018) show that aggregate effort can be increased by favoring the loser of battle one, thereby extending the logic of leveling the playing field from a static to a dynamic setting. While we share with Barbieri and Serena (2018) the finding that, in battle 2, efforts are maximal when winning probabilities are equalized, in our setting, maximization of effort on aggregate requires the playing field to be "unleveled". Private information acts differently than a multiplicative bias because it influences the players' valuations rather than their probabilities of winning.

[10]Klein and Schmutzler (2017) provide a rationale for why competition amongst workers may take the format of a best-of-N contest akin to our model. They show that aggregate effort is maximized when no intermediate prizes are awarded and performance at every stage receives a positive weight in the determination of the overall winner.

mental when contestants are very poorly or very well informed but can improve incentives when information is of moderate quality.

Finally, on a more abstract level, our results resonate well with the general idea that, in strategic common-value settings, dynamics and private information, although detrimental when considered separately, can be beneficial when combined. For example, in a preemption game, where players aim to be the first to invest when investment is lucrative and not to invest at all when investment is wasteful, private information can be welfare improving by counteracting the players preemption motive (Hopenhayn and Squintani, 2011; Bobtcheff et al., 2021). Similarly, in a strategic experimentation setting, where players can learn from the experimentation of others, private information can mitigate the players' free-riding problem (Heidhues et al., 2015; Dong, 2016; Klein and Wagner, 2019). In our dynamic contest framework the incentive-improving role of private information derives from the fact that battle outcomes induce competitors to update their beliefs in opposite directions which helps to level the playing field.

2 Model

We consider two homogeneous, risk-neutral players engaged in a dynamic contest for a single prize of common value. The prize can take two values, $V \in \{0, 1\}$, and we denote by $\omega \in (0, 1)$ the likelihood that $V = 0$ and by $\mathbb{E}[V] = 1 - \omega$ the expected prize.[11] The contest consists of three identical, consecutive battles and the prize is awarded to the first player achieving a total number of two battle victories. In each battle $t \in \{1, 2, 3\}$, the two players $i \in \{1, 2\}$ choose their efforts $e_{it} \geq 0$ simultaneously. Player i's performance in battle t is then determined by the product of his effort e_{it} and an individual noise component $x_{it} > 0$. A player's payoff equals his prize winnings minus his effort costs aggregated over all battles, i.e. we abstract from discounting. The costs of

[11] While $V = 1$ is just a normalization, the assumption that the prize may have zero value greatly simplifies our analysis, as will become clear below.

effort are identical across players and battles and are assumed to be linear, i.e. $C(e_{it}) = e_{it}$. With linear costs, expected aggregate effort in the static competition benchmark becomes independent of the players' information (see Section 3). This enables us to focus on the effect of information on incentives that arises from the dynamic nature of competition rather than the shape of the players' cost functions. A discussion of the effects of extending the contest to more than three battles is postponed until Section 8.

Competition. Each battle is won by the player with the highest performance, i.e. player i wins battle t if and only if $e_{it}x_{it} > e_{jt}x_{jt}$ or $\frac{x_{jt}}{x_{it}} < \frac{e_{it}}{e_{jt}}$. Individual noise is distributed identically and independently across battles and players. Denoting by $H(.)$ the cumulative distribution function of the ratio of individual noise $y_t = \frac{x_{jt}}{x_{it}}$, player i's probability of winning battle t is thus given by $H(\frac{e_{it}}{e_{jt}})$. As equilibrium will be fully determined by the distribution of the ratio of individual noise, we make assumptions directly on the corresponding probability density $h = H'$.[12] By symmetry, we have $H(y) = 1 - H(\frac{1}{y})$ and differentiating both sides leads to $yh(y) = \frac{1}{y}h(\frac{1}{y})$. It follows that the function $yh(y)$ must have a minimum or a maximum at $y = 1$. We assume that $yh(y)$ is unimodal and converges to zero for $y \to 0$, which guarantees that $y = 1$ constitutes a global maximum.[13] In order to guarantee the existence of a pure-strategy equilibrium we also assume that h is differentiable and strictly decreasing. Our distributional assumptions are summarized as follows:

Assumption 1. *The density h of the ratio of individual noise is continuously differen-*

[12]Note that two different individual noise distributions, f and $\tilde{f}$, can give rise to the same ratio distribution h, even when f and $\tilde{f}$ differ in their "shape". For example, the distribution of $\frac{x_1}{x_2}$ is given by $h(\frac{x_1}{x_2}) = \frac{1}{(1+\frac{x_1}{x_2})^2}$ when x_1, x_2 are distributed according to $f(x_i) = \exp(-x_i)$ *and* when x_1, x_2 are distributed according to $\tilde{f}(x_i) = \frac{1}{x_i^2}\exp(-\frac{1}{x_i})$, although f is monotone decreasing whereas $\tilde{f}$ has a unique positive mode. It it therefore sensible to consider h as the primitive of our model and to make assumptions about the shape of h rather than the shape of f.

[13]Hodges and Lehmann (1954) show that the distribution of the difference of two unimodal noise distributions must itself be unimodal. Using this result, a straight forward logarithmic transformation shows that $yh(y)$ must be unimodal when the underlying distribution of individual noise is unimodal. Unimodality is a common assumption in models where performance is additive in effort and noise (Lazear and Rosen, 1981).

tiable and strictly decreasing. Moreover, the function $yh(y)$ is unimodal with $\lim_{y\to 0} yh(y) = 0$.

Example. A family of densities that satisfy our distributional assumptions is given by

$$h_{(d,r)}(y) = \frac{r\Gamma(2\frac{d}{r})}{\Gamma(\frac{d}{r})^2} \frac{y^{-d-1}}{(1+y^{-r})^{2\frac{d}{r}}}. \tag{1}$$

These ratio distributions arise when individual noise follows a generalized Gamma distribution $f_{(d,r)}(x) = \frac{r}{\Gamma(\frac{d}{r})} x^{d-1} \exp(-x^r)$ with parameters $d, r > 0$ (Malik, 1967). $h_{(d,r)}$ is unimodal. It satisfies Assumption 1 if and only if $d \leq 1$. Note that this family accommodates as special cases ($d = r$) the ratio distributions $h_r(y) = \frac{ry^{-r-1}}{(1+y^{-r})^2}$ generating the most frequently employed (generalized) Tullock contest success function $H_r(\frac{e_1}{e_2}) = \frac{e_1^r}{e_1^r + \tilde{e}_2^r}$ (Jia, 2008). Besides the Tullock function with parameter $r \leq 1$, which originates when individual noise follows an exponential ($d = r = 1$) or Weibull distribution ($d = r < 1$), Assumption 1 allows individual noise to be Chi-distributed ($r = 2, d < 1$), Chi-squared-distributed ($r = 1, d < 1$), or folded-normal distributed ($r = 2, d = 1$), to name just a few.

Information. Our model captures situations in which contestants have private information about the (common) value of a contested prize, and, while unable to observe their rival's efforts, may learn about their rival's information via their observation of a battle's outcome. For example, in an R&D race firms' market research may generate private information about the value of an invention, and, while information about the rivals' R&D spending is not available, firms may update their beliefs about their rival's market evaluation by observing its technological advancement. Similarly, in a labor tournament, where efforts are commonly considered as unobservable, relative performance feedback in form of a midterm review may inform employees about the value their rival attaches to a promotion. In line with these examples, we thus assume that players' efforts are unobservable and that, prior to the first battle, each player obtains a private signal, $s_i \in \{B, G\}$, that is informative about the value of V. Signals are independent draws from the same conditional probability distribution $\text{Prob}(s_i|V)$ specified by Table 1. The

$\text{Prob}(s_i\|V)$	$V = 0$	$V = 1$
$s_i = B$	σ	0
$s_i = G$	$1 - \sigma$	1

Table 1: Information structure.

parameter $\sigma \in (0, 1)$ measures the informativeness of the players' signals. In particular, for $\sigma \to 1$ players become perfectly informed about the value of the prize, whereas for $\sigma \to 0$ signals become completely uninformative. Note that implicit in this formulation is the assumption that a "bad" signal $s_i = B$ is conclusive, as it can only be received when $V = 0$. For example, workers competing for a promotion may learn that the position will be filled with an outsider. This assumption, together with the fact that, in this state of the world, the prize has *zero* value, greatly simplifies the analysis because it implies that efforts must be zero upon the observation of a bad signal. Hence, our analysis can concentrate on the players' behavior conditional on receiving a "good" signal, $s_i = G$. Our results are robust to the introduction of pre-play communication if we assume that signals are non-verifiable. With non-verifiable signals, players have an incentive to claim to have received a bad signal, independently of their true signal, making all communication uninformative.[14]

Equilibrium. Our setting constitutes a dynamic Bayesian game, with players' "types" given by their signals. We use Perfect Bayesian equilibrium as our solution concept and focus our analysis on symmetric equilibria in pure strategies. In our model, a symmetric, pure-strategy Perfect Bayesian equilibrium – in the remainder simply denoted as "an equilibrium" – can be described by a vector of efforts $(e_1^*, e_L^*, e_F^*, e_3^*)$ which players exert conditional on having observed a good signal. Here e_1^* and e_3^* denote a player's efforts during the first and the third battle, respectively, whereas e_L^* and e_F^* denote a player's effort in the intermediate battle depending on whether the player has become the leader

[14]With verifiable signals, private information would unravel, because only players with a good signal have an incentive to conceal.

(L) or follower (F) by winning or losing the previous battle. Note that a player's effort in the third battle is independent of the sequencing of past-outcomes (win-loss versus loss-win) because in equilibrium the third battle can be reached only when *both* players have observed a good signal, giving players identical beliefs about the value of the prize.[15]

3 Benchmarking

Before we start the analysis of our model it is instructive to consider as benchmarks the cases where either competition is static rather than dynamic or signals are observable publicly rather than privately.

3.1 Static competition

Suppose that, instead of sequentially, all three battles take place at the same time, once players have received their private signals. Players must choose an effort level for each battle simultaneously. By symmetry, players should distribute their efforts evenly across battles. A player will choose a non-zero effort $e^S > 0$ only when he received a good signal. As a player wins the prize when he is victorious either in two or in three battles, and the prize can have non-zero value only when also the rival's signal was good, in equilibrium e^S must solve

$$e^S \in \arg\max_{e \geq 0} \beta_1 [H(\frac{e}{e^S})^3 + 3H(\frac{e}{e^S})^2(1 - H(\frac{e}{e^S}))]V^G - 3e. \tag{2}$$

Here we have denoted by

$$\beta_1 \equiv \text{Prob}(s_j = G | s_i = G) = \frac{1 - \omega + \omega(1 - \sigma)^2}{1 - \omega + \omega(1 - \sigma)}, \tag{3}$$

[15] In our model, Bayesian updating is greatly simplified by the fact that a player with a zero effort cannot win against a player with a positive effort. While this property constitutes an implicit assumption of the frequently employed Tullock model and seems a realistic feature of many settings (e.g. innovation requiring investment), it distinguishes our framework from those models where effort and noise are substitutes rather than complements (e.g. Lazear and Rosen, 1981).

the player's belief about the likelihood with which his rival has also received a good signal and by

$$V^G \equiv \mathbb{E}[V|s_1 = s_2 = G] = \frac{1-\omega}{1-\omega+\omega(1-\sigma)^2} > \mathbb{E}[V] \tag{4}$$

the expected value of the contest's prize, conditional on both players' signals being good. Taking the first order condition of (2) and setting $e = e^S$ gives $e^S = \frac{1}{2}\beta_1 V^G h(1)$ as the unique candidate for a pure-strategy equilibrium.[16] Summing efforts over both players and all battles, and multiplying with the probability that $s_i = G$ gives the corresponding expected aggregate effort:

$$E^S = \min\{3h(1), 1\} \cdot \mathbb{E}[V]. \tag{5}$$

Note that this expression is independent of the quality σ of players' information. Also note that if signals were observed publicly rather than privately, then, conditional on $s_1 = s_2 = G$ both players would exert efforts $e = \frac{1}{2}V^G h(1)$ and it follows from the fact that $\text{Prob}(s_1 = s_2 = G) = \text{Prob}(s_i = G)\beta_1$ that expected aggregate effort would be the same as in (5). We summarize these findings in the following:

Lemma 1 (Benchmark: Static competition)**.** *When the contest is static rather than dynamic, aggregate incentives are independent of the contest's information structure. In particular, expected aggregate effort is independent of σ and given by (5), no matter whether signals are observed privately or publicly.*

On the one hand, when signals are private, a player is more likely to exert positive effort, because effort requires only his own signal to be good. On the other hand, the observation of only one good signal leaves the player less optimistic about the prize's value, making players choose a lower effort level than when signals are public. When

[16]Note that the corresponding equilibrium payoff of each player is $\frac{1}{2}\beta_1 V^G[1-3h(1)]$, i.e. existence of a pure-strategy equilibrium in the static contest requires $h(1) < \frac{1}{3}$. For larger values of $h(1)$, equilibrium must be in mixed strategies and, in analogy to Klumpp and Polborn (2006), equilibrium features full rent-dissipation, i.e. expected aggregate effort must equal the expected prize $\mathbb{E}[V] = 1-\omega$.

cost functions are linear, both effects cancel out. Lemma 1 is therefore a consequence of players having linear costs of effort and the reason why we assumed cost functions to be linear in the first place. It implies that, in our setting, any dependence of aggregate incentives on information must have its origin, not in the shape of the effort cost function, but in the dynamic nature of competition.

3.2 Public signals

We now return to the dynamic model but suppose that, instead of being observed privately, both players' signals are publicly available.[17] If $s_i = B$ for some $i \in \{1, 2\}$, both players will exert zero effort because the contest's prize is commonly known to have zero value. If $s_1 = s_2 = G$, players' expectations of the contest's prize are given by V^G . In the Appendix, we determine the unique pure-strategy Subgame Perfect equilibrium $(e_1^P, e_L^P, e_F^P, e_3^P)$, describing players' effort levels conditional on $s_1 = s_2 = G$. Letting U_3 denote a player's continuation value of reaching battle 3, our characterization makes use of the variable

$$\rho = \frac{U_3}{V^G - U_3} = \frac{1 - 2h(1)}{1 + 2h(1)} \in (0, 1) \tag{6}$$

representing the ratio of players' valuations of winning battle 2. With the help of this variable, the equilibrium can be expressed in closed form and we can formulate the following result:

Lemma 2 (Benchmark: Public information)**.** *When signals are public rather than private, players exert effort only upon observation of two good signals and there exists a unique pure-strategy subgame perfect equilibrium* $(e_1^P, e_L^P, e_F^P, e_3^P)$ *where* $e_1^P = [H(\frac{1}{\rho}) - 2h(1)\rho h(\rho)]h(1)V^G$, $e_F^P = \frac{1}{1+\rho}h(\frac{1}{\rho})V^G$, $e_L^P = \frac{1}{\rho}e_F^P$, *and* $e_3^P = h(1)V^G$. *Expected aggregate effort is* $E^P = \{2h(1) + \rho h(\rho)[1 - 4h(1)^2]\}\mathbb{E}[V]$ *and thus independent of the informativeness,* σ, *of players' signals.*

[17]An alternative benchmark has players observe only one and the same signal. Whether there are one or two signals available has no influence on expected aggregate effort.

Proof: See Appendix.

A comparison of expected aggregate effort E^P with the static equivalent E^S serves as a measure of the severity of the discouragement effect. In the dynamic setting, the follower becomes discouraged by the leader's advantage in battle 2, leading to a reduction of aggregate incentives. Indeed, for $h(1) < \frac{1}{3}$, the relative loss in incentives due to discouragement is

$$\Delta E^{dis} = \frac{E^S - E^P}{E^S} = \frac{1}{3} - \rho h(\rho)[\frac{1}{3}h(1)^{-1} - \frac{4}{3}h(1)] > \frac{4}{3}h(1)^2, \tag{7}$$

where we have used the fact that $\rho h(\rho) < h(1)$ by unimodality. Note that the lower bound on the loss in incentives is increasing in $h(1)$ and thus proportional to the rate of rent dissipation of the final battle given by

$$R \equiv 1 - \frac{2U_3}{V^G} = 2h(1). \tag{8}$$

The larger the battles' rate of rent dissipation, the greater the loss in aggregate incentives due to the discouragement effect. This is intuitive, because when rent dissipation is high, the additional cost of having to win an extra battle weighs heavily. Figure 1 depicts ΔE^{dis} for the distribution h_r generating Tullock's contest success function where $R = \frac{r}{2}$. In the example, the incentive loss due to discouragement ranges up to 14 percent and is monotonically increasing in Tullock's parameter r, measuring the sensitivity with which battle outcomes depend on players' efforts.

4 The encouragement effect

In this section we begin our analysis of the dynamic contest with private signals. As before, our information structure allows us to focus on the efforts that a player exerts after having observed a good signal. After receiving a bad signal, a player knows the contest's prize to have no value, making zero effort his optimal choice. However, in comparison to the benchmarks analyzed in the previous section, the analysis is complicated by the fact that

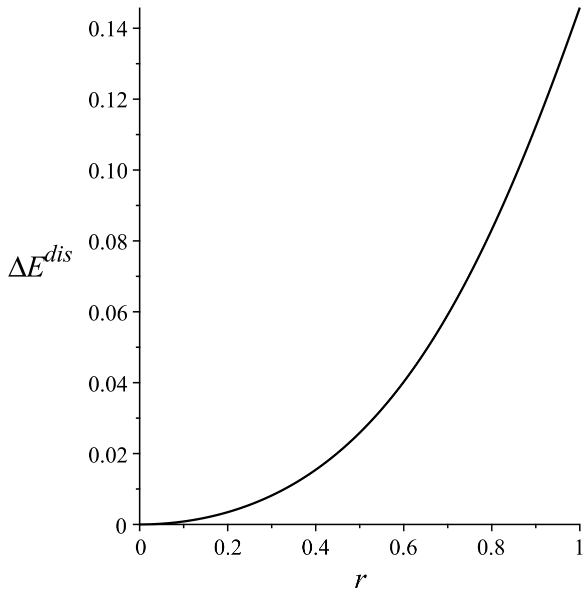

Figure 1: **Discouragement Effect**: Relative loss in aggregate incentives, ΔE^{dis}, as a function of Tullock's parameter $r \in (0, 1]$. In the example, the ratio of noise distribution is given by $h_r(y) = \frac{ry^{-r-1}}{(1+y^{-r})^2}$.

players must update their beliefs about their rival's information based on their observation of the contest's history.

Consider a player $i \in \{1, 2\}$ with signal $s_i = G$ in battle 3. Note that battle 3 can be reached only when each player has lost one battle and that a player exerting positive effort may loose a battle only when his rival also exerted positive effort. This means that when reaching battle 3 after having exerted effort, player i can be certain that $s_j = G$. Hence, whether signals are observed privately or publicly makes no differences for battle 3, i.e. effort must be the same as in the Subgame Perfect equilibrium characterized by Lemma 2:

$$e_3^* = h(1)V^G. \tag{9}$$

Moreover, a player's continuation value from reaching battle 3, is thus as determined in the proof of Lemma 2, i.e. $U_3 = [\frac{1}{2} - h(1)]V^G > 0$. In battle 3, the players' beliefs are identical because the contest's history is symmetric, in that it features one battle win for each player. In contrast, as we will see next, updating differs across players in battle 2, because there exists a leader and a follower.

Assuming that the follower exerted effort in battle 1, in battle 2 he must conclude from having lost the previous battle that his opponent has observed a good signal. Had his opponent observed a bad signal he would have exerted zero effort and would not have defeated him. In contrast to the follower, the leader does not know whether he won the first battle because he was lucky or because his opponent failed to provide effort after observation of a bad signal. Moreover, the distinction between these two cases depends on the effort the leader has taken in battle 1. In particular, suppose the leader chose effort $e_1 > 0$ and the follower employed the equilibrium strategy of exerting effort e_1^* upon observation of a good signal and zero effort after observation of a bad signal. Then the leader would have won the first battle with probability $H(\frac{e_1}{e_1^*})$ in the case where $s_j = s_i = G$, which occurs with likelihood $1-\omega+\omega(1-\sigma)^2$. In contrast, the leader would have won with certainty in the case where $s_j = B \neq G = s_i$, which occurs with likelihood $\omega\sigma(1-\sigma)$. Bayesian updating thus implies that from the viewpoint of the leader, the likelihood with which the follower has observed a good signal is given by

$$\mathrm{Prob}(s_j = G | i = L, s_i = G) = \frac{[1-\omega+\omega(1-\sigma)^2]H(\frac{e_1}{e_1^*})}{[1-\omega+\omega(1-\sigma)^2]H(\frac{e_1}{e_1^*})+\omega\sigma(1-\sigma)} \equiv \beta_2(e_1). \quad (10)$$

It is important to note that $\beta_2(e_1) < \beta_1$, i.e. winning the first battle represents "bad news" about the rival's signal.[18] In particular, in battle 2, the follower updates his belief about his rival's signal upwards to $1 > \beta_1$, whereas the leader updates his belief downwards to $\beta_2(e_1) < \beta_1$. In equilibrium, effort choices (e_L^*, e_F^*) must satisfy:

$$e_L^* \in \arg\max_{e_L \geq 0} \beta_2^* \left[U_3 + H(\frac{e_L}{e_F^*})(V^G - U_3)\right] - e_L \quad (11)$$

$$e_F^* \in \arg\max_{e_F \geq 0} H(\frac{e_F}{e_L^*})U_3 - e_F, \quad (12)$$

[18]The fact that a deviation from e_1^* to $e_1 \neq e_1^*$ influences the informativeness of the first battle's outcome must be taken into account in the determination of the equilibrium effort level e_1^* in Section 5.

where we have abbreviated notation by letting

$$\beta_2^* \equiv \beta_2(e_1^*) = \frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)^2+2\omega\sigma(1-\sigma)}. \tag{13}$$

By Assumption 1, the above objectives are concave and the corresponding first order conditions lead to the equilibrium values

$$\begin{aligned} e_F^* &= \frac{1}{1+\rho}\beta_2^* h(\frac{\beta_2^*}{\rho})V^G & (14)\\ e_L^* &= \frac{\beta_2^*}{\rho}e_F^*. & (15)\end{aligned}$$

An important feature of the equilibrium is that the ratio of the leader's and the follower's efforts takes the following simple form:

$$\frac{e_L^*}{e_F^*} = \frac{\beta_2^*}{\rho}. \tag{16}$$

This means that information, in the form of the players' signal quality, can be employed to fine-tune the ratio of efforts and hence the winning probabilities in battle 2. In order to formulate our first result, we define the following threshold:

$$\hat{R}(\omega) \equiv \frac{2-\omega-2\sqrt{1-\omega}}{\omega}. \tag{17}$$

Proposition 1 (Encouragement Effect.)**.** *Private information increases the probability that the follower catches up with the leader. In particular, $\frac{e_L^*}{e_F^*}$ has U-shape with a minimum at*

$$\sigma = \hat{\sigma}(\omega) \equiv \frac{1-\sqrt{1-\omega}}{\omega} \in (0,1) \tag{18}$$

and $\lim_{\sigma\to 0}\frac{e_L^*}{e_F^*} = \lim_{\sigma\to 1}\frac{e_L^*}{e_F^*} = \frac{e_L^P}{e_F^P} = \frac{1}{\rho} > 1$. Moreover, if the rate of rent dissipation is not too high, private information can make the follower more likely to win the second battle than the leader. Formally, if $R < \hat{R}(\omega)$ then $\frac{e_L^*}{e_F^*} < 1$ for all $\sigma \in (\sigma_-, \sigma_+)$ where $0 < \sigma_- < \hat{\sigma}(\omega) < \sigma_+ < 1$.

Proof: See Appendix.

The intuition for Proposition 1 is as follows. As argued above, the presence of private information induces the leader and the follower to update their beliefs about their rival's signal in opposite directions. Since loosing the first battle represents positive news whereas winning the first battle represents negative news about the rival's signal, the follower becomes more confident than the leader that the contest's prize is of positive value. This divergence of the players' believes counteracts the fact that the leader is required to win only one more battle whereas the follower is required to win twice. Private information thus mitigates the discouragement effect, and, as the second part of the proposition shows, can even *encourage* the follower to provide a larger effort than the leader.

5 Aggregate incentives

Our results in the previous section suggest that, in a dynamic contest, private information may have a *positive* effect on incentives. By raising the probability that the final battle is reached, and by balancing players' valuations of winning the intermediate battle, private information increases expected efforts in battles 2 and 3. However, to fully understand how aggregate incentives vary with the players' information, one has to consider how these changes affect the players' incentives to exert effort in battle 1. In the following, we first complete our characterization of equilibrium by determining the players' effort choice e_1^*, before comparing aggregate incentives with the benchmarks of public information and static competition.

In battle 1, a player who observed a good signal believes that his rival observed a good signal with probability β_1. With probability β_1 the rival will thus exert the equilibrium effort e_1^* whereas with probability $1-\beta_1$ the rival's effort will be zero. Denoting the continuation values of the leader and the follower, *conditional on the rival's signal* $s \in \{G, B\}$, by U_L^s and U_F^s, respectively, the players' equilibrium effort in battle 1 must therefore satisfy:

$$e_1^* \in \arg\max_{e_1>0} \beta_1 \left\{ H(\frac{e_1}{e_1^*})U_L^G(e_1) + [1 - H(\frac{e_1}{e_1^*})]U_F^G \right\} + (1-\beta_1)U_L^B(e_1) - e_1. \tag{19}$$

Here we have used the fact that, conditional on his opponent having observed a bad signal, a player exerting a strictly positive effort must establish himself as the leader with certainty.[19] Moreover, it is important to note that the continuation values of becoming the leader, depend on the player's effort choice e_1 through its influence on the player's belief $\beta_2(e_1)$ in battle 2. A variation in e_1 changes the belief the player must hold about the rival's signal after winning battle 1 and will thus lead him to adjust his effort e_L in battle 2 optimally. In the proof of Lemma 3 we can thus employ the envelope theorem to show that the first-order condition that corresponds to (19) takes the following simple form:

$$e_1^* = \beta_1 h(1)[U_L^G - U_F^G]. \tag{20}$$

Comparing e_1^* with its public information analog $e_1^P = h(1)[U_L^G - U_F^G]$ we see that private information has two effects. It reduces battle 1 efforts by a factor $\beta_1 \in (0,1)$ because a player is more likely to win the contest when his rival's signal is bad and the contest's prize has no value. However, in our setting, this so-called winner's curse has no effect on expected aggregate effort because with private signals a player exerts e_1^* when his own signal is good whereas with public signals a player exerts e_1^P only when *both* signals are good and it holds that $\text{Prob}(s_1 = s_2 = G) = \beta_1 \text{Prob}(s_i = G)$.

The second effect of private information is to influence the players' continuation values U_L^G and U_F^G.[20] More specifically, first period incentives depend on the players' updated beliefs β_2^* via the difference in continuation values

$$U_L^G - U_F^G = H(\frac{e_L^*}{e_F^*})V^G - e_L^* + e_F^* = V^G \left[H(\frac{\beta_2^*}{\rho}) - [\frac{\beta_2^*}{\rho} - 1]\frac{1}{1+\rho}\beta_2^* h(\frac{\beta_2^*}{\rho}) \right]. \tag{21}$$

[19]The possibility of a deviation to $e_1 = 0$ must be checked separately, because in that case a player will loose against a rival with a bad signal with probability $\frac{1}{2}$. See the proof of Lemma 3 for details.

[20]Note that in equilibrium, the continuation value U_L^G does not depend on e_1^* because for $e_1 = e_1^*$, $\beta_2(e_1)$ becomes equal to β_2^* given by (13) which is independent of e_1^*.

Battle 1 incentives derive from the fact that an early success leads to the opportunity to secure overall victory already in the intermediate battle, which happens with probability $H(\frac{e_L^*}{e_F^*})$ and comes at the expense of the effort differential $e_L^* - e_F^*$. Before we analyze the effect of information on incentives in battle 1 and on aggregate in detail, the following lemma summarizes our analysis and proves existence of equilibrium:

Lemma 3 (Equilibrium existence)**.** *In a symmetric, pure-strategy Perfect Bayesian equilibrium, players exert effort only when their private signal is good and effort levels are $(e_1^*, e_L^*, e_F^*, e_3^*)$ given by (9), (14), (15), and (20). For the ratio distribution h_r generating the Tullock contest success function, such an equilibrium exists and it is unique. For general distributions satisfying Assumption 1, existence is guaranteed when players are sufficiently informed or uninformed, i.e. when σ is sufficiently close to* 1 *or* 0.

Proof: See Appendix.

5.1 Comparison with public information benchmark

To understand the effect of private information on aggregate incentives, it is instructive to aggregate efforts over battles 1 and 3, before adding efforts in the intermediate battle. The expected sum of players' efforts in battle 1 is $E_1^* = 2\text{Prob}(s_1 = G)e_1^*$. As battle 3 is reached only when both players observe a good signal and when the follower wins the second battle, the expected sum of efforts in battle 3 is given by $E_3^* = 2\text{Prob}(s_1 = s_2 = G)[1 - H(\frac{e_L^*}{e_F^*})]e_3^*$. Substituting efforts and using the fact that the contest's expected prize is $\mathbb{E}[V] = \text{Prob}(s_1 = s_2 = G)V^G = 1 - \omega$ and the final battle's rate of rent dissipation is $R = 2h(1)$, the expected sum of efforts in battles 1 and 3 can be expressed as follows:

$$E_1^* + E_3^* = R \cdot \mathbb{E}[V] \left\{1 - \frac{1}{V^G}(e_L^* - e_F^*)\right\}. \tag{22}$$

Note that effort aggregated over battles 1 and 3 is entirely determined by the difference between the leader's and the follower's effort in the intermediate battle. Intuitively, a change in battle 2 efforts affects the likelihood that the last battle is reached $[1 - H(\frac{e_L^*}{e_F^*})]$ by the same absolute amount as it influences the likelihood $H(\frac{e_L^*}{e_F^*})$ that securing leadership in battle 1 results in overall victory already in battle 2. In particular, any potential gain in aggregate effort that is due to a higher likelihood of the final battle being reached is exactly compensated by a loss in incentives due to a reduction of the benefits of becoming the contest's leader. $E_1^* + E_3^*$ thus consists of a constant term and the contribution of the effort differential $e_L^* - e_F^*$ to battle 1 incentives. A decrease in $e_L^* - e_F^*$ increases battle 1 incentives because it makes it less costly, in terms of future effort, to become the leader.

It remains to consider the expected sum of players' efforts in battle 2. If $s_1 = s_2 = G$ then both leader and follower exert effort in battle 2, whereas for $s_1 = G, s_2 = B$ or $s_1 = B, s_2 = G$ only the leader exerts effort. From

$$E_2^* = (1-\omega)\frac{1+\rho}{\rho}\frac{e_F^*}{V^G} = \mathbb{E}[V]\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}) \tag{23}$$

and the fact that $yh(y)$ is unimodal with mode at $y = 1$ it follows that battle 2 incentives are maximal when $e_L^* = e_F^*$, i.e. when "the playing field is leveled". This result sounds familiar from the literature on static contests. However, in our dynamic setting, leveling the playing field in battle 2 has the additional effect of reducing the effort cost differential $e_L^* - e_F^*$ which, as we noted earlier, has a positive effect on the sum of efforts in battle 1 and 3.

Letting $E^*(\sigma) = E_1^* + E_2^* + E_3^*$ denote the expected sum of efforts, aggregated over all players and battles, the above analysis leads to the following:

Proposition 2 (Private vs. Public Information)**.** *In the dynamic contest with private signals, aggregate incentives are higher than in the public information benchmark. In particular, if $R \geq \hat{R}(\omega)$ then $E^*(\sigma) > E^P$ for all $\sigma \in (0,1)$, while if $R < \hat{R}(\omega)$ then $E^*(\sigma) > E^P$ for all $\sigma \in (0, \sigma_-) \cup (\sigma_+, 1)$.*

Proof: See Appendix.

When the rate of rent dissipation is high, private information cannot level the playing field in battle 2. In this case, the advantage of requiring only one rather than two battle wins for overall victory is too large to be offset by the bad news about the contest's prize associated with a win in battle 1. Private information can then reduce the difference between e_L^* and e_F^* but will never induce the follower to exert as much effort as the leader. Because private information can only make the playing field more leveled, aggregate effort is larger than in the public information benchmark, independently of the signals' precision.

In contrast, if the rate of rent dissipation is low, then the encouragement effect can overcome the discouragement effect and private signals can make the follower exert a higher effort than the leader. In this case, information can happen to be "too asymmetric" and only sufficiently informative or sufficiently uninformative signals are guaranteed to increase aggregate incentives above the public information benchmark.

5.2 Comparison with static competition benchmark

Based on the insight of Proposition 2 that private information can be employed to increase aggregate incentives, a natural question to ask if whether the corresponding gain in incentives due to encouragement can be sufficient to overcome the discouragement effect that is commonly associated with the dynamic nature of competition. More specifically, we now investigate whether it is possible to choose the signals' precision σ in a way that increases aggregate effort in the dynamic competition with private signals, $E^*(\sigma)$, beyond the benchmark provided by static competition, E^S. For this purpose, Figure 2 depicts the ratio $\frac{E^*(\sigma)}{E^S}$, together with the leader's and the follower's efforts for a Tullock contest with parameter $r = 2R = \frac{1}{4}$. Note that $\frac{E^*(\sigma)}{E^S}$ reaches its maximum at a σ^* for which the encouragement effect outweighs the discouragement effect in that the follower exerts a *larger* effort than the leader, i.e. $e_F^*(\sigma^*) > e_L^*(\sigma^*)$. Also note that, in the example, at its maximized value, expected aggregate effort is *higher* than in the static benchmark, i.e. $\frac{E^*(\sigma)}{E^S} > 1$. This contrasts with the common wisdom that the dynamic nature of compe-

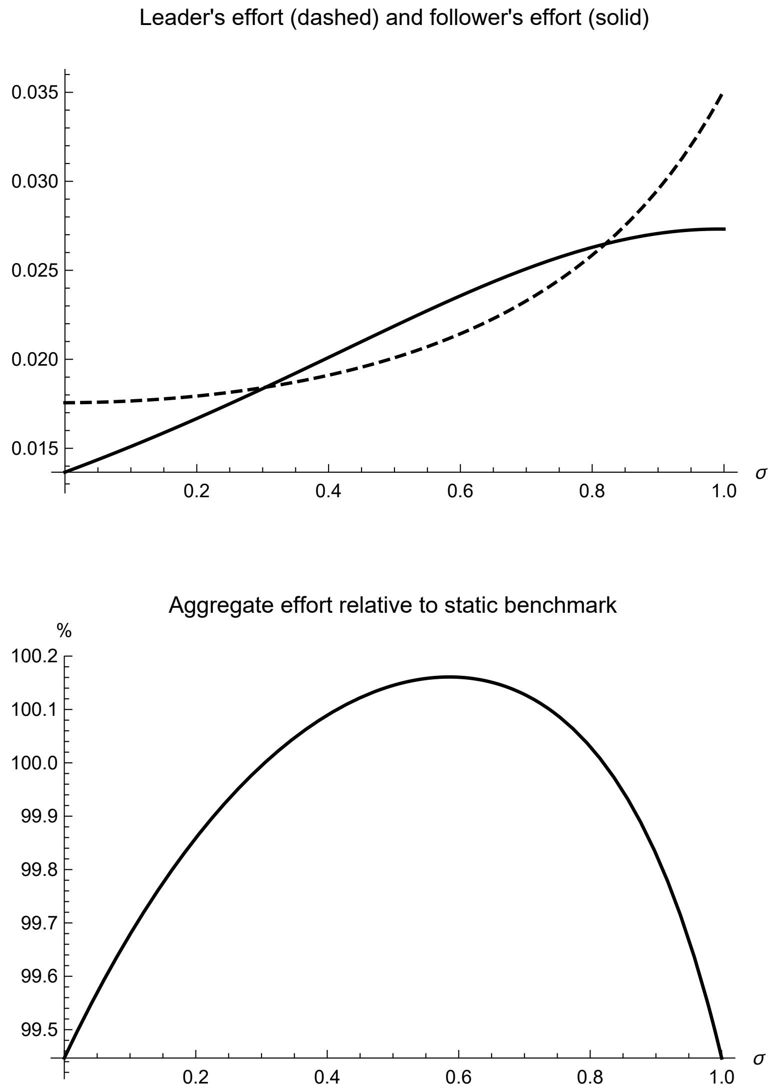

Figure 2: **Dynamic Incentives with Private Signals**: Efforts $e^*_L(\sigma)$ and $e^*_F(\sigma)$ in battle 2 (upper panel), and expected effort aggregated over all battles, relative to the static benchmark $E^*(\sigma)/E^S$ (lower panel), in dependence of the signals' informativeness σ. In the example, the ratio of noise distribution is given by $h_r(y) = \frac{ry^{-r-1}}{(1+y^{-r})^2}$, with the Tullock parameter set to $r = 0.25$, and the prior is assumed to be symmetric, i.e. $\omega = \frac{1}{2}$.

tition must be harmful for incentives (e.g. Klumpp and Polborn, 2006). Our benchmark analysis in Section 3 has confirmed that this intuition applies to our setting when players are symmetrically informed about the contest's prize. However, as shown by the example, dynamics must not necessarily be harmful for incentives when players are endowed with private information. When signals are private rather than public, the discouragement effect can be overcome by the encouragement effect, leading not only to an increase in the incentives of a follower beyond the incentives of a leader but to an increase in *aggregate* incentives beyond the static benchmark. In the Tullock example, this happens for all those values of σ for which the follower's effort exceeds the leader's. For general distributions, our assumptions do not guarantee that dynamic incentives exceed the static benchmark for *all* such σ. However, the following result confirms that, whenever the follower can be induced to exert higher effort than the leader for *some* σ, then at its maximized value, aggregate effort must be higher than in the static benchmark.

Proposition 3 (Dynamic vs. Static Competition)**.** *If the rate of rent dissipation is not too high, then the players' signal quality can be chosen such that aggregate incentives are strictly higher in the dynamic contest with private signals than in the static benchmark. Formally, if $R < \hat{R}(\omega)$, then there exist a $\sigma^* \in (0, 1)$ such that $E^*(\sigma^*) > E^S$.*

Proof: See Appendix.

To understand this result, remember that for low rates of rent dissipation, the discouragement effect can be compensated by the encouragement effect, and the follower can be induced to exert the same effort as the leader. If σ is chosen such that $\frac{e_L^*}{e_F^*} = 1$, then the contest is equally likely to be decided after two or three battles. Hence, winning battle 1 has no effect beyond the resulting increase in the winner's score. Incentives at every stage of the dynamic contest thus become equal to static incentives, leading to $E^*(\sigma) = E^S$. Decreasing $\frac{e_L^*}{e_F^*}$ marginally below 1 has a zero first-order effect on E_2^* (because E_2^* is maximized at $\frac{e_L^*}{e_F^*} = 1$) but a positive first-order effect on $E_1^* + E_3^*$, making aggregate incentives in the dynamic contest strictly larger than in the static benchmark. At an intuitive level, the static contest can be improved upon, because it forces players to invest in all three battles even when two battles turn out to be decisive.

6 Information design

In this section, we characterize the signal quality that maximizes aggregate incentives in dependence of the contest's rate of rent dissipation, R, and the contestants' prior, ω. While our results contribute to the nascent but growing literature on information design in contests (discussed below), it is important to note that our assumption of partly conclusive signals poses a restriction on the set of posteriors that can be induced.[21] In particular, besides the usual requirement of Bayes plausibility, posteriors must satisfy $Pr(V = 1|s_i = B) = 0$.[22] In our setting an "information structure" is therefore fully determined by the posterior $Pr(V = 1|s_i = G) = \frac{1-\omega}{1-\omega+\omega(1-\sigma)}$, parametrized by the signal quality σ, and our following result characterizes the information structure that maximizes aggregate incentives within the set of all such partially conclusive information structures:

Proposition 4 (Information design)**.** *In the dynamic contest with private signals, the signal quality σ^* that maximizes aggregate incentives, $E^*(\sigma)$, depends on the contest's rate of rent dissipation, $R = 2h(1)$ and the contestants' prior $\omega = Prob(V = 0)$ as follows:*

- *If $R \geq \hat{R}(\omega)$ then $E^*(\sigma)$ has inverted U-shape. The optimal signal is $\sigma^* = \hat{\sigma}$ as defined in (18) and it induces more effort from the leader than from the follower, i.e. $e^*_L > e^*_F$. More pessimistic priors require more accurate information, i.e. $\hat{\sigma}(\omega)$ is strictly increasing with $\lim_{\omega\to 0} \hat{\sigma}(\omega) = \frac{1}{2}$ and $\lim_{\omega\to 1} \hat{\sigma}(\omega) = 1$.*

- *If $R < \hat{R}(\omega)$ then $E^*(\sigma)$ is strictly increasing in $(0, \sigma_-]$ and strictly decreasing in $[\sigma_+, 1)$. The optimal signal $\sigma^* \in (\sigma_-, \sigma_+)$ induces more effort from the follower than from the leader, i.e. $e^*_F > e^*_L$.*

[21] An alternative simplification of the information design problem can be achieved by assuming information to be verifiable which allows to focus on the choice between disclosure and concealment (e.g. Serena, 2021).

[22] In the seminal contribution of Kamenica and Gentzkow (2011) and related articles, the optimal information structure turns out to be partially conclusive, i.e. the restriction to such structures might be less restrictive than it appears.

The threshold $\hat{R}(\omega)$ *is strictly increasing with* $\lim_{\omega\to 0}\hat{R}(\omega)=0$ *and* $\lim_{\omega\to 1}\hat{R}(\omega)=1$*.*

Proof: See Appendix.

Proposition 4 shows that, independently of the contest's rate of rent dissipation and the contestants' prior, dynamic incentives are maximized when private information is neither perfectly informative nor perfectly uninformative.

This contrasts with the findings of Zhang and Zhou (2016) who consider a static two-player Tullock contest with linear costs in which one player's valuation of the contest's prize is common knowledge whereas the other player's valuation constitutes the player's private information. Using a Bayesian persuasion approach, they show that when, as in our setup, valuations are binary, the information structure that maximizes expected aggregate efforts is either full or no disclosure. They also show that partially revealing signals can become optimal when valuations can take more than two values. In our dynamic setting, partially informative signals are optimal even for binary valuations, because the encouragement effect can mitigate the discouragement effect only when information is asymmetric, which is ruled out when signals are perfectly informative or perfectly uninformative.

Partially informative signals are also optimal in Antsygina and Teteryatnikova (2021) who consider a two-player static all-pay auction with linear costs where both players' valuations are binary and ex ante uncertain. They allow for information technologies that send messages to players privately or publicly. They show that the optimal information structure features private signals and induces symmetric beliefs. This structure reveals the state whenever both players valuations are identical but employs noisy and correlated signals when valuations differ. Intuitively, the designer tries to make players believe that their valuations are likely to be equal, because effort is largest when valuations are identical. As in our setting, information is thus used to "level the playing field", but the incentive-deteriorating heterogeneity emerges from exogenous differences in prize valuations rather than endogenous differences in intermediate performance. Other studies reporting the optimality of private and partially informative signals include Chen (2021), Kuang et al. (2019), and Melo-Ponce (2020) but all of them focus on static settings.

Finally, for high rates of rent dissipation, Proposition 4 identifies an inverse relationship between the contestants' prior expectations of the contested prize and the incentive-maximizing accuracy of their private information. In fact, when the contest's prize is zero with near certainty, the optimal partially conclusive signal converges to the signal that is perfectly revealing. The reason for this result is that for high rates of rent dissipation, private information can improve the competitive balance between the leader and the follower but cannot restore it completely. This means that the bad news of a win should be made as bad a news as possible. To see that this requires the players' signals to be perfectly revealing in the limit, note that, for $\omega \to 1$, the leader's updated belief converges to $\beta_2^* \to \frac{\frac{1}{2}(1-\sigma)^2}{\frac{1}{2}(1-\sigma)^2+\sigma(1-\sigma)} = \frac{1-\sigma}{1+\sigma}$. The nominator represents the probability with which, player i receives a good signal and wins the first battle, conditional on the other player j having obtained a good signal $s_j = G$ whereas the denominator represents the unconditional probability of the same event. The leader's belief that the follower has obtained a good signal and hence that the contest's prize may have some value, in spite of a zero value being so likely, is minimized when the signal's quality is chosen as high as possible.

7 Selective efficiency

In this section, we extend our analysis to allow for possible differences in the contestants' prize valuations. This enables us to consider the effect of private information on the contest's selective efficiency, i.e. the probability with which the contest's prize is allocated to the highest valuing contestant. Selective efficiency is a valid concern in light of our result that, under dynamic competition, private information encourages contestants who lag behind (Proposition 1). As a low-valuation contestant is more likely to be lagging behind than a high-valuation contestant, one may expect that private information has an adverse effect on selective efficiency. However, as we show in this section, this intuition is incomplete, as it neglects the fact that the intensity with which private information improves a contestant's chances also depends on the contestant's valuation. More specifically, we identify conditions under which private information can have a positive effect on selective efficiency and thus conclude that the gain in aggregate incentives from private information identified in the previous section (Proposition 2) must not necessarily come at the cost of a reduction in selective efficiency.

Selective efficiency is especially relevant in promotional contests where, besides the provision of incentives, the selection of the most "able" candidate constitutes an important objective. In such settings, "ability" is commonly interpreted as the inverse of a contestant's marginal cost of effort. In this section, we thus introduce heterogeneity by allowing contestants to differ in their constant marginal costs of effort but it should be noted, that, because prizes and costs enter linearly into our model, our approach is equivalent to allowing for differences in the contestants' valuation of the contest's prize.

We thus extend our model by assuming that costs of effort are $C^i(e_t^i) = c^i e_t^i$ and that one contestant has a lower marginal cost than the other, i.e. we let $\frac{c^h}{c^l} \equiv \gamma > 1$. A super-index will be used throughout the analysis to denote the contestants' cost-types. To keep our model tractable, we assume that contestants observe whether they are the low-cost contestant l or the high-cost contestant h only *after* they have competed once by exerting effort in the first battle.[23] In some applications, such as promotion tournaments, where workers are ignorant of their abilities relative to their rivals, this assumption may be a reasonable starting point. In other settings, where abilities are known right from the start, our subsequent results remain valid when ability differences are sufficiently small.

[23]When contestants are heterogeneous in battle 1 they will exert differing efforts, so that equilibrium beliefs in battle 2 will depend on past efforts, which means that the model can no longer be solved recursively.

Selective efficiency, i.e. the probability that the low-cost (high-ability) contestant wins the contest is given by

$$S \equiv \frac{1}{2} \cdot [H(\frac{e_L^l}{e_F^h}) + H(\frac{e_F^h}{e_L^l})H(\frac{e_3^l}{e_3^h})] + \frac{1}{2} \cdot H(\frac{e_F^l}{e_L^h})H(\frac{e_3^l}{e_3^h}). \tag{24}$$

The two terms represent the cases where the low-cost type has won or lost the first battle, respectively. Both cases are equally likely because, given our assumptions, contestants will exert identical efforts in the first battle.

Efforts and expected payoffs in battle three are straightforward to calculate and given by

$$e_3^l = \gamma \cdot \frac{V^G}{c^l} h(\gamma) > \frac{V^G}{c^l} h(\gamma) = e_3^h \tag{25}$$

$$U_3^l = [H(\gamma) - \gamma h(\gamma)]V^G > [H(\frac{1}{\gamma}) - \gamma h(\gamma)]V^G = U_3^h. \tag{26}$$

In the second battle, we have to distinguish between two cases. If the low-cost contestant has become the leader, equilibrium efforts must solve

$$e_L^l \in \arg\max_e U_3^l + \beta_2^*(V^G - U_3^l)H(\frac{e}{e_F^h}) - c^l e \tag{27}$$

$$e_F^h \in \arg\max_e U_3^h H(\frac{e}{e_L^l}) - c^h e \tag{28}$$

and it follows that

$$\frac{e_L^l}{e_F^h} = \gamma \beta_2^* \frac{V^G - U_3^l}{U_3^h}. \tag{29}$$

If, instead, the high-cost contestant has become the leader, equilibrium efforts solve

$$e_L^h \in \arg\max_e U_3^h + \beta_2^*(V^G - U_3^h)H(\frac{e}{e_F^l}) - c^h e \tag{30}$$

$$e_F^l \in \arg\max_e U_3^l H(\frac{e}{e_L^h}) - c^l e, \tag{31}$$

and we get

$$\frac{e_F^l}{e_L^h} = \gamma \frac{1}{\beta_2^*} \frac{U_3^l}{V^G - U_3^h}. \tag{32}$$

Substitution of (26), (29), and (32) into (24) gives a closed form expression for selective efficiency $S(\gamma, \sigma)$ in dependence of the contestants' cost differential γ and their signals' informativeness σ:

$$S(\gamma,\sigma) = \frac{1}{2}[H\left(\beta_2^*(\sigma)\gamma\frac{V^G - U_3^l}{U_3^h}\right)H(\frac{1}{\gamma}) + H(\gamma)] + \frac{1}{2}H\left(\frac{\gamma}{\beta_2^*(\sigma)}\frac{U_3^l}{V^G - U_3^h}\right)H(\gamma). \quad (33)$$

We obtain the following result:

Proposition 5 (Selective Efficiency)**.** *Private information must not have a deteriorating effect on a dynamic contest's selective efficiency. In particular, for all $\gamma > 1$ and all $\sigma \in (0, \sigma^{max}) \cup (\sigma^{min}, 1)$, selective efficiency $S(\gamma, \sigma)$ is strictly larger than in the public information benchmark.*

Proof: See Appendix.

To understand the intuition for this result consider the effect of lowering the leader's equilibrium belief β_2^* starting from its public information benchmark value $\beta_2^* = 1$ via the introduction of private information. Lowering the leader's equilibrium belief raises the likelihood with which the second battle is won by the lagging contestant. This decreases selective efficiency when the low-cost contestant is in the lead but increases selective efficiency when the low-cost contestant has fallen behind. From (33), the decrease in selective efficiency is given by

$$\Delta S^- = \frac{1}{2}\gamma\frac{V^G - U_3^l}{U_3^h}h(\gamma\frac{V^G - U_3^l}{U_3^h})H(\frac{1}{\gamma}) \quad (34)$$

whereas the corresponding increase in selective efficiency is

$$\Delta S^+ = \frac{1}{2}\gamma\frac{U_3^l}{V^G - U_3^h}h(\gamma\frac{U_3^l}{V^G - U_3^h})H(\gamma). \quad (35)$$

In the Appendix we show that $\Delta S^+ > \Delta S^-$. Intuitively, the encouragement effect is stronger for a lagging low-cost contestant than for a lagging high-cost contestant. When the likelihood with which the low-cost contestant is lagging is (approximately) equal to the likelihood that the high-cost contestant is lagging, the overall effect is thus an increase in selective efficiency.

8 Discussion and conclusion

In this article, we have identified the encouragement effect as a novel aspect of dynamic competition with private information. Before we summarize our main message, a discussion of the model's assumptions is in order. While our model has put few restrictions on the "shape of competition" by allowing for rather generic contest success functions, the assumed information structure and the focus on a best-of-three contest deserve some comments.

To lend tractability to our model, we have assumed that a bad signal is conclusive, in that it allows players to conclude that the contest's prize has no value. As an alternative, one could consider a model with "good news" where players can conclude from the observation of a good signal that the contest's prize must be valuable. In strategic experimentation settings, good news models (e.g. Keller et al., 2005) produce different investment and learning dynamics than bad news models (e.g. Keller and Rady, 2015; Bonatti and Hörner, 2017). Although a "bad news" model appears to be the most conservative in light of our result that private information *improves* incentives under dynamic competition, a thorough investigation of dynamic incentives in a contest with good news is important. Similar to strategic experimentation settings, good news may lead to novel equilibrium features, whose analysis is beyond the scope of the present paper and is thus left for future research.

While our analysis has focused on a best-of-three contest, where the gap between the leader and the follower can take only one value, we know from empirical studies that the discouragement effect can become more pronounced when this gap is widened. As a consequence, one would expect the discouragement effect to have a heavier toll on incentives in contests with longer horizons. Given that, in equilibrium, players must conclude that their rival's signal must (also) be good as soon as they have lost a single battle, our model maintains its tractability when extended to more than three battles.

In a best-of-five Tullock contest, the effect of private information on aggregate incentives turns out to be even more positive than in a best-of-three contest. More specifically, we have confirmed numerically that the relative gain in incentives due to information being private rather than public is larger in a best-of-five contest than in a best-of-three contest, *independently* of the signals' informativeness.

We thus conclude, that in a dynamic contest, private information about the contest's prize, must have a positive effect on aggregate incentives. Under private information, the discouraging effect of falling behind is offset by the the encouraging effect of learning about the rival's information. As an important consequence, the common wisdom that dynamics must be harmful for incentives, may not be correct. In the presence of private information, aggregate incentives in a dynamic contest can be even greater than in the static benchmark. This result contrasts with the existing literature on dynamic contests that has mostly abstracted from the potential privacy of information and thus sheds a new light on applications such as R&D competition, presidential primaries, and labor tournaments.

Appendix

Proof of Lemma 2. Our characterization of the unique pure-strategy subgame perfect equilibrium can restrict attention to first order conditions because our assumption that h is decreasing guarantees the concavity of players' objectives. Using backward induction, in battle 3 equilibrium efforts must solve

$$e_3^P = \arg\max_{e_3 \geq 0} H(\frac{e_3}{e_3^P})V_G - e_3. \tag{36}$$

Setting $e_3 = e_3^P$ in the corresponding first order condition gives

$$e_3^P = h(1)V^G \tag{37}$$

and a player's continuation payoff from reaching battle 3 is thus as follows:[24]

$$U_3 = [\frac{1}{2} - h(1)]V^G > 0. \tag{38}$$

In battle 2, the leader and the follower differ in their valuation of winning. The follower's valuation of winning battle 2 is given by U_3 whereas the leader's valuation of winning battle 2 is $V^G - U_3 = [\frac{1}{2} + h(1)]V^G > U_3$. In a Subgame Perfect equilibrium (e_L^P, e_F^P) must therefore solve

$$\begin{aligned} e_L^P &\in \arg\max_{e_L \geq 0} U_3 + (V^G - U_3)H(\frac{e_L}{e_F^P}) - e_L & (39)\\ e_F^P &\in \arg\max_{e_F \geq 0} U_3[1 - H(\frac{e_L^P}{e_F})] - e_F. & (40) \end{aligned}$$

The first order conditions following from (39) and (40) have a unique solution given by

$$\begin{aligned} e_F^P &= \frac{1}{1+\rho} h(\frac{1}{\rho})V^G & (41)\\ e_L^P &= \frac{1}{\rho} e_F^P. & (42) \end{aligned}$$

The corresponding continuation payoffs from entering battle 2 as the leader or the follower are

$$\begin{aligned} U_L^G &= U_3 + H(\frac{1}{\rho})(V^G - U_3) - e_L^P = U_3 + [H(\frac{1}{\rho}) - \rho h(\rho)](V^G - U_3) > U_3 & (43)\\ U_F^G &= H(\rho)U_3 - e_F^P = [H(\rho) - \rho h(\rho)]U_3 > 0, & (44) \end{aligned}$$

where the inequalities follow from the fact that $H(\frac{1}{\rho}) > H(\rho)$ and because $H(y) > yh(y)$ for all $y > 0$ by Assumption 1 (see footnote 24). Finally, in battle 1 players' have identical valuations of winning, $U_L^G - U_F^G$, and choose their effort to solve

$$e_1^P \in \arg\max_{e_1 \geq 0} U_F^G + H(\frac{e_1}{e_1^P})(U_L^G - U_F^G) - e_1 \tag{45}$$

[24] Note that this payoff is positive because it follows from Assumption 1 that the function $H(y) - yh(y)$ is strictly increasing, converges to zero for $y \to 0$, and equals $\frac{1}{2} - h(1)$ for $y = 1$.

leading to

$$e_1^P = (U_L^G - U_F^G)h(1) = [H(\frac{1}{\rho}) - 2h(1)\rho h(\rho)]h(1)V^G > 0. \tag{46}$$

The corresponding equilibrium payoff is strictly positive because each player can guarantee himself a payoff of $U_F^G > 0$ by choosing $e_1 = 0$. Aggregating expected efforts over all three battles and both players gives

$$\begin{aligned} E^P &= \text{Prob}(s_1 = s_2 = G)[2e_1^P + e_L^P + e_F^P + 2H(\rho)e_3^P] \\ &= 2(1-\omega)\{h(1) + \rho h(\rho)[\frac{1}{2} - 2h(1)^2]\} \end{aligned} \tag{47}$$

which is independent of σ. □

Proof of Proposition 1. From

$$\frac{e_L^*}{e_F^*} = \frac{\beta_2^*}{\rho} = \frac{1}{\rho}\frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)^2+2\omega\sigma(1-\sigma)} \tag{48}$$

it follows that

$$\lim_{\sigma\to 0}\frac{e_L^*}{e_F^*} = \lim_{\sigma\to 1}\frac{e_L^*}{e_F^*} = \frac{e_L^P}{e_F^P} = \frac{1}{\rho} > 1. \tag{49}$$

Moreover, the derivative

$$\frac{d}{d\sigma}[\frac{e_L^*}{e_F^*}] = \frac{1}{\rho}\frac{2\omega(2\sigma-1-\omega\sigma^2)}{(1-\omega\sigma^2)^2} \tag{50}$$

has a unique root in $(0,1)$ at $\sigma = \hat{\sigma}(\omega)$ defined in (18), is negative for $\sigma \in (0, \hat{\sigma}(\omega))$ and positive for $\sigma \in (\hat{\sigma}(\omega), 1)$. Hence $\frac{e_L^*}{e_F^*}$ has U-shape with a minimum at $\sigma = \hat{\sigma}(\omega)$. Its minimized value is

$$\frac{e_L^*}{e_F^*}|_{\sigma=\hat{\sigma}(\omega)} = \frac{1}{\rho}\frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}}. \tag{51}$$

Using the fact that $\rho = \frac{1-R}{1+R}$, it follows that $\frac{e_L^*}{e_F^*} < 1$ for a non-empty interval (σ_-, σ_+) if and only if

$$\frac{1}{\rho}\frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}} < 1 \Leftrightarrow R < \hat{R}(\omega). \tag{52}$$

The thresholds σ_- and σ_+ are given by the solutions of the equation $\frac{e_L^*}{e_F^*} = 1$. □

Proof of Lemma 3. We first show that the first order condition corresponding to the battle 1 objective in (19) takes the simple form in (20). For this purpose, note that we can substitute continuation values $U_L^G(e_1)$ and $U_L^B(e_1)$ to rewrite the term $\beta_1 H(\frac{e_1}{e_1^*})U_L^G(e_1) + (1-\beta_1)U_L^B(e_1)$ as

$$[\beta_1 H(\frac{e_1}{e_1^*}) + 1 - \beta_1]\{\beta_2(e_1)[U_3 + H(\frac{e_L(e_1)}{e_F^*})(V^G - U_3)] - e_L(e_1)\}. \tag{53}$$

Here we have made use of the fact that under Bayesian updating it holds that $\beta_2(e_1) = \frac{\beta_1 H(\frac{e_1}{e_1^*})}{\beta_1 H(\frac{e_1}{e_1^*})+1-\beta_1}$. The term in parentheses equals the battle 2 objective of a player who deviated in battle 1 by choosing e_1 and happened to become the leader. More precisely, such a player will choose

$$e_L(e_1) \in \arg\max_{e_L} \beta_2(e_1)[U_3 + H(\frac{e_L}{e_F^*})(V^G - U_3)] - e_L \tag{54}$$

in battle 2. Since $e_L(e_1)$ maximizes the above objective, it follows from the envelope theorem that the derivative with respect to e_1 of the term in parentheses in (53) must be zero. Hence, the derivative of the battle 1 objective in (19) with respect to e_1 is given by

$$\beta_1 h(\frac{e_1}{e_1}^{*})\frac{1}{e_1^*}[U_L^G(e_1) - U_F^G] - 1 \tag{55}$$

and evaluation at $e_1 = e_1^*$ leads to the simple first order condition in (20). Together with the analysis contained in Section 4, this shows that $(e_1^*, e_L^*, e_F^*, e_3^*)$ defined by (9), (14), (15), and (20), is the unique candidate for a symmetric pure-strategy Perfect Bayesian equilibrium.

A comment is in order concerning the fact that the maximization program in (19) restricts the players' choice to strictly positive effort levels $e_1 > 0$. We now show that a deviation to $e_1 = 0$ is dominated by a deviation to $e_1 = \epsilon$ for $\epsilon > 0$ sufficiently small, which implies that neglecting the possibility of zero effort in (19) comes without loss of generality. Treating the possibility of zero effort separately is necessary because Bayesian updating in the case where $e_1 = 0$ differs from Bayesian updating in the case where $e_1 > 0$. More precisely, consider an equilibrium with $e_1^* > 0$, and suppose a player deviates to $e_1 = 0$. If the deviating player wins the first battle he learns that his rival must have received the signal B. Instead, if the deviating player loses the first battle, he will update his belief to $\beta_2^0 = \frac{\beta_1}{\beta_1 + \frac{1}{2}(1-\beta_1)}$ and then choose an effort $e_F^0 \in \arg\max_{e_F} \beta_2^0 H(\frac{e_F}{e_L^*})U_3 - e_F$. The payoff from a deviation to zero effort in battle 1 is thus given by

$$U_1^0 = \beta_1[H(\frac{e_F^0}{e_L^*})U_3 - e_F^0] - (1-\beta_1)\frac{1}{2}e_F^0. \tag{56}$$

Instead, a deviation to $e_1 = \epsilon$ gives the payoff

$$U_1^\epsilon = \beta_1\{H(\frac{\epsilon}{e_1^*})U_L^G(\epsilon) + [1 - H(\frac{\epsilon}{e_1^*})]U_F^G\} + (1-\beta_1)U_L^B(\epsilon) - \epsilon. \tag{57}$$

After winning battle 1, a player who deviated from an equilibrium $e_1^* > 0$ by exerting only a small effort in battle 1 must be nearly certain that his rival has observed a bad signal. Formally, for $\epsilon \to 0$ it holds that $\beta_2(\epsilon) \to 0$ and thus $e_L(\epsilon) \to 0$. Hence, for $\epsilon \to 0$, it holds that

$$U_1^\epsilon \to \beta_1 U_F^G = \beta_1[H(\frac{e_F^*}{e_L^*})U_3 - e_F^*] \geq U_1^0, \tag{58}$$

and the inequality follows from the fact that $e_F^* \in \arg\max_{e_F} H(\frac{e_F}{e_L^*})U_3 - e_F$. Intuitively, although a player can achieve that a win in battle 1 reveals the rival's signal perfectly by choosing $e_1 = 0$, the player can do even better because when choosing an infinitesimal effort $e_1 = \epsilon$, the rival's signal becomes revealed not only by a win (approximately) but also by a loss in battle 1.

Finally, to prove existence of equilibrium it remains to consider second order conditions. We first consider the case where the distribution of the ratio of noise is given by $h_r = \frac{ry^{-r-1}}{(1+y^{-r})^2}$ generating the generalized Tullock contest success function with parameter r. Nti (1999) shows that in a static Tullock contest a pure strategy equilibrium exists if and only if $r \leq 1 + v^r$ where $v \in (0, 1]$ denotes the contestants' ratio of valuations of winning. Our contest is dynamic rather than static, but using continuation values we were able to write each battle in the form of a static Tullock contest. The contestants have identical valuations of winning in battles 1 and 3, i.e. valuations differ only in battle 2 where $v = \frac{U_3}{\beta_2^*(V^G - U_3)}$. v is minimized when signals are public, i.e. for $\beta_2^* = 1$. Note that in contrast to Nti (1999), our contest features imperfect information. However, because contestants exert zero efforts after observing a bad signal, the conditions for a pure strategy Perfect Bayesian equilibrium are just an analogue of the equilibrium conditions in Nti (1999). Since for h_r we find $U_3 = (\frac{1}{2} - \frac{r}{4})V^G$ and $V^G - U_3 = (\frac{1}{2} + \frac{r}{4})V^G$, a pure strategy Perfect Bayesian equilibrium thus exists for all σ if and only if

$$r \leq 1 + (\frac{2-r}{2+r})^r. \tag{59}$$

As this inequality is satisfied for all $r \leq 1$ we have thus shown existence of equilibrium for the family of Tullock contest success functions with parameters $r \leq 1$. The equilibrium is unique and can be determined in closed form as:

$$\begin{aligned} e_3^* &= \frac{rV_G}{4} && (60) \\ e_L^* &= \frac{rV_G}{4}\beta_2(2+r)\chi && (61) \\ e_F^* &= \frac{rV_G}{4}(2-r)\chi && (62) \\ e_1^* &= \frac{rV_G}{4}\beta_1\chi\{(\beta_2\frac{2+r}{2-r})^r + 1 - \frac{r}{4}[\beta_2(2+r) - 2 + r]\} && (63) \end{aligned}$$

where we abbreviated notation by defining $\chi \equiv \frac{\beta_2^r(2+r)^r(2-r)^r}{[\beta_2^r(2+r)^r + (2-r)^r]^2}$.

While for the Tullock family, equilibrium existence is guaranteed for all $\sigma \in [0, 1]$, that is, *independently* of the informativeness of the contestants' signals, for general distributions of the ratio of noise, existence is harder to establish. In the remainder of this proof we show that, under the conditions of Assumption 1, an equilibrium exists when contestants' information is "sufficiently public", that is when σ is sufficiently close to 0 or 1.

To see this, first note that the players' objective in battle 3, given by (36), as well as the leader's and the follower's objectives in battle 2, given by (11) and (12), are globally concave because $h = H'$ is assumed to be strictly decreasing. For the remaining battle 1, the second order condition which guarantees that e_1^* constitutes a maximizer can be obtained by calculating the derivative of (55) with respect to e_1 at $e_1 = e_1^*$. It is satisfied if

$$\frac{h'(1)}{\beta_1 h(1)} + h(1)\frac{dU_L^G(e_1^*)}{de_1} < 0 \tag{64}$$

with $U_L^G - U_F^G > 0$ given by (21) and

$$\begin{aligned} U_L^G(e_1) &= U_3 + H(\frac{e_L(e_1)}{e_F^*})(V^G - U_3) - e_L(e_1), &(65)\\ e_L(e_1) &\in \arg\max_{e_L \geq 0} \beta_2(e_1)[U_3 + (V^G - U_3)H(\frac{e_L}{e_F^*})] - e_L. &(66)\end{aligned}$$

We get

$$\frac{dU_L^G(e_1^*)}{de_1} = [h(\frac{e_L^*}{e_F^*})(V^G - U_3) - 1]\frac{de_L(e_1^*)}{de_1} = \frac{1-\beta_2^*}{\beta_2^*}\frac{de_L(e_1^*)}{de_1} \tag{67}$$

where we have used the fact that e_L^* solves the first order condition $\beta_2^* h(\frac{e_L^*}{e_F^*})(V^G - U_3) = 1$. As $e_L(e_1)$ satisfies an analogue first order condition with β_2^* substituted by $\beta_2(e_1)$, we can employ the Implicit Function Theorem to get

$$\frac{de_L(e_1^*)}{de_1} = -\frac{h(\frac{e_L^*}{e_F^*})e_F^*}{h'(\frac{e_L^*}{e_F^*})\beta_2^*}\frac{d\beta_2(e_1^*)}{de_1} \tag{68}$$

Note from (10) that $\frac{d\beta_2(e_1^*)}{de_1}$ is positive but tends to zero for $\sigma \to 0$ and for $\sigma \to 1$. As $\beta_2^* \to 1$ in both cases, we can thus conclude from $h' < 0$ that the second order condition in (64) must be satisfied when σ is sufficiently close to 0 or 1.

□

Proof of Proposition 2. Consider

$$E^* = E_1^* + E_3^* + E_2^* = 2(1-\omega)h(1)[1 - \frac{1}{V^G}(e_L^* - e_F^*)] + (1-\omega)\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}). \tag{69}$$

Substitution of e_F^* and e_L^* from (14) and (15) gives

$$E^* = 2(1-\omega)\{h(1) + \frac{\beta_2^*}{\rho}h(\frac{\beta_2^*}{\rho})[\frac{1}{2} - \frac{\beta_2^* - \rho}{1+\rho}h(1)]\}. \tag{70}$$

From $\frac{1-\rho}{1+\rho} = 2h(1)$ it follows that $\lim_{\beta_2^* \to 1} E^* = 2(1-\omega)\{h(1) + \rho h(\rho)[\frac{1}{2} - 2h(1)^2]\} = E^P$ where we have used the fact that by symmetry $\frac{1}{\rho}h(\frac{1}{\rho}) = \rho h(\rho)$. Defining the function $g(y) \equiv yh(y)$ and denoting its derivative by g' we can write

$$\frac{1}{2(1-\omega)}\frac{dE^*}{d\beta_2^*} = \frac{1}{\rho}g'(\frac{\beta_2^*}{\rho})[\frac{1}{2} - \frac{\beta_2^* - \rho}{1+\rho}h(1)] - \frac{1}{1+\rho}g(\frac{\beta_2^*}{\rho})h(1). \tag{71}$$

Note that it follows from $\beta_2^* \leq 1$ and from $h(1) < \frac{1}{2}$ that

$$[\frac{1}{2} - \frac{\beta_2^* - \rho}{1+\rho}h(1)] \geq [\frac{1}{2} - \frac{1-\rho}{1+\rho}h(1)] = \frac{1}{2} - 2h(1)^2 > 0. \tag{72}$$

As $g(.)$ is unimodal with a mode at 1, it must therefore hold that $\frac{dE^*}{d\beta_2^*} < 0$ for all $\beta_2^* > \rho$. The Proposition then follows from the fact that $\beta_2^*(\sigma)$ is U-shaped with $\lim_{\sigma\to 0}\beta_2^* = \lim_{\sigma \to 1}\beta_2^* = 1$ and takes its minimum value $\min_{\sigma\in(0,1)}\beta_2^* = \frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}}$ at $\sigma = \hat{\sigma}(\omega)$ and that this minimum value is smaller than ρ if and only if $R < \hat{R}(\omega)$. $\square$

Proof of Proposition 3. Suppose that $R < \hat{R}(\omega)$. Then according to the proof of Proposition 2 it holds that $\frac{dE^*}{d\beta_2^*} < 0$ for all $\beta_2^* > \rho$, or equivalently $\frac{e_L^*}{e_F^*} > 1$, i.e. E^* is strictly increasing in σ for all $\sigma \in (0, \sigma_-]$ and strictly decreasing in σ for all $\sigma \in [\sigma_+, 1)$ where $0 < \sigma_- < \sigma_+ < 1$ are the thresholds defined in the proof of Proposition 1. Hence, there must exist a $\sigma^* \in (\sigma_-, \sigma_+)$ such that $E^*(\sigma^*) > \max(E^*(\sigma_-), E^*(\sigma_+))$. Note that, as $R < \hat{R}(\omega)$ implies that $h(1) < \frac{1}{3}$, we can write

$$E^*(\sigma) - E^S = \frac{1}{2}[\frac{\beta_2^*}{\rho}h(\frac{\beta_2^*}{\rho}) - h(1)] - \frac{\beta_2^*}{\rho}h(\frac{\beta_2^*}{\rho})h(1)\frac{\beta_2^* - \rho}{1+\rho}. \tag{73}$$

The result then follows from the fact that $E^*(\sigma_-) = E^*(\sigma_+) = E^S$ which holds because by the definition of the thresholds, at $\sigma = \sigma_-$ and $\sigma = \sigma_+$ it holds that $e_L^* = e_F^*$, or equivalently $\beta_2^* = \rho$. $\square$

Proof of Proposition 4. For $R \geq \hat{R}(\omega)$, $E^*(\sigma)$ inherits its shape from $\beta_2^*(\sigma)$, because $\frac{dE^*}{d\beta_2^*} < 0$ for $\beta_2^* > \rho \Leftrightarrow \frac{e_L^*}{e_F^*} > 1$ and because the follower cannot be induced to exert higher effort than the leader, independently of σ, as shown in the proof of Proposition 2. For $R < \hat{R}(\omega)$, the proof of Proposition 3 has shown that $E^*(\sigma)$ must be maximized at a $\sigma^* \in (\sigma_-, \sigma_+)$ and since the thresholds σ_- and σ_+ are defined by the requirement that $e^*_L = e^*_F$, at $\sigma = \sigma^*$ it must hold that $e^*_F > e^*_L$. It thus remains to consider the comparative statics:

$$\frac{d\hat{R}}{d\omega} = \frac{2 - \omega - 2\sqrt{1-\omega}}{\omega^2\sqrt{1-\omega}} > 0 \tag{74}$$

because the nominator is increasing in ω for $\omega \in (0, 1)$ and converges to zero for $\omega \to 0$. For the same reason it holds that

$$\frac{d\hat{\sigma}}{d\omega} = \frac{2 - \omega - 2\sqrt{1-\omega}}{2\omega^2\sqrt{1-\omega}} > 0. \tag{75}$$

□

Proof of Proposition 5. Note first that $\gamma > 1$ implies that $H(\gamma) > H(\frac{1}{\gamma})$. Remember that the function $yh(y)$ is unimodal with a unique maximum at $y = 1$ and that $yh(y) = \frac{1}{y}h(\frac{1}{y})$. As

$$\frac{V^G - U^l_3}{U^h_3} = \gamma\frac{H(\frac{1}{\gamma}) + \gamma h(\gamma)}{H(\frac{1}{\gamma}) - \gamma h(\gamma)} > 1 \tag{76}$$

it holds that $\gamma\frac{V^G - U^l_3}{U^h_3} > 1$ and it is thus sufficient for $\Delta S^+ > \Delta S^-$ that

$$[\gamma\frac{V^G - U^l_3}{U^h_3}]^{-1} < \gamma\frac{U^l_3}{V^G - U^h_3} < \gamma\frac{V^G - U^l_3}{U^h_3}. \tag{77}$$

The second inequality follows directly from

$$\frac{U^l_3}{V^G - U^h_3} = \frac{H(\gamma) - \gamma h(\gamma)}{H(\gamma) + \gamma h(\gamma)} < 1. \tag{78}$$

For the first inequality note that $\frac{U^l_3}{V^G - U^h_3} > \frac{U^h_3}{V^G - U^l_3}$ if and only if

$$H(\gamma) - \gamma h(\gamma) - [H(\gamma) - \gamma h(\gamma)]^2 > H(\frac{1}{\gamma}) - \gamma h(\gamma) - [H(\frac{1}{\gamma}) - \gamma h(\gamma)]^2. \tag{79}$$

This inequality is satisfied because the terms $H(\gamma) - \gamma h(\gamma)$ and $H(\frac{1}{\gamma}) - \gamma h(\gamma)$ lie between zero and one and the former is closer to $\frac{1}{2}$ than the latter. We have thus shown that $\Delta S^+ > \Delta S^-$, or more formally that

$$\frac{\partial S}{\partial \beta^*_2}|_{\beta^*_2 = 1} < 0. \tag{80}$$

Using private information to reduce the leader's belief marginally below his belief $\beta^* = 1$ in the public information benchmark has a positive effect on selective efficiency. $\square$

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