

Markups, Taxes, and Rising Inequality*

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Abstract

How to explain rising income and wealth inequality? We build an original heterogeneous-agent model with three key features: (i) an explicit link between firm's market power and top income shares, (ii) a granular representation of the tax and transfer system, and (iii) three assets with endogenous portfolio decisions. Using France as an illustration, we look at how changes in markups, taxes, factor productivity, and asset prices affect inequality dynamics over the 1984-2018 period. Rising markups account for the bulk of rising income inequality. Wealth inequality dynamics result mostly from changes in saving rate inequality but only in response to the exogenous changes in taxation and markups. Our results point to the critical importance of endogenous saving decisions in response to exogenous shocks as a key driver of wealth inequality.

Keywords: Heterogeneous Agents, Taxes, Market Power, Income Inequality, Wealth Inequality

JEL Class.: D4, E2, H2, O4, O52.

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1 Introduction

How to explain income and wealth inequality dynamics? While the empirical literature has documented the rise in inequalities in many countries during the past decades ([Alvaredo et al. \(2017\)](#)), we still lack a clear understanding of the mechanisms behind this rise. In this paper, we study how income inequality is formed and translates into wealth inequality dynamics. To do so, we build an original heterogeneous-agent (HA) model with three key features: *(i)* an explicit link between firm's market power and top income shares, *(ii)* a granular representation of the complex tax and transfer system, and *(iii)* endogenous portfolio decisions leading to heterogeneity in wealth composition. We look at how changes in market power, taxes, factor productivity, and asset prices affect inequality dynamics. Since our framework considers all the potential behavioral effects (direct, indirect, general equilibrium) on the relevant margins (savings, consumption, and labor), it allows us to identify the key mechanisms at work.

We study the rise in income and wealth inequality dynamics in France since 1984, the time period before which inequalities start rising. First, we carefully calibrate the model to match the level of income and wealth inequality in 1984. We then show that changes in market power, taxes, and asset prices allow to replicate the observed dynamics of inequality. Second, we run counterfactual simulations to quantify the contribution of the various factors (technology, capital gains, market power, taxes and transfers) to the dynamics of income and wealth inequality. Third, we decompose wealth inequality dynamics into various endogenous mechanisms.¹

We find that the increase in income inequality between 1984 and 2019 is mostly driven by rising markups. Wealth inequality dynamics are more complex than income inequality dynamics. They are driven by the increase in saving rate inequality and rising pretax income inequality induced by changes in taxation and markups. Absent changes in taxation, markups and capital gains, income and wealth inequality would have been stable. As a result, changes in saving rate inequality are the main determinant of wealth inequality dynamics, but only in response to exogenous variables (asset prices, taxation, and markup). More generally, our results point to the critical importance of modeling endogenous saving decisions because their endogenous response to exogenous forces is a key driver of wealth inequality.

We develop an original heterogeneous-agent (HA) model that accounts for all the aforementioned sources of heterogeneity. We extend the classical structure of [Krusell and Smith \(1998\)](#) with uninsurable idiosyncratic income risk, heterogeneous discount factors, and wealth accumulation along three dimensions.

First, we introduce two assets in addition to capital: indivisible housing and deposits. We make the composition of individual portfolios endogenous. This allows us to *(i)* match the observed composition of portfolios along the wealth distribution, *(ii)* provide a microfounded

¹We consider the following endogenous mechanisms: changes in pretax income inequality, tax progressivity, saving rate inequality, capital gain inequality, and aggregate pretax income.

explanation for increasing returns along the wealth distribution², and (iii) account for the potentially heterogeneous effects of capital gains in the dynamic simulations.

Second, we introduce a key relation between market power and top income shares.³ A small proportion of households (henceforth called “entrepreneurs”) receive large per-capita monopolistic profits, which helps match observed top income shares. Because entrepreneurs face large risks of losing their status, they secure large precautionary savings and become top wealth owners.

Third, we take into account the complexity of the French tax and transfer system using a very rich and realistic set of time-varying flat and progressive taxes and transfers that apply to the relevant tax bases (payroll taxes on labor income, corporate taxes on profits, income taxes, taxes on consumption, taxes on wealth, and monetary transfers). Since our model jointly matches key aggregate ratios (labor share, aggregate wealth portfolios, the wealth-income ratio, etc...) and their distributions, the granular tax system replicates the mapping between pretax, disposable, and post-tax income inequalities. In addition, the resulting model offers a framework to take into account all the potential behavioral effects (direct, indirect, general equilibrium) of changes in taxes and transfers on the relevant margins (savings, consumption, and labor).

We calibrate the model’s steady state in 1984 and feed it with exogenous time-varying forces using newly available wealth and income inequality series and fiscal data. These series are part of the “Distributional National Accounts” (DINA) project, and consist in long-term series of wealth, pretax and post-tax national income that (i) are fully consistent with national accounts, (ii) cover the entire distribution, and (iii) provide detailed information on income, asset detention, portfolios as well as all taxes and transfers at the individual level (see [Saez and Zucman \(2016\)](#); [Piketty, Saez, and Zucman \(2018\)](#) for the U.S.; [Garbinti, Goupille-Lebret, and Piketty \(2018\)](#), [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#) and [Bozio et al. \(2021\)](#) for France). We also exploit the richness of these time series to gauge the quality of our dynamic simulations on multiple dimensions (macroeconomic aggregates, income and wealth inequalities, the aggregate and distributional tax structure) from 1985 to 2018.

Our model’s stationary distribution matches the 1984 characteristics of income and wealth distributions from the bottom 50% to the top 1%, the composition of individual and aggregate wealth, the aggregate wealth-income ratio, as well as the usual macroeconomic ratios. Alternative specifications show that the introduction of entrepreneurs and of a granular and progressive tax system are both key to match the top and the bottom of the income and wealth distributions. In addition, endogenous labor supply matters for the bottom of the income distribution, and the consideration of three assets matters to match the aggregate wealth-income ratio (on top of being critical for the distributional effects of capital gains through portfolios compositions in the dynamic simulations).

²See [Cao and Luo \(2017\)](#) or [Xavier \(2020\)](#) for evidence based on U.S. data and [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#) for similar evidence based on French data.

³We build on the intuition of [De Loecker, Eeckhout, and Unger \(2020\)](#) for the U.S. and documented globally by [De Loecker and Eeckhout \(2018\)](#) incorporated in a macroeconomic framework by [Eggertsson, Robbins, and Wold \(2021\)](#).

Starting from a stationary distribution in 1984, we then feed the model with a series of exogenous variables: changes in capital gains, taxation (taxes and transfers), markups, and others market forces such as the rate of capital depreciation or total factor productivity (TFP hereafter). Our dynamic simulations track the data very closely and reproduce two main facts regarding inequality dynamics: (i) the rise in income inequality is mostly driven by a significant increase in the top 1% income share and (ii) the rise in wealth inequality is mostly driven by a significant increase in the top 1% wealth share at the expense of the bottom 50%.

The great fit of our baseline dynamic simulations allows us to implement counterfactual experiments that shed light on the respective contributions of the different driving forces. First, we neutralize the effects of changes in capital gains, taxation, and markups. When others market forces — capital depreciation and TFP — are the only drivers of our counterfactual economy, income and wealth inequalities are basically stable, and the aggregate wealth-income ratio remains flat over the 1984-2019 period. Second, changes in markups are the main factor behind rising pretax income inequality (accounting for 85% of the rise), while changes in taxation play a more limited role (15% of the rise). Third, changes in capital gains, taxation, and markups all play a significant role in explaining wealth inequality dynamics. In particular, changes in taxation and markups reduce the wealth growth rate of the bottom 90% and increase that of the top 10% in the same proportion while keeping the growth rate of aggregate wealth unchanged. Changes in capital gains dramatically increase the wealth growth rate of housing owners and equity holders (middle 40% and top 10% wealth groups) but have no effects on liquidity (deposits), which is the main asset held by the bottom 50%.

We then investigate the mechanisms through which the above driving forces operate, focusing on the dynamics of the top 1% wealth share. For the different driving forces at play, we decompose the evolution of the top 1% wealth share as resulting from five complementary mechanisms: changes in pretax income inequality, tax progressivity, saving rate inequality, capital gain inequality, and aggregate pretax income. We study these mechanisms over time and quantify their respective contributions to the changes in the top 1% wealth share between 1984 and 2018. Wealth inequality would have remained stable absent changes in taxation, markups, and capital gains. Introducing changes in markups and taxation explains most of the rise in wealth inequality dynamics via rising pretax income inequality among wealth groups and saving rate inequality. In particular, the effects of changes in taxation operate mostly through behavioral and general equilibrium effects on saving rate inequality, and not through their mechanical effect — which affects the gap between pretax and disposable income inequality. Finally, adding changes in capital gains on top of changes in markups and taxation has a marginal effect on the top 1% wealth share: the strong negative mechanical effect of capital gains — higher housing *vs.* equity prices benefits relatively less the top 1% wealth group — is almost entirely offset by changes in saving rate inequality. More generally, our results highlight the key contribution of saving rate inequality to the dynamics of wealth inequality and calls for a thorough modeling of endogenous saving decisions to various driving forces.

Literature. Since the pioneering works of [Bewley \(1977\)](#), [Huggett \(1993\)](#), and [Aiyagari \(1994\)](#), a macroeconomic literature has developed and improved general equilibrium models with heterogeneous agents to reproduce the wealth and income inequality *at a given point in time* and explain their determinants (see [De Nardi and Fella \(2017\)](#); [Benhabib and Bisin \(2018\)](#) for a literature review). In particular, these contributions consider switching discount factors ([Krusell and Smith \(1998\)](#)), bequest motives ([De Nardi \(2004\)](#)), entrepreneurs ([Cagetti and De Nardi \(2006\)](#)), wealth in the utility function ([Francis \(2009\)](#)), original labor-income processes ([Ferriere et al. \(2021\)](#)), and stochastic jumps affecting the returns to assets ([Benhabib, Cui, and Miao \(2021\)](#)) as potential drivers of wealth inequality.⁴ Recently, a new line of papers investigates the *dynamics* of income inequality ([Gabaix et al. \(2016\)](#)) and wealth inequality ([Kaymak and Poschke \(2016\)](#) and [Hubmer, Krusell, and Smith \(2020\)](#)) for the U.S. We make progress on both fronts by developing a unified HA model combining heterogeneous discount factors, entrepreneurs, three assets, and wealth in utility that fits both the *level* and *dynamics* of income and wealth inequality in France along the distribution. We also use our model to quantify the contribution of various mechanisms to the dynamics of wealth inequality depending on the driving forces.

Along the way, we make use of the recent technical advances in solving HA models based on continuous-time formulation of the heterogeneous-agent problem in solving the Hamilton-Jacobi-Bellman and Kolmogorov forward equations (see [Achdou et al. \(2022\)](#)), and rely on fast and fully non-linear dynamic simulations. An additional contribution to the literature is to further extend the approach of [Berger et al. \(2018\)](#) used by [Fagereng et al. \(2019\)](#), and solve a three-asset model including deposits, indivisible housing, and equity capital using a one-asset formulation. While multiple assets have been used to study the short-run effects of monetary or fiscal policy within HA models (see [Kaplan, Moll, and Violante \(2018\)](#) and [Kaplan and Violante \(2021\)](#) among others), we show that having various classes of assets is key to account for wealth inequality dynamics through differential asset price dynamics.⁵

Our paper relates to the recent literature that empirically documents the rise of markups in the U.S. economy ([De Loecker, Eeckhout, and Unger \(2020\)](#)) and around the world ([De Loecker and Eeckhout \(2018\)](#)), and its implication for optimal regulation ([Boar and Midrigan \(2019\)](#) and [Eeckhout et al. \(2021\)](#)). We contribute to this literature by introducing a link between market power, firms' profits, and top income shares through entrepreneurs and by quantifying the impact of rising market power on income and wealth inequality.⁶ [Eggertsson, Robbins, and Wold](#)

⁴See [Saez and Stantcheva \(2018\)](#) for additional references on wealth in utility and potential microfoundations, including bequests and services from wealth.

⁵[Kaplan, Moll, and Violante \(2018\)](#) and subsequent papers rely on a liquid/illiquid divide among assets and adjustment costs because they seek to replicate the pattern of marginal propensities to consume in the event of short-lived shocks. In contrast, we consider different classes of assets (deposits, housing, and equity) that feature different rates of returns and capital gains both on average and over time, with potentially large effects on wealth inequality dynamics.

⁶The determinants of rising market power in the literature include the development of increasing returns from widening markets through trade or technologies ([Autor et al. \(2020\)](#)), the associated reallocation of market shares towards larger and more efficient firms ([Baqaee and Farhi \(2020\)](#)) or the falling demand elasticity driven by consumers becoming less price-sensitive ([Döppler et al. \(2021\)](#)).

(2021) follow a similar path but focus on the aggregate macroeconomic implications of rising market power, while we consider both aggregate and distributional effects of rising markups. In addition, we incorporate additional driving forces such as taxes and transfers, TFP, and housing with housing capital gains.

Finally, our paper relates to the recent applied literature dedicated to the construction of "Distributional National Accounts" for pretax and post-tax income (see in particular [Piketty, Saez, and Zucman \(2018\)](#) for the U.S and [Garbinti, Goupille-Lebret, and Piketty \(2018\)](#); [Bozio et al. \(2021\)](#) for France) and to the evolution and the determinants of wealth inequality ([Saez and Zucman \(2016\)](#); [Martínez-Toledano \(2020\)](#); [Kuhn, Schularick, and Steins \(2020\)](#); [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#)). Our paper provides an illustration of how such rich data can be used to improve the ability of HA models to reproduce distributional dynamics over time. In turn, it contributes to this literature by evaluating the contribution of market forces and institutional factors to the dynamics of inequality, taking into account all the potential behavioral effects (direct, indirect, and general equilibrium).

The paper is structured as follows. Section 2 presents the model and discusses the main assumptions. Section 3 details the calibration, presents the characteristics of the initial stationary equilibrium and presents some counterfactual stationary distributions. Section 4 presents the predictions of the dynamic model when driven by a set of exogenous driving forces such as changes in productivity, capital depreciation, market power or the tax and transfer system and shows a great fit with the data. Section 5 offers a decomposition of the main drivers of income and wealth inequality relying on counterfactual experiments. Section 6 offers a more detailed investigation of the mechanisms underlying the large increase of the top 1% share of wealth, highlighting the key role of behavioral (savings) responses to changes in exogenous factors such as taxation or capital gains.

2 Model

The model features heterogeneous agents with uninsurable earnings and status risk and a realistic taxation system that incorporates an extensive set of proportional and progressive taxes and transfers. Households can be workers or entrepreneurs, patient or impatient. They switch types exogenously. When they are workers, they face standard labor earning risk and supply labor endogenously. When they are entrepreneurs, they receive monopolistic profits and face a risk of losing the status, which pushes them to precautionary-save. Last, we also introduce wealth in the utility of patient households to account for additional saving motives, such as dynastic altruism. Households have access to three types of assets: deposits and savings accounts, indivisible housing, and equity capital. Since deposits and housing provide utility directly from liquidity and housing services, all households hold deposit savings, and demand housing as their first asset despite the lower returns it provides. Whether they actually hold housing depends on whether their demand exceeds the indivisible amount of housing. In equilibrium, the

share of assets with higher returns increases with wealth . This generates increasing returns along the wealth distribution as observed in the data. The distributional profile of these returns also contributes to the equilibrium dispersion of wealth.

2.1 Households

The model is presented and solved in continuous time. For the sake of clarity, time subscripts are omitted. The economy is populated by a unit-size continuum of heterogeneous households $j \in [0, 1]$. All variables are expressed per-capita and deflated by a labor productivity index that grows at an exogenous rate γ .

2.1.1 Income processes

Households can be either workers or entrepreneurs.

When working, households supply ℓ^j units of labor and receive a real wage $w^j = we^{z^j}$, where w is the aggregate real wage resulting from the equilibrium of labor markets and e^{z^j} is a lognormal process:⁷

$$\dot{z}^j = -\rho_z z^j + \sigma_z \epsilon_z^j \quad (1)$$

Parameter σ_z governs the dispersion of earnings and ρ_z the persistence of earning shocks ϵ_z^j .

When households are entrepreneurs, they supply $\ell^j = 0$ units of labor and receive a fraction of the aggregate profits $\pi^j = \pi/e$, where π are the aggregate profits and e is the number of entrepreneurs. We consider these profits as mixed-income and split them into labor and capital income. Following standard convention, 70% is classified as labor income (and taxed as such) and 30% as capital income (and then subject to capital income taxes after the payment of corporate taxes).⁸ The status of entrepreneur is risky. Entrepreneurs face a larger probability of losing their status than the probability for workers to become entrepreneurs.

Any household j can hold three assets: a housing asset in quantity $h^j \in \{0, [h^{\min}, \infty)\}$, deposits in quantity m^j , and equity capital k^j . Housing is net of debt and indivisible in the sense that households cannot buy housing unless their demand exceeds the minimal size of housing h^{\min} , the minimum down payment required to take out a loan and buy housing. The total wealth of household j writes $a^j = p^k k^j + p^h h^j + m^j$, where p^k and p^h are the relative prices of equity capital and housing and the price of non-durable goods is used as *numéraire*.

⁷Variable z_t^j denotes the relative productivity of worker j and aggregate productivity is normalized in the steady state so that the sum of labor income received by workers equals the aggregate labor income paid by firms.

⁸This is the standard convention used in the income inequality literature ([Alvaredo et al. \(2020\)](#), [Bachas et al. \(2022\)](#)) and it is closed to the estimates by [Smith et al. \(2019\)](#) that show that three-quarters of top pass-through profit in the U.S can be classified as human capital income.

As a result, labor and capital income of household j are given by

$$\Phi_\ell^j = (1 - \tau_\ell^j) \left((1 + \nu) w^j (1 - \mathbb{1}_e^j) \ell^j + \mathbb{1}_e^j 0.7 \pi^j \right) \quad (2)$$

$$Y_k^j = r^k k^j + r^h p^h h^j + r^m m^j + \mathbb{1}_e^j (1 - \tau_\pi) 0.3 \pi^j \quad (3)$$

where $\mathbb{1}_e^j$ is an indicator function that equals 1 if household j is an entrepreneur and zero otherwise, and τ_π denotes the corporate tax rate, as detailed in the section describing firms' behavior. r^k , r^h , and r^m are the returns for equity capital, housing, and deposits, respectively. Note that a fraction ν of the real wage w^j is added to capture the net balance of the unemployment and pension system.⁹ τ_ℓ corresponds to the individual rate of non-contributive payroll taxes, i.e. the payroll taxes that do not finance unemployment and pension benefits.

2.1.2 Preferences and optimization problem

Following [Krusell and Smith \(1998\)](#), households are also heterogeneous in terms of their discount factor and can be patient or impatient. The discount factor follows a two-states Markov process. Patient households feature an additional savings motive as wealth enters into their utility. The optimization problem of household j with discount factor $\rho^j = \{\rho_L, \rho_H\}$ is thus:

$$\max_{k^j, h^j, m^j, c^j} \int_0^\infty e^{-\rho^j t} \left\{ \log \Lambda^j - \frac{(\ell^j)^{1+\zeta}}{1+\zeta} + \mathbb{1}_{\rho^j=\rho_L} \log(a^j + \psi) \right\} dt \quad (4)$$

$$\begin{aligned} \text{s.t.} \quad & p^k k^j + p^h h^j + m^j + \gamma a^j + (1 + \tau_c) c^j = (1 - \tau^j) (\Phi_\ell^j + Y_k^j) - \phi^j a^j + T^j \\ & k^j \geq 0, h^j \in \{0, [h^{\min}, \infty)\}, m^j \geq 0 \end{aligned} \quad (5)$$

Households derive utility from their individual expenditure on goods and services Λ^j , disutility from supplying labor ℓ^j and, when patient, utility from holding wealth a^j . In addition, ζ denotes the inverse of the Frisch elasticity on labor supply, ψ is a scale parameter for wealth in utility and $\mathbb{1}_{\rho^j=\rho_L}$ is an indicator function that equals one if household j is patient and zero otherwise. In the budget constraint, on the right-hand side, τ^j is the individual income tax rate, Φ_ℓ^j denotes labor income, and Y_k^j capital income. Wealth is taxed at the individual rate ϕ^j , and T^j denotes the individual monetary transfer received from the government. On the left-hand side, household j allocates its income in non-durable goods c^j taxed at the rate τ_c and durable goods k^j (equity capital), h^j (housing) and m^j (deposits). Last, remember that level quantities, including individual wealth, increase at the rate γ so that γa^j has to be saved each period to keep the productivity-deflated level of wealth a^j at least constant in the stationary equilibrium.

In Equation (4), Λ^j is a bundle of expenditures on non-durable goods c^j , housing h^j and

⁹The model features employed households or dynasties with infinite lives. While the model does not explicitly account for the situation of unemployed and retired household, ν is introduced to account for the discrepancy between pension and unemployment contributions, and pension and unemployment benefits. See Appendix A for more details on this aspect.

deposits m^j :

$$\Lambda^j = \left((1 - \chi)^{\frac{1}{\mu}} (C^j)^{\frac{\mu-1}{\mu}} + \chi^{\frac{1}{\mu}} (m^j)^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} \quad (6)$$

$$C^j = \left((1 - \kappa)^{\frac{1}{\lambda}} (c^j)^{\frac{\lambda-1}{\lambda}} + \kappa^{\frac{1}{\lambda}} (h^j)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}} \quad (7)$$

In these bundles, μ is the elasticity of substitution between deposits m^j and other types of expenditure — the consumption of non-durable and housing — and χ a weight parameter. The elasticity of substitution between expenditure on non-durable c^j and housing h^j is λ and κ is a weight parameter.

2.1.3 Reformulation

The above problem can be reformulated as a one-asset problem while preserving the endogenous composition of household portfolios. As explained below, the problem can be split into a dynamic problem that consists in choosing the level of consumption expenditure and savings, and thus the total amount of wealth, and a static expenditure minimization problem that allocates total consumption expenditure into three main categories: non-durable consumption, housing services, and liquidity services. Assuming that households are willing to accept lower returns on housing and deposits relative to equity capital because holding these assets generates utility gains, foregone returns on housing and deposits can be rewritten as opportunity costs of holding housing and deposits.

To see this, start by imposing $a^j = p^k k^j + p^h h^j + m^j$, with $0 \leq m^j \leq a^j$ and $h^{\min} \leq h^j \leq (a^j - m^j) / p^h$, which both imply $k^j \geq 0$. The budget constraint can be rewritten as:

$$\dot{a}^j + \gamma a^j + \overbrace{(1 + \tau_c) c^j + R^{hj} h^j + R^{mj} m^j}^{p^{\Lambda j} \Lambda^j} = (1 - \tau^j) (\Phi_\ell^j + \Phi_k^j) - \phi^j a^j + \Xi^j + T^j \quad (8)$$

where

$$R^{hj} = \max \left(p^h (1 - \tau^j) \left(\frac{r^k}{p^k} - r^h \right), 0 \right) \quad (9)$$

$$R^{mj} = \max \left((1 - \tau^j) \left(\frac{r^k}{p^k} - r^m \right), 0 \right) \quad (10)$$

$$\Phi_k^j = \frac{r^k}{p^k} a^j + \mathbb{1}_c^j (1 - \tau_\pi) 0.3 \pi^j \quad (11)$$

respectively denote the opportunity cost of holding housing (R^{hj}) or deposits (R^{mj}) rather than equity capital, and Φ_k^j is an alternative measure of capital income. In addition, $\Xi^j = \dot{p}^k k^j + \dot{p}^h h^j$

captures the equity and housing valuation gains.¹⁰ The treatment of capital gains only matters in the dynamic setting and is explained later on. This reformulation has the major advantage of reducing the program of household j to the following one-asset problem:

$$\begin{aligned} \max_{\Lambda^j, a^j} \int_0^\infty e^{-\rho_j t} \left\{ \log \Lambda^j - \frac{(\ell^j)^{1+\zeta}}{1+\zeta} + \mathbb{1}_{\rho^j=\rho_L} \log(a^j + \psi) \right\} dt \\ \text{s.t. } \dot{a}^j + P^{\Lambda j} \Lambda^j = (1 - \tau^j) \left(\Phi_\ell^j + \Phi_k^j \right) - (\phi^j + \gamma) a^j + \Xi^j + T^j \\ a^j > 0 \end{aligned} \quad (12)$$

The impact of changes in the environment on portfolio compositions is entirely encapsulated in P_Λ^j and P^j while, as explained below, m^j and h^j are determined *after* Λ^j , ℓ^j , and \dot{a}^j have been chosen by households. Note that the above dynamic problem also solves for the endogenous labor supply decisions of workers, implying

$$\ell^j = \left[\frac{(1 - \tau^j) (1 - \tau_\ell^j) (1 + \nu) (1 - \mathbb{1}_e^j) w^j}{P^{\Lambda j} \Lambda^j} \right]^{\frac{1}{\zeta}} \quad (13)$$

Workers' labor supply depends positively on the after-tax real wage and negatively on individual taxes through the substitution effect, where $1/\zeta$ is the elasticity of labor supply. Labor supply also varies negatively with aggregate expenditure Λ^j through a standard wealth effect.

2.1.4 Endogenous portfolio choice

Once the above dynamic problem is solved, solving the problem of choosing the composition of Λ^j subject to the relative costs of the three expenditure categories is static and writes:

$$\begin{aligned} \min_{c^j, h^j, m^j} (1 + \tau_c) c^j + R^{hj} h^j + R^{mj} m^j \\ \text{s.t. } \left((1 - \chi)^{\frac{1}{\mu}} \left((1 - \kappa)^{\frac{1}{\lambda}} (c^j)^{\frac{\lambda-1}{\lambda}} + \kappa^{\frac{1}{\lambda}} (h^j)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1} \frac{\mu-1}{\mu}} + \chi^{\frac{1}{\mu}} (m^j)^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} = \bar{\Lambda}^j = \text{cst} \end{aligned}$$

¹⁰This term stems from the fact that $\dot{a}^j = \dot{p}^k k^j + p^k \dot{k}^j + p^h \dot{h}^j + \dot{p}^h h^j + \dot{m}^j$.

and gives the following decision rules for deposits m^j , the stock of housing h^j and c^j , the consumption of non-durable goods:

$$m^{dj} = \max \left(\min \left(\chi \left(\frac{P_{\Lambda}^j}{R^{mj}} \right)^{\mu} \Lambda^j, a^j \right), 0 \right) \quad (14)$$

$$h^{dj} = \min \left(\kappa (1 - \chi) \left(\frac{P^j}{R^{hj}} \right)^{\lambda} \left(\frac{P_{\Lambda}^j}{P^j} \right)^{\mu} \Lambda^j, \frac{a^j - m^{dj}}{p^h} \right) \quad (15)$$

$$h^j = h^{dj} \text{ if } h^{dj} \geq h^{\min} \text{ and } 0 \text{ if } h^{dj} < h^{\min} \quad (16)$$

$$m^j = m^{dj} + \max \left(h^{\min} - h^{dj}, 0 \right) \quad (17)$$

$$c^j = (1 - \kappa) (1 - \chi) \left(\frac{P^j}{1 + \tau_c} \right)^{\lambda} \left(\frac{P_{\Lambda}^j}{P^j} \right)^{\mu} \Lambda^j \quad (18)$$

where $P_{\Lambda}^j = \left((1 - \chi) (P^j)^{1-\mu} + \chi^{\frac{1}{\mu}} (R^{mj})^{1-\mu} \right)^{\frac{1}{1-\mu}}$ and $P^j = \left((1 - \kappa) (1 + \tau_c)^{1-\lambda} + \kappa (R^{hj})^{1-\lambda} \right)^{\frac{1}{1-\lambda}}$. Equation (14) gives the demand for deposits before the housing decision m^{dj} and is bounded above by a^j and below by zero. Equation (15) gives the demand for housing h^{dj} and is bounded above by the amount of expenditure remaining to allocate after the demand for deposits has been satisfied, and below by zero. As described by Equation (16), whenever this housing demand is less than the minimal size of housing units, *i.e.* $h^{dj} < h^{\min}$, the effective housing demand is zero and the corresponding expenditure is allocated to deposits m^j . Finally, Equation (18) gives the demand for the consumption of non-durable goods. Overall, the above decision rules show that expenditure or demand in the three categories are increasing in aggregate expenditure Λ^j , decreasing in their weight in preferences and decreasing in their relative prices. For instance, a fall in the relative price of deposits or housing — a fall in the opportunity cost R^{mj} or R^{hj} triggered by a drop in capital equity returns r^k — raises households demand for these items: all else equal, households rebalance their portfolios and choose to hold more deposits or demand more housing. Finally, using the definition of $a^j = p^k k^j + p^h h^j + m^j$ gives the individual amounts of equity capital holdings k^j .

2.2 Firms

A representative firm produces an intermediate good y^m under perfect competition with the following technology

$$y^m = \xi k^{\alpha} \ell^{1-\alpha} \quad (19)$$

where ξ is a measure of total factor productivity, and sells it to the retailers at price φ . Let us define intermediate goods producer profits as

$$\pi^m = (1 - \tau_{\pi}) \left(\varphi \xi k^{\alpha} \ell^{1-\alpha} - w \ell - \delta k \right) - r^k k \quad (20)$$

where r^k is the rental rate of physical capital and $\delta \in [0, 1]$ is the depreciation rate of capital. Corporate profits are taxed at rate τ_π based on their total sales minus their wage bill with an allowance for depreciated capital. Their maximization implies the following optimal factor demands:

$$\alpha \frac{\varphi y^m}{k} = \frac{r^k}{1 - \tau_\pi} + \delta \text{ and } (1 - \alpha) \frac{\varphi y^m}{\ell} = w \quad (21)$$

A unit-size continuum of retailers indexed in i buys the intermediate good at price φ and differentiates it into varieties. Let $p(i)$ be the nominal price set by retailer i for its variety and $y^d(i) = (p(i)/p)^{-\theta} y$ the demand of this variety, where $\theta > 1$ is the (potentially varying) elasticity of substitution between varieties, and y the total demand for the final good. We omit the taxation rate of corporate profits since the latter does not affect first-order conditions of retailers. The optimal price $p(i)$ solves

$$\max_{p(i)} \pi(i) = \left(\frac{p(\omega)}{p} - \varphi \right) \left(\frac{p(\omega)}{p} \right)^{-\theta} y \quad (22)$$

Assuming symmetry across retailers ($p(\omega) = p$ and $y^d(\omega) = y = y^m$), the optimal pricing condition gives:

$$\frac{\theta}{\theta - 1} \varphi = 1 \quad (23)$$

2.3 Modeling the tax and transfers system: functional forms

The tradition in macroeconomic models is to pool the different taxes and monetary transfers together and consider a progressive tax schedule (see [Heathcote, Storesletten, and Violante \(2017\)](#) for a discussion). The latter usually features a level parameter and a progressivity parameter that both apply to the entire distribution of the tax base.¹¹

Leveraging the exceptional richness of the DINA series, we extend the standard approach as follows. First, we apply this approach for each type of tax and for monetary transfers *separately* based on their relevant tax bases (payroll taxes on labor income, income taxes on fiscal income, wealth taxes on wealth, and monetary transfers on fiscal income). Second, we consider varying level and progressivity parameters over the distribution of tax bases. More precisely, we split the distribution of each tax base in several segments and estimate parameters for each segment. Third, we estimate all the parameters of all the tax and transfer schedules for the year 1984 and each subsequent year.

Each progressive tax rate \mathcal{T} (income taxes, payroll taxes, wealth taxes, and monetary trans-

¹¹[Heathcote, Storesletten, and Violante \(2017\)](#) show that this simple functional form offers a good approximation of the tax and transfer system in the U.S. See also [Cagetti and De Nardi \(2009\)](#), [Kaymak and Poschke \(2016\)](#) or [Hubmer, Krusell, and Smith \(2020\)](#), among others, for applications focusing on inequality dynamics. [Hubmer, Krusell, and Smith \(2020\)](#) propose a more granular approach that fits a step-wise tax function on the distribution of personal income.

fers) is household-specific, and we assume the following functional form for each segment s :

$$\mathcal{T}_s^j = 1 - (1 - \bar{\mathcal{T}}_s) \left(\frac{\mathcal{B}^j}{\bar{\mathcal{B}}_s} \right)^{-\eta_s} \quad (24)$$

For each type of tax or transfer $\mathcal{T} = \{\tau, \tau_\ell, \phi, T\}$, the individual tax rate or transfer is described by a level parameter $\bar{\mathcal{T}}_s$ and a progressivity parameter η_s on segment s , where \mathcal{B} is the relevant tax base and $\bar{\mathcal{B}}_s$ its average value on the segment s .¹²

Appendix B presents the complete methodology and the fit for each tax and transfer of the model. It shows that the French tax and transfer system features very different and non-linear progressivity patterns for different tax bases and time periods.¹³

While the traditional approach may be successful in matching the overall level of tax progressivity, it overlooks its granularity. As a consequence, it may fail to account for the effects of changes in taxes over time on the relevant margins (consumption, savings and labor supply). In Section 3.3, we show that using our approach over the traditional approach improves the mapping between pretax and post-tax income distributions and is instrumental in matching the wealth distribution.

2.4 Government and market clearing

Next, we present the budget constraint of the government and the market clearing conditions.

First, the government uses the revenues from the different taxes to finance monetary transfers, the deficit of the pension and unemployment systems, as well as an exogenous amount of public good and services g . Its budget constraint yields

$$\begin{aligned} g + \nu w \ell + \underbrace{\int_j \Omega^j T^j dj}_{\text{Transfers}} &= \underbrace{\int_j \Omega^j m^j dj}_{\text{Deposits (supply)}} + \underbrace{\int_j \Omega^j \phi^j a^j dj}_{\text{Capital tax}} + \underbrace{\tau_\pi \left(r^k k + \int_j \mathbb{1}_e^j 0.3 \pi^j dj \right)}_{\text{Corporate tax}} + \underbrace{\tau_c \int_j \Omega^j c^j dj}_{\text{Consumption tax}} \\ &+ \underbrace{\int_j \Omega^j \tau_\ell^j \left((1 + \nu) w^j \left(1 - \mathbb{1}_e^j \right) \ell^j + \mathbb{1}_e^j 0.7 \pi^j \right) dj}_{\text{Social security}} + \underbrace{\int_j \Omega^j \tau^j \left(\Phi_\ell^j + Y_k^j \right) dj}_{\text{Income tax}} \end{aligned} \quad (25)$$

where Ω^j is the distribution of households with $\int_j \Omega^j = 1$

Second, the market clearing conditions of the asset and labor markets are

$$k = \int_j \Omega^j k^j dj = \int_j \Omega^j \left(\frac{a^j - p^h h^j - m^j}{p^k} \right) dj \quad (26)$$

$$\ell = \int_j \Omega^j \left(1 - \mathbb{1}_e^j \right) \left(w^j / w \right) \ell^j dj \quad (27)$$

¹²Payroll tax rates τ_ℓ^j are computed on labor income (see Equation (2)), income tax rates τ^j and monetary transfers T^j are computed on the total fiscal income $\Phi_\ell^j + Y_k^j$, and wealth tax rates ϕ^j are computed on a^j .

¹³See Figures 14 to 17 in Appendix B.

and $e = \int_j \Omega^j \mathbb{I}_e^j dj$ is the proportion of entrepreneurs in the economy. These conditions ensure that the goods market clearing condition is met by Walras' law.¹⁴ Total pretax profits are given by $\pi = y/\theta$, and are fully redistributed to the entrepreneurs so that the amount of profit per entrepreneur is

$$\pi^j = \frac{y}{e\theta} \quad (28)$$

Finally, aggregate output, the real wage and the return on capital can be respectively expressed as:

$$y = \xi k^\alpha \ell^{1-\alpha}, w = (1-\alpha) \frac{\theta-1}{\theta} \frac{y}{\ell} \text{ and } r^k = (1-\tau_\pi) \left(\alpha \frac{\theta-1}{\theta} \frac{y}{k} - \delta \right) \quad (29)$$

2.5 Income concepts

Before we turn to the calibration and results, we define the different concepts of income we use, since their distribution are the basis of many objects we track in the paper. In line with the Distributional National Account literature, we use three basic income concepts in our analysis: pretax income, disposable income, and post-tax income. By definition, aggregate pretax and post-tax income are both equal to national income.¹⁵ A full description of the income concepts is presented in Appendix A.

Pretax income is our benchmark concept to study the distribution of income before government intervention. It is defined as the sum of all income flows going to labor and capital, after taking into account the operation of the pension and unemployment insurance systems, but before taking into account other taxes and transfers. That is, we deduct pension and unemployment contributions and add pension and unemployment distributions. To recover the concept of pretax income in our model, we reassign corporate taxes, non-contributive payroll taxes, and consumptions taxes to the labor and capital incomes of households.¹⁶

Disposable income is defined as pretax income minus all forms of taxes plus monetary transfers (T^j).

Post-tax income is defined as the sum of all income flows going to labor and capital, after considering all forms of government interventions. It is equal to disposable income plus in-kind transfers and collective consumption expenditure net of the government balance budget (g in our model, rebated on a lump-sum basis).

¹⁴The latter reads:

$$c + \dot{k} + \delta p^k k + \dot{h} + g = y + r^h p^h h + r^m m$$

¹⁵National income is defined as GDP minus capital depreciation plus net foreign income, following standard national accounts guidelines (SNA 2008).

¹⁶Following the convention of national accounts, consumption taxes are already deducted before the value added is used to remunerate factors of production (unlike direct taxes). Therefore, they must be added to labor and capital income in order to reach a consistent pretax income concept. See Appendix A for more details.

3 Calibration and stationary equilibrium

We solve the model in two steps. The first step consists in finding a stationary equilibrium, including a stationary distribution of asset holdings, a composition of portfolios, and policy functions over an asset grid a^j where we have imposed that all variables of the model are constant.¹⁷ We consider the economy to be in the stationary equilibrium in 1984 and use French data for this year to calibrate the model. The second step, used in the dynamic simulations, solves for the transition dynamics using a non-linear algorithm with a variety of exogenous drivers to analyze their effects on aggregate and distributional dynamics. The details of both steps are given in Appendix C.

3.1 Calibration

The model is calibrated at an annual frequency using data or targeting data moments pertaining to the French economy in 1984, the time period before which inequalities start rising in the data. For some parameters, there is a direct mapping between the model's moment and the data. For other parameters, the mapping is too complex and we use a minimum distance method to set the parameters to the values that best fit the moments. In any case, the stationary equilibrium involves constant housing and equity prices, *i.e.* $p^h = p^k = 1$.

Earnings and transition probabilities. The growth rate of productivity is set to capture the average growth rate of national income per capita over the period, *i.e.* $\gamma = 0.01$. The AR1 process for labor log-earnings z^j is discretized over three states using Rouwenhorst's method. The persistence and volatility parameters are set to match key moments of the distribution of pretax income using minimum distance methods (see dedicated paragraph below). This yields $\rho_z = 0.6757$, which is the equivalent of an AR(1) process of 0.3243 and a volatility of $\sigma_z = 0.3270$.¹⁸ In addition, the presence of endogenous labor supply increases the dispersion of labor income in comparison of models with inelastic labor supply. We assume that a dynasty can switch from patient to impatient with a symmetric probability of 1/75, which corresponds to the life expectancy (75 years) observed in France in 1984. Symmetry in switching patience types implies that the population — workers and entrepreneurs — is split in half between patient and impatient households. The two discount factors ρ_L and ρ_H , the probability of becoming a entrepreneur and the probability of losing the status do not map directly into observable moments, so they are also chosen to match distributional and aggregate moments.

Preferences. We assume a unit elasticity of substitution between deposits and other types of expenditure, $\mu = 1$, and a unit elasticity of substitution between housing and non-durable goods, $\lambda = 1$. The weights of deposits χ , housing κ and the scale parameter for wealth in utility

¹⁷With balanced growth, this implies that all quantities grow at the exogenous rate of labor productivity γ .

¹⁸While these values suggest less persistence and more dispersion than usual values in the macro literature they should not be directly compared to the values used by models without entrepreneurs, as the introduction of entrepreneurs changes the distribution of income and wealth in significant ways, as shown by our counterfactual experiments.

ψ are set to match observed moments using a minimum distance method. As shown by Francis (2009), the latter is critical in matching top percentiles of the wealth distribution. Keeping in mind that h^j denotes the amount of housing net of debt, the minimum size of a housing unit is h^{min} and represents the minimal amount of savings required to take a mortgage and actually buy a housing unit. As such, we set $h^{min} = 0.5y$, that is, half the average yearly income. Finally, in line with evidence by Chetty et al. (2011) on intensive margin adjustments of labor supply, we impose a Frisch elasticity of $1/\zeta = 1/2.5 = 0.4$.

Firms and pretax asset returns. Recent micro evidence for the manufacturing sector in France by Bauer and Boussard (2020) suggest high markups, around 38.4% in 1984. However, aggregate markups also take into account (lower) markups in the service sector, and no markup at all in the public sector.¹⁹ Hence, we assume a lower aggregate markup of 10%, implying $\theta = 11$, which is close to the estimates of Kaplan, Moll, and Violante (2018). Data from national accounts point to an aggregate labor share of 0.75 in 1984 which implies a capital elasticity $\alpha = 0.25$. The depreciation rate of capital is taken directly from national accounts data: $\delta = 0.1128$. The real interest rates on deposits and housing in 1984 were $r^m = 0.009$ and $r^h = 0.028$.²⁰ As already mentioned, the initial relative prices of housing and equity are normalized to $p^h = p^k = 1$. In equilibrium, the resulting net rental rate of capital is $r^k = 0.0415$.

Government. Our calibration focuses on effective — not statutory — tax rates. Using data from national accounts, the effective consumption tax (VAT) rate is $\tau_c = 0.2234$. Similarly, the effective corporate tax rate is $\tau_\pi = 0.0902$, the amount of government expenditure on goods and services is $g/y = 0.2934$ and the excess in replacement income is $\nu = 0.0488$. To model the monetary transfers and each progressive taxes (payroll taxes, wealth taxes, income taxes), we rely on Equation (24) and estimate a level parameter and a progressivity parameter for each segment of the corresponding tax base distribution. This estimation relies on the French DINA series by Bozio et al. (2021), which provide detailed annual series of the joint distribution of pretax income, post-tax income and wealth, and is broken down by income and tax categories. See Appendix B for a complete presentation of the methodology and the fit for each tax and transfer of the model.

Moments matching. The following parameters of the model are set to match key distributional and aggregate moments from the data. The discount factors for patient and impatient households ρ_L and ρ_H , the probability of becoming an entrepreneur and the probability of losing the status, the persistence and variance of the labor income risk process and the following preference parameters χ (deposits), κ (housing), and ψ (wealth in utility) are all set together to match empirical moments. Our target moments are the following: the bottom 50%, middle 40%, top 10%, and top 1% shares of both pretax income and wealth, the aggregate shares of deposits (m/a) and housing (h/a) in total wealth and the wealth-income ratio a/y . The resulting dis-

¹⁹National accounting values public goods and services at their production costs.

²⁰For the return on housing, we consider the 5-years average return since housing decisions are made over a medium-term horizon.

count factors of households are $\rho_L = 0.0188$ and $\rho_H = 0.0752$. The probability of becoming a entrepreneur is 0.0008 every year and the probability of losing the status is 0.0446. Both values imply large precautionary savings for entrepreneurs at the top of the income distribution and a stationary proportion $e = 1.9\%$ of entrepreneurs in the economy. The remaining fraction of households $1 - e = 98.1\%$ are workers. The values of $\chi = 0.0194$ and $\kappa = 0.0394$ imply that deposits represent $m/a = 0.152$ of aggregate wealth (the exact value of the data) and housing $p^h h/a = 0.437$ (against 0.429 in the data). Finally, the wealth in utility scale parameter is $\psi = 67.1$. This calibration delivers a wealth-income ratio of 3.214, very close to the observed ratio (3.241). Parameter values are summarized in Table 1.

Table 1: Parameter values and initial values of exogenous variables.

Parameters		
Steady-state growth rate	$\gamma = 0.01$	(fixed)
Discount rate patient	$\rho_L = 0.0188$	(moments matching)
Discount rate impatient	$\rho_H = 0.0752$	(moments matching)
Persistence of labor earnings shock	$\rho_z = 0.3243$	(moments matching)
Variance of labor earnings shock	$\sigma_z^2 = 0.327$	(moments matching)
Annual prob. of switching $\rho_L \rightarrow \rho_H$	1/75	(life expectancy of 75 years)
Annual prob. of switching $\rho_H \rightarrow \rho_L$	1/75	(life expectancy of 75 years)
Annual prob. of becoming a entrepreneur	0.0008	(moments matching)
Annual prob. of becoming a worker	0.0446	(moments matching)
Weight of housing in utility	$\kappa = 0.0394$	(moments matching)
Elast. of subs. between housing and cons.	$\lambda = 1$	(fixed)
Indivisible housing parameter	$h^{\min} = 0.5 \times y$	(fixed)
Weight of deposits in utility	$\chi = 0.0194$	(moments matching)
Elast. of subs. between deposits and other exp.	$\mu = 1$	(fixed)
Elast. of labor supply	$1/\zeta = 1/2.5 = 0.4$	(CGDW (2011))
Wealth in utility scale parameter	$\psi = 67.1$	(moment matching)
Capital intensity parameter	$\alpha = 0.25$	(data – labor share)
Initial values of exogenous variables		
Capital depreciation	$\delta = 0.1128$	(data)
Markups	$\theta/(\theta - 1) = 1.1$	(KMV (2018))
Return on equity	$r^k = 0.0415$	(result)
Return on housing (excl. capital gains)	$r^h = 0.028$	(data)
Return on deposits (incl. capital gains)	$r^m = 0.009$	(data)
Gov. spending to output	$g/y = 0.2934$	(data)
Consumption tax rate	$\tau_c = 0.2234$	(data)
Corporate tax rate	$\tau_\pi = 0.0902$	(data)
Excess in replacement income	$\nu = 0.0488$	(data)
Progressive tax rates and transfers	See Figure 4	(see Appendix B)

Our calibration of the model delivers stationary distributions of income and wealth that reproduce several features of the data in 1984, as shown in Table 2. The model delivers an almost perfect match of the bottom 50%, middle 40%, top 10%, and top 1% shares of the pretax income and wealth. Further, while the model does not target any of the post-tax shares of income, it matches them remarkably well, highlighting our ability to capture key redistributive features of

the French tax and transfer system.

Table 2: Moments from the data (1984) vs. model.

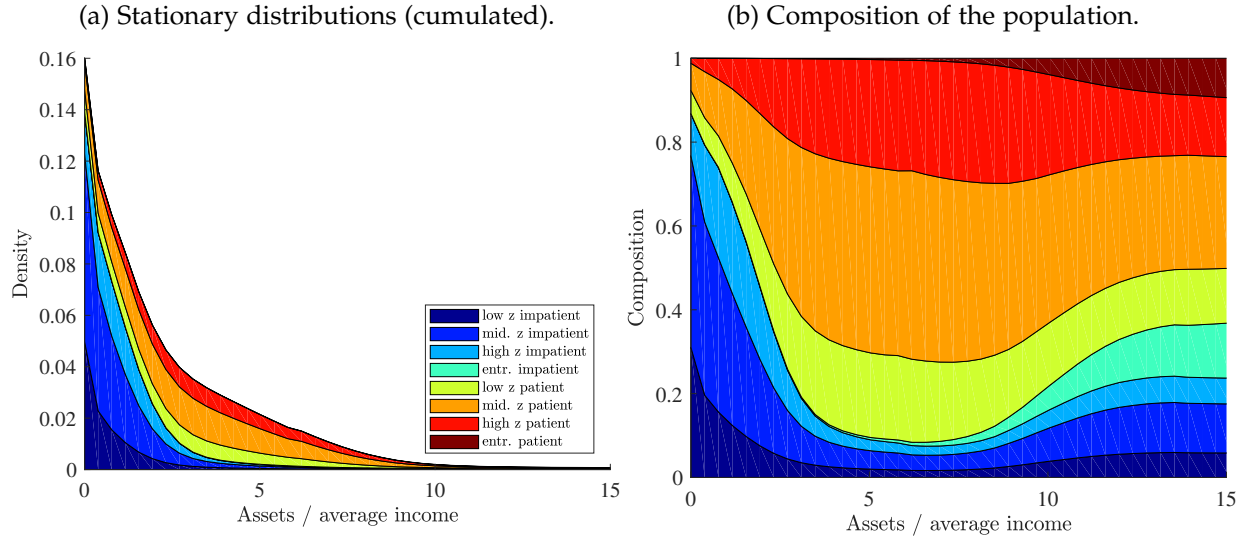
	Data (1984)			Model (1984)		
	Pretax	Post-tax	Wealth	Pretax	Post-tax	Wealth
Bottom 50%	0.230	0.337	0.081	0.231	0.331	0.081
Middle 40%	0.486	0.450	0.406	0.487	0.460	0.405
Top 10%	0.283	0.214	0.513	0.283	0.210	0.514
Top 1%	0.070	0.046	0.160	0.070	0.045	0.160
Share of deposits in agg. wealth		0.152			0.152	
Share of housing in agg. wealth		0.429			0.437	
Wealth to income ratio		3.241			3.214	

Note: Bold numbers are not targeted.

3.2 Initial stationary equilibrium

The calibration gives rise to the stationary distribution of households depicted in Figure 1.

Figure 1: Stationary distributions

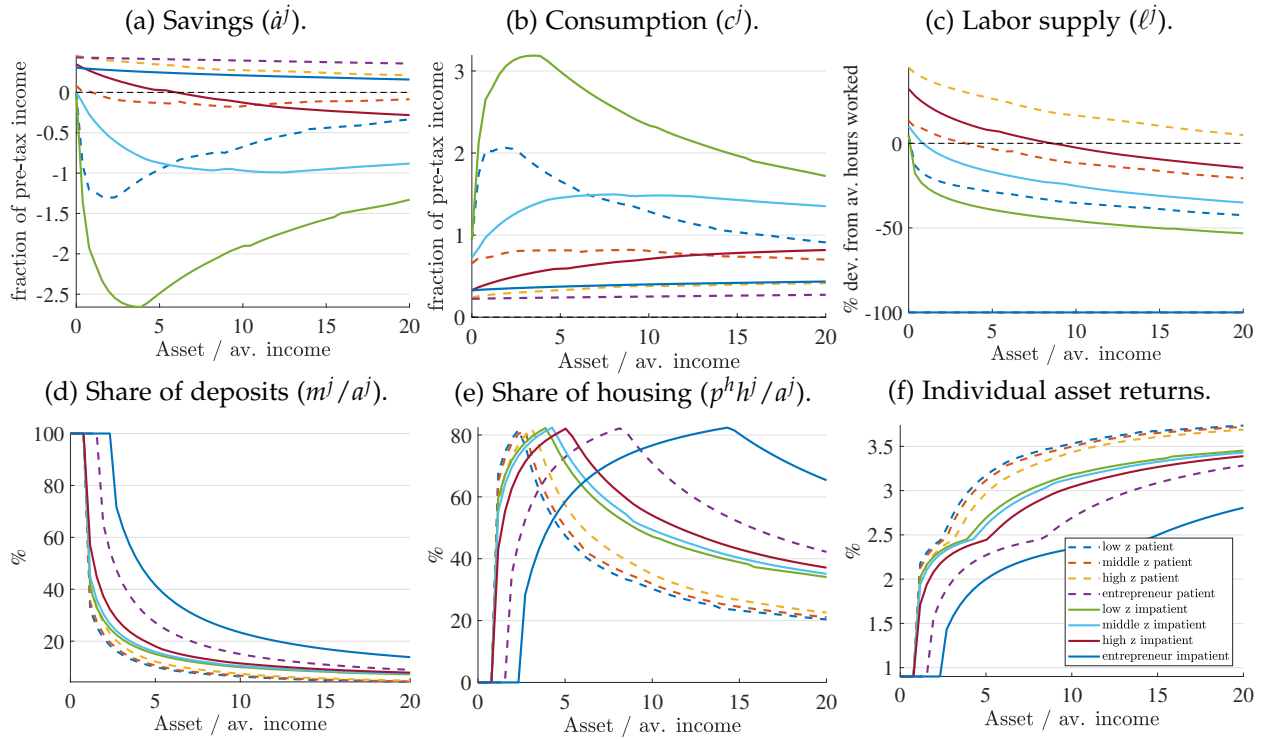


As expected, the left panel of Figure 1 shows that the stationary distribution is highly skewed to the left and fat-tailed at the right. The detailed distribution shows that asset holdings increase with the level of productivity, and with the degree of patience. Low-wage households hold less asset than middle-wage households, who hold less than high-wage households and entrepreneurs. Further, within a given type, impatient households are more concentrated at the left of the distribution — hold fewer assets — than patient households. The transition probabilities implied by our labor income process and the transition probabilities towards the status of entrepreneur determine the respective stationary proportions of household types: low-wage and

high-wage households each represent 24.5% of the population, middle-wage households make 49%, and entrepreneurs represent the remaining 1.9%. While they are small in numbers, entrepreneurs are extremely rich in terms of income because they receive the aggregate profits of firms. In addition, they precautionary-save a lot because the probability of losing the status is much larger than the probability of gaining the status. Hence, they represent a large share of top wealth owners, as shown by the right panel of Figure 1.

The model generates the policy functions depicted in Figure 2. Panel (a), (b) and (c) show the savings, consumption and labor supply schedules, Panel (d) and (e) the household-specific holdings of deposits and housing and Panel (f) the household-specific returns on wealth.

Figure 2: Policy functions



Panel (a) of Figure 2 shows that the largest savers in the model are entrepreneurs. Given the low probability of becoming an entrepreneur and the relatively large probability of losing the status, a strong precautionary motive drives them to save up to 50% of their (large) disposable income. Workers also save to self-insure against earning risk, that is, the risk of transiting to lower levels of productivity and income. Hence, the second largest savers are the high and middle productivity workers. Saving rates are decreasing in wealth levels because once households have reached their target amount of precautionary saving, they stop saving. In addition, the graphs clearly show that within productivity types, saving rates are larger for patient households than for impatient households. Hours worked are driven by the substitution effect — higher wages through higher productivity induce more labor supply — and by the wealth effect —

higher income and wealth induce less labor supply. Panel (c) shows that the substitution effect dominates for any given level of wealth and high-productivity workers supply more labor than middle and low productivity workers. Second, labor supply decreases with wealth for all types of workers. Third, due to the higher convexity of the marginal utility of Λ^j at low levels of Λ^j , low-and middle-productivity workers supply almost as much labor as high-productivity workers at low levels of assets, but the wealth effect is stronger for these workers, as their supply of labor decreases faster with wealth compared to high-productivity workers.

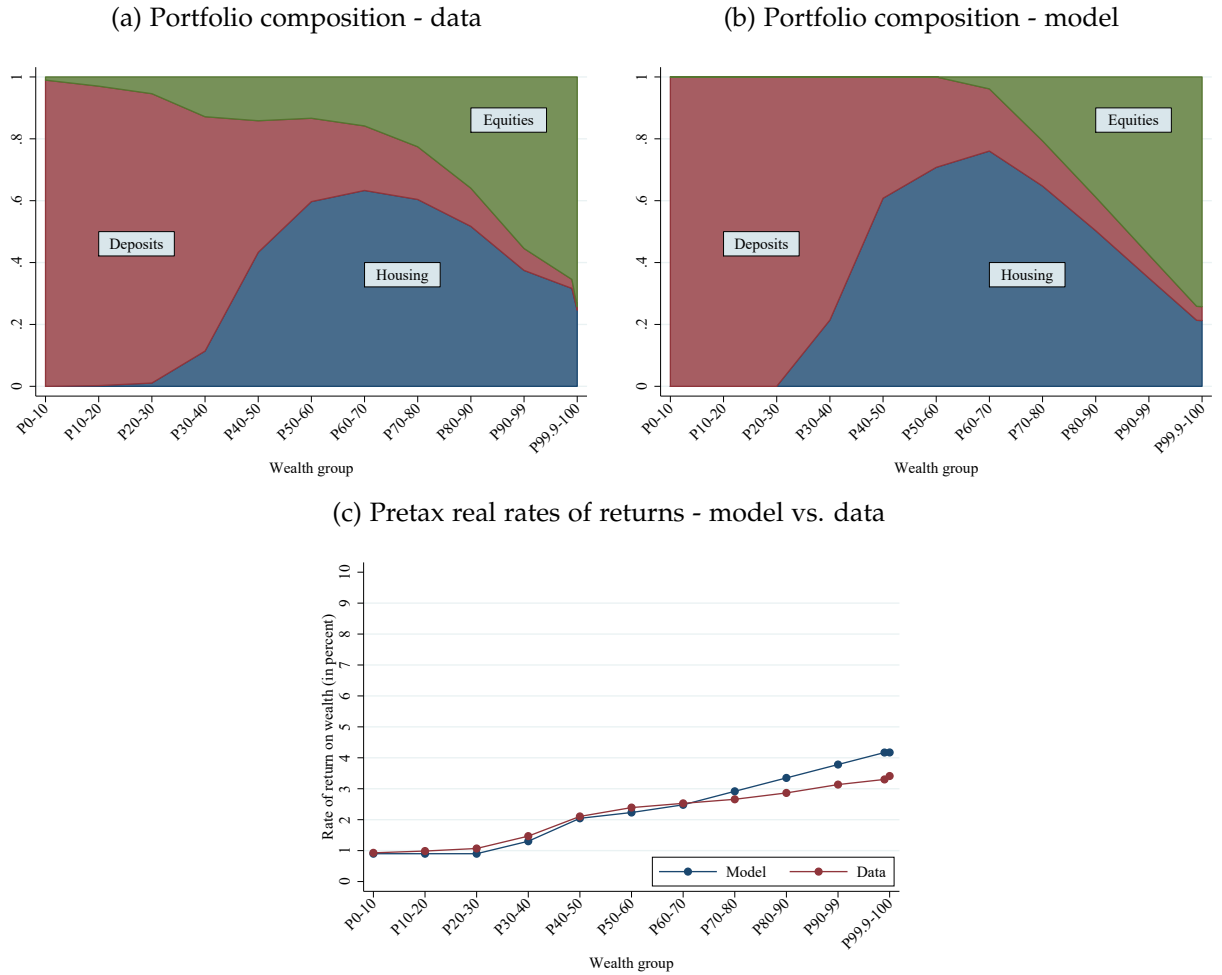
Our model not only considers wealth accumulation but also features three assets, which allows us to track the individual composition of portfolios. Panels (d) and (e) of Figure 2 report the shares of deposit and housing by wealth levels. They show that households at the bottom of the asset grid do not save enough to reach the housing threshold, and therefore keep their wealth in the form of liquid deposits. When households save enough to buy housing, they allocate almost all of their wealth to net housing before diversifying their portfolios and buying equity capital. As a result of the varying composition of portfolios along the distribution of wealth, individual pretax returns are increasing in wealth (panel (f)). However, these individual policy functions are not yet weighted by the stationary distributions of households, and thus do not take into account the fact that there are almost no low-wage workers holding large amounts of assets, and almost no entrepreneurs at the bottom of the asset grid. By aggregating over households at the different levels of the asset grid, we obtain the aggregate patterns of returns and the portfolio compositions over the distribution of wealth depicted in Figure 3.

Figure 3 shows how portfolio composition varies along the wealth distribution. Clearly, households at the bottom of the wealth distribution hold only deposits. Above the bottom 20th percentile, households hold an increasingly large (up to 75%) share of their wealth in the form of housing, fewer deposits and the rest in equity. Above the 70th, the share of housing starts declining, the share of deposits continues to shrink, and the share of equity increases to reach 75% for the top 0.1% of the wealth distribution. These equilibrium portfolio compositions produce increasing returns along the wealth distribution, as has been found in the data.²¹ Since deposits carry the lowest returns (0.9%), households holding only deposits receive low returns. When portfolios start including housing, which carries a larger return (2.8%), portfolio returns start increasing. Returns further increase when households start holding equity, which carries the largest return (4.15%). The differential composition of portfolios explains that top wealth owners receive returns that exceed those of bottom wealth owners by more than 3 percentage points.

Figure 4 reports the amount and composition of taxes paid by households ranked by income and wealth groups as a fraction of their pretax income. The consumption tax is regressive given that poorer households consume a larger fraction of their disposable income. Social security taxes are also regressive at the top of the income distribution. Our results further show a strong

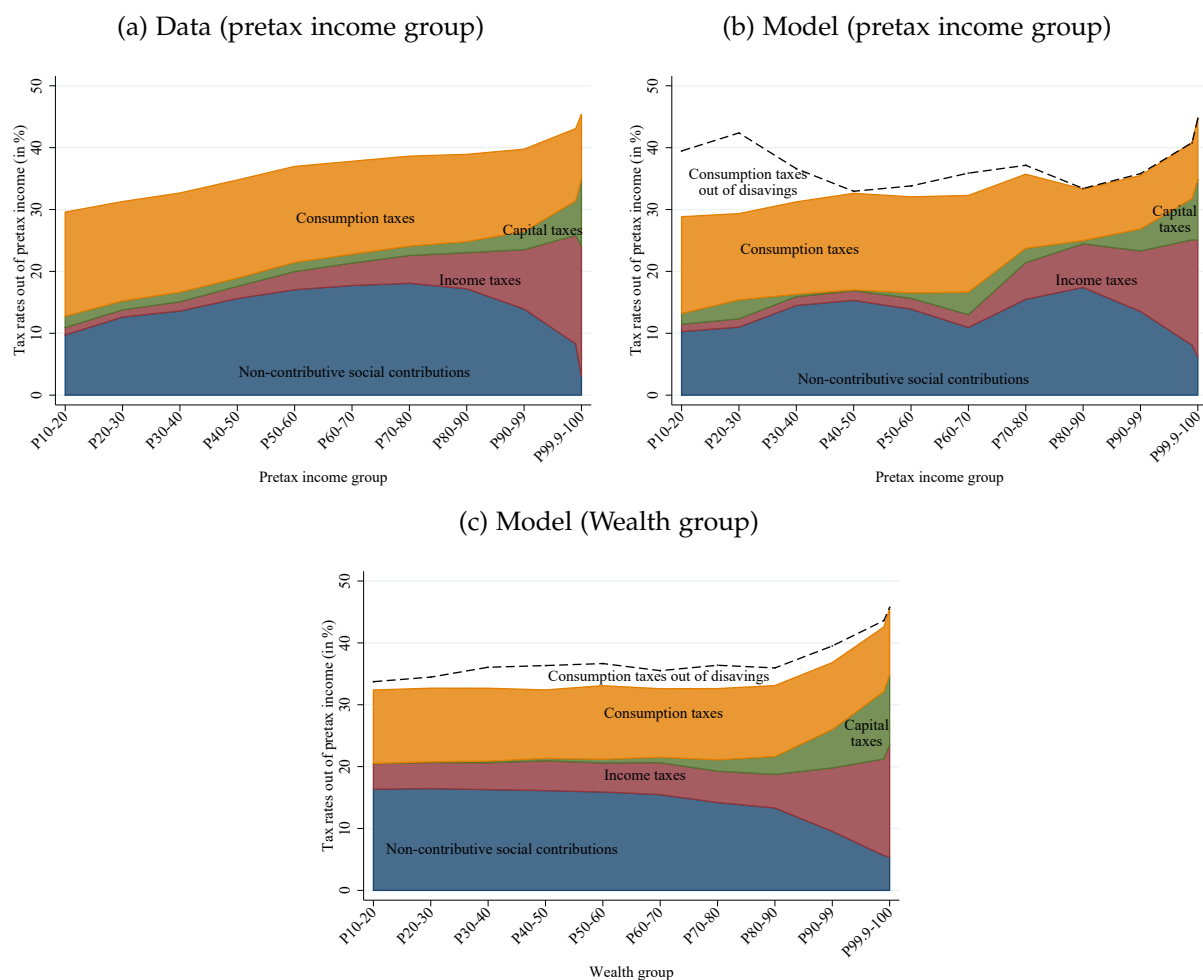
²¹See [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#) for similar evidence from France, and [Cao and Luo \(2017\)](#) or [Xavier \(2020\)](#) for evidence based on U.S. data.

Figure 3: Portfolio composition and returns among wealth groups in 1984.



Notes: Series from Panel (a) come from [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#). In Panel (c), real rates of returns are computed by weighting each asset-specific real rate of returns (housing, equities, and deposits) by the proportion of each asset in the wealth of the group. Returns on deposits and housing are taken from the national accounts. Returns on equities come from the national accounts in the data series.

Figure 4: Taxes paid (in % of pretax income) by wealth or pretax income groups, France 1984.



Notes: Data from Panel (a) taken from [Bozio et al. \(2021\)](#).

Table 3: Alternative specifications

	Base.	Alternative (Δ with baseline level)							
	(1)	$\rho^L = \rho^H$ (2)	$\sigma_z^2 \sim 0$ (3)	$\theta \rightarrow \infty$ (4)	No WIU (5)	$\zeta \rightarrow \infty$ (6)	Flat (7)	One tax (8)	One asset (9)
Pretax inc.									
B50	0.23	+0.00	+0.16	+0.02	+0.00	+0.04	-0.00	+0.00	+0.02
M40	0.49	+0.00	-0.12	+0.04	-0.00	-0.02	-0.01	-0.00	-0.00
T10	0.28	-0.00	-0.04	-0.06	-0.00	-0.01	+0.01	+0.00	-0.02
T1	0.07	+0.00	+0.00	-0.05	-0.00	+0.00	+0.01	+0.00	-0.00
Post-tax inc.									
B50	0.33	+0.00	+0.10	+0.01	+0.00	+0.02	-0.03	-0.03	+0.01
M40	0.46	+0.00	-0.07	+0.02	+0.00	-0.01	-0.01	-0.00	+0.00
T10	0.21	-0.01	-0.03	-0.03	-0.00	-0.01	+0.04	+0.04	-0.01
T1	0.05	+0.00	+0.00	-0.03	-0.00	+0.00	+0.01	+0.01	-0.00
Wealth									
B50	0.08	+0.03	-0.06	+0.03	-0.00	-0.01	-0.02	-0.02	-0.00
M40	0.40	-0.05	-0.15	+0.15	+0.04	-0.03	-0.12	-0.11	-0.00
T10	0.51	+0.03	+0.22	-0.18	-0.04	+0.04	+0.14	+0.13	+0.00
T1	0.16	+0.02	+0.06	-0.11	-0.01	+0.01	+0.07	+0.07	+0.03
Agg. ratios									
m/W	0.15	+0.01	-0.03	-0.01	-0.00	+0.00	+0.00	+0.02	-0.15
h/W	0.43	-0.06	-0.08	-0.04	-0.03	-0.02	+0.06	-0.01	-0.43
k/W	0.41	+0.05	+0.12	+0.05	+0.03	+0.02	-0.06	-0.02	+0.59
W/Y	3.21	-0.56	-0.75	-0.07	-0.25	-0.15	+0.34	-0.62	-1.64

Note: The table expresses the key moments targeted by our model. The first column refers to the baseline model and results, moments are expressed in levels. In the remaining columns, moments are expressed in difference from the baseline case. Column 2: no heterogeneity in discount factors. Column 3: no labor income risk. Column 4: no markups and thus no entrepreneurs. Column 5: no wealth as an argument of the utility function. Column 6: inelastic labor supply. Column 7: all progressive taxes and transfers are replaced by flat rates that equal the average rates paid in the baseline model. Column 8: all progressive taxes and corporate taxes are replaced by a single progressive income tax as usually done in macro models and we estimate unique parameters from the data for the level of taxation and the degree of progressivity; monetary transfers are rebated lump-sum. Column 9: one asset (capital) is considered and thus $\chi = \kappa = 0$ in the utility function.

progressivity of the income and asset tax schedule. The latter is especially progressive at the very top because of large levels of wealth and because of large returns on wealth, driven by larger shares of equity capital in portfolios.

3.3 Alternative specifications

Let us now investigate the contribution of our various assumptions to the results by reporting the distributions of income and wealth when we simplify or abstract from some of our assumptions. Table 3 reports the baseline moments in the first column and the difference between the alternative distribution and the baseline in the remaining columns. In particular, it allows us to study the role of the saving motive and of a progressive tax system in explaining the level of income and wealth inequality in 1984. Positive numbers indicate that the alternative overshoots the moments while negative numbers signal undershooting.

First, consider an alternative economy with identical discount factors, *i.e.* $\rho_L = \rho_H$ (Column (2) of Table 3), to quantify the importance of having heterogeneous discount factors. In this case, the discount factor is the average of the two discount factors of the baseline model. This alternative has little effect on the distribution of income, regardless of whether considering pretax or post-tax income, but has important implications for the wealth distribution. The middle 40% wealth share is lower than the baseline (and the data) by 5 percentage points (pp) while the top 10% and top 1% shares are respectively larger by 3pp and 2pp. With homogeneous discount factor rates, the saving rate of households in the upper middle of the distribution of income — mostly high-wage households — is too low, and higher income households — entrepreneurs — save too much. Heterogeneity in discount factors thus allows for a greater and more realistic dispersion of saving rates and offers a better match of the wealth distribution.

Second, Column (3) of Table 3 considers an economy with a negligible labor income risk by setting $\sigma_z^2 \sim 0$. This alternative economy shows the critical importance of labor income risk in matching the bottom 50% and middle 40% shares of income — the largest part of income for these households is labor income — and its role in shaping savings and the wealth distribution: since labor risk is negligible, workers save too little, which implies that the bottom 50% share of wealth is almost zero and that the middle 40% share of wealth is 15pp below its baseline value. In contrast, a much larger share of wealth is held by top earners (entrepreneurs) who face income risk. Without labor income risk, the top wealth shares are massively overstated while the bottom wealth shares are much too low.

The next alternative economy (Column (4) of Table 3) takes a seemingly opposite stance and considers the impact of having zero markups assuming $\theta/(\theta - 1) = 1$ by imposing $\theta \rightarrow \infty$. Doing so drives aggregate profits to zero, and thus extinguishes the primary source of income of entrepreneurs. Removing markups from the model redistributes income and wealth from the top to the bottom of the distribution. The bottom 50% and middle 40% pretax income shares are much larger (+2pp and +4pp respectively) than their baseline values, and the top 10% and 1% shares are much lower (-6pp and -5pp respectively). Because the status of entrepreneurs is risky and implies large precautionary savings, this movement is amplified for the wealth shares. We conclude that the presence of markups and entrepreneurs in the baseline model is central to match top income and wealth shares.

What if utility does not depend on wealth and therefore a saving motive is shut down? Column (5) of Table 3 reports the corresponding results. As expected, wealth in utility affects the distribution of wealth but not of income. The assumption of wealth in utility prevents the saving rate from decreasing too fast along with wealth. Its absence results in lower saving rates at the top of the wealth distribution and lowers the top 10% wealth share by 4pp and the top 1% wealth share by 1pp.

Assuming an inelastic supply of labor by setting $\zeta \rightarrow \infty$ (Column (6) of Table 3) slightly reduces pretax income inequalities among workers: the bottom 50% share of pretax income is

4pp higher while the middle 40% and top 10% shares are respectively 2pp and 1pp lower. When labor supply is elastic, all else equal, the substitution effect implies that workers with low wages — in the bottom 50% of the pretax income distribution — supply less labor than workers facing large wages — in the middle 40% and top 10%. Endogenous labor supply thus increases the dispersion of labor income. Shutting down this channel thus reduces labor income inequalities. The impact on post-tax income inequalities stems from the impact on pretax income inequalities, and the effect on wealth inequalities is relatively minor.

The next alternative answers a longstanding question: What is the effect of tax and transfer progressivity on pretax income inequality? Our alternative shows not much. However, the effects on wealth inequality are much larger. Column (7) of Table 3 reports the results when we apply the (weighted) average tax or transfer rate to all households uniformly. Switching to a flat system of taxation raises the top 10% and top 1% shares of pretax income by 1pp, lowers the middle 40% share by 1pp, and leave the bottom 50% share relatively constant. Overall, it implies a mild increase in pretax income inequality. Quite naturally, a flat tax system in the model fails to match the distribution of post-tax income. This is expected since the distribution of post-tax income results from a progressive system of taxation in the data. Noticeably, a flat tax and transfer system has much stronger effects on wealth inequality: the middle 40% wealth share falls by 12pp while the top 10% and top 1% wealth share are respectively 14pp and 7pp higher. These results are driven by a flat taxation of capital, which favors wealth accumulation for top savers.

The two last alternative distributions highlight our contribution against standard macroeconomic models by respectively looking at a less granular tax schedule and at a model with equity capital as the only asset. In the former case, all progressive taxes — except transfers — are pooled with the corporate tax into a single income tax. A single average rate $\tau = 0.3075$ and a single progressivity parameter $\eta = -0.0589$ are estimated from French fiscal data in 1984. In the latter case, we make equity capital the only possible vehicle of savings by imposing $\chi = \kappa = 0$. With a single income tax including all the sources of taxation, the model fails to replicate the distribution of pretax income, especially for the bottom 90% of the distribution. In addition, with only one tax, the progressivity of capital and capital income taxes is not accounted for properly, which substantially increases wealth inequality. Last, by definition, a one-asset model — abstracting from deposits and housing — can not capture the composition of portfolios. It also spectacularly fails to match the wealth-income ratio. Further, returns on wealth become uniform along the wealth distribution. This makes households at the bottom of the income and wealth distributions richer in terms of income through higher returns on savings. It also makes them poorer in terms of assets, as less savings are needed — given higher returns compared to the baseline model — to reach a given amount of self-insurance. Overall, regarding the income distribution, both effects compensate each other and the net effects are relatively negligible. Regarding the wealth distribution, it makes top wealth owners wealthier because these households save in any case, and the top 1% wealth share is 3pp above its baseline value.

4 Dynamic simulations

We now simulate our model using the following exogenous variables from 1984 to 2018. Because the Hamilton-Jacobi-Bellman equation is forward-looking, several current-period variables depend on their expected future path. We thus solve the model for an additional 30 years after 2018.

4.1 Exogenous variables

First, we consider a time-varying depreciation rate of capital δ , derived using national account data. The level of TFP is taken from the long-term productivity database of [Bergeaud, Cetto, and Lecat \(2016\)](#). Since the model already features labor productivity growth in the steady-state at rate γ , we feed the model with log-deviations of TFP from an HP-filtered trend with $\lambda = 5000$. We assume constant returns on housing (excluding capital gains) and deposits (including capital gains).²²

Second, we introduce exogenous variations in aggregate markups between 1985 and 2016 based on the estimated variations of [Bauer and Boussard \(2020\)](#). We take their reported percentage variations and apply them to the 1984 level of markups. As in [Eggertsson, Robbins, and Wold \(2021\)](#), rising market power can account for key observed aggregate dynamics such as changes in the labor share. In a model featuring heterogeneous agents, it can also account for a substantial fraction of the rise in income and wealth inequality.

Third, we introduce time-varying parameters in all taxes, transfers, government spending, and excess replacement income. For 1984, 1988, and 1994-2018, a set of estimates for all the tax schedules (or rates for the flat corporate profits and consumption taxes) are fed into the model, along with time-varying values for the government spending ratio g/y and the excess replacement income ratio ν . As our static analysis has already shown, both in the baseline case and through alternative specifications, the progressivity of French tax and transfer system is quantitatively important and shapes income and wealth inequality. Changes in taxes and transfers may thus substantially affect inequality dynamics over this period.

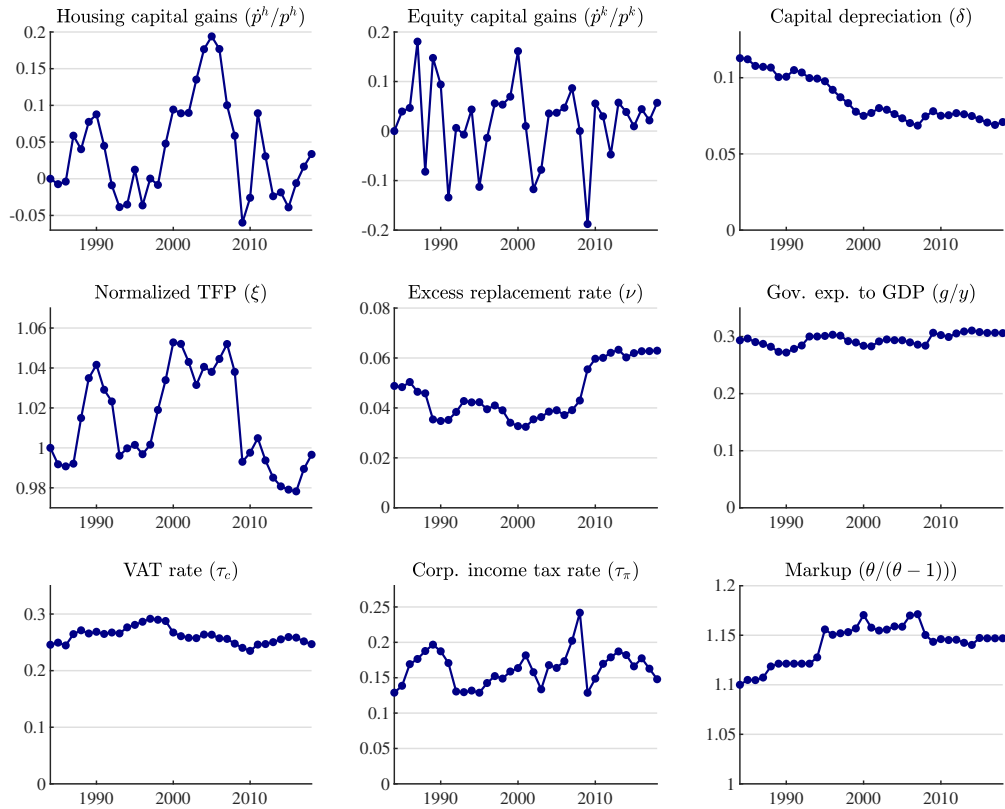
Last, we feed the model with housing capital gains \dot{p}^h/p^h , and equity capital gains \dot{p}^k/p^k . The treatment of capital gains is the following. The level of equity prices is taken as constant in the value function of households, while only a fraction (25%) of the increase in housing prices, reflecting the turnover of housing units, is taken into account by households. In addition, pure valuation effects, captured by Ξ^j in our model, are treated in line with what [Fagereng et al. \(2019\)](#) coin as “saving by holding”. That is, we assume $\Xi^j = 0$ in the optimization problem of households and reflate housing and equity amounts *after* optimization by the observed housing and equity capital gains based on their individual asset quantities and types. This increases both

²²Our results are robust to the introduction of time-varying paths for these two exogenous variables. Given their negligible role however, we choose not to include them in our simulations.

the left-hand side (\dot{a}^j) and right-hand side (Ξ^j) of the budget constraint of each household. Our approach thus neutralizes the potential wealth effects stemming from unrealized capital gains.

These four types of exogenous variables — except the tax progressive tax and transfer schedules, for which there are too many parameters to track — are all graphed in Figure 5. The model is solved for an additional 30 years after 2018. After 2018, when there are no data for exogenous variables anymore, the latter are assumed to remain equal at their 2018 values. Post-2018 equity and housing capital gains are assumed to be zero, *i.e.* asset prices are stabilized at their 2018 levels from 2018 onwards.

Figure 5: Exogenous variables



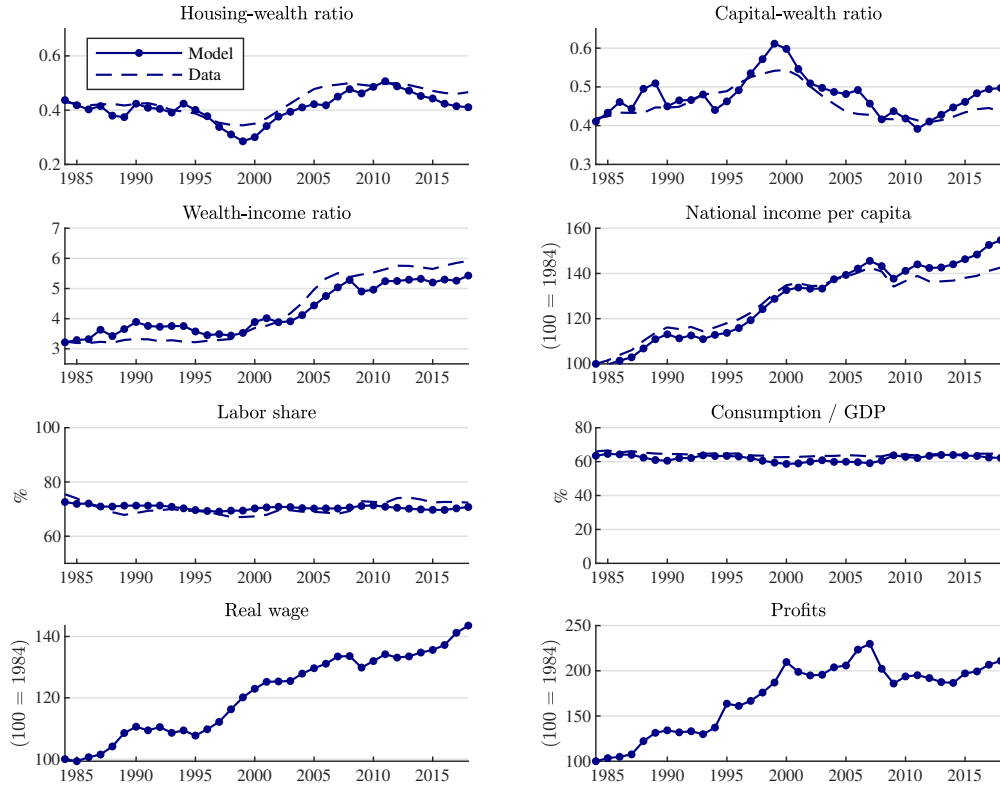
4.2 Results

Let us start by looking at the performance of our simulated model in replicating aggregate features of the data.

The top panels of Figure 6 report the aggregate housing-wealth ratio, the aggregate capital-wealth ratio, the aggregate wealth-to-income ratio, and national income per capita. The model accounts very well for the fall of the housing-wealth ratio from 1984 to 2000 and for its rise after 2000. An opposite movement — a rise until 2000 and then a fall — of the capital-wealth ratio is observed and well reproduced by the model. Further, the observed wealth-income ratio rises

from 3.2 in 1984 to almost 6 in 2018, an overall increase that our model matches closely. Finally, the dynamics of the national income are also matched very closely, an additional indication of the excellent performance of our model simulations.

Figure 6: Macroeconomic variables

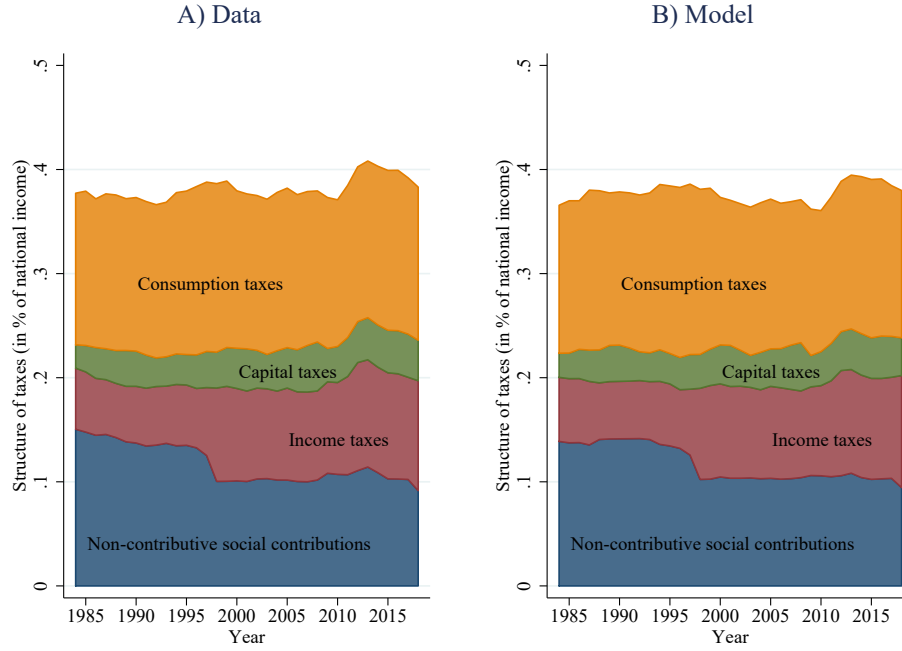


The bottom panels of Figure 6 report the dynamics of the labor share, the consumption-income ratio, and the normalized levels of the average real wage and aggregate profits. The model simulations track the observed dynamics of the labor share and of the consumption-income ratio very well. Finally, since our simulations are driven by a significant increase in markups, they display a larger increase in profits than in the average real wage.

Figure 7 reports the simulated and observed decomposition of taxes in percentage of national income over time. The model reproduces almost perfectly the evolution of the aggregate tax structure and of the aggregate tax level.

Finally, Figures 8 to 10 depict the performance of our simulated model in reproducing inequality dynamics. The model fits almost perfectly the evolution of pretax income and post-tax income shares for all groups (bottom 50%, middle 40%, top 10% and top 1%). The observed dynamics of wealth shares are also very well accounted by our model simulations for the bottom 50%, middle 40%, and top 10% wealth groups. For the top 1% wealth share, the overall increasing pattern is qualitatively well-reproduced but the model undershoots the large variations observed around 2000, resulting in a lower simulated top 1% wealth share in 2014 (21% vs. 24% in the

Figure 7: Structure of taxes



data).

Figures 8 to 10 highlight two main facts regarding inequality dynamics: (i) a rise in income inequality driven mostly by a significant increase in the top 1% income share from 7% in 1984 to 10% in 2019 (+46% over the 1984-2019 period) and (ii) a rise in wealth inequality driven mostly by a significant increase in the top 1% wealth share from 16% in 1984 to 23% in 2019 (+41% over the 1984-2019 period) at the expense of the bottom 50% wealth share, which decreases from 8% in 1984 to 3% in 2019 (-61% over the 1984-2019 period). We provide a deeper understanding of the key forces and driving mechanisms in the next section.

Figure 8: Pretax income inequalities

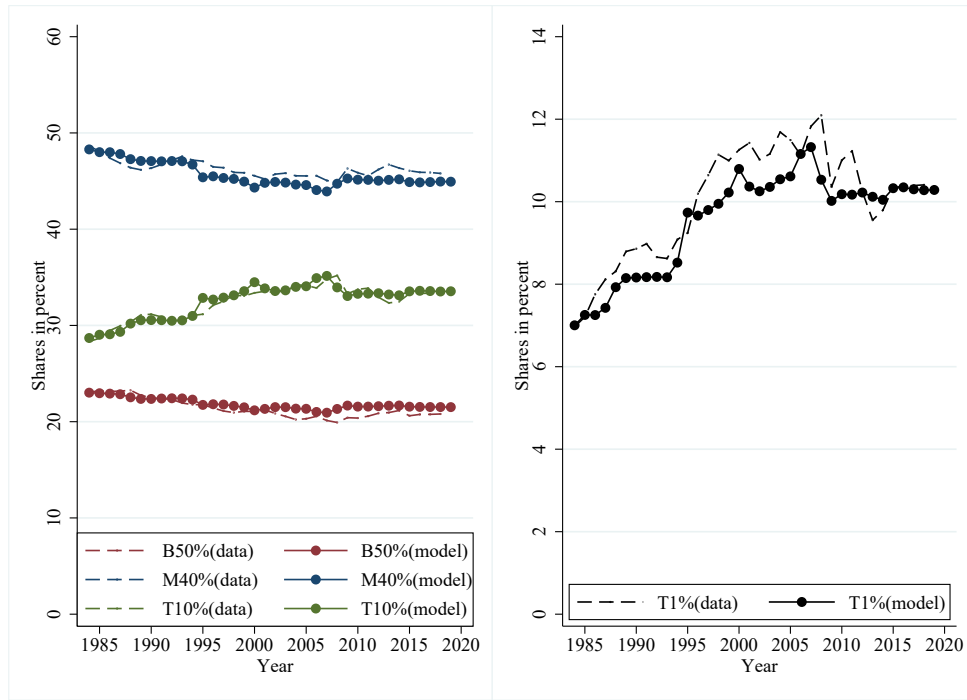


Figure 9: Post-tax income inequalities

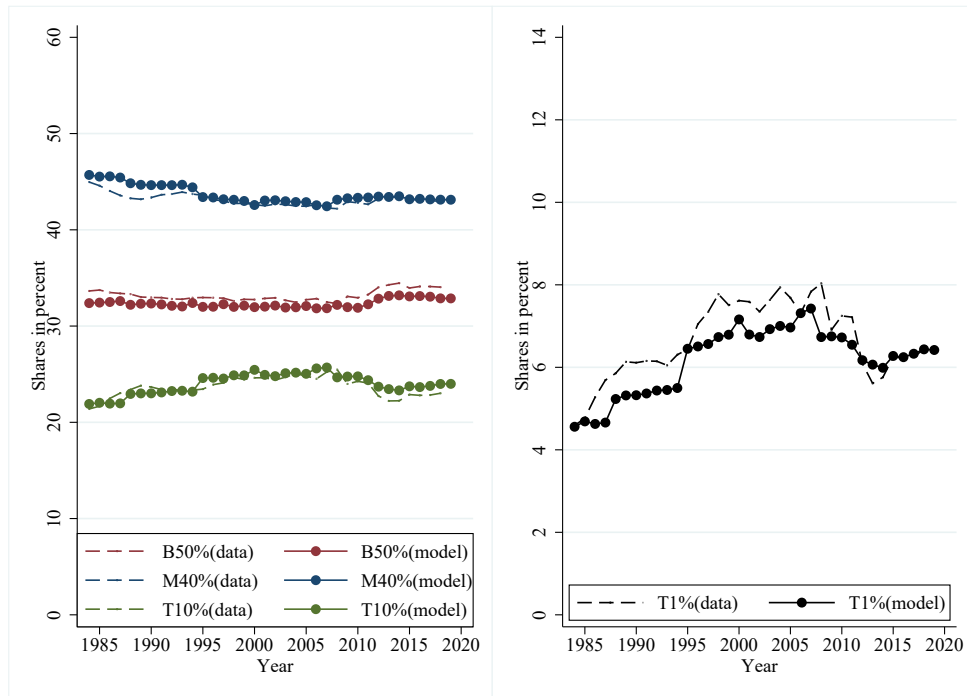
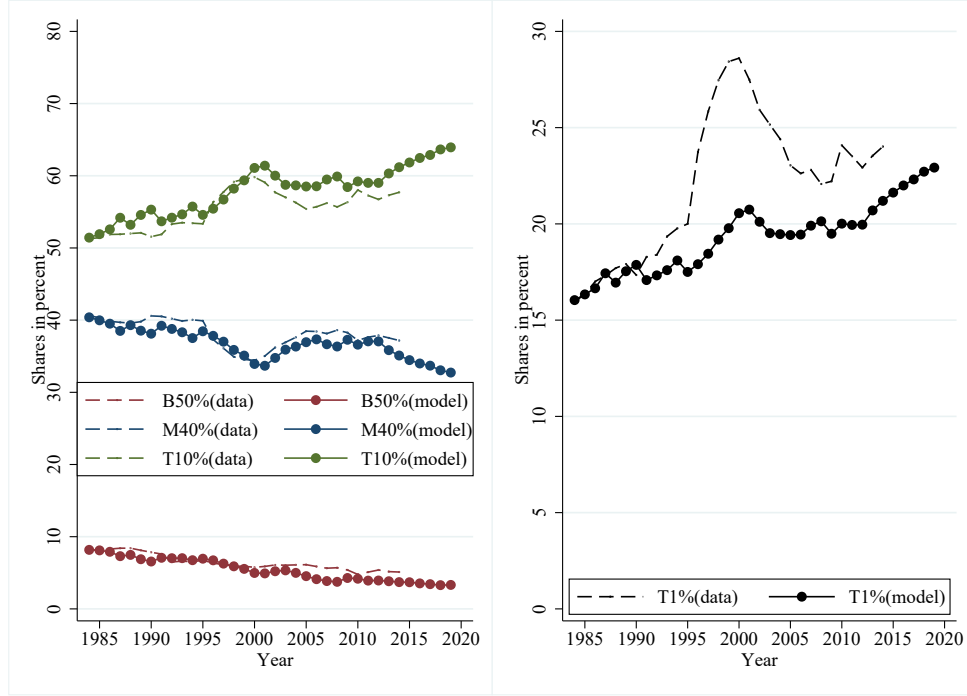


Figure 10: Wealth inequalities



5 Counterfactual dynamics

We now run a counterfactual analysis to shed light on how the different exogenous variables shape income and wealth inequality dynamics. To do so, we group exogenous variables in four different categories: capital gains, taxes and transfers, markups, and others market forces (capital depreciation rate and TFP). We then run counterfactual simulations assuming that one or several groups of driving forces remain constant and equal to their 1984 level over the 1984-2019 period. Finally, we compare the resulting income and wealth shares with our benchmark series to quantify the respective contributions of each group of driving forces to the evolution of income and wealth inequalities.

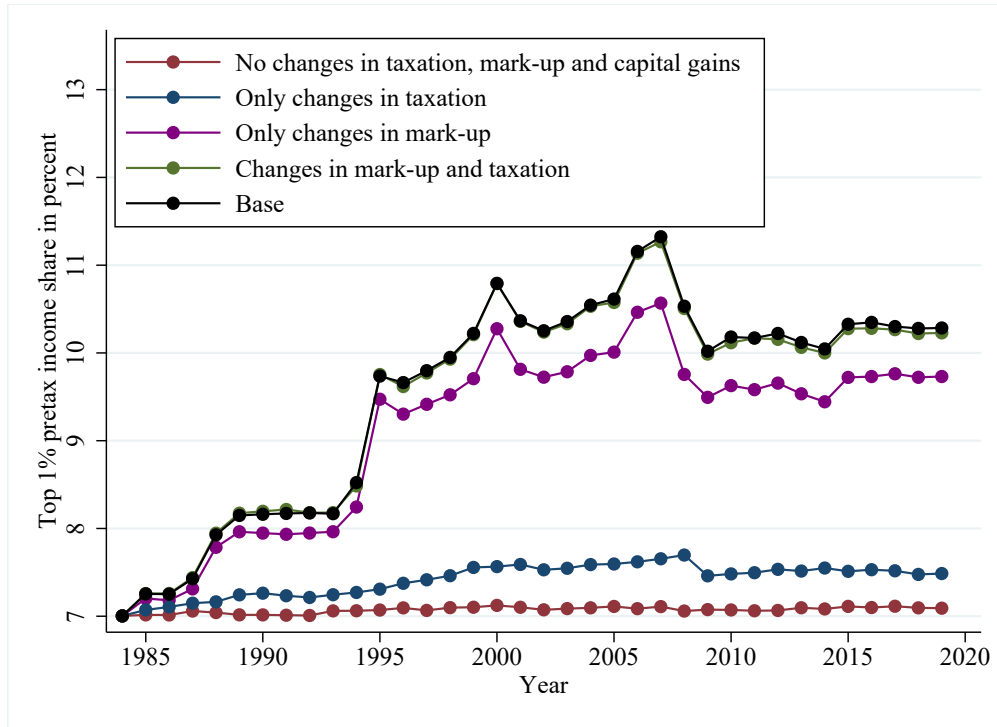
5.1 Understanding income inequality dynamics

For the analysis of income inequality dynamics, we focus on top 1% income share because it is the main driver of income inequality. Figure 11 displays the evolution of alternative top 1% income shares by scenario over the 1984-2019 period. The evolution of all income shares (B50, M40, T10 and T1) by scenario is shown in Appendix D (Figures 18-21).

Figure 11 shows that in absence of changes in taxation, markups, and capital gains (red curve), the top 1% income share would have remained constant at its 1984 level (7%). Changes in other market forces (capital depreciation rate and TFP growth) thus have no impact on income

inequality in our model. Considering changes in taxation on top of changes in other market forces (blue curve) increases the top 1% income share very mildly: from 7.0% in 1984 to 7.5% in 2019. In contrast, adding changes in markups (the purple curve in Figure 11) induces a dramatic rise in the top 1% share of income. The latter reaches 9.6% in 2019, just below the value produced by our benchmark scenario (10.2%). Finally, combining the changes in markups with the changes in taxes and excluding changes in capital gains (green curve) produces a top 1% income share that is virtually identical to our benchmark scenario.

Figure 11: Counterfactual top 1% pretax income share



Summing up, our counterfactual exercise shows that (i) changes in markups are the main factor behind rising top 1% income share (accounting for 83% of the rise), (ii) changes in taxation play a significant though more limited role (15% of the rise), and (iii) changes in capital gains and other market forces have virtually no impact on income inequality.

Table 4 reports the average annual growth rate for the full population and the different income groups by scenario over the 1984-2019 period. It provides complementary insights to the counterfactual analysis by showing how the different scenarios affect the performance of the economy and income growth rates along the income distribution. If the average income growth of a given group is lower (higher) than the aggregate growth, its income share decreases (increases) over the period.

Table 4 shows that aggregate income growth is close to 1.3% in all scenarios. The distribution of income growth by income group varies dramatically depending on the group of driving forces

Table 4: Average annual growth rate by scenario, pretax income, 1984-2019

Group	Income Shares in 1984	Baseline	Without changes in markups, taxation and capital gains	Only changes in capital gains	Only changes in taxation and markups	Only changes in taxation	Only changes in markups
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full Population	100%	1,3%	1,3%	1,3%	1,3%	1,3%	1,3%
Bottom 50%	23%	1,1%	1,3%	1,3%	1,1%	1,3%	1,1%
Middle 40%	48%	1,1%	1,3%	1,3%	1,1%	1,3%	1,1%
Top 10%	29%	1,7%	1,3%	1,3%	1,8%	1,4%	1,7%
<i>Incl. Top 10-1%</i>	22%	1,5%	1,3%	1,3%	1,5%	1,4%	1,5%
<i>Incl. Top 1%</i>	7%	2,4%	1,4%	1,3%	2,4%	1,5%	2,3%

considered. In particular, when we neutralize changes in markups, taxation, and capital gains (3rd column), income growth rates are almost identical along the income distribution, which leads to a stability of income inequality over the 1984-2019 period. Adding changes in taxation on top of other market forces (6th column) slightly increases the income growth of the top 1% from 1.35% to 1.5% but leaves the growth rate of the bottom 99% almost unchanged. Considering changes in markups (7th column) induces a moderate decrease in the income growth of the bottom 90% income earners but raises significantly the income growth rate of the top 1% of the distribution (from 1.35% to 2.3%) , and to a lesser extent income growth rate of the top 10-1%. Finally, changes in capital gains (4th column) have almost no impact on the income growth of all groups and thus leave income inequality unchanged.

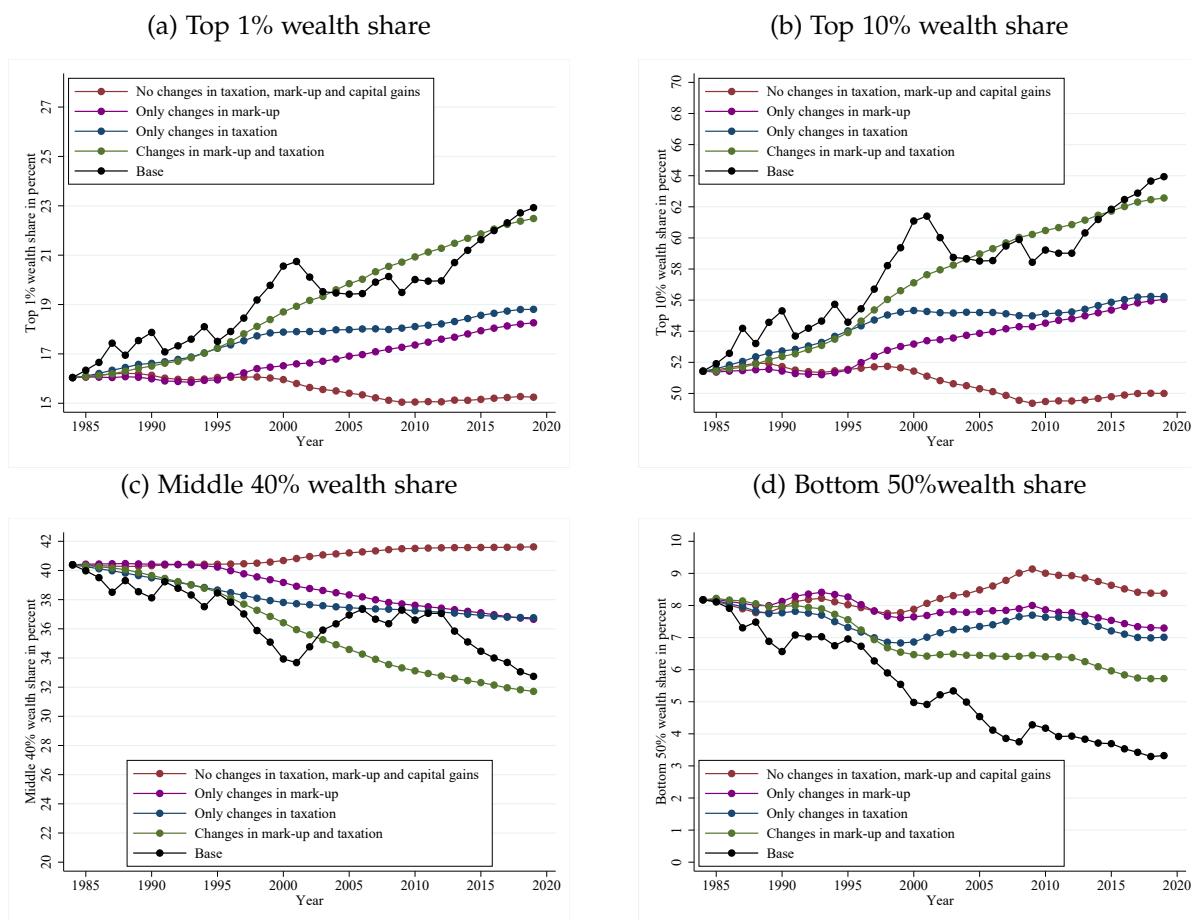
5.2 Understanding wealth inequality dynamics

For the analysis of wealth inequality dynamics, we focus on the evolution of the top 1%, the middle 40%, and the bottom 50% wealth shares by scenario (Figure 12), as well as the average annual wealth growth rate for the full population and the different wealth groups by scenario over the 1984-2019 period (Table 5). The evolution of all wealth shares (B50, M40, T10 and T1) by scenario is shown in Appendix D (Figures 22-25).

Figure 12 shows that wealth inequality would have remained almost stable (or slightly decreased) over the 1984-2019 period in absence of changes in taxation, markups, and capital gains (red curves). As for the distribution of income, other market forces do not affect the distribution of wealth much. Considering changes in taxation in addition to changes in other market forces (blue curve in the figure), the top 1% wealth share increases continuously and significantly over the period from 16% in 1984 to 19% in 2019. Changes in markups (purple curve) induce a slightly lower increase in the top 1% wealth share, especially between 1997 and 2019. As a consequence, when combining changes in taxation and markups (green curve) — and therefore excluding only changes in capital gains — the top 1% wealth share increases dramatically, up to 22.5% in 2019, a level almost similar to that produced by our benchmark scenario. This rise occurs at the expense

of both the bottom 50% and the middle 40% wealth shares.

Figure 12: Counterfactual wealth shares



When considering changes in capital gains on top of all other changes, both the top 1% and the middle 40% wealth shares increase very slightly at the expense of the bottom 50% wealth share, which experience a dramatic decreases from 5.7% to 3.3% in 2019. Digging deeper into the dynamics, changes in capital gains have a continuous negative impact on bottom 50% wealth shares. In contrast, they have ambiguous and opposing effects on the top 1% and middle 40% wealth shares depending on the period. Capital gains increase inequality between the top 1% and middle 40% between 1984 and 2000, decrease it between 2001 and 2012, and raise it again since 2013. The differential impact of capital gains on wealth group by time period can be explained easily by variations in relative asset prices (housing *vs.* equity prices, see Figure 5) and stark differences in portfolio compositions among wealth groups (deposits for the bottom 50%, mostly housing for the middle 40% and equity capital for the top 10% and top 1%, see Panel (b) of Figure 3). In particular, the large fluctuations in wealth inequality around 2000 is due to rising stock prices in the late 1990s, peaking in 2000. In contrast, housing prices rise strongly during the 2000s while stock prices contemporaneously decline.

Table 5 shows that changes in the driving forces have not only an impact on the distribution of wealth but also on the growth rates of wealth accruing to the different wealth groups. Absent changes in capital gains, markups and taxes (3rd column), the average annual growth rate of wealth would have been almost similar in all groups, around 1.5%, just slightly higher than the growth rate of national income. Consequently, the wealth-income ratio would have increased slightly from 320% in 1984 to 340% in 2019, keeping both wealth and income inequalities quasi stable over time. In contrast, our benchmark scenario (2nd column) depicts a much higher wealth-income ratio (540%) and aggregate wealth growth rate (3.0% vs. 1.5%) as well as stark differences among wealth groups. In particular, the wealth growth rate is increasing in wealth: 0.2% for the bottom 50%, 2.2% for the middle 40%, 3.2% for the top 10-1% and 3.9% for the top 1%.

Table 5: Average annual growth rate by scenario, wealth, 1984-2019

Wealth Group	Shares in 1984	Baseline	Without changes in markups, taxation and capital gains	Only changes in capital gains	Only changes in taxation and markups	Only changes in taxation	Only changes in markups
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full Population	100%	2,8%	1,5%	2,9%	1,3%	1,3%	1,5%
Bottom 50%	8%	0,2%	1,6%	2,1%	0,3%	0,8%	1,2%
Middle 40%	40%	2,2%	1,6%	3,2%	0,6%	1,0%	1,2%
Top 10%	51%	3,4%	1,4%	2,9%	1,9%	1,5%	1,8%
Incl. Top 10-1%	35%	3,2%	1,4%	2,9%	1,7%	1,4%	1,7%
Incl. Top 1%	16%	3,9%	1,3%	2,8%	2,3%	1,7%	1,9%
Aggregate Wealth-Income ratio		540%	340%	567%	325%	317%	348%

Changes in taxation and markups increase the growth rate of the top 10% wealth group at the expense of the bottom 90%, keeping the growth rate of aggregate wealth and the wealth-income ratio almost unchanged (5th, 6th and 7th column). When considering the impact of capital gains on top of changes in taxation and markups (2nd - 5th column), the average wealth growth rate increases significantly from 1.3% to 2.8%, which benefits the middle 40% and the top 10% wealth groups almost exclusively, leaving the growth rate of the bottom 50% almost unchanged (0.3% vs. 0.2%).

Overall, changes in taxation and markups reduce the wealth growth rate of the bottom 90% and increase that of the top 10%. However they do not affect the aggregate wealth growth rate. Changes in capital gains raise dramatically the wealth growth rate of households holding housing and equity capital (middle 40% and top 10% wealth groups) but have no effect on the liquid asset (deposits), which is mostly held by the bottom 50% wealth group. As a result, changes in taxes, markups and capital gains induce a quasi-stability of the *amount* of wealth owned by the bottom 50%, which leads to a strong reduction of their wealth share over the 1984-2019 period. The top 10% and top 1% wealth groups benefit from all these changes, which leads to a dramatic

increase in their wealth share. While changes in taxes and markups strongly reduce the wealth growth rate of the middle 40% wealth group, large capital gains on housing assets more than offset this reduction. As a result, the wealth of the middle 40% wealth group grows at an average annual rate of 2.2%. Because this growth rate is lower than that of aggregate wealth (2.8%), their wealth share declines over the period.

6 How do the driving forces affect the top 1% wealth share?

This section investigates the mechanisms through which the driving forces operate. First, we present a simple wealth accumulation equation that highlights the key mechanisms behind wealth inequality dynamics. Second, we use this equation to decompose the evolution of the top 1% wealth share by mechanisms and counterfactual scenarios. We thus quantify the channels through which the different driving forces contribute to wealth inequality dynamics and disentangle mechanical from behavioral and general equilibrium effects. Third, we present the results of this exercise along with the evolution of these mechanisms by counterfactual scenario over the 1984-2019 period.²³

We rely on the following wealth accumulation equation to study the mechanisms behind the evolution of the top 1% wealth share

$$W_{it+1} = (1 + q_{it})(W_{it} + S_{it}) = (1 + q_{it})(W_{it} + s_{it}(1 - \tau_{it})sh_{it}^Y Y_t) \quad (30)$$

where W_{it} is the amount of wealth owned by wealth group i at time t , S_{it} the amount of savings and $(1 + q_{it})$ is the rate of capital gains of wealth group i . Further, savings S_{it} can be split into four components: (i) the saving rate out of disposable income (s_{it}), (ii) the net-of-tax rate $(1 - \tau_{it})$, which account for both taxes and monetary transfers, (iii) the share of pretax income accruing to wealth group i (sh_{it}^Y), (iv) and aggregate pretax income (Y_t). As a consequence, $(1 - \tau_{it})sh_{it}^Y Y_t$ is the disposable income of wealth group i . Incidentally, we define S_{it} and s_{it} in the same way as [Saez and Zucman \(2016\)](#); [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#). That is, we compute S_{it} and s_{it} as the synthetic savings and saving rates that account for the evolution of wealth of group i from t to $t + 1$ given the observed values of the remaining variables (W_{it+1} , W_{it} , q_{it} , τ_{it} , sh_{it}^Y and Y_t).

Using Equation (31), the evolution of the top 1% wealth share is given by

$$sh_{it+1}^W = \frac{W_{it+1}}{W_{t+1}} = \frac{(1 + q_{it})}{(1 + q_t)} \cdot \frac{(W_{it} + s_{it}(1 - \tau_{it})sh_{it}^Y Y_t)}{(W_t + s_t(1 - \tau_t)Y_t)} \quad (31)$$

This equation highlights that wealth inequality dynamics result from five complementary

²³Our five counterfactual scenarios are the same as those presented in [subsection 5.1](#): (1) without any changes from 1984 in taxation, markups, and capital gains, (2) with only changes in capital gains, (3) with only changes in taxation, (4) with only changes in markups, and (5) with only changes in taxation and markups.

mechanisms: changes in (i) pretax income inequality, (ii) tax progressivity, (iii) saving rate inequality, (iv) capital gains inequality, and (v) aggregate pretax income. The next step consists in studying the evolution of these mechanisms and their impact on wealth inequality dynamics by counterfactual scenario over time.

Our methodology consists in three steps for each counterfactual scenario. First, we fix all parameters except for aggregate pretax income in Equation (31) to their 1984 values for each group. The resulting wealth shares are computed and the results provide the relative contribution of aggregate pretax income to the evolution of the top 1% wealth share. Second, we allow each of the three following mechanisms — saving rate, pretax income share and tax rate — to vary over time according to our simulated values. In this case, we keep the two other mechanisms constant and equal to their 1984 value. We compute the resulting wealth shares, which then gives the first-order contribution of each of the mechanisms to the evolution of the top 1% wealth share. Then, we let two of the three mechanisms vary over time according to our simulated values, compute the wealth shares, and remove the first-order contributions to obtain the second-order contributions (interactions), which are then allocated to each of the two mechanisms proportionally to their respective first-order contributions. Last, we let the three mechanisms vary and proceed similarly to determine the third-order contributions. The total contribution of each mechanism is the sum of its first, second, and third-order contribution. Third, we consider time-varying capital gains (only in the baseline scenario) along with all the remaining mechanisms, compute the wealth shares and quantify the contribution of capital gains to wealth inequality dynamics.

Table 6 reports the relative variation of the top 1% wealth share and the respective contributions of our five mechanisms to this evolution over the 1984-2019 period.²⁴ As such, it quantifies the contribution of changes in each mechanism (aggregate pretax income, pretax income shares, effective tax rates, savings rates, and rates of capital gains) to the changes in the top 1% wealth share depending on the driving forces at play. Each row reports this decomposition for a different counterfactual scenario. Figure 13 complements Table 6 by graphing the annual evolution of the mechanisms of wealth inequality by counterfactual scenario over the 1984-2019 period.

In the counterfactual scenario with no changes in taxation, no markups, and no capital gains (last row of Table 6), the top 1% wealth share decreases slightly (-5%). Our decomposition explains this small variation by the negative impact of the rise in aggregate pretax income (-4%) and to a lesser extent by the changes in saving rate inequality (-2%). Under this scenario, the contributions of pretax income inequality and tax progressivity to wealth inequality dynamics are negligible. Figure 13 confirms these results: it shows stable shares of pretax income and disposable income accruing to the top 1% wealth group (panel (a) and (c), red curve) as well as a stability of tax progressivity (panel (b)), measured as the ratio between the effective tax rate of the top 1% and the aggregate tax rate. Further, panel (h) shows that the aggregate pretax income exhibits similar dynamics under all scenarios, echoing the last column of Table 6, which indicates

²⁴The detailed table containing all the first, second and third-order contributions is available upon request.

Table 6: Mechanisms of the top 1% wealth share evolution by counterfactual scenario, 1984-2019

Counterfactual Scenarios	Top 1% Wealth Share Variation	Variation due to changes in				
		Pretax Income Inequality	Tax progressivity	Saving rate inequality	Capital gains inequality	Aggregate income
Base: Changes in markup, taxation and capital gains	43%	14%	0%	67%	-34%	-4%
Changes in markup and taxation	40%	14%	1%	29%	0%	-4%
Changes in markup	14%	10%	0%	8%	0%	-4%
Changes in taxation	17%	4%	1%	17%	0%	-4%
No changes in taxation, markup and capital gains	-5%	1%	0%	-2%	0%	-4%

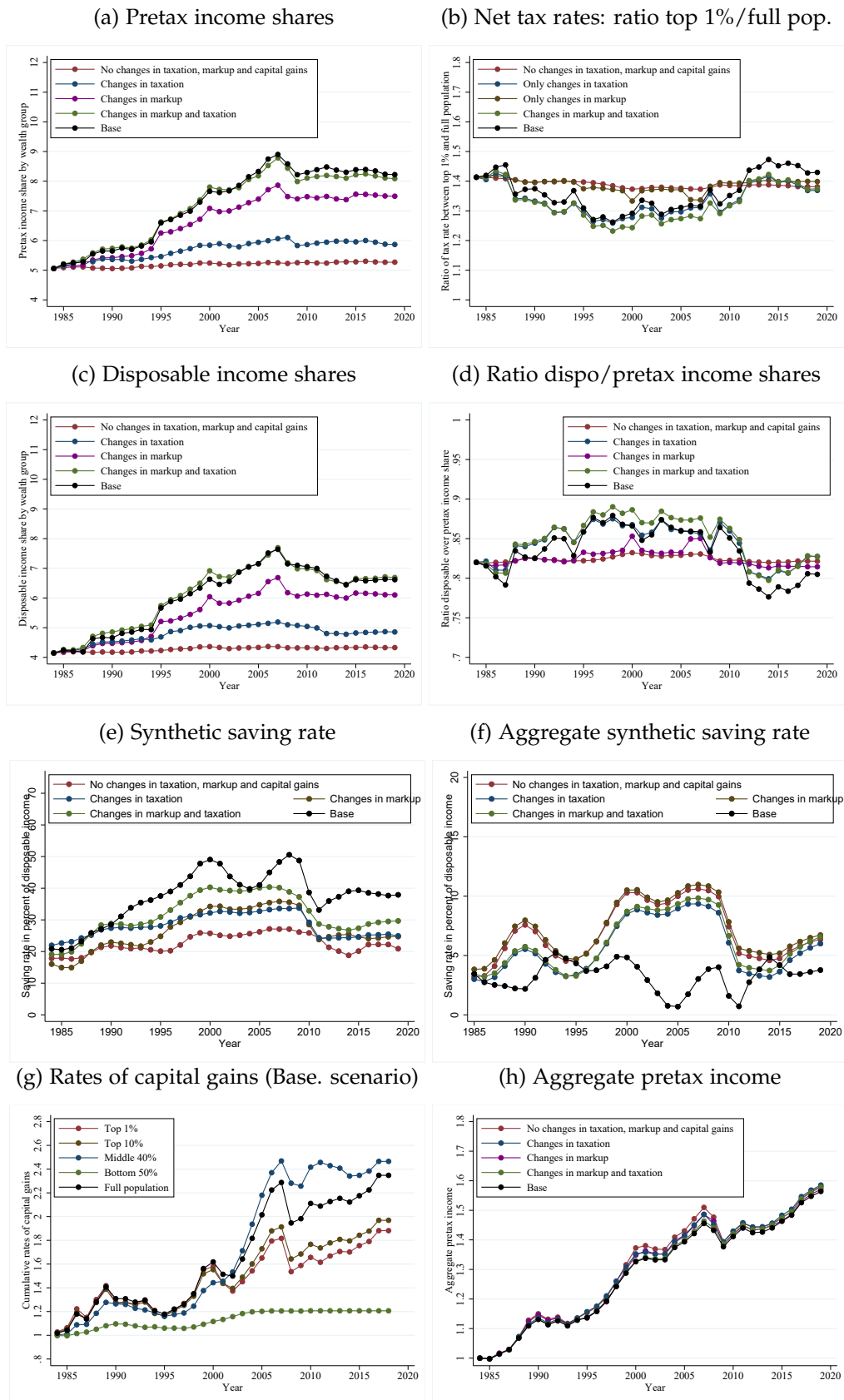
Note: In the baseline scenario, the top 1% wealth share increases by 43% over the 1984-2019 period, of which 14 percentage points are due to changes in pretax income inequality.

that it decreases the top 1% wealth share by 4% under all counterfactual simulations. The impact of the other mechanisms, however, varies markedly depending on the driving forces at play.

While tax and transfer parameters were constant in the previous counterfactual scenario, we now look at the specific impact of changes in tax and transfers parameters over time (fourth row of Table 6). Doing so affects the dynamics of the economy directly through changes in tax progressivity, *i.e.* the mechanical effect of taxation, but it also potentially affects all other mechanisms (pretax income and saving rate inequality, aggregate income) through behavioral and general equilibrium effects.²⁵ When introducing changes in tax and transfers parameters, the top 1% wealth share now increases by 17%. Interestingly, the impact of changes in tax and transfer parameters on wealth inequality is not driven by the mechanical effect of taxation but by rising saving rate inequality (+17%) and to a lesser extent by rising pretax income inequality (+4%). When considering changes in markups (third row of Table 6) instead of changes in taxation, the top 1% wealth share increases by 14%. This increase is driven by the rise in pretax income inequality (+10%) and by the rise in saving rate inequality (+8%).

²⁵The mechanical effect of taxation corresponds to the impact of taxation on the gap between pretax and disposable income inequality among wealth groups.

Figure 13: Mechanisms of wealth inequality for the top 1% wealth group by scenario



When we introduce both changes in taxation and markups (second row of Table 6), the top 1% wealth share increases by 40%, almost its baseline value. The evolution of pretax income inequality accounts for 14 percentage points while saving rate inequality accounts for 29 percentage points of this 40% increase. In contrast, the mechanical effect of the evolution of taxation on the top 1% wealth share is negligible (+1%). Figure 13 digs further into these results. Panel (a) and (c) show that changes in taxation and markups induce a continuous and substantial rise in the share of pretax and disposable income accruing to the top 1% wealth group between 1984 and 2019 (+60% for pretax income and +61% for disposable income). Panel (b) and (d) shed light on the small contribution of tax progressivity. Panel (b) reports the evolution of our measure of tax progressivity, which displays a U-shaped pattern. Tax progressivity evolves from 1.41 in 1984, *i.e.* the top 1% wealth group faces an effective tax rate 41% higher than the aggregate tax rate, to 1.23 in 1998, before reverting to its initial level in 2019. The resulting reduction in income inequality among wealth group is relatively moderate, as shown by panel (d).²⁶ Hence, the large rise in disposable income inequality among wealth group is driven by the rise in pretax income inequality rather than by the mechanical effect of taxation. In turn, the contribution of tax progressivity to the evolution of the top 1% wealth share is negligible as compared to the contribution of pretax income inequality. Panels (e) and (f) report the evolution of the synthetic saving rate (out of disposable income) for the top 1% wealth group and for the total population, respectively. Changes in taxation and markups induce a substantial rise in the saving rate of the top 1% wealth group and a small decline in the aggregate saving rate. This raises saving rate inequality among wealth groups and therefore triggers a large surge in the top 1% wealth share over time, accounting for a 29% increase in the top 1% wealth share for a total increase of 40% in this counterfactual scenario.

Finally, when we introduce changes in capital gains on top of changes in markup and taxation, the top 1% wealth share increases only marginally (+43% instead of +40%). Table 6 shows that capital gains have a strong mechanical effect and decrease the top 1% wealth share by 34%. However, this strong negative mechanical effect of capital gains is almost entirely offset by the positive impact of changes in saving rate inequality on the top 1% wealth share (+67% instead of +29% in absence of capital gains). Therefore, our results imply that capital gains play only a moderate role in increasing the top 1% wealth share, once all the effects (mechanical, behavioral and general equilibrium) of capital gains on the various mechanisms of wealth inequality are taken into account. Panels (e), (f) and (g) in Figure 13 highlight the forces at play. Panel (g) reports the cumulative rate of capital gains by wealth group over time. It shows that all wealth groups except the bottom 50% have benefitted from large capital gains since the end 1990s. However, the cumulative rate of capital gains is much lower for the the top 1% wealth group than for the total

²⁶Panel (d) depicts the ratio between the share of disposable and pretax income accruing to the top 1% wealth group over the 1984-2019. This ratio shows how the mechanical effect of taxation impacts the share of income accruing to the top 1% wealth group (from pretax to disposable) and can be seen as a measure of tax progressivity between wealth groups. Panel (d) shows that this ratio was equal to 0.82 in 1984, *i.e.* the mechanical effect of taxation decreases the share of income accruing to the top 1% wealth group by 18% from pretax to disposable income. This ratio increases only moderately up to 0.89 in 1998 — tax progressivity decreases — before returning to its initial level in 2019.

population, leading to a negative contribution of capital gains (mechanical effect) to the evolution of the top 1% wealth share. Indeed, the large increase in housing *vs.* equity prices over the period is more beneficial to the middle 40% wealth group, as housing represents a larger proportion of their wealth. This explains the difference in capital gains dynamics among wealth groups. Panels (e) and (f) further show that changes in capital gains increase slightly the saving rate of the top 1% but reduce dramatically the aggregate saving rate, resulting in rising saving rate inequality. This is especially the case between 1998 and 2008 where housing prices rose sharply.²⁷

In a nutshell, the large increase in the top 1% wealth share over the 1984-2019 period is driven by the increase in saving rate inequality and pretax income inequality induced by changes in taxation and markup. The first row of Table 6 is consistent with previous empirical work based on simple simulation exercises and stressing the key role of asset prices, saving rate inequality and pretax income inequality on the dynamic of wealth inequality (see [Saez and Zucman \(2016\)](#), [Kuhn, Schularick, and Steins \(2020\)](#), [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#), [Martínez-Toledano \(2020\)](#)). However, we argue that our framework carries deeper conclusions and three important contributions to this literature. First, although the mechanical effect of capital gains reduces top wealth shares, it is almost entirely offset by behavioral and general equilibrium effects on saving rate inequality over the medium run. Second, although the mechanical effect of taxation on top wealth shares is negligible, changes in taxation have strong effects on wealth inequality dynamics through the indirect and general equilibrium effects on saving rate inequality. Third, in absence of changes in taxation, markups and capital gains, wealth inequality would have been stable. As a result, changes in saving rate inequality are the main mechanisms of wealth inequality dynamics but only in response to the institutional and economic context (asset prices, taxation and markup). More generally, our results point to the critical importance of endogenous saving decisions as a key driver of wealth inequality.

7 Conclusion

Unifying micro and macro remains an area of vast research potential. We build a rich, micro-founded macro model: an original heterogeneous-agent model with three assets (deposits, housing and equity), labor-income risk, entrepreneurs, and a rich and realistic set of flat and progressive taxes and transfers. Thanks to newly available wealth and income inequality series and fiscal data, we calibrate our model and test its ability to fit the level and dynamics of wealth and income inequalities, the aggregate and distributional tax structure, the composition of wealth along the distribution, and key macroeconomic aggregates from 1984 to 2018. We show the im-

²⁷Note that while changes in capital gains have almost no impact on the evolution of the top 1%, top 10%, and middle 40% wealth shares, they have a strong impact on the bottom 50% wealth share (see Figure 12 and Table 5). Indeed, for the bottom 50% wealth group, changes in capital gains imply a negative mechanical effect of capital gains and a negative effect on saving rate. The evolution of the middle 40% wealth share is not affected by changes in capital gains because the high and positive mechanical impact of capital gains are offset by a corresponding decline in the saving rate.

portance of (i) markups in explaining the dynamics of income inequality and of (ii) endogenous saving decisions in response to exogenous shocks as a key driver of wealth inequality.

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A Concept of pretax income

This work relies extensively on the long-term series of pretax income, post-tax income and wealth developed for France within the “Distributional National Accounts” project. This project aims at combining national accounts, tax, and survey data in a comprehensive and consistent manner to build long-term series of inequalities that are unified over time and across countries, cover the entire distribution and are fully consistent with the National Accounts.

Complete methodological details about the construction of these series are provided in [Garbinti, Goupille-Lebret, and Piketty \(2021\)](#) for the wealth series, and [Bozio et al. \(2021\)](#) for pretax and post-tax income series, along with a wide set of tabulated series, data files and computer codes. A complete presentation of the concepts and the general methodology to construct “Distributional National Accounts” series is provided by [Alvaredo et al. \(2020\)](#).

In this Appendix, we present the concept of pretax income and discuss its implications for the model.

Pretax vs. factor national income Pretax income is our benchmark concept to study the distribution of income before government intervention. It is defined as the sum of all income flows going to labor and capital, after taking into account the operation of the pension and unemployment insurance systems, but before taking into account other taxes and transfers. This concept should be benchmarked against the definition of factor income, which is equal to the sum of all income flows going to labor and capital, before considering the operation of the pension and unemployment system. The key difference between factor income and pretax income is the treatment of pensions, which are counted on a contribution basis for factor income and on a distribution basis for pretax income. In other words, pretax income includes pension and unemployment benefits, while it excludes contributive payroll taxes, *i.e.* the fraction of payroll taxes dedicated to the financing of the pension and unemployment system.

The main limitation of factor income is that retired individuals typically have very small factor income in countries using pay-as-you pension systems. As a result inequality of factor income tends to look artificially large in countries and time periods with an older population. In contrast, pretax income inequality will not be affected by ageing population nor by the design of the pension system.²⁸ However, the limitation of the concept of pretax income is that it does not incorporate the redistribution carried out by the pension and UI systems over the life-cycle.

Using the concept of pretax income yields three main implications for our model that we now discuss.

The incidence of taxes Computing pretax income requires to assign taxes that are not directly paid by households (corporate taxes and payroll taxes) into their income using tax incidence

²⁸Note that pretax income is broader but conceptually similar to what most tax administrations attempt to tax, as pensions and unemployment benefits are largely taxable, while contributions are largely tax deductible.

assumptions. As pointed out by [Saez and Zucman \(2019\)](#), one need to distinguish current distributional analysis from tax reform distributional analysis. Current distributional analysis shows the current tax burden by income groups and should assign taxes on each economic factor without including behavioral responses. As such, taxes based on labor income (payroll taxes) should fall on the corresponding workers. Taxes based on wealth or capital income should fall on the owners of the corresponding assets. In contrast, tax reform distributional analysis shows the impact of a tax reform and should describe the effect on pretax incomes, post-tax incomes, and taxes paid by group separately and factoring in potential behavioral responses.

Therefore, when we compute pretax income, we assign current payroll taxes and corporate taxes paid to the corresponding household income without incorporating any behavioral response (current distributional analysis). In contrast, we use our model and rely on a counterfactual analysis to study how changes in taxes affect the aggregates and the distributions of pretax income, post-tax income and wealth relative to baseline through potential behavioral effects (direct, indirect, general equilibrium).

Pension and payroll taxes Because pretax income includes pension and unemployment benefits, the model should only include the fraction of payroll taxes that do not finance the pension and unemployment system to avoid double counting. Indeed, if we include all payroll taxes in the model and reassign them to pretax income, pretax income will include pension and unemployment contributions but also the corresponding benefits. Disposable and post-tax income will also be inconsistent as pension and unemployment contributions will be subtracted but the corresponding benefits will not be added when going from pretax to disposable and post-tax income.

Note that our model also includes a parameter ν (see Equation (2)) that accounts for the potential discrepancy between aggregate pension and unemployment contributions, and aggregate pension and unemployment benefits. If benefits exceed contributions ($\nu > 0$), the government will use a part of its tax revenues to finance this deficit. The introduction of the parameter ν allows to disconnect the deficit/surplus of the pension and unemployment system from the remuneration of labor on the production side.

The specific case of consumption taxes Consumption taxes constitute the majority of the primary income of the government. These taxes are already deducted before the value added is used to remunerate factors of production (unlike direct taxes).

Following the convention of national accounts, production taxes must be added to household income levels to reach a consistent pretax income concept, rather than subtracted from it. This convention is somewhat at odds with intuition. However, in practice, this is the only way of providing a treatment that is consistent with direct taxes (which are included within household primary income), and which avoids double-counting. If we were to remove consumption taxes

from household income, we would effectively be removing them twice. Another reason for including them in pretax income is that the frontier between consumption taxes and direct income and wealth taxes is somewhat arbitrary, so that it is unclear why we should deduct the former and not the latter. Thus, for the purpose of making comparisons over time and across countries, it makes more sense to look at the distribution of income before the deduction of any tax, be they consumption taxes or direct taxes. Consumption taxes also constitute an important source of revenue for governments: excluding them from analysis would bias the comparison of tax levels and progressivity among countries with different tax systems. This is why we choose to distribute them as part of pretax income.

There are several ways of doing so, and we choose to follow the DINA guidelines [Alvaredo et al. \(2020\)](#) for comparability purposes. We make a distinction between the distribution of consumption taxes in pretax income and their distribution when moving from pretax to disposable and post-tax income. To compute pretax income, we distribute consumption taxes to pretax labor income and pretax capital income on a proportional basis. In contrast, we will remove the amount of consumption taxes effectively paid by each household when we consider moving from pretax income to disposable and post-tax income

As explained by [Alvaredo et al. \(2020\)](#) p.59, the rationale behind this choice is the following. "The VAT acts as the wedge between factor prices and market prices: therefore, its direct, mechanical effect is on prices. Factor price national income (national income excluding consumption taxes) can buy the full production at pretax prices (prices received by producers that do not include consumption taxes). Market price national income (national income including consumption taxes) can buy the entire production at post-tax prices (prices paid by consumers, which include consumption taxes). In national accounts, prices are always measured post-tax (*i.e.*, including VATs, sales taxes, etc.) which is why standard national income includes consumption taxes. Factor price income cannot buy full production at post-tax prices precisely because consumption taxes create a wedge between pretax and post-tax prices. Therefore, to compute pretax income, labor and capital incomes should be inflated uniformly to line up with the national income aggregate. That way, they reflect the purchasing power of pretax income at the post-tax prices that exist in the economy. Because this is pretax (before any consumption decision is made), it makes the most sense to do a uniform rescaling, so as to preserve the same distribution as labor and capital income. In other words, going from factor price to market price national income is about changing the price index, and not about distributing taxes to individuals."

B Modeling the tax and transfer system

This Appendix provide a brief overview of the French tax and transfer system and presents in details the methodology used to estimate the different tax parameters.

B.1 Overview of the French tax and transfer system

The French tax system includes a large variety of taxes that we can regroup into five categories depending on the relevant tax basis: (i) taxes borne by pretax labor income (“non-contributive social contributions”), (ii) by total income (pretax labor and capital income) that we thus call “income tax”, (iii) by capital (“wealth taxes”), (iv) by corporate profits (“tax on corporate profits”), and (v) by consumption (“consumption taxes”).

Government spending can be decomposed into three distinct categories: monetary transfers, in-kind transfers, and collective consumption expenditure. Monetary transfers amount to about 4% of national income and include various types of housing benefits, family benefits, and social benefits. In-kind transfers are all transfers that are not monetary (or quasi-monetary) and can be individualized. They correspond to individual goods and services produced directly or reimbursed by government. In-kind transfers make up to 20% of national income (including 12.5% for health and 6.5% for education expenditure). Collective consumption expenditure regroups all consumption services that benefit to the community in general and cannot be individualized (spending on defense, police, the justice system, public infrastructure, etc.). It amounts to 10% of national income.

B.2 Fitting the progressivity of taxes and transfers

The DINA series contain a full decomposition of taxes and transfers that we exploit to compute the different tax rates that are used in our model.

We assume the functional forms given by Equation (24). Our approach consists in estimating two parameters for each tax: a level parameter ($\bar{T}, \tau_\ell, \tau, \phi$) and a progressivity parameter η . We refine this approach in two ways. First, to describe the effective tax schedule as accurately as possible, we compute aside the average tax rate for the top 0.1% of the distribution of the tax basis. As we will see, it is particularly relevant for the progressive income tax. Second, to allow for more flexibility and for a better fit to the actual effective tax rates, we fit the functional form on different segments of the distribution. That is, we estimate potentially different level and progressivity parameters on several subsets of the distribution of tax rates along the tax base. We proceed identically for transfers. We detail the tax concepts and results below.

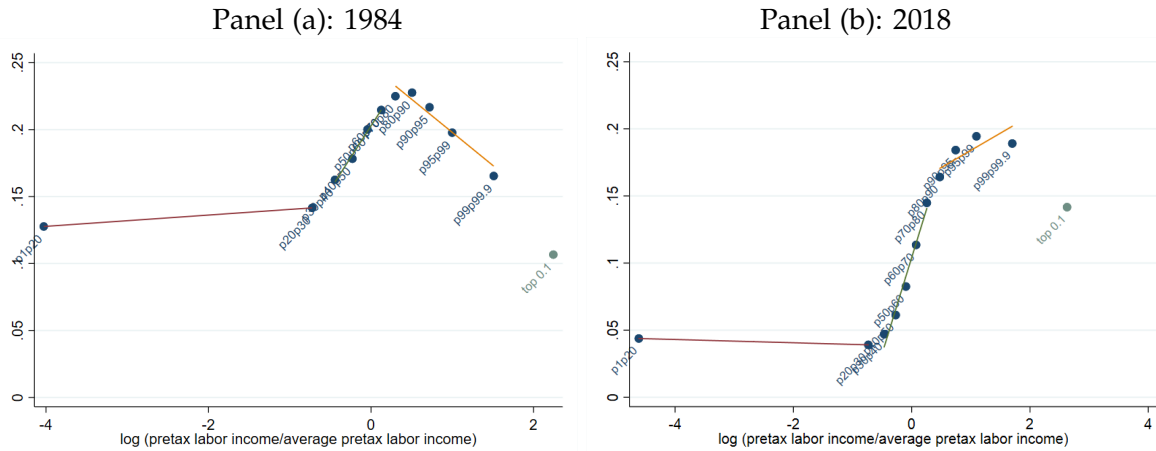
B.3 Taxes & their progressivity

We gather taxes depending on the relevant tax basis. We start from the detailed categories of taxes (presented in detail in [Bozio et al. \(2021\)](#)) and we classify them in 5 broad categories: non-contributive payroll taxes, income taxes, wealth taxes, tax on corporate profits, and consumption

taxes.²⁹ Hereafter, for taxes and transfers, we present our method for the year 1984. A similar approach is used for all other years for which we have data (1988 and each year from 1994 onwards).

Payroll taxes τ_ℓ^j include all social security contributions that are not dedicated to the financing of the pension and unemployment systems as well as taxes on wages. Altogether, they make up to 11% of national income in 2018. They are applied to pretax labor income. For the different years for which we have data (1984, 1988 and each year from 1994), we estimate the different parameters on three segments of the distribution of pretax labor income (in addition to the average tax rate computed for the top 0.1%). Figure 14 shows how we fit the distribution of the non-contributive SSC for the year 1984 and 2018. It illustrates that our flexible non-linear specification allows for an excellent fit of the distribution of tax rates and improves significantly our ability to model tax rates as close as possible as those observed in the data. This goodness of the fit is similar for all subsequent years for which we have data (1984, 1988 and all years after 1994). Comparing panels (a) and (b) show the crucial importance of having time-varying parameters.

Figure 14: Individual SSC contributions τ_ℓ^j (% pretax labor income)

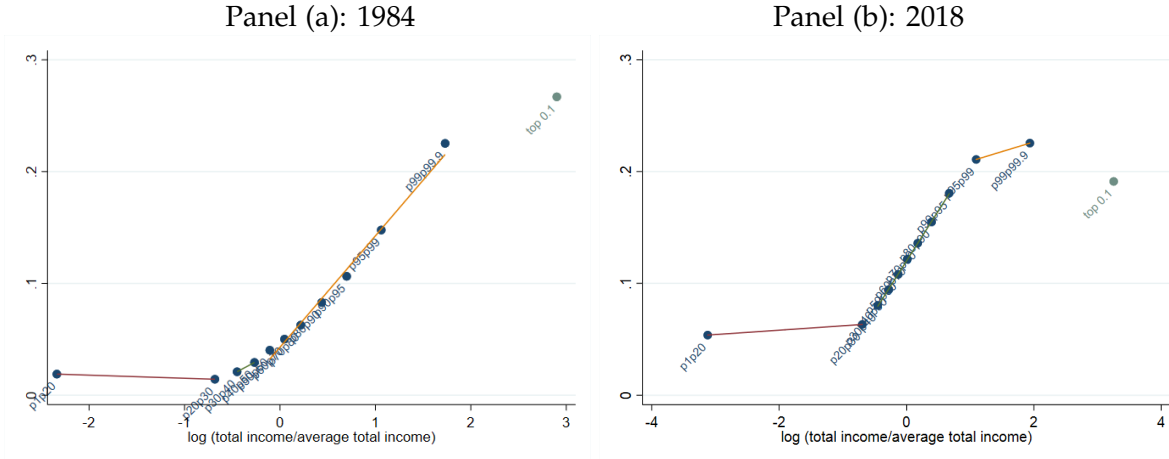


Income tax. We gather in the income tax τ^j taxes that are borne by total pretax income (labor and capital income, including profits). It thus includes both the income tax (“impôt sur le revenu des personnes physiques”) and the “CSG” (“Contribution Sociale Généralisée”, a flat tax) on capital and labor incomes. As for non-contributive SSC, we perform the estimation of the different level and progressivity parameters for three segments of the distribution of total pretax

²⁹Note that we add pension and unemployment benefits to labor and capital incomes. Consequently, we do not add contributive payroll taxes to the analysis to avoid double-counting since these payroll taxes fund these benefits. In [Garbinti, Goupille-Lebret, and Piketty \(2018\)](#), we present another concept of income (factor income) where pension and unemployment benefits are not added to labor and capital incomes, and that allows to investigate the role of payroll taxes. One problem of that measure is that retired individuals typically have very small factor income in countries using pay-as-you pension systems such as France. As a result, inequality of factor income tends to rise mechanically with the fraction of old-age individuals in the population, which biases comparisons over time and across countries.

income. Figure 15 shows the fit of the distribution of the income tax SSC for the year 1984 and 2018. Here again, it clearly shows that our specification provides a great fit of the distribution of the tax rates, a feature that is similar for all other years of our sample.

Figure 15: Individual income tax rates τ^j (% total pretax income)



Wealth tax. The wealth tax ϕ^j includes all taxes borne by assets. This corresponds to the wealth tax, the property tax and the estate tax. Note that it applies to the *level* of wealth. Further, note that in France, there exists a “tax shelter” (bouclier fiscal) to insure that the total tax burden cannot exceed 60 to 70% (depending on the period) of total income. This mechanism has been set to avoid that wealth taxes lead to a tax burden deemed too high. We deduct this tax shelter from the wealth tax. Figure 16 shows the fit of the wealth tax for 1984 and 2018. The tax rates appear aligned with a linear trend along the net wealth distribution, except for the bump observed from the 40th to the 60th percentiles. This is explained by the fact that property taxes apply to *gross* rather than *net* housing wealth. Here, we use the concept of *net* wealth (net of liabilities). It thus raises mechanically the tax rates for indebted individuals that are over-represented between the 40th and 60th percentiles of the net wealth distribution. In an alternative specification, we take into account this bump by allowing for non-linearity but this does not change our results. This is likely due to the fact that the magnitude of the gap between the bump and the rest of the surrounding tax rates is small. We thus opt for the simplest specification and chose a linear one (up to the top 0.1% which is still considered aside).

Tax on corporate profits. This tax directly applies to profits (Equation 20) and is a flat tax. We thus simply apply the same flat tax rate to firms profits. It is taken directly from national account every year.

Consumption taxes. These taxes are borne by consumption (Equation 5). We consider here consumption taxes as a flat tax. This is not a significant departure from reality since the value added tax (VAT) represents the bulk of these taxes. Although the VAT has four different rates (ranging from 2.5 to 20%), the vast majority of goods are taxed at 20%. Consequently, as for the

Figure 16: Individual capital tax rates ϕ^i (% net wealth)



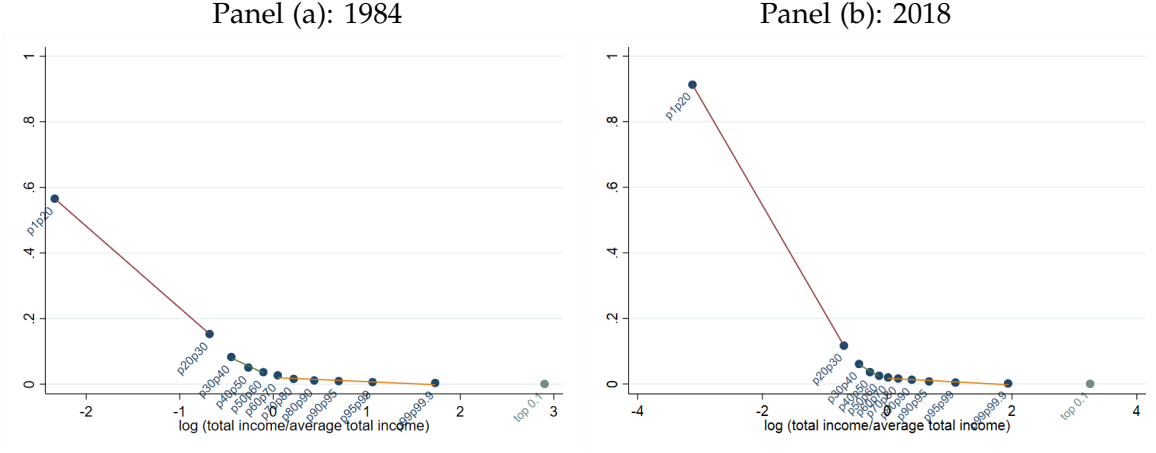
tax on corporate profits, the value of this flat tax is taken directly from national account every year.

B.4 Individual transfers & their progressivity

Individual transfers (or monetary transfers) T^i represent about 4% of national income and include the various types of housing benefits, family benefits, and social benefits.³⁰ Here again, we perform the estimation of the level and progressivity parameters for three segments of the distribution of total pretax income. Figure 17 shows how we fit the distribution of monetary transfers expressed as a percentage of pretax income for the year 1984 and 2018. Here, as for taxes, our non-linear specification allows for a very accurate representation of the transfers actually observed in the data over time.

³⁰The housing benefits regroup “Allocation de Logement Familiale” (ALF), “Allocation de Logement Personnalisée” (APL), and “Allocation de logement sociale” (ALS). The family benefits include “Allocation Familiale” (AF), “Complément Familial” (CF), “Allocation Pour Jeune Enfant” (APJE), “Prestation d’Accueil du Jeune Enfant” (PAJE), “Allocation de Rentrée Scolaire” (ARS), “Allocation d’Education de l’Enfant Handicapé” (AEEH), and “Allocation de Soutien Familial” (ASF). The social benefits regroup “Revenu de Solidarité Active”/“Prime d’Activité” (RSA/PPA), “Allocation Adulte Handicapé” (AAH), and “Allocation de Solidarité aux Personnes Agées” (ASPA).

Figure 17: Individual transfers T^j (% total pretax income, $Y^{pretax,j}$)



C Solution method

Our solution method is fully non-linear and takes advantage of the continuous-time formulation of the heterogeneous-agent problem solving the Hamilton-Jacobi-Bellman and Kolmogorov forward equations. Our codes are adapted from those of Bence Bardoczy taken from the HACT project page maintained by Benjamin Moll: <https://benjaminmoll.com/codes/>.

C.1 Stationary equilibrium

The solution method uses an asset grid with 8 states (3 types of workers + entrepreneurs, either patient or impatient) and 501 grid points over an asset grid $a^j \in [0, 200]$. The algorithm solving for the steady state is the following. Starting from initial guesses for the steady-state level of capital k , total labor ℓ and taxes:

1. Compute output, y , capital rental rate r^k , aggregate real wage w and firms' aggregate profits π
2. Given the income tax schedule τ^j , the consumption tax rate τ_c , the rental rate of housing r^h and the rate on deposits r^m , compute the (household-specific) opportunity costs of housing R^j and deposits R^{mj} and the price indices P^j and P_Λ^j
3. Given the labor tax schedule τ_ℓ^j and the productivity levels z^j , compute individual labor income Φ_ℓ^j over the asset grid
4. Given the capital rental rate r^k and profits $\pi^j = \pi/e$, compute capital income $\Phi_k^j = r^k a^j + \mathbb{1}_\ell^j 0.3\pi^j$ over the asset grid
5. Given the income tax schedule τ^j , the capital tax schedule ϕ^j and the transfer schedule T^j , compute income consistently with the budget constraint: $(1 - \tau^j) (\Phi_\ell^j + \Phi_k^j) - (\phi^j + \gamma) a^j + T^j$

6. Solve the Hamilton-Jacobi-Bellman equation based on the utility function to determine the individual saving rules a^j and the individual expenditure rules Λ^j
7. Given Λ^j , P_Λ^j and R^{mj} compute the optimal rule for deposit demand m^{dj}
8. Determine the residual expenditure $C^j = P_\Lambda^j \Lambda^j - R^{mj} m^{dj}$
9. Given C^j , P^j , τ_c , p^h and R^j , compute the optimal rule for housing demand h^{dj} , and impose $h^j = h^{dj}$ if $h^{dj} > h^{\min}$ and $h^j = 0$ otherwise
10. Adjust $m^j = \min(m^{dj} + h^{\min} - h^{dj}, a^j)$
11. Determine the residual expenditure on non-durables $c^j = P^j C^j - p^h R^j h^j$
12. Solve the Kolmogorov forward equation to get the distribution of households Ω^j over the asset grid
13. Update the number of entrepreneurs e
14. Update the distributions of labor income Φ_ℓ^j and capital income Φ_k^j and Y_k^j and all the relevant measures of income
15. Update the progressive tax and transfer schedules τ_ℓ^j , τ^j , ϕ^j and T^j
16. Update aggregate labor $\ell = \int_j \Omega^j \left(1 - \mathbb{1}_e^j\right) (w^j/w) \ell^j dj$ and update the average level of labor productivity that guarantees $\int_j \Omega^j \left(1 - \mathbb{1}_e^j\right) w^j dj = w$
17. Compute the residual of capital-market clearing condition as the difference between the sum of individual capital detention $\int_j \Omega^j k^j dj$ and the aggregate stock of capital k
18. Adjust aggregate capital k using the above residual and iterate from 1. until the residual of the capital-market clearing condition is less than 0.01% of the aggregate capital stock

Solving for the steady state takes a few seconds.

C.2 Transition dynamics

The algorithm solving for the transitional dynamics is the following. Starting from a steady state sequence of aggregate capital $\{k_t\}_{t=1}^{t=T} = k$, and all the relevant variables, given the path of exogenous variables $\left\{\theta_t, \delta_t, \dot{p}_t^h / p_t^h, p_t^h, g_t / y_t, \tau_{ct}, \tau_{\pi t}, v_t, \xi_t, \Gamma_t\right\}_{t=1}^{t=T}$ where Γ_t captures all the parameters governing the progressive tax and transfer schedules:

1. For $t = \{1 : T\}$, compute output, y_t , capital rental rate r_t^k , the aggregate real wage w_t and firms' aggregate profits π_t
2. For $t = \{1 : T\}$, update the distributions of labor income $\Phi_{\ell t}^j$, capital income Φ_{kt}^j and Y_{kt}^j , and update the tax and transfer schedules $\tau_{\ell t}^j$, τ_t^j , ϕ_t^j and T_t^j

3. For $t = \{1 : T\}$, update the measures of income consistently with the budget constraint $(1 - \tau_t^j) (\Phi_{\ell t}^j + \Phi_{kt}^j) - (\phi_t^j + \gamma) a_t^j + T_t^j$
4. For $t = \{1 : T\}$, update the (household-specific) opportunity costs of housing R_t^j and deposits R_t^{mj} and the price indices P_t^j and $P_{\Lambda t}^j$
5. For $t = \{T : 1\}$, **for each t starting from the last period** solve the Hamilton-Jacobi-Bellman equation and update the saving rule \dot{a}_t^j
6. For $t = \{T : 1\}$, update aggregate expenditure Λ_t^j , deposits m_t^j , housing h_t^j , consumption of non-durable c_t^j and labor supply ℓ^j according to the decision rules described in the main text
7. For $t = \{1 : T\}$, solve the Kolmogorov equation to obtain the distribution of households over the asset grid
8. For $t = \{1 : T\}$, update aggregate labor ℓ_t and update the average level of labor productivity
9. For $t = \{1 : T\}$, update the distributions of labor income $\Phi_{\ell t}^j$, capital income Φ_{kt}^j and Y_{kt}^j , and reflate asset detentions h_t^j and k_t^j with housing and equity capital gains, respectively
10. For $t = \{1 : T\}$, update the tax and transfer schedules $\tau_{\ell t}^j$, τ_t^j , ϕ_t^j and T_t^j
11. Compute the vector of residuals of capital-market clearing conditions as the difference between aggregate capital and the sum of individual productive capital detention
12. Update the path of k_t using the above (time-varying) residuals and iterate from 1. using the new sequence of k_t as initial guess until the maximum excess capital holding between $\{2 : T\}$ is strictly less than tolerance (0.01% of total capital stock). By definition, since capital is predetermined, errors at time $t = 1$ can not be brought to zero.

Solving for the transition dynamics takes a few minutes depending on the exercise, nature of the exogenous drivers and length of the simulation.

D Additional Tables and Figures

Figure 18: Counterfactual pretax income shares: constant taxes & markups & capital gains

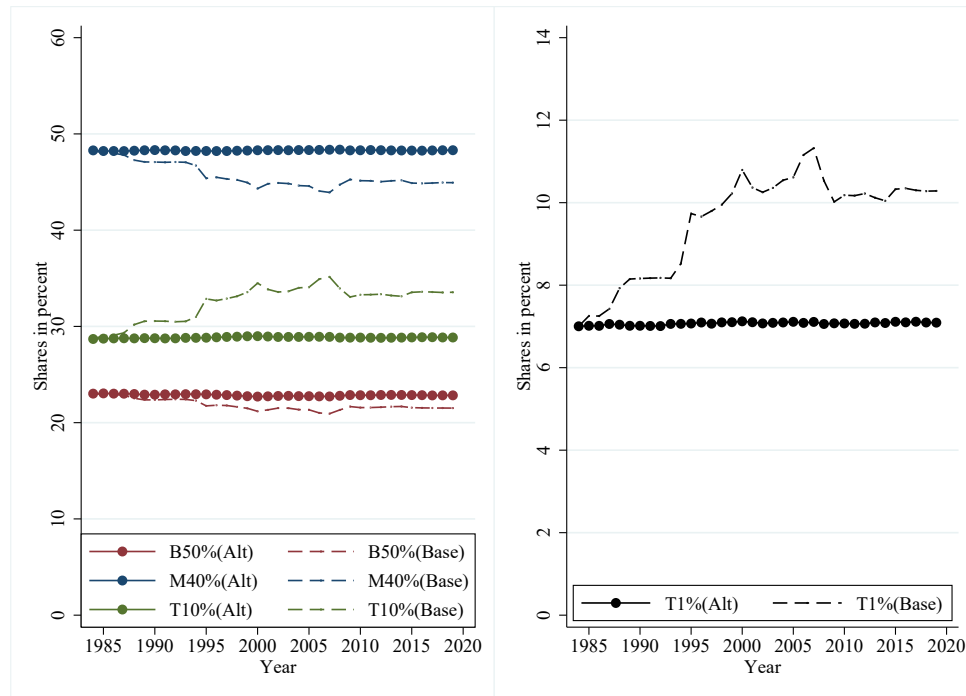


Figure 19: Counterfactual pretax income shares: only changes in taxes and transfers (constant markups & capital gains)

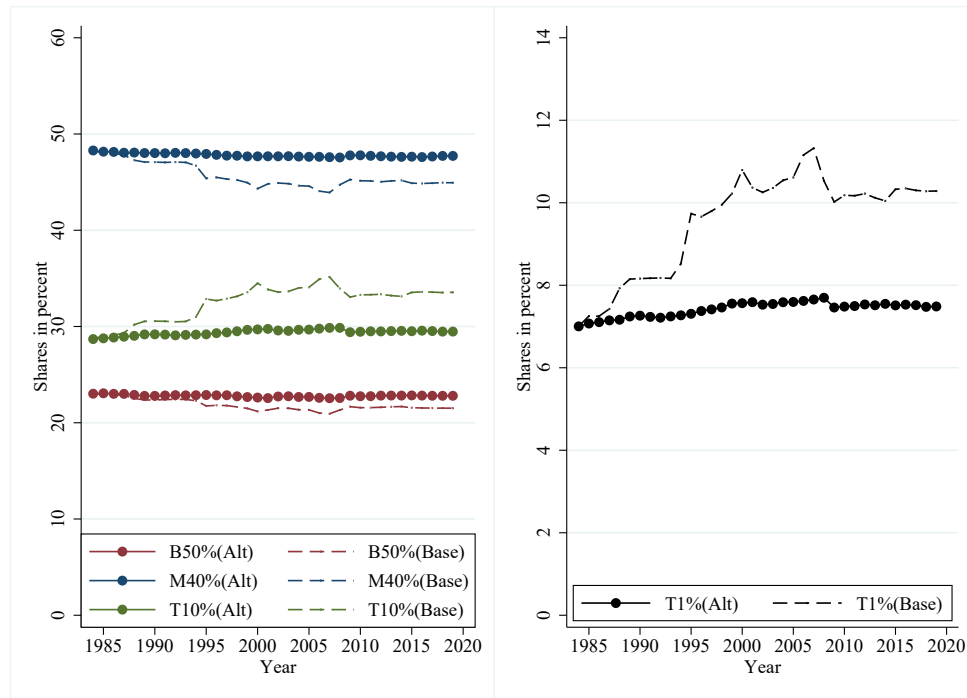


Figure 20: Counterfactual pretax income shares: only changes in markups (constant taxes and transfers & capital gains)

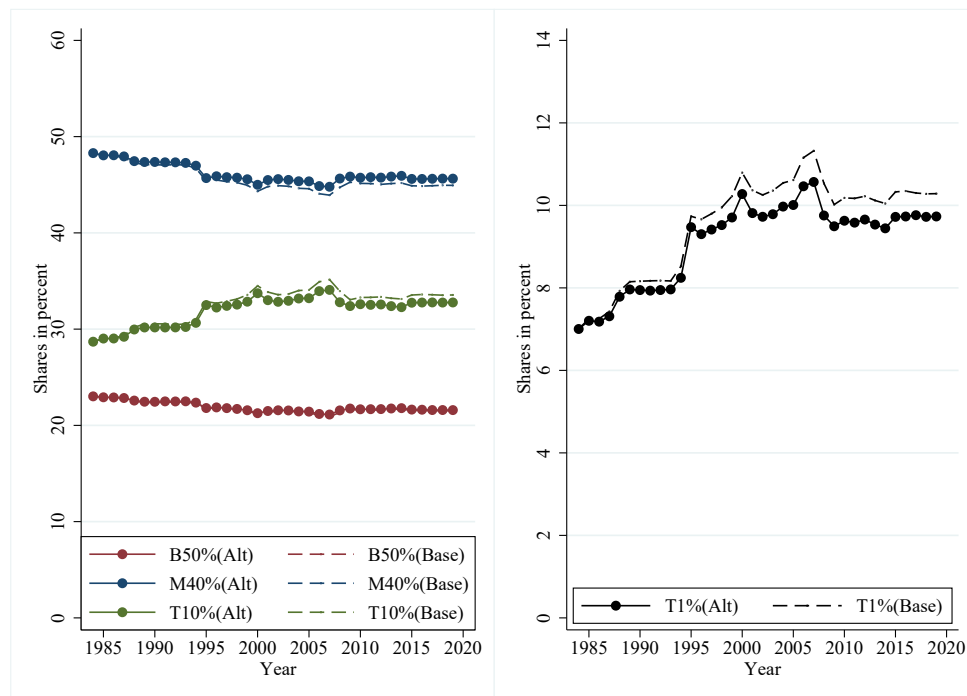


Figure 21: Counterfactual pretax income shares: Changes in both taxes and transfers & markups (constant capital gains)

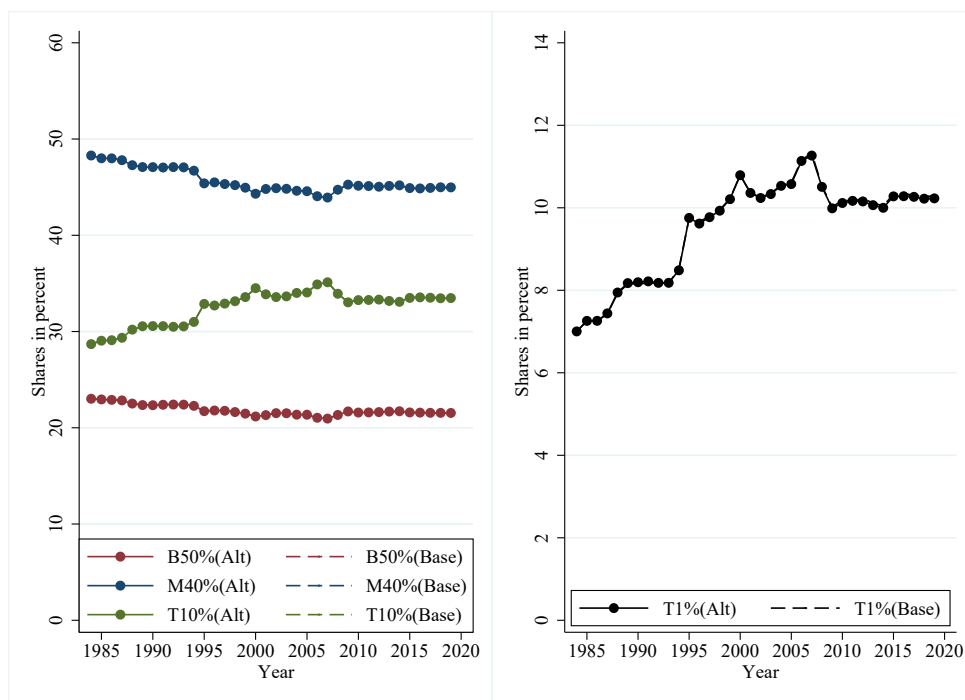


Figure 22: Counterfactual wealth shares: constant taxes & markups & capital gains

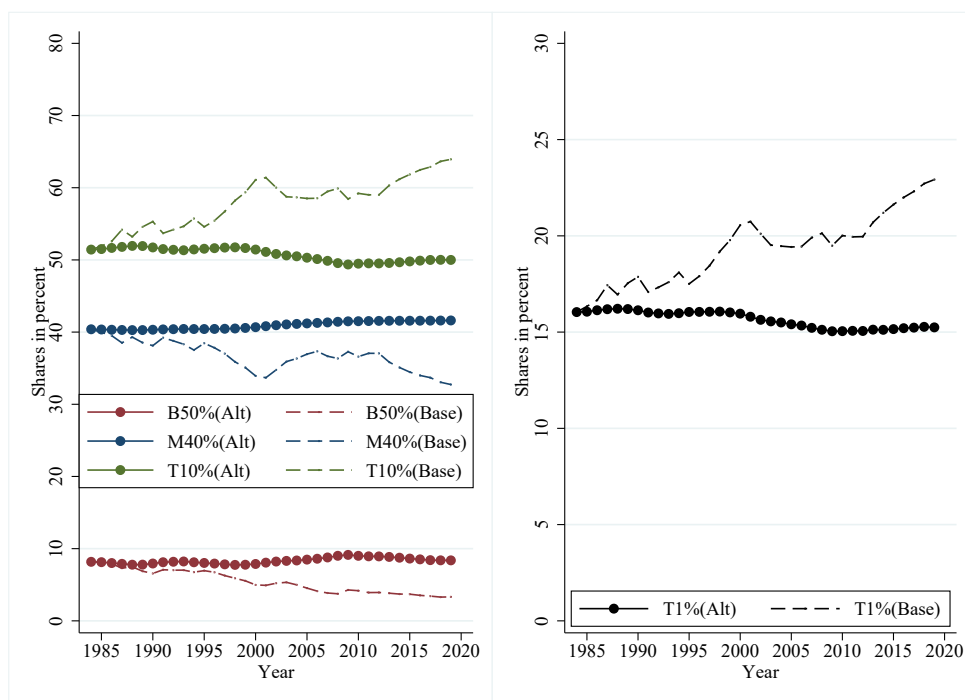


Figure 23: Counterfactual wealth shares: only changes in taxes and transfers (constant markups & capital gains)

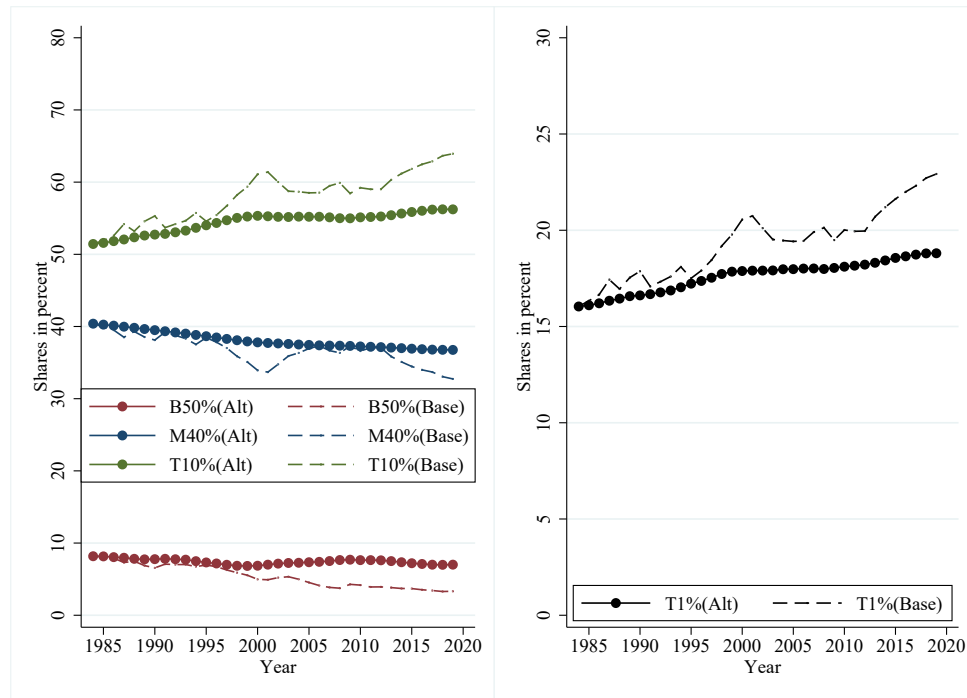


Figure 24: Counterfactual wealth shares: only changes in markups (constant taxes and transfers & capital gains)

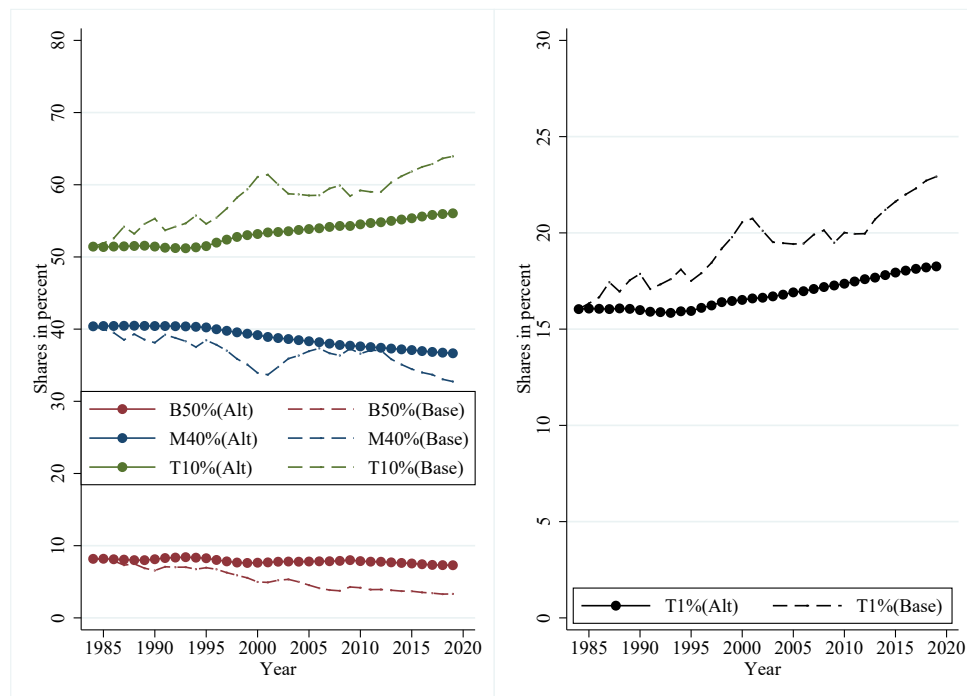


Figure 25: Counterfactual wealth shares: Changes in both taxes and transfers & markups (constant capital gains)

