Falling Interest Rates and Credit Reallocation: Lessons from General Equilibrium

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Abstract

We show that in a canonical model with heterogeneous entrepreneurs, financial frictions, and an imperfectly elastic supply of capital, a fall in the interest rate has an ambiguous effect on aggregate economic activity. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing financial constraints and by prompting relatively less productive entrepreneurs to invest. In general equilibrium, however, this higher demand for capital raises its price and crowds out investment by more productive entrepreneurs. When this general-equilibrium induced reallocation is strong enough, a fall in the interest rate reduces aggregate output. We show that this reallocation effect is of the same order of magnitude as the balance-sheet channel, and that the interaction of both gives rise to boom-bust dynamics in response to a fall in the interest rate. Our novel mechanism contributes to the debate on whether and how low-interest environments may foster the proliferation of *socially* unproductive activities.

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1 Introduction

One distinctive feature of recent decades has been the sustained and significant decline in real interest rates across the globe. Although the conventional wisdom holds that declining interest rates should stimulate economic activity, there are mounting concerns that declines in interest rates – at least when they are persistent – may have undesirable side effects, such as endangering financial stability (Rajan, 2015; Martinez-Miera and Repullo, 2017; Coimbra and Rey, 2017; Brunnermeier and Koby, 2018; Bolton et al., 2021) or slowing-down the pace of technological innovation and long-term growth (Liu et al., 2019; Quadrini, 2020; Benigno et al., 2020). Moreover, recent evidence suggests that periods of fast credit growth fueled by low interest rates can also result in lackluster productivity performance (Reis, 2013; Gopinath et al., 2017; Doerr, 2018; García-Santana et al., 2020).

In this paper, we contribute to this debate by proposing a novel mechanism through which declining interest rates can foster the proliferation of *socially* unproductive activities. We consider a canonical economy populated by entrepreneurs who have the ability to invest in capital. We make three assumptions. First, entrepreneurs are heterogeneous in their productivity, i.e., they differ in their effectiveness at using capital to produce consumption goods. Second, entrepreneurs face financial frictions, i.e., they cannot pledge the entire surplus from their activities to outsiders. Finally, the supply of capital is imperfectly elastic. We show that, in this environment, a fall in the interest rate distorts the allocation of capital across entrepreneurs and, therefore, has an ambiguous effect on aggregate output.[1]

Our findings challenge the conventional view that lower interest rates stimulate economic activity by raising both entrepreneurs' willingness and ability to invest. In our canonical framework of heterogeneous productivity and financial frictions, the conventional view holds in partial equilibrium. In general equilibrium, however, declining interest rates may stimulate investment by the wrong mix of entrepreneurs. The logic goes as follows. Lower interest rates make it attractive for less productive entrepreneurs to invest, which raises the price of capital and crowds out investment by more productive (albeit financially constrained) entrepreneurs. As a result, capital is reallocated from more to less productive entrepreneurs. We formalize this general-equilibrium *reallocation effect* of declining rates and show that it attenuates their stimulative effect on output. When this general-equilibrium effect is strong enough, a fall in the interest rate reduces aggregate output!

[1] We focus throughout most of the paper on a small-open economy that takes the world interest rate as given. Our results extend to a closed economy where interest rate changes are driven by changes in fundamentals; see Section 3.1.

Our mechanism relies on the three ingredients highlighted above: heterogeneous productivity, financial frictions, and an imperfectly elastic supply of capital. Since the capital supply is not perfectly elastic, entrepreneurial investment affects the price of capital. Yet, individual entrepreneurs do not internalize this effect, which gives rise to a *pecuniary externality*. In the absence of heterogeneity or financial frictions, the return to investment would be equalized across entrepreneurs and this pecuniary externality would have no first-order effect on output or welfare. With heterogeneity and financial frictions, however, this is no longer the case. Simply put, less productive entrepreneurs do not internalize the crowding-out effect of their investment on that of more productive entrepreneurs.

The strength of the reallocation effect depends on the elasticity of the capital supply – which determines the magnitude of general-equilibrium effects – and on the severity of the financial friction. In particular, when the supply of capital is sufficiently inelastic and the financial friction is severe enough, the reallocation effect becomes so strong that a fall in the interest rate is contractionary. Moreover, we show that the reallocation effect is tied to an inefficiency: some entrepreneurs invest excessively. A benevolent social planner, even if she is also subject to financial frictions, would limit the investment of less productive entrepreneurs in order to reduce the equilibrium price of capital and foster investment by more productive entrepreneurs. Thus, a fall in the interest rate is always expansionary in the constrained efficient allocation.

We view our main contribution as theoretical: namely, that in the presence of financial frictions the expansionary effect of a decline in the interest rate is weakened (or even overturned!) by general-equilibrium reallocation effects. Our mechanism requires that the price of capital depend on local economic conditions (i.e., domestic demand and supply for capital), which is a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets (e.g., Kiyotaki and Moore (1997); Krishnamurthy (2003); Lorenzoni (2008); Brunnermeier and Sannikov (2014)). Indeed, just as balance-sheet effects would vanish if the price of capital were independent of local economic conditions, so would the reallocation effects that we emphasize in this paper.

Going beyond the conceptual insight, what is the real-world interpretation of capital in the model? The literature has often thought of physical assets, which are owned by firms and give rise to balance-sheet effects (e.g., Kiyotaki and Moore (1997)). Within these assets, moreover, the literature has often focused on land or real estate, where the importance of general-equilibrium effects (e.g., as induced by an imperfectly elastic supply) seems most natural. This is in opposition to machinery, for instance, which is tradable internationally and whose prices are less dependent on local economic conditions. Our key insight, instead, is not limited to physical assets, and can arise in relation to any factor of production that requires credit.[2]

We extend our analysis to an infinite-horizon economy to incorporate balance-sheet effects and show how they interact with our reallocation effects. In doing so, we follow the literature and interpret capital as land or real estate, whose prices affect entrepreneurial net worth. In this environment, rising asset prices boost entrepreneurial wealth and thus relax the financial constraints of more productive entrepreneurs. This channel should intuitively amplify the stimulative effect of declining rates. We show that this intuition is only partially correct. The reason is that balance-sheet effects are driven by unexpected changes in the price of capital and are thus inherently transitory, whereas the general-equilibrium forces that drive reallocation last as long as interest rates remain low. As a result, the response of output to a persistent fall in the interest rate features a transitory boom, followed by a persistent bust: the balance-sheet channel temporarily raises the investment of more productive entrepreneurs, but its effect gradually wears-off as the contractionary reallocation effect begins to dominate.

Lastly, we perform a numerical exploration by calibrating the model to US data. We use available measures of real-estate supply elasticities to proxy for the strength of general-equilibrium effects. We calibrate the elasticity of the capital supply to its empirical counterpart for OECD economies, and find that the reallocation effect is quantitatively large. Whereas a permanent decline in the interest rate from 2% to 1% would raise output by 4% if productivity were constant, it actually reduces output by 2% once the adverse general-equilibrium effects on productivity are taken into account. The key takeaway is not so much the ultimate size of the reallocation effect – which may be overestimated by assuming that all capital in the economy takes the form of land/real estate – but rather that it is of the same order of magnitude as balance-sheet effects. Given that macroeconomists have ample evidence for the former (Peek and Rosengren, 2000; Gan, 2007; Chaney et al., 2012), our findings suggest that they should also pay close attention to the latter.

Our theory is consistent with recent evidence on the macroeconomic effects of credit booms driven by low interest rates. In particular, it sheds further light on the experience of several Southern European economies during the early 2000s, when a reduction in interest rates coincided with local booms in credit and asset prices and with a decline in aggregate productivity (Gopinath et al., 2017; García-Santana et al., 2020). Our findings suggests that this decline in productivity may have been driven not just by the expansion of less productive activities –

[2]One example is skilled labor, which (i) requires working capital insofar as firms pay wages before output is realized, and (ii) is relatively scarce, at least in the medium-term. Indeed, "skilled labor shortages" have received much attention in the press recently: `https://www.forbes.com/sites/carolinecastrillon/2021/09/22/why-us-talent-shortages-are-at-a-ten-year-high/?sh=46cdc65879c2`.

which, once again, is not bad in itself – but also by the crowding-out of more productive activities due to general-equilibrium effects. Indeed, evidence of such adverse reallocation is found by Banerjee and Hofmann (2018), who show that – for a set of advanced economies during recent decades – an increase in the share of "zombie" (i.e., less productive) firms in a given sector has been associated with a decline in investment and employment by "non-zombie" (i.e., more productive) firms in that same sector.[3]

We complement this evidence by analyzing the effects of interest rate changes across geographical regions and sectors in the US using data from the Bureau of Economic Analysis (BEA). We test the following key prediction of our theory: the stimulative effect of declining interest rates should be weaker when the reallocation induced by general-equilibrium effects is stronger. We again interpret capital in the model as real estate/land, which enables us to use existing measures of real-estate supply elasticities at the local-level for the US. We measure the response of sectoral output at the local-level to changes in the interest rate, and analyze how this response correlates with the local elasticity of real-estate supply and with the real-estate intensity of the sector.[4] Our findings are in line with the theory's prediction: a decline in the interest rate is less stimulative in (i) regions where the supply of real estate is less elastic (i.e., where general-equilibrium effects are strong), and (ii) sectors that use real estate more intensively as an input (i.e., where general-equilibrium effects are relevant for production costs).

Our paper contributes to the literature that studies mechanisms through which credit booms can adversely affect aggregate productivity (Reis, 2013; Gopinath et al., 2017; Gorton and Ordonez, 2020). Gopinath et al. (2017), in particular, have argued that declining interest rates can lead to a fall in productivity if the ensuing rise in credit is channeled to less productive firms. However, in their setting – as in our partial-equilibrium analysis – such an expansion in credit is beneficial because even less productive firms add value from a social perspective. Our contribution is to show how declines in interest rates can actually destroy social value – and even reduce aggregate output – once general equilibrium effects are taken into account.[5]

Our paper is also related to the growing literature on macroeconomics with heterogeneous agents. Much of this literature has studied how heterogeneity shapes an economy's response to monetary policy. Although this research focuses mostly on heterogeneity among households

[3]For related work on "zombie lending", see also Caballero et al. (2008); Adalet McGowan et al. (2018); Tracey (2019); Acharya et al. (2021).

[4]We use local (MSA level) measures of real-estate supply elasticities (Saiz, 2010), and we follow Vom Lehn and Winberry (2019) to construct a sector-level measure of real-estate intensity using BEA data.

[5]Recently, Kiyotaki et al. (2021) develop a model in which – due to financial frictions – entrepreneurs borrow predominantly against their near-term incomes, which means that a fall in the interest rate may actually tighten financial constraints.

(e.g., Cloyne et al. (2020), Kaplan et al. (2018), Slacalek et al. (2020)), a growing body of work also analyzes heterogeneity among firms (e.g., Anderson and Cesa-Bianchi (2020), Cloyne et al. (2018), Manea (2020), Ottonello and Winberry (2020), Jeenas (2020)).[6] Within this work, we are closest to Monacelli et al. (2018) and González et al. (2020), who study how interest rates shape productivity through their effects on the incentives and financial constraints of heterogeneous firms: differently from them, however, our focus is on the reallocation of resources across firms through general-equilibrium effects.

Finally, our model is related to work that stresses inefficiencies in the allocation of factors of production due to financial frictions. A recurring theme in this work is that individual firms do not internalize the effect of their demand on factor prices, which may lead to inefficient outcomes in the presence of financial constraints. Biais and Mariotti (2009), Ventura and Voth (2015), Martin et al. (2018), Asriyan et al. (2021), Buera et al. (2021) and Lanteri and Rampini (2021) provide examples of this work. In a related vein, Coimbra and Rey (2017) study the allocation of risky capital across financial intermediaries that are subject to financial constraints and are heterogeneous in their tolerance for risk. They show how, by reallocating capital towards riskier intermediaries, a decline in the interest rate may increase financial instability.

The paper is organized as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium effects of declining interest rates, as well as the normative properties of equilibria. Section 4 extends the analysis to an infinite-horizon economy and performs a numerical exploration. Section 5 provides supporting evidence, and Section 6 concludes.

2 Baseline model

Consider an economy that lasts for two periods, $t = 0, 1$. There are two goods: a perishable consumption good (c) and capital (k). There are two sets of risk-neutral agents, entrepreneurs and capitalists, each of unit mass.

Preferences. The preferences of all agents are given by:

$$U = E_0[c_1],$$

where c_1 is the consumption at $t = 1$ and $E_0[\cdot]$ is the expectations operator at $t = 0$.

[6] Relatedly, Leahy and Thapar (2019) study how age-structure shapes the impact of monetary policy: they find that monetary policy is most potent in regions with a larger share of middle-aged due to their higher propensity for entrepreneurship. On the other hand, Caggese and Pérez-Orive (2020) show how lower interest rates may become less expansionary in economies where intangible investments become more important.

Endowments. Each entrepreneur is endowed with w units of the consumption good at $t = 0$, while capitalists have no endowment.

Technology. Each capitalist has access to a production technology that converts $\chi(k)$ units of the consumption good into k units of capital at $t = 0$, where χ is quasi-convex and weakly increasing in k.[7] Capital can be used for production by entrepreneurs. Specifically, each entrepreneur has access to a production technology that converts one unit of capital at $t = 0$ into A units of the consumption good at $t = 1$. We refer to A as entrepreneurial productivity and assume that it is distributed independently across entrepreneurs, with distribution G that has an associated density g with full support on the interval $[0, 1]$.

Markets. The economy is small and open and there is an international financial market where agents can borrow and lend consumption goods at a world interest rate R. Here, we introduce a central friction of our analysis by supposing that an entrepreneur can always walk away with a fraction $1 - \lambda$ of her output at $t = 1$. This pledgeability friction will endogenously limit the borrowing and investment that each entrepreneur can undertake. There is also a competitive capital market, where agents can trade capital at a unit price q in period $t = 0$. Note that, as the economy ends at $t = 1$, capital is no longer valuable after production at that date.

2.1 Optimization and equilibrium

Since agents can borrow and lend consumption goods in the international financial market at the interest rate R, only the clearing of the capital market is crucial for equilibrium. To characterize this market clearing condition, we analyze next the demand and supply of capital.

Capital demand. Let b_A and k_A respectively denote total borrowing and capital demand by an entrepreneur with productivity A. Given prices $\{q, R\}$, the entrepreneur makes her optimal borrowing and investment decisions to maximize her expected consumption:

$$\max_{\{b_A, k_A\}} \quad A \cdot k_A - R \cdot b_A, \tag{1}$$

[7] The case where capitalists are simply endowed with $\bar{K}$ units of capital is captured by the following cost function: $\chi(k) = 0$ for $k \leq \bar{K}$ and $\chi(k)$ is infinite thereafter.

subject to budget, borrowing and feasibility constraints:

$$q \cdot k_A \leq \; w + b_A, \tag{2}$$

$$R \cdot b_A \leq \; \lambda \cdot A \cdot k_A, \tag{3}$$

$$0 \leq k_A \; . \tag{4}$$

Note that the price of capital q enters the budget constraint (2) but not the borrowing constraint (3): the reason, as already explained, is that the price of capital at $t = 1$ equals zero. This will no longer be true when we extend the analysis to an infinite-horizon economy in Section 4.

Optimization leads to the following capital demand:

$$k_A(q, R) \begin{cases} = 0 & \text{if } \frac{A}{q} < R \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } R = \frac{A}{q} \\ = \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w & \text{if } \frac{\lambda \cdot A}{q} < R < \frac{A}{q} \\ = \infty & \text{if } R \leq \frac{\lambda \cdot A}{q} \end{cases}, \tag{5}$$

which has an associated level of borrowing:

$$b_A(q, R) = q \cdot k_A(q, R) - w. \tag{6}$$

Equation (5) states that an entrepreneur's demand of capital is decreasing in the interest rate, R. When the interest rate is smaller than her return to capital, A/q, an entrepreneur finds it optimal to invest in financial markets and demands no capital. When both returns are equalized, the entrepreneur is indifferent between investing in capital and not doing so. When the interest rate is smaller than the return to capital but greater than its pledgeable return, the entrepreneur demands capital until her borrowing constraint binds.[8] Finally, when the interest rate is below the pledgeable return to capital, the entrepreneur's demand of capital is unbounded.

Equation (5) also implies that the demand function $k_A(q, R)$ is decreasing in q. For an entrepreneur who is unconstrained, i.e., $A \leq q \cdot R$, lower values of q raise the return to investing in capital. For an entrepreneur who is constrained, lower values of q relax the borrowing constraint and enable her to expand her borrowing and her purchases of capital. Finally, the demand function $k_A(q, R)$ is weakly increasing in λ, because a higher pledgeability of output

[8] Note that $q - \lambda \cdot A \cdot R^{-1}$ represents the "down payment" that is required to purchase a unit of capital: the price of each unit is q, but a part $\lambda \cdot A \cdot R^{-1}$ can be financed by borrowing against the unit's future output.

enables constrained entrepreneurs to expand their borrowing and thus their purchases of capital.

We denote the aggregate demand for capital by entrepreneurs by:

$$K^D(q,R) \equiv \int_0^1 k_A(q,R) \cdot dG(A). \tag{7}$$

Capital supply. Given the price of capital, each capitalist chooses his supply of capital to maximize profits. Formally, we use $K^S(q)$ to denote a solution to:

$$\max_{k \geq 0} \; q \cdot k - \chi(k). \tag{8}$$

Since all capitalists are identical, $K^S(q)$ denotes both the individual and the aggregate supply of capital, which is weakly increasing in q.[9]

Market clearing. The price of capital, q, ensures that the capital market clears:

$$K^S(q) = K^D(q,R). \tag{9}$$

Aggregate domestic output is given by:

$$Y(q,R) = \int_0^1 A \cdot k_A(q,R) \cdot dG. \tag{10}$$

Throughout, our main objective is to characterize the effect of changes in the interest rate on aggregate output and, in particular, on the aggregate stock of capital and its allocation among entrepreneurs. As we shall see, this latter allocative effect will drive the changes in the aggregate productivity of capital, defined as the ratio $Y(q,R)/K^S(q)$.

Equilibrium. An equilibrium consists of prices $\{q,R\}$ and allocations $\{\{k_A,b_A\}_A, K^S, Y\}$, such that, given prices, $\{k_A,b_A\}_A$ satisfy Equations (5) and (6), K^S is a solution to (8), the capital market clears according to Equation (9), and Y satisfies Equation (10).

3 Equilibrium effects of changes in interest rates

We want to understand the equilibrium effects of changes in the interest rate R. For now, we interpret changes in R as being induced by exogenous factors, such as changes in the world

[9]E.g. if $\chi(\cdot)$ is increasing and convex with $\chi(0) = \chi'(0) = 0$, then $K^S(q) = \chi'^{-1}(q)$ and is increasing in q.

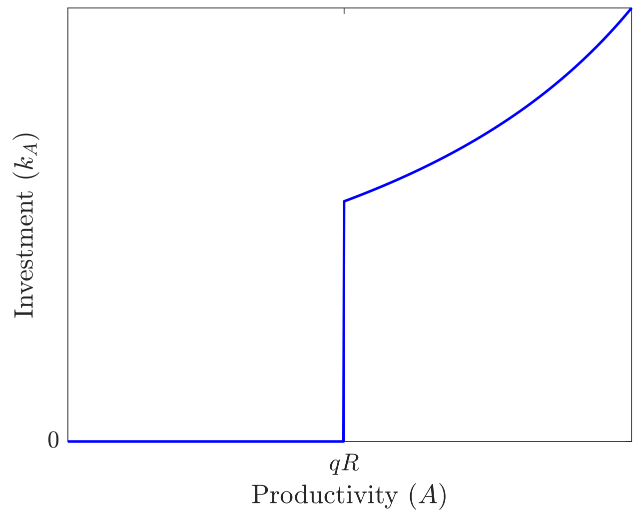


Figure 1: **Distribution of capital across entrepreneurs given prices** $\{q, R\}$**.**

interest rate or in capital inflows. We later show that our results also hold when R is an endogenous variable and its decline is driven by changes in model fundamentals.

Figure 1 depicts the distribution of capital across entrepreneurs for given prices $\{q, R\}$. To determine the aggregate effects of a change in R, we need to understand how this distribution of capital responds to such a change. In what follows, we refer to those entrepreneurs who find it optimal not to invest (i.e., $A < q \cdot R$) as "infra-marginal", to those that invest until their borrowing constraint binds (i.e., $A > q \cdot R$) as "supra-marginal", and to those that are indifferent (i.e., $A = q \cdot R$) as "marginal" entrepreneurs.

At first sight, the effect of a change in R on investment and output seems trivial. It follows immediately from Equation (5) that, for a given price q, all entrepreneurs raise their demand of capital when R falls. Supra-marginal entrepreneurs demand more capital because a lower value of R raises the present value of pledgeable output, thus relaxing their borrowing constraints. Moreover, some infra-marginal entrepreneurs start investing because a lower value of R raises the present value of their investment. This partial-equilibrium effect of a decline in R is depicted through a shift from the solid-blue to the dashed curve in panels (a) and (b) of Figure 2.

As long as the supply of capital is not perfectly elastic, however, the effects of a decline in R do not end here. There is also a general-equilibrium effect because the price of capital q must increase to ensure capital-market clearing. This general-equilibrium effect makes capital less attractive and reduces investment along all margins, but it cannot be so strong as to raise the productivity of the marginal entrepreneur, $q \cdot R$: otherwise, all entrepreneurs would reduce

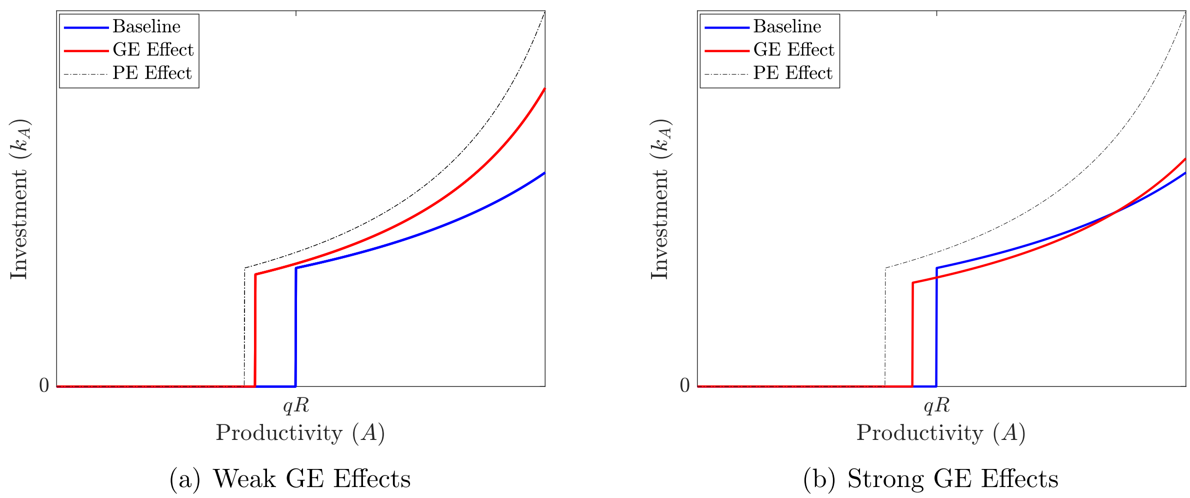


Figure 2: **Partial- and general-equilibrium effects of a fall in the interest rate on k_A.**

their demand of capital, which is a contradiction. This implies that a decline in the interest rate must necessarily raise the investment of some infra-marginal entrepreneurs, although it may reduce the investment of some supra-marginal entrepreneurs.

Formally, the change in the investment of a supra-marginal entrepreneur with productivity A is given by:

$$\frac{dk_A}{dR} = \frac{\left|\frac{dq}{dR}\right| - \frac{\lambda \cdot A}{R^2}}{q - \frac{\lambda \cdot A}{R}} \cdot k_A. \tag{11}$$

Equation (11) shows that the change in investment induced by a change in the interest rate has a partial- and a general-equilibrium component. The second term in the numerator represents the partial-equilibrium effect, by which a decline in R increases the pledgeable return to capital and thus entrepreneurs' ability to invest. The first term in the numerator captures instead the general-equilibrium effect, by which a decline in R raises the price of capital thereby reducing entrepreneurial demand of capital.

Equation (11) suggests that the investment of supra-marginal entrepreneurs may decline when the interest rate falls, provided that the general-equilibrium effect is strong and the partial-equilibrium effect is weak enough. The strength of the general-equilibrium effect depends largely on the elasticity of capital supply, while the strength of the partial-equilibrium effect depends on the tightness of the financial friction. As $\lambda \to 0$, for instance, the partial-equilibrium effect disappears altogether. Panels (a) and (b) in Figure 2 depict the effects of a fall in the interest rate on entrepreneurial investment, which is captured by the shift from the dashed to the solid-red curve. In panel (a), general-equilibrium effects are weak and all supra-marginal entrepreneurs invest more when the interest rate falls; in panel (b), instead, general-equilibrium effects are strong and some supra-marginal entrepreneurs invest less when the interest rate declines.

The effect of interest rates on output is shaped by the behavior of investment across entrepreneurs. Formally, we can combine Equations (9) and (10) to obtain:

$$\frac{dY}{dR} = \underbrace{\int_{q\cdot R}^{1} (A - q \cdot R) \cdot \frac{dk_A}{dR} \cdot dG(A)}_{\text{Capital-reallocation effect } \equiv\, \mathcal{R}} + \underbrace{q \cdot R \cdot \frac{dK^S(q)}{dR}}_{\text{Capital-supply effect } \equiv\, \mathcal{K}}, \tag{12}$$

where dk_A/dR is given by Equation (11). The first term in Equation (12), which we denote by $\mathcal{R}$, captures the capital-reallocation effect: the change in output driven by changes in the investment of supra-marginal entrepreneurs. As we have already noted, this effect can be positive or negative depending on the relative strength of the general- and partial- equilibrium effects. In what follows, we say that the capital-reallocation effect is *stronger* when $\mathcal{R}$ is more positive. The second term in Equation (12), which we denote by $\mathcal{K}$, captures instead the capital-supply effect: the change in output driven by adjustments in the aggregate capital stock. This effect is always (weakly) negative, since lower interest rates raise the demand for capital and thus the equilibrium stock of capital. In what follows, we say that the capital-supply effect is *stronger* when $\mathcal{K}$ is more negative.

Equation (12) illustrates the role of both heterogeneity and financial frictions in shaping the aggregate response of output to changes in the interest rate. In the absence of heterogeneity, all entrepreneurs would have the same productivity; in the absence of financial frictions, only the most productive entrepreneur would invest. In either case, the capital-reallocation effect would disappear and the response of output to the interest rate would be negative and driven only by the capital-supply effect, i.e., on the economy's ability to adjust the capital supply to the shifting demand. With heterogeneous productivity and financial frictions, however, the response of output to changes in the interest rate depends not just on the behavior of aggregate investment but also on its reallocation across entrepreneurs. In fact, the capital reallocation effect can be so strong that falling interest rates may become contractionary!

To illustrate this, consider a simple example in which there is no borrowing and lending (i.e., $\lambda = 0$) and the capital stock is fixed (i.e., $K^S(q) = \bar{K}$). The lack of borrowing and lending means that the investment of supra-marginal entrepreneurs is equal to w/q, and thus independent of the interest rate: this maximizes the strength of the reallocation effect, $\mathcal{R}$. The fixed capital supply, in turn, eliminates the capital supply effect, $\mathcal{K}$, altogether. In this case, a decline of the interest rate must necessarily reduce aggregate output. By boosting the investment of infra-marginal entrepreneurs, a lower interest rate raises the equilibrium price of capital and thus reduces supra-marginal investment (which is more productive).

This example is of course stark but, as the next proposition shows, the result is more general and does not rely on such extreme scenarios.

Proposition 1 *Consider two economies that have the same equilibrium allocations and are identical in all respects except for the capital supply schedule. Let ε denote the elasticity of capital supply with respect to the price of capital q in equilibrium. Then, in the economy with lower ε:*

- *the capital-reallocation effect, $\mathcal{R}$, is stronger;*
- *the capital-supply effect, $\mathcal{K}$, is weaker;*
- *the response of output to a change in the interest rate, dY/dR, is less negative;*

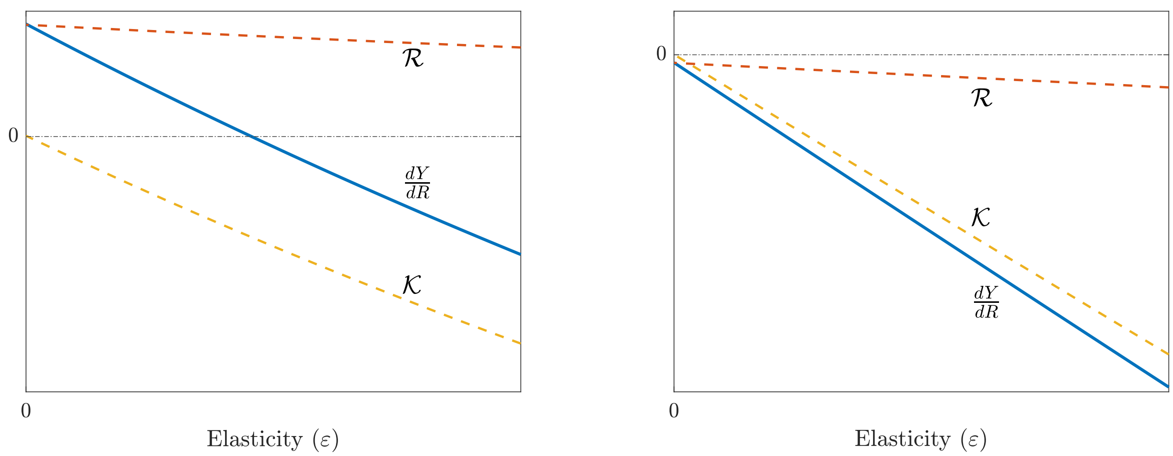
moreover, for low enough ε, there is a threshold $\bar{\lambda}_\varepsilon > 0$ such that dY/dR is positive if $\lambda < \bar{\lambda}_\varepsilon$.

Proposition 1 states that the response of output to the interest rate is decreasing in the elasticity of the capital supply, ε, which governs the strength of the general-equilibrium effect.[10] This is illustrated in Figure 3, which plots dY/dR against ε for low and high values of λ, respectively. Both panels show that dY/dR increases as ε decreases. Lower values of ε weaken the capital-supply effect and, by reinforcing the general-equilibrium response of the price of capital, strengthen the capital-reallocation effect. When λ is low, moreover, the reallocation effect becomes positive and – for low values of ε – a fall in the interest rate leads to a decline in aggregate output (see panel (a)).[11]

Proposition 1 shows that, in general-equilibrium, the interaction of heterogeneous productivity and financial frictions gives rise to capital-reallocation effects that can mitigate or even overturn the expansionary effects of declining interest rates. Although it has been derived in a fairly stylized environment, we discuss next how it extends to more general settings.

[10] The parameterization of the cost function $\chi(\cdot)$ used for Figure 3 is provided in the proof of Proposition 1.

[11] We want to note that result in Proposition 1 is local, in the sense that it characterizes dY/dR at a given equilibrium. In particular, if the equilibrium changes – e.g. due to a change in R – so does the strength of the reallocation effect and thus the threshold $\bar{\lambda}_\varepsilon$. This implies that, for given parameter values, the sign of dY/dR need not be the same for all levels of R.



(a) Low λ (b) High λ

Figure 3: **Effect of the elasticity of capital supply on dY/dR.**

3.1 Robustness

Unconstrained firms. We have assumed throughout that all entrepreneurs are subject to financial constraints. One may wonder how our results change if, as in recent macroeconomic models (e.g., Cloyne et al. (2018); Ottonello and Winberry (2020)), constrained and unconstrained entrepreneurs coexist. To this end, Online Appendix C.1 modifies our baseline setup by introducing a subset of unconstrained entrepreneurs. The key result is that, as long as unconstrained entrepreneurs are active in equilibrium, our main results do not change and they may even be strengthened.[12] The reason is that these entrepreneurs can freely adjust their capital demand to changes in the interest rate, which strengthens the capital reallocation effect and raises dY/dR.

Diminishing returns at the entrepreneur level. In our baseline model, entrepreneurs operate a linear production technology. Thus, investment displays "bang-bang" behavior, i.e., an entrepreneur either finds it unattractive to invest or invests as much as possible. Online Appendix C.2 shows that our results remain unchanged if, as is commonly assumed in the firm-dynamics literature (e.g., Hopenhayn (1992)), there are diminishing returns at the entrepreneur level. The key difference is that the set of marginal entrepreneurs has a positive measure under diminishing returns and, if Inada conditions are satisfied, it includes all entrepreneurs below a threshold level of productivity A. In equilibrium, marginal entrepreneurs are unconstrained because – given their lower marginal product of capital – they operate at a smaller scale

[12] This requires that the unconstrained entrepreneurs not be too productive; otherwise, they would absorb the entire capital stock and the economy would effectively be frictionless.

than the constrained, more productive entrepreneurs.[13] Under similar conditions as in our baseline model, moreover, a fall in the interest rate generates a reallocation of capital from more productive (high-MPK) to less productive (low-MPK) entrepreneurs, thus reducing output.

Default risk and credit spreads. In our baseline model, even though access to credit is limited by a financial friction, all entrepreneurs are able to borrow at the risk-free interest rate. Micro-evidence suggests, however, that there is substantial heterogeneity in corporate funding costs or credit spreads (e.g., Cavalcanti et al. (2021)). Though a complete analysis of corporate credit spreads is beyond the scope of the paper, we incorporate an important driver of such spreads – risk of default – in Online Appendix C.3. We do so through a simple extension of our model in which entrepreneurs differ both in their probability of success and in their productivity upon success. Moreover, the probability of success and the productivity upon success may be positively or negatively correlated. In this setting, we show that an entrepreneur's funding cost is equal to the risk-free rate plus a spread, which depends both on the risk-free rate and on the entrepreneur's probability of success. Despite the multi-dimensional heterogeneity among entrepreneurs, our main results remain qualitatively unchanged. The key difference from our baseline setting is that it is now the heterogeneity in entrepreneurs' *expected* productivity that matters for the capital-reallocation effect.

Closed economy and savings gluts. We have considered throughout a small, open economy that takes the world interest rate as given. One may wonder what would change in a closed economy, where the interest rate is determined endogenously. To this end, Online Appendix C.4 shows that our results remain unchanged if the interest rate is determined endogenously and its fall is prompted by an increased desire to save (i.e., a savings glut). In a closed-economy version of our baseline model, such an increase in savings is triggered either by a shift in preferences (higher patience) or in endowments (capitalists become richer). Thus, our findings are consistent with one of the most popular hypotheses to explain the sustained decline in interest rates over the past several decades (e.g., Bernanke et al. (2005); Caballero et al. (2008)).

Dynamics of wealth accumulation. Perhaps the main limitation of the baseline economy is that it is essentially static, as it lasts for only two periods and entrepreneurial wealth is exogenous. In a fully dynamic economy, however, entrepreneurial wealth would naturally be endogenous to (i) productivity, as more productive entrepreneurs may accumulate wealth faster; and potentially (ii) asset prices, due to the well-known balance-sheet effects à la Kiyotaki and Moore (1997). This raises the question of whether our results would hold in such an economy.

[13] Note that, since now there is a mass of marginal entrepreneurs who are unconstrained in equilibrium, this extension is conceptually similar to the previous one where some firms are assumed to be unconstrained.

Section 4 addresses this question by extending the analysis to an infinite-horizon economy and showing how the reallocation effect interacts with wealth accumulation and balance sheet effects. Before moving to the fully dynamic model, however, we turn to the normative properties of equilibrium.

3.2 Normative properties

We now analyze the extent to which the competitive equilibrium is constrained (in)efficient. As we will show, the general-equilibrium induced reallocation that we identified in the previous section gives rise to an externality that renders the investment of some relatively less productive entrepreneurs excessive from the social point of view. This externality turns out to be at the heart of why a fall in R has ambiguous effects on economic activity.

Consider the problem of a social planner whose objective is to maximize aggregate consumption at $t = 1$.[14] The planner is constrained to only choosing investment $\{k_A\}$ for entrepreneurs, but agents make all the other decisions on their own. This implies, in particular, that the planner must respect individual budget and financial constraints. To simplify the exposition, we assume throughout that $\chi(\cdot)$ is strictly convex.

Formally, the social planner solves the following maximization problem:

$$\max_{\{k_A\}} \int_0^1 A \cdot k_A \cdot dG(A) - R \cdot [\chi(K^S) - w], \tag{13}$$

subject to:

$$R \cdot (q \cdot k_A - w) \leq \lambda \cdot A \cdot k_A \quad \text{and} \quad 0 \leq k_A \quad \forall A, \tag{14}$$

and to capitalists' optimization and market clearing:

$$\chi'^{-1}(q) = K^S = \int_0^1 k_A \cdot dG(A). \tag{15}$$

The objective function of the planner in (13) says that aggregate consumption at $t = 1$ is equal to aggregate output net of repayments to international lenders, which are given by R times the difference between the cost of capital production and aggregate endowment at $t = 0$. Equations (14) and (15) state that the planner must respect individual budget and financial constraints, feasibility, and market clearing. In particular, the planner is not able to make transfers so as to overcome financial frictions.

[14] Since preferences are linear, such an objective is equivalent to maximizing the equally-weighted aggregate welfare. We thus abstract from distributional considerations.

To understand the solution to the planner's problem, consider the (social) net present value (NPV_A^{SP}) of a unit of investment, k_A, by entrepreneur with productivity A:

$$\text{NPV}_A^{SP} \equiv \frac{A}{R} - q - \left[\chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG(\widehat{A})\right] \tag{16}$$

where $\gamma_{\widehat{A}}$ denotes the multiplier on the borrowing constraints of entrepreneurs with productivity $\widehat{A}$. The first observation is that, since NPV_A^{SP} is linear and increasing in A, there exists a marginal entrepreneur $\widetilde{A}$ with $\text{NPV}_{\widetilde{A}}^{SP} = 0$, such that only entrepreneurs with productivity above $\widetilde{A}$ invest and they do so until their borrowing constraints bind. The second observation is that, since the term in brackets is positive, the planner perceives a higher social cost (or a lower social benefit) of investment than individual entrepreneurs, who only compare A/R to q. This is because the planner internalizes that each additional unit of investment raises the equilibrium price of capital (as $\chi''(K^S) > 0$) and thus tightens the borrowing constraints of all entrepreneurs with productivity above $\widetilde{A}$. Since the borrowing constraints bind for all such entrepreneurs (as $\gamma_A > 0 \ \forall A > \widetilde{A}$), this entails a first-order welfare loss. Consequently, the planner restricts investment by some entrepreneurs by setting $\widetilde{A} > q \cdot R$.

The following proposition summarizes the above discussion and also states its main implication for the response of output to changes in interest rates.

Proposition 2 *Let $\widetilde{A}$ denote the productivity of the marginal entrepreneur at the social planner allocation, and q^{CE} and q^{SP} respectively denote the price of capital in the competitive equilibrium and in the social planner's allocation. Then:*

$$\widetilde{A} > R \cdot q^{CE} > R \cdot q^{SP},$$

i.e., relative to the competitive equilibrium, the planner restricts investment by some supra-marginal entrepreneurs thereby depressing asset prices. Moreover, letting Y^{SP} denote output in the social planner allocation, it holds that:

$$\frac{dY^{SP}}{dR} < 0,$$

i.e., a fall in the interest rate is always expansionary in the social planner's allocation.

The first part of Proposition 2 follows directly from our previous discussion. It only illustrates why the planner forbids some entrepreneurs from investing altogether: by doing so, she reduces the price of capital relative to the competitive equilibrium, thus enabling more productive entrepreneurs to expand their investment. The second part of Proposition 2 follows directly from the first. To see this, simply note that a fall in the interest rate can only reduce output if it reallocates capital from supra- to infra-marginal entrepreneurs (see Equation (12)). But the planner can always keep these reallocation effects under control by adjusting the productivity of the marginal entrepreneur, $\widetilde{A}$, when the interest rate changes.

This type of planner intervention, which prevents some entrepreneurs from investing altogether, may seem far-fetched and informationally demanding for the planner (i.e., she needs to know which entrepreneurs to exclude). However, it is straightforward to show that the planner allocations can be decentralized through a simple Pigouvian subsidy τ on savings, financed with lump-sum taxes on capitalists. By choosing the subsidy appropriately, the planner can ensure that all entrepreneurs with productivity lower than $\widetilde{A}$ prefer to save their endowments at the market interest rate and collect the subsidy rather than investing in capital.

These results are reminiscent of the literature on "zombie" firms, which emphasizes that low interest rates incentivize unproductive activities (Caballero et al., 2008; Adalet McGowan et al., 2018; Banerjee and Hofmann, 2018; Tracey, 2019; Acharya et al., 2021). In much of that literature, however, the emphasis is on distortions that provide incentives to keep some firms operational even though they are not profitable (e.g. evergreening by banks). We show instead that investment can be socially excessive despite having a positive net present value from a private standpoint, because individual entrepreneurs do not internalize the crowding-out effect that they have on more productive investment.

3.3 How do we interpret capital?

The main insight of our paper is that in the presence of financial frictions the expansionary effect of a decline in the interest rate is weakened (or even overturned!) by general-equilibrium reallocation effects. Our mechanism requires the price of capital to depend on local economic conditions (i.e., domestic demand and supply for capital), which is a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets (e.g., Kiyotaki and Moore (1997); Krishnamurthy (2003); Lorenzoni (2008); Brunnermeier and Sannikov (2014)). Indeed, just as balance-sheet effects would vanish if the price of capital were independent of local economic conditions, so would our reallocation effects.

Going beyond the conceptual insight, how can we interpret capital in the data? The literature has often thought of physical assets, which are owned by firms and give rise to balance sheet effects (e.g., Kiyotaki and Moore (1997)). Within these assets, moreover, the literature has mostly focused on land or real estate, where the importance of general equilibrium effects (e.g.

as induced by an imperfectly elastic supply) seems most natural. This is in opposition, for instance, to machinery that is tradable internationally, whose prices are less dependent on local economic conditions. In the remainder of the paper, we follow the literature and interpret capital as land or real estate. By doing so, we can incorporate the classic balance-sheet effects and show how they interact with the reallocation effects that we emphasize here.

Our key insight is not limited to physical assets, however, and could arise in relation to any factor of production that requires credit. One example is skilled labor, which (i) requires working capital insofar as firms pay wages before output is realized, and (ii) is relatively scarce, at least in the medium-term. Under this interpretation, there are no balance sheet effects (as firms do not own their workers!) but there still are reallocation effects: by increasing the demand for skilled labor by relatively less productive firms, a decline in interest rates would raise skilled wages thereby crowding out labor demand by relatively more productive firms.[15]

4 Dynamics of wealth accumulation

There are two main reasons for extending the analysis to a dynamic economy. The first is conceptual. In a dynamic setting, changes in the interest rate are likely to have additional effects through wealth accumulation. In particular, productive entrepreneurs may stand to gain from a falling interest rate, as they benefit both through lower costs of borrowing and – potentially – through higher asset prices and their associated balance-sheet effects. The second reason is quantitative: a dynamic setting lends itself to a numerical exploration that can inform us about the quantitative relevance of reallocation effects.

Throughout this section, we follow the literature and adopt a continuous-time setting. The advantage of doing so is that it enables us to obtain analytical expressions for the stationary distribution of wealth, which is a key component of equilibrium characterization. We show, however, that all of our results go through in a discrete-time economy à la Kiyotaki and Moore (1997) in Online Appendix C.5.

[15] Moreover, these two interpretations are not mutually exclusive. Even firms that do not use real estate directly as an input of production do so indirectly as the value of residential real estate is a key determinant of wages. Under this interpretation, a decline in the interest rate raises labor demand by relatively less productive firms, boosting the local demand for residential real estate and prompting an increase in wage that crowds out relatively more productive firms.

4.1 A dynamic economy

Suppose that time t is continuous. To simplify notation, we omit time subscripts whenever possible. As in the baseline model of Section 3, the economy is populated by a continuum of entrepreneurs with mass one. The key difference relative to the baseline is that entrepreneurs differ not only in their productivity, which varies stochastically, but also in their wealth, which evolves endogenously.

Individual productivity fluctuates stochastically over time according to an idiosyncratic Poisson process with common arrival rate Θ. If an entrepreneur with productivity A is hit by the shock, she draws a new productivity A' from the distribution $g(A'|A)$, where we denote $g(A)$ as the corresponding stationary density that has full support on some interval $[\underline{A}, \bar{A}]$. Otherwise, her productivity remains unchanged. An individual's wealth w in turn evolves endogenously according to her equilibrium rate of return and her consumption choices. Absent the Poisson shock, the law of motion of entrepreneurial wealth is given by:

$$\dot{w} = y + \dot{q} \cdot k - \delta \cdot q \cdot k - r \cdot b - c, \tag{17}$$

where $y = A \cdot k$ is the output flow obtained from operating capital stock $k \geq 0$; $\dot{q}$ is the rate of change of the price of capital $q > 0$; $\delta \geq 0$ is the depreciation rate of physical capital; $r > 0$ is the interest rate on debt b, where $b < 0$ denotes savings; and $c \geq 0$ is the consumption flow.

As in the baseline economy, entrepreneurs may be financially constrained. We follow the literature (e.g., Moll (2014)) and introduce a financial friction by assuming that, immediately after issuing debt, an entrepreneur can default, walk away with a portion $1 - \lambda \in (0, 1)$ of her capital and re-enter financial markets.[16] Formally, this gives rise to the following financial constraint:

$$b \leq \lambda \cdot q \cdot k. \tag{18}$$

As in the baseline model, capital is supplied by a unit mass of capitalists. In particular, they have the ability to produce $I \geq 0$ units of capital per unit of time at a cost of $\chi(I) \geq 0$ in terms of the consumption good, where $\chi(\cdot)$ is increasing and quasi-convex. Thus, the aggregate capital stock K evolves according to:

$$\dot{K} = I(q) - \delta \cdot K, \tag{19}$$

[16] As in the literature, switching to a continuous-time setting requires a slight change in the nature of the financial friction in order to maximize tractability. As we mentioned earlier, however, Online Appendix C.5 extends our results to a discrete-time setting à la Kiyotaki and Moore (1997) where the financial constraint is analogous to the one in the baseline model of Section 2.

where $I(q)$ is derived from the capitalists' optimization problem and is given by:

$$I(q) \equiv \arg\max_{I \geq 0} \left\{ q \cdot I - \chi\left(I\right) \right\} . \tag{20}$$

All agents have logarithmic preferences for consumption and discount future consumption at rate $\rho > r$.[17] Note that, in this small open economy, capitalist behavior only affects equilibrium dynamics through the supply of capital. We thus ignore capitalist optimal consumption-savings choice in what follows.

4.2 Equilibrium

In any period t, entrepreneurs choose their consumption c, their capital stock k, and their debt b, given the path of asset prices and the interest rate. The optimal choice of entrepreneurs with productivity A is:

$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ 0 & \text{otherwise} \end{cases}, \tag{21}$$

and

$$b = q \cdot k - w, \tag{22}$$

$$c = \rho \cdot w. \tag{23}$$

Equation (21) says that, just as in the baseline model, there is a threshold entrepreneur who is indifferent between saving or investing in capital. The only difference is that this threshold is now given by $(r+\delta) \cdot q - \dot{q}$, as part of the cost of capital is in the form of depreciation and part of the return to capital accrues in the form of capital gains. Entrepreneurs above the threshold (i.e., supra-marginals) borrow and invest as much as possible, whereas those below (i.e., infra-marginals) save at the interest rate r. Equation (23) says that, due to log-preferences, entrepreneurs consume a portion ρ of their wealth at each instant.

From substituting these optimal choices into law of motion (17), it follows that individual wealth w evolves according to:

$$\dot{w} = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \delta - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho\right] \cdot w & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ (r-\rho) \cdot w & \text{otherwise} \end{cases}. \tag{24}$$

[17] As is standard, entrepreneurs must be impatient relative to international lenders because otherwise they would eventually outgrow financial constraints.

Equation (24) captures the endogeneity of wealth accumulation in the dynamic economy, which depends on the interest rate and the return to capital. In particular, more productive entrepreneurs accumulate wealth at a faster pace due to their higher return on capital. Moreover, a lower interest rate has a positive effect on the wealth accumulation of supra-marginal entrepreneurs (who are borrowers) and a negative effect on that of infra-marginal entrepreneurs (who are savers). Lastly, higher capital gains boost the wealth accumulation of those entrepreneurs that invest more and thus have a higher exposure to capital.

To characterize the equilibrium, it is convenient to aggregate all entrepreneurs with the same productivity A. We thus define the aggregate wealth of entrepreneurs with productivity A as:

$$W_A \equiv \int w \cdot f\left(A, w\right) \cdot dw, \tag{25}$$

where $f\left(A, w\right)$ is the share of entrepreneurs with productivity A and wealth w, while we denote aggregate entrepreneurial wealth by:

$$W \equiv \int W_A \cdot dA. \tag{26}$$

By combining Equation (25) with the stochastic structure of productivity shocks, it follows that W_A evolves according to:

$$\dot{W}_A = \int \dot{w} \cdot f\left(A, w\right) \cdot dw + \Theta \cdot \int \left[g\left(A|A'\right) \cdot W_{A'} - g\left(A'|A\right) \cdot W_A\right] \cdot dA' \,.$$

The first term in this expression aggregates the evolution of individual wealth across all entrepreneurs with productivity A. The second term reflects how this pool of entrepreneurs evolves due to productivity shocks. Together with Equation (24), we obtain:

$$\dot{W}_A = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \delta - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho - \Theta\right] \cdot W_A + \Theta \cdot \int g\left(A|A'\right) \cdot W_{A'} \cdot dA' & \text{if } A + \dot{q} \geq \left(r + \delta\right) \cdot q \\ \left(r - \rho - \Theta\right) \cdot W_A + \Theta \cdot \int g\left(A|A'\right) \cdot W_{A'} \cdot dA' & \text{otherwise} \end{cases} . \tag{27}$$

Per-capita investment of entrepreneurs with productivity A is then given by:

$$k_A = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{W_A}{q} \cdot \frac{1}{g(A)} & \text{if } A + \dot{q} \geq \left(r + \delta\right) \cdot q \\ 0 & \text{otherwise} \end{cases} , \tag{28}$$

so that the market-clearing condition for capital can be expressed as:

$$K = \int_{A \geq (r+\delta)\cdot q - \dot{q}} k_A \cdot g(A) \cdot dA, \tag{29}$$

where K satisfies Equation (19), and aggregate output is:

$$Y = \int_{A \geq (r+\delta)\cdot q - \dot{q}} A \cdot k_A \cdot g(A) \cdot dA. \tag{30}$$

An equilibrium consists of paths of prices $\{q, r\}$ and of allocations $\{\{W_A, k_A\}_A, W, K, Y, I\}$ such that Equations (19)-(20) and (26)-(30) are satisfied in all periods.

4.3 An illustration: reallocation effects in steady state

To illustrate how this dynamic economy relates to the baseline, we focus on the special case in which the capital supply is fixed at $\bar{K}$ (thus $\delta = 0$) and the productivity process satisfies $g(A'|A) = g(A')$ for all A' and A, i.e., the likelihood that an entrepreneur transitions to productivity A' is independent of her current productivity. This enables us to characterize analytically the steady-state effects of a fall in the interest rate r.

In steady state, prices and aggregate variables are constant over time, i.e., $\dot{q} = 0$ and $\dot{W}_A = 0$ for all A. Given our assumptions on productivity, we can express the wealth of entrepreneurs with productivity A as a share of aggregate wealth W:

$$\frac{W_A}{W} = \begin{cases} \frac{\Theta}{\Theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)} \cdot g(A) & \text{if } A \geq r \cdot q \\ \frac{\Theta}{\Theta+\rho-r} \cdot g(A) & \text{otherwise} \end{cases}. \tag{31}$$

Equation (31) shows that the positive link between productivity and wealth extends to the steady state. Since wealth shares must add up to one, it follows that:

$$\int_{r\cdot q}^{1} \frac{\Theta}{\Theta + \rho - \frac{1}{1-\lambda} \cdot \left(\frac{A}{q} - \lambda \cdot r\right)} \cdot g(A) \cdot dA = 1 - \int_{0}^{r\cdot q} \frac{\Theta}{\Theta + \rho - r} \cdot g(A) \cdot dA, \tag{32}$$

while the market clearing condition for capital can be written as:

$$\bar{K} = \left[\int_{r\cdot q}^{1} \frac{\Theta}{\Theta + \rho - \frac{1}{1-\lambda} \cdot \left(\frac{A}{q} - \lambda \cdot r\right)} \cdot g(A) \cdot dA\right] \cdot \frac{1}{1-\lambda} \cdot \frac{W}{q}. \tag{33}$$

Equations (31)-(33) jointly determine the steady-state values of $\{W_A\}$, W, and q as a function of r in this simplified economy. Equation (31) shows that, for given aggregate wealth W and price of capital q, a fall in r reduces the wealth of infra-marginal entrepreneurs and raises the wealth of supra-marginal entrepreneurs: in a nutshell, lower interest rates transfer wealth from creditors to debtors. Together with Equation (32), this implies that the productivity of the marginal entrepreneur, $r \cdot q$, must fall: intuitively, savings become less attractive relative to investment. Finally, since a fall in r increases the share of wealth that is directed towards investment, Equation (33) implies that aggregate wealth in terms of capital (W/q) must fall to preserve market clearing. This illustrates that the mechanics of the reallocation effect are similar to those of the baseline economy, i.e., it is driven by a general equilibrium effect associated to the clearing of the capital market. The main difference is that, since aggregate wealth W is endogenous in the dynamic economy, the reallocation effect is not just driven by the price of capital q but also by the ratio W/q.[18]

The steady-state effect of r on output is also as in the baseline economy. By combining Equations (29) and (30), we obtain:

$$\frac{dY}{dr} = \int_{r \cdot q}^{1} (A - r \cdot q) \cdot \frac{dk_A}{dr} \cdot g(A) \cdot dA, \tag{34}$$

which is the equivalent of Equation (12). Since the capital stock is fixed at $\bar{K}$ in this simplified economy, there is only a capital-reallocation effect. As in the baseline model, this effect can be positive or negative because it captures both the reallocation of capital from supra- to infra-marginal entrepreneurs (which reduces aggregate output) and the reallocation of capital among supra-marginal entrepreneurs (which may increase aggregate output).

Figure 4 illustrates this discussion by depicting the steady-state effect of a fall in the interest rate. Panel (a) shows the per-capita wealth shares held by entrepreneurs as a function of their productivity, at the initial and final steady states. In line with our discussion, a fall in the interest rate raises the wealth shares of supra-marginal entrepreneurs and reduces the productivity of the marginal entrepreneur. Panel (b) shows per-capita investment at the initial and final steady states. In this illustration, all supra-marginal entrepreneurs reduce their investment while some infra-marginal entrepreneurs increase theirs; since the capital supply is fixed, it follows that aggregate TFP and output decline in response to the fall in r.

[18]Note, in fact, that the effect of a fall in r on the price of capital q is in general ambiguous. E.g., if the losses inflicted on infra-marginal entrepreneurs by a lower value of r are not compensated by the gains of supra-marginals (e.g. if λ is small), then Equation (32) implies that q must fall in response to a fall in r. Online Appendix C.6 displays the steady state in closed form for a simpler case in which productivity is uniformly distributed on the unit interval across entrepreneurs.

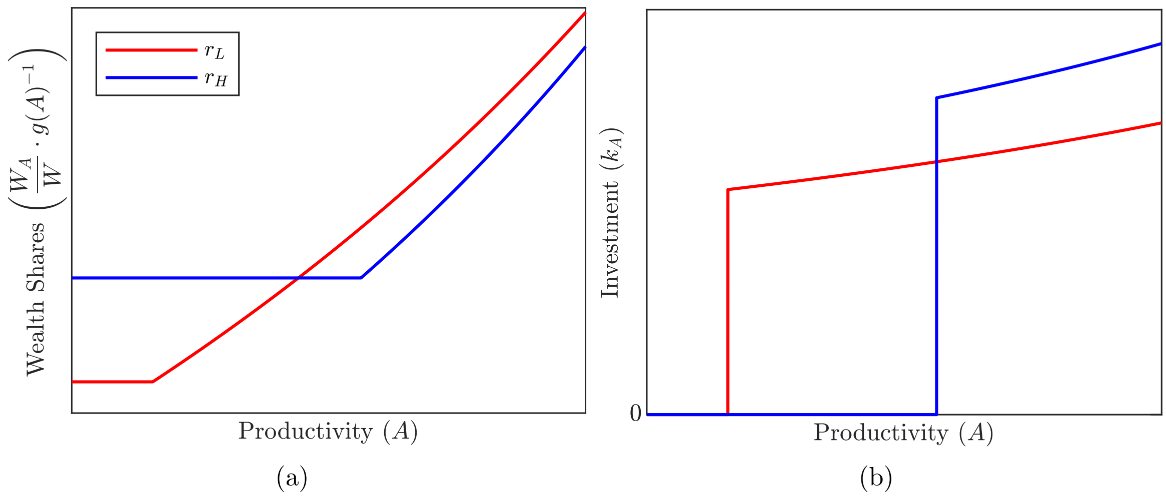


Figure 4: **Steady-state effect of a fall in the interest rate on per capita wealth shares and investment of entrepreneurs.**

This simplified economy was useful to illustrate that the reallocation effects that we have emphasized throughout are also present in a dynamic world. To see whether these effects are quantitatively significant, we perform next a numerical exercise on the full dynamic model.

4.4 A numerical exploration

We explore the model's quantitative implications by calibrating it to an annual frequency. The functional form for the production costs of capital is:

$$\chi(I) = \frac{\delta^{-\frac{1}{\varepsilon}}}{1 + \frac{1}{\varepsilon}} \cdot I^{1+\frac{1}{\varepsilon}},$$

where $\varepsilon \geq 0$ is the price-elasticity of the capital supply. Under this functional form, $I(q) = \delta \cdot q^{\varepsilon}$ and thus, in steady state, $K = q^{\varepsilon}$.

Table 1 reports the parameter values used. These values are standard in the literature, except perhaps for those of the productivity process and the elasticity of capital supply.

In the model, idiosyncratic productivity changes when a Poisson shock arrives, in which case productivity evolves according to the transition density $g(A'|A)$. We set the grid of idiosyncratic productivity and its transition density to match the evolution of firm-level productivity in the data, which according to Gilchrist et al. (2014) is well captured in logarithmic form by an $AR(1)$

Parameter	Value	Source / Target
Subjective discount rate	$\rho = 4\%$	Literature
International interest rate	$r = 2\%$	Literature
Elasticity of capital-supply	$\varepsilon = 1$	Cavalleri et al. (2019) and Saiz (2010)
Depreciation rate of capital	$\delta = 5\%$	Literature
Financial friction	$\lambda = 0.75$	Leverage multiple of entrepreneurs who invest (=4)
Frequency of transition of productivity	$\Theta = 1$	Yearly
Autocorrelation of productivity	$\rho_A = 0.40$	Gilchrist et al. (2014)
Innovation of productivity	$\sigma_A = 0.65$	Gilchrist et al. (2014)

Table 1: **The parameter values used for the numerical exercise.**

process with autocorrelation coefficient $\rho_A = 0.4$ and innovation of disturbance $\sigma_A = 0.65$.[19] The arrival rate of the Poisson shock is set to one, so that productivity transitions every year. For the elasticity of the capital supply, we follow the literature and think of capital as representing land or real estate. We set the supply elasticity to one, which lies within the range of empirically plausible values for OECD economies.[20] We follow the same approach for the depreciation rate of capital, and thus set $\delta = 5\%$. Lastly, we set $\lambda = 0.75$ to match a leverage multiple of 4 for entrepreneurs who borrow to invest in capital. This target for the multiple is also standard in the literature (e.g., Moll (2014)).

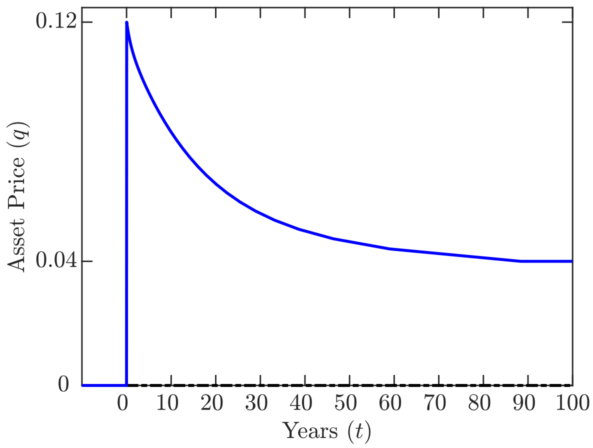
Given these parameter values, we compute the effect of a permanent and unanticipated one-percentage-point fall in the interest rate, from 2% to 1%. The main finding is that the reallocation effect is quantitatively significant. If productivity were constant, for instance, steady-state output would expand by 4%. Once the reallocation effect is taken into account, however, output contracts by 2%! These steady-state effects fail to capture transitional dynamics, however, which are non-monotonic due to the presence of balance-sheet effects.

When the fall in r materializes, the price of capital q rises on impact. This raises the wealth of supra-marginal entrepreneurs because they own the capital.[21] Letting t_0 and t_0^+ respectively denote the instants before and after the interest-rate shock, the wealth of entrepreneurs

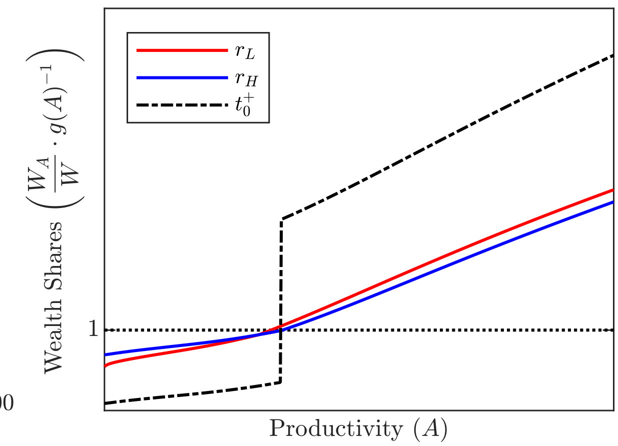
[19] This autocorrelation coefficient is lower than some other estimates in the literature (e.g., Asker et al. (2014); Midrigan and Xu (2014)) which find $\rho_A \geq 0.8$. The quantitative significance of the reallocation effect between steady states in our numerical exercise, however, is not significantly influenced within this plausible range of values. With $\rho_A = 0.4$, for instance, we find that – in response to a permanent one-percentage-point decline in the interest rate – the reallocation effect leads to a 6% fall in productivity (Figure 6, dashed-red line). If we instead set $\rho_A = 0.8$ or $\rho_A = 0.9$, the reallocation effect leads to a 7% fall in steady-state productivity.

[20] See, e.g., Cavalleri et al. (2019) and Saiz (2010).

[21] As in the literature (e.g., Kiyotaki and Moore (1997)), we are assuming in the background that debt contracts are not indexed to the price of capital. This assumption is inconsequential for the rest of the analysis but it is important to allow for balance-sheet effects along the transition.



(a)



(b)

Figure 5: **Balance-sheet effects induced by a fall in the interest rate.**

immediately after the shock is given by:

$$W_{A,t_0^+} = \begin{cases} \left(\frac{q_{t_0^+}}{q_{t_0}} - \lambda\right) \cdot \frac{1}{1-\lambda} \cdot W_{A,t_0} & \text{if } A \geq (r_{t_0} + \delta) \cdot q_{t_0} \\ W_{A,t_0} & \text{otherwise} \end{cases} . \tag{35}$$

Equation (35) says that, on impact, the wealth of supra-marginal entrepreneurs increases with their capital holdings and with the rise in the price of capital. The wealth of infra-marginal entrepreneurs is instead unaffected. From then on, the evolution of aggregate variables $\{\{W_A, k_A\}_A, W, q, K, Y, I\}$ is characterized as before by Equations (19)-(20) and (26)-(30).

The transition of the economy to its new steady state is depicted in Figures 5 and 6. Panel (a) of Figure 5 shows the evolution of the price of capital q relative to its initial value. As the figure shows, q jumps by 12% on impact and then gradually declines to its new, higher steady-state value. Panel (b) shows that, on impact, this rise in q boosts the wealth share of all supra-marginal entrepreneurs (see shift from the solid-blue curve to the dashed-black curve). This has implications for the evolution of output, as illustrated in Figure 6. If productivity were constant, output would be driven solely by the capital supply effect (depicted by the dashed-green curve), and it would rise gradually to its new steady state as the economy accumulates capital. Due to the reallocation effect, however, output exhibits a strong non-monotonic behavior. On impact, output increases by 8% due to balance-sheet effects, which benefit the most productive entrepreneurs and reallocate capital towards them. From then on, however, output declines monotonically to its new – lower – steady-state level. The reason is that balance-sheet effects

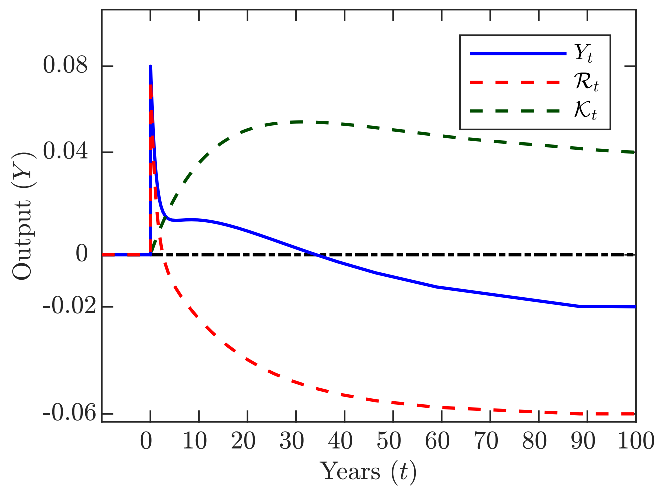


Figure 6: **Evolution of aggregate output after a fall in the interest rate.**

operate exactly as in the literature, i.e., a one-time transfer of wealth that takes place at the time of the shock. The capital-reallocation effect that we have emphasized throughout (depicted by the dashed-red curve) is instead long-lasting, just like the fall in r. As the figure shows, it is already dominant shortly after the shock and – despite the expansion of the capital stock – eventually leads to a 2% contraction in output relative to the original steady state.

The key takeaway of this numerical exercise is that the same mechanism that gives rise to balance-sheet effects – namely, heterogeneity and financial frictions, coupled with an imperfectly elastic capital supply – also gives rise to substantial reallocation. Moreover, our emphasis is not on the size of the reallocation effect – which could be overestimated in our exercise by assuming that the entire capital stock is in the form of land or real estate – but rather on the fact that it is of the same order of magnitude as the balance-sheet effect. Thus, given that macroeconomists have ample evidence for the former (Peek and Rosengren, 2000; Gan, 2007; Chaney et al., 2012), our findings suggest that they should also pay close attention to the latter.

Another implication of this exercise is that the duration of interest-rate changes is potentially central to understanding their dynamic effects on investment and output. A key driver of the dynamics depicted in Figure 6 is that the decline in the interest rate is persistent: once the balance-sheet effect dissipates, the economy is left with the adverse reallocation effect of low rates, which continue to prompt investment by relatively unproductive entrepreneurs. The theory thus provides one potential justification for the view that persistent declines in the interest rate may be specially harmful for economic activity.[22]

[22] This is related to the recent "low-for-long" debate on the potentially adverse consequences of persistently low interest rates (see, for instance, Bean et al. (2015)).

5 Supporting evidence

The main insight of our theory is that, in the presence of financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned!) by general-equilibrium reallocation effects. In particular, the expansion of relatively unproductive investment tends to raise the price of capital thereby crowding-out more productive investment. Albeit we view the main contribution of the paper as being conceptual, it is nonetheless important to highlight that the theory is consistent with several strands of evidence.

First, it has been shown that credit booms fuelled by persistent declines in interest rates, such as in Spain in the early 2000s, are associated with a decline in aggregate TFP (Gopinath et al., 2017; García-Santana et al., 2020). Second, there is evidence that low-interest environments are conducive to the expansion of low-productivity or "zombie" firms (Caballero et al., 2008; Adalet McGowan et al., 2018; Banerjee and Hofmann, 2018; Tracey, 2019; Schivardi et al., 2020; Acharya et al., 2021). Moreover, and more directly in line with our theory, Banerjee and Hofmann (2018) find direct traces of reallocation effects for advanced economies: specifically, an increase in the share of "zombie" (i.e., less productive) firms in a given sector is associated with a decline in investment and employment by "non-zombie" (i.e., more productive) firms in that same sector.

In this section, we complement existing evidence by testing a more direct implication of the theory on US data: namely, the stimulative effect of declining interest rates should be weaker when general-equilibrium effects are stronger. To test this, we need to take a stance on two issues: how to measure interest-rate shocks and what is the real-world counterpart of capital in the model.

We follow the literature and proxy for interest-rate shocks by using various measures of monetary policy shocks. Our baseline measure is the high-frequency monetary shock from Jarociński and Karadi (2020), who identify monetary surprises based on high-frequency data around monetary policy announcements, combined with sign restrictions to take out the component of the announcement that reflects the economic outlook. We aggregate their shocks over annual periods to construct yearly measures of changes in monetary policy. Admittedly, this type of monetary policy shock does not exactly capture the type of persistent changes that we have in mind. It would be preferable, for instance, to have well-identified, long-lasting shocks to the interest rate associated with structural changes such as financial liberalization or a shift in the discount rate. Although imperfect, there is ample evidence that monetary policy shocks are persistent and, to the extent that they are not, using these shocks should work against finding evidence for the type of reallocation effects that we have emphasized in the theory.[23]

Regarding the proxy of capital in the model, we focus throughout on real estate/land. The main advantage of doing so is that there are readily available measures of real-estate supply elasticities for the US, which we can use to proxy for the strength of general-equilibrium effects. We specifically rely on local-level (Metropolitan Statistical Area - MSA) measures of real-estate supply elasticities commonly used in the literature (Saiz, 2010). To allow for sectoral-level variation in the intensity of general-equilibrium effects, we follow Vom Lehn and Winberry (2019) and construct a measure of real-estate intensity by computing the share of real estate in total fixed assets at the sector level.

Finally, to make sure that we cover a representative sample of the overall economy, we use data on the activity of US firms by region and sector from the US Bureau of Economic Analysis (BEA) Regional Accounts. To the best of our knowledge, these accounts provide the widest coverage of disaggregated data on firm output. Appendix B provides a detailed explanation of all variables and of other measures of interest rate changes that we use for robustness.

We assess whether the expansionary effect of a fall in the interest rate in the US is weaker (i) in regions where the supply of real estate is less elastic (i.e., where general-equilibrium effects are strong), and (ii) in sectors that use real estate more intensively as an input (i.e., where general-equilibrium effects are relevant for production costs).

5.1 Empirical strategy

To assess whether the expansionary effects of interest-rate declines is weaker in regions with a lower elasticity of real-estate supply, we estimate – for sector i in region j at date t – the following equation:

$$\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \varepsilon_{ijt} \tag{36}$$

where Δy_{ijt} is the real GDP growth rate in sector i in Metropolitan Statistical Area (MSA) j in year t, α_{ij} is a sector i by region j fixed effect, α_{it} is a sector i by year t fixed effect, y_{ijt-1} is lagged GDP in sector i in MSA j, r_t is the annual monetary policy shock in year t, and H_j is the elasticity of land supply in MSA j. Δy_{ijt} and y_{ijt-1} are respectively computed as $ln(Y_{ijt}/Y_{ijt-1})$ and $ln(Y_{ijt-1})$, where Y is output expressed in constant (chained 2012) US dollars. The sector-region fixed effects capture permanent differences in output dynamics across sectors and regions, and the sector-year fixed effects control for sectoral differences in exposure to aggregate shocks. We include the lagged level of real GDP to capture growth convergence

[23]For evidence of the persistence of interest rates, see Neely et al. (2008) and references therein.

effects, although doing so does not affect our main results.

The theory predicts that $\beta_1 < 0$. In other words, the effect of a monetary expansion on economic activity is weaker in regions with a lower elasticity of real-estate supply. Of course, there may be multiple channels through which real-estate prices affect local economic activity (e.g., they may boost local demand through wealth effects), which may confound the reallocation effects identified in the theory.

Thus, we test whether the differential effect of interest rates across regions is stronger for sectors that are more real-estate intensive. Specifically, we estimate variations of the following equation:

$$\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \Gamma' Z_{ijt-1} + \varepsilon_{ijt} \tag{37}$$

where $RE_{i,t-1}$ is the average real-estate intensity of sector i in year $t-1$ (one year lagged) and Z_{ijt-1} is a vector of controls. As described in Appendix B, real-estate intensity is alternatively computed as (i) the share of real-estate related fixed assets in total fixed assets, or (ii) the share of real-estate related investment in total investment.

The theory predicts that $\beta_2 < 0$ and significant, implying that the differential effect of interest rates across regions should be stronger for sectors that are more real-estate intensive. This is our main coefficient of interest in what follows.

5.2 Empirical results

Our main results are reported in Table 2. Column 1 presents estimates of Equation (36). For ease of interpretation, we invert the sign of r_t and H_j such that higher values of r_t denote an easing of interest rates and higher values of H_j denote a more inelastic land supply. All explanatory variables except for the monetary policy shock are standardized. In all regressions, we cluster standard errors two-ways, by sector-region and year. This increases the standard errors compared to not clustering and should therefore be seen as a conservative estimate. The estimate of β_1 is not statistically significant, i.e., we do not find the expansionary effects of real interest rates to be weaker in regions with a lower elasticity of real-estate supply. As we anticipated, however, this is a weak test of the theory as there may be multiple mechanisms at work (Mian and Sufi, 2011).

Column 2 reports estimates of Equation (37). The estimated value of β_2 is negative and statistically significant, which is consistent with a strong reallocation effect. The estimated effect is economically substantial, moreover: the semi-elasticity of GDP growth to monetary

Table 2: **Monetary transmission, real-estate supply, and real-estate intensity**

VARIABLES	(1) $\Delta y_{i,j,t}$	(2) $\Delta y_{i,j,t}$	(3) $\Delta y_{i,j,t}$	(4) $\Delta y_{i,j,t}$
mp_med_t				-0.019
				(0.064)
$mp\_med_t * H_j$	0.011	0.017		0.025*
	(1.010)	(0.010)		(0.010)
$mp\_med_t * H_j * RE_{i,t-1}$		-0.035**	-0.041***	-0.032***
		(0.009)	(0.009)	(0.008)
$mp\_med_t * RE_{i,t-1}$				0.011
				(0.026)
$y_{i,j,t-1}$	-0.225***	-0.225***	-0.237***	-0.192***
	(0.026)	(0.026)	(0.027)	(0.027)
Sector-Year FE	Yes	Yes	Yes	No
Sector-Region FE	Yes	Yes	Yes	Yes
Region-Year FE	No	No	Yes	No
Observations	17756	17756	17752	17756
R-squared	0.276	0.277	0.345	0.168
p>F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: Regression results from estimating the following specification: $\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \varepsilon_{it}$. Δy_{ijt} is the real GDP growth rate at the sector-MSA level, and y_{ijt-1} is the one-period lag of log of real GDP. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over each year. H_j is the elasticity of real-estate supply at the MSA level. We invert the sign of the mp_med_t and H_j variables to ease interpretation, so that higher values of mp_med_t denote an easing of monetary policy and higher values of H_j denote a more inelastic housing supply. RE_{it} is the ratio of real-estate assets to total fixed assets of the sector. All explanatory variables except the monetary shock are standardized. Standard errors are clustered two-ways by sector-region and year.

easing is reduced by 0.035 units when the elasticity of real estate and the real-estate intensity of the sector are respectively reduced and raised by one standard deviation relative to the average in the sample. This is a meaningful effect, taking into account that the average growth rate of real GDP of 0.018.

Column 3 includes region-year fixed effects. The estimated value of β_2 now increases to -0.041, and it is statistically significant at the 1% level. Column 4 in turn drops both the sector-year and region-year fixed effects, and shows that the estimate of β_2 remains basically unchanged.

Table 4 in Appendix B shows that these results are robust to the use of alternative definitions of the monetary policy shocks, to the use of the flow measure of real-estate intensity, and to the inclusion of financial leverage in the regression as a way to control for balance-sheet effects. The reallocation effect that we have emphasized in the theory occurs over the long run. Consistent with this, Appendix B shows that our main result is persistent and remains significant up to five years after the monetary policy shock (see Figure 7).

Taken together, these findings are consistent with the presence of strong reallocation effects: namely, the expansionary effect of declining interest rates is weaker in regions with a lower elasticity of real-estate supply and for sectors that are more real-estate intensive.

6 Conclusions

This paper makes a conceptual point. In a canonical economy with heterogenous entrepreneurs, financial frictions, and an imperfectly elastic supply of capital, a fall in the interest rate has ambiguous effects on aggregate economic activity. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing borrowing constraints and by prompting relatively less productive entrepreneurs to increase their investment. In general equilibrium, however, this higher demand for capital raises its price and crowds-out investment by more productive (albeit financially constrained) entrepreneurs: ultimately, there is a reallocation of capital from more to less productive entrepreneurs. When this general-equilibrium effect is strong enough, a fall in the interest rate becomes contractionary.

Our mechanism requires that the price of capital depend on local economic conditions, a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets. Indeed, the reallocation effects that we study here are bound to appear in any model that gives rise to the traditional balance-sheet effects. We show that both effects coexist and, in a numerical exploration of the model, that they are of the same order of magnitude. Thus, given that macroeconomists have ample evidence for the importance of balance-sheet effects, our findings suggest that they should also pay attention to the adverse reallocation effects that we have studied here.

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A Proofs

Proof of Proposition 1. The capital market clearing condition is:

$$K^S(q) = \int_{q \cdot R}^{1} \frac{R}{q \cdot R - \lambda \cdot A} \cdot g(A) \cdot dA \cdot w, \tag{38}$$

and aggregate output, Y, is given by:

$$Y = \int_{q \cdot R}^{1} A \cdot \frac{R}{q \cdot R - \lambda \cdot A} \cdot g(A) \cdot dA \cdot w. \tag{39}$$

The derivative of aggregate output, Y, with respect to the interest rate, R, is:

$$\begin{aligned} \frac{dY}{dR} = & \int_{q \cdot R}^{1} \frac{A}{(q \cdot R - \lambda \cdot A)^2} \cdot \left(\left| \frac{dq}{dR} \right| - \frac{\lambda \cdot A}{R^2} \right) \cdot g(A) \cdot dA \cdot w \\ & + \frac{R}{1-\lambda} \cdot g(q \cdot R) \cdot w \cdot \left(R \cdot \left| \frac{dq}{dR} \right| - q \right), \end{aligned} \tag{40}$$

which, all else equal, is increasing in $|dq/dR|$. To arrive at Equation (12), we totally differentiate the capital market clearing condition and replace the last term in Equation (40).

Assume $\chi(\cdot)$ is locally twice differentiable, so that the capital supply elasticity is given by:

$$\varepsilon = \frac{q}{\chi''(\chi'^{-1}(q)) \cdot \chi'^{-1}(q)}. \tag{41}$$

From the capital market clearing condition (38), therefore, we have:

$$\left| \frac{dq}{dR} \right| = \frac{q \cdot \int_{q \cdot R}^{1} \frac{1}{(q \cdot R - \lambda \cdot A)^2} \cdot \frac{\lambda \cdot A}{R^2} \cdot g(A) \cdot dA \cdot w + \frac{q \cdot R}{1-\lambda} \cdot g(q \cdot R) \cdot w}{\varepsilon \cdot K^S(q) + q \cdot \int_{q \cdot R}^{1} \frac{1}{(q \cdot R - \lambda \cdot A)^2} \cdot g(A) \cdot dA \cdot w + \frac{R}{1-\lambda} \cdot g(q \cdot R) \cdot w}. \tag{42}$$

Thus, observe that, all else equal, $\left| \frac{dq}{dR} \right|$ is decreasing in the capital supply elasticity, ε, in equilibrium. It follows that dY/dR is decreasing in ε. Below, we will verify that it is indeed possible to change ε (by adjusting $\chi(\cdot)$) without affecting the equilibrium allocations.

Next, dY/dR is continuous in λ and, when $\lambda = 0$, aggregate output is given by:

$$Y = \left(\int_{q \cdot R}^{1} A \cdot \frac{g(A)}{1 - G(q \cdot R)} \cdot dA \right) \cdot K^S(q), \tag{43}$$

where:

$$K^S(q) = \frac{w}{q} \cdot (1 - G(q \cdot R)). \tag{44}$$

From the capital market clearing condition, it is clear that $q \cdot R$ must rise with R. Hence, it follows that:

$$\frac{d\left(\int_{q\cdot R}^{1} A\cdot\frac{g(A)}{1-G(q\cdot R)}\cdot dA\right)}{dR}>0, \tag{45}$$

that is, aggregate TFP is increasing in R. Moreover, since:

$$\frac{dY}{dR}=\frac{d}{dR}\left(\int_{q\cdot R}^{1} A\cdot\frac{g\left(A\right)}{1-G\left(q\cdot R\right)}\cdot dA\right)\cdot K^{S}(q)+\left(\int_{q\cdot R}^{1} A\cdot\frac{g\left(A\right)}{1-G\left(q\cdot R\right)}\cdot dA\right)\cdot\frac{dK^{S}(q)}{dR}, \tag{46}$$

we have that $dY/dR>0$ provided that the capital supply elasticity, ε, is small enough, i.e., ε is below some $\bar{\varepsilon}>0$. Fix such an $\varepsilon<\bar{\varepsilon}$. By continuity, there exists a threshold $\bar{\lambda}_{\varepsilon}$ such that $dY/dR>0$ for all $\lambda<\bar{\lambda}_{\varepsilon}$.

Finally, to produce Figure 3, we consider the following parameterization of the capital supply schedule (which is equivalent to parametrizing the cost of capital production). Suppose that the interest rate is equal to R and let γ be defined as follows:

$$\bar{K}=\int_{\gamma\cdot R}^{1}\frac{1}{\gamma-\frac{\lambda\cdot A}{R}}\cdot w\cdot dG(A). \tag{47}$$

Consider the following family of capital supply schedules:

$$K^{S}(q;\varepsilon)=\max\left\{0,\ \bar{K}\cdot\left(1+\varepsilon\cdot\frac{q-\gamma}{\gamma}\right)\right\},\ \varepsilon\in[0,\infty). \tag{48}$$

Note that, at the interest rate R, the equilibrium allocations are independent of ε, the elasticity of the capital supply. In particular, $K^{S}=\bar{K}$ and $q=\gamma$. But, as the interest rate changes, the equilibrium allocations will change since γ is a parameter and q will no longer equal γ. ■

Proof of Proposition 2. Consider the problem of the social planner:

$$\max_{\{k_A\}}\int_0^1 A\cdot k_A\cdot dG\left(A\right)-R\cdot\left(\chi\left(\int_0^1 k_A\cdot dG\left(A\right)\right)-w\right)$$

subject to:

$$R\cdot\left(\chi'\left(\int_0^1 k_A\cdot dG\left(A\right)\right)\cdot k-w\right)\leq\lambda\cdot A\cdot k_A\quad\left(\gamma_A\cdot g\left(A\right)\right),$$

$$0\leq k_A\quad\left(\omega_A\cdot g\left(A\right)\right).$$

In parentheses, we denote the multipliers on the constraints. We also suppose that the cost of capital production, $\chi(\cdot)$, is strictly convex (see below for the case of inelastic capital supply).

The first-order conditions to the planner's problem are given by:

$$\frac{A}{R}-q-\chi''\left(K^{S}\right)\cdot\int_0^1\gamma_{\widehat{A}}\cdot k_{\widehat{A}}\cdot dG\left(\widehat{A}\right)=\gamma_A\cdot\left(q-\frac{\lambda\cdot A}{R}\right)-\omega_A\quad\forall A, \tag{49}$$

which together with the Kuhn-Tucker conditions characterize the solution to the problem. Since at the planner's allocation it must be that $q=\chi'\left(K^{S}\right)\geq\frac{\lambda}{R}$, it follows that there exists $0<\widetilde{A}\leq 1$ such that $\omega_A=0$ for all $A<\widetilde{A}$. There are thus two possibilities.

Case 1: $\widetilde{A} = 1$. In this case, $\omega_A > 0 = \gamma_A$ for $A < 1$ and, therefore, $\widetilde{A} > q^{CE} \cdot R$. Further, market clearing requires that $q^{SP} = \frac{\lambda}{R}$ and the capital stock and output are given by:

$$Y = K^{SP} = \chi^{-1}\left(\frac{\lambda}{R}\right),$$

and note that $q^{SP} < q^{CE}$.

Case 2: $\widetilde{A} < 1$. In this case, $\gamma_A > 0 = \omega_A$ for all $A > \widetilde{A}$ and after some algebra we have:

$$\widetilde{A} = q^{SP} \cdot R + \frac{\chi''\left(K^S\right) \cdot \int_{\widetilde{A}}^{1} \left(A - q^{SP} \cdot R\right) \cdot \frac{1}{\left(q^{SP} - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot dG\left(A\right)}{1 + \chi''\left(K^S\right) \cdot \int_{\widetilde{A}}^{1} \frac{1}{\left(q^{SP} - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot dG\left(A\right)}, \tag{50}$$

where

$$q^{SP} = \chi'\left(K^S\right) = \chi'\left(\int_{\widetilde{A}}^{1} \frac{1}{q^{SP} - \frac{\lambda \cdot A}{R}} \cdot w \cdot dG\left(A\right)\right). \tag{51}$$

Clearly, again, $\widetilde{A} > q^{SP} \cdot R$ since the RHS in Equation (50) is positive. Note that, from Equation (51), the capital price q^{SP} is depressed below q^{CE} since the entrepreneurs who invest do so until their financial constraints bind, but there are fewer such entrepreneurs.

Next, note that when R falls, the planner can always ensure the same equilibrium allocations (with unchanged equilibrium price of capital, q^{SP}), in which case all supra-marginal entrepreneurs' financial constraints become slack. Moreover, it is clear that the planner would never want to reduce both $K^S(q^{SP})$ and Y^{SP} in response to a fall in R, since the aggregate productivity of capital (TFP) is higher than R times the marginal cost of producing a unit of capital, χ'^{S})), both before and after the fall in R. Finally, it follows that the decline in R must be expansionary. For instance, the planner can always increase $\widetilde{A}$ and k_A for $A > \widetilde{A}$ so that K^S is unchanged but TFP increases. ■

B Empirical Appendix

This Appendix describes the dataset and definitions of the variables used in the empirical analysis in Section 5 and presents several robustness checks of our main results.

B.1 Dataset construction and definition of variables

The dataset is created at the sector-region-year level primarily using data on the economic activity of firms from the US Bureau of Economic Analysis (BEA), combined with information on the real-estate component of fixed asset holdings and investments at the sectoral level from the BEA. We supplement this data using information on monetary policy changes and real-estate supply elasticities at the Metropolitan Statistical Area (MSA) level. The sample period is 2001 to 2019. We use MSA as unit for regions using the latest MSA classification available from the US Census, and we define industry groupings based on the BEA's sector classification, which is based on the most recent 2016 North American Industry Classification System (NAICS) of industry codes. The BEA's classification of sectors considers a total of 17 sectors, at either the two-digit or three-digit NAICS level: Agriculture, forestry, fishing and hunting (NAICS 11), Mining, quarrying, and oil and gas extraction (NAICS 21), Utilities (NAICS 22), Durable goods manufacturing (NAICS 321,327-339), Nondurable goods manufacturing (NAICS 311-316, 322-326), Wholesale trade (NAICS 42), Retail trade (NAICS 44-45), Transportation and warehousing (NAICS 48-49), Information (NAICS 51), Professional, scientific, and technical services (NAICS 54), Management of companies and enterprises (NAICS 55), Administrative and support and waste management and remediation services (NAICS 56), Educational services (NAICS 61), Health care and social assistance (NAICS 62), Arts, entertainment, and recreation (NAICS 71), Accommodation and food services (NAICS 72), Other services except government and government enterprises (NAICS 81). Given our focus on the use of real estate in production, we exclude firms operating in the construction (NAICS 23), finance and insurance (NAICS 52), and real-estate agents (NAICS 53) sectors. We limit the sample to those 95 MSAs for which we have information on the housing supply elasticity. This leaves a sample of 1,512 region-year observations and 107,390 sector-region-year observations for the period 2001 to 2019.

Economic activity. We measure the level of real GDP, y_{ijt}, and the growth rate of real GDP, Δy_{ijt}, at the sector-MSA level using the Local Area Gross Domestic Product from the US BEA Regional Economic Accounts, available at `https://www.bea.gov/data/economic-accounts/regional`. We use the Real GDP by county and metropolitan area (CAGDP9) data files contained in the CAGDP9.zip file for the period 2001-2019. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes, using annual data.

Real-estate intensity. We compute two sectoral measures of real estate intensity at the NAICS sector-level using the Nonresidential detailed estimates from the BEA Nonresidential Industry Fixed Assets tables. These can be downloaded at: `https://www.bea.gov/data/investment-fixed-assets/industry`.

The first measure, RE_{it}, is constructed on a stock basis, as the ratio of nonresidential real-estate assets to total fixed assets (at fixed cost). The second measure, RE_inv_{it}, is constructed on a flow basis, as the ratio of nonresidential real-estate investment to total fixed investments (at fixed cost). Both measures are computed using the BEA Fixed Assets Table, available from: `https://apps.bea.gov/national/FA2004/Details/xls/detailnonres_stk2.xlsx` and `https://apps.bea.gov/national/FA2004/Details/xls/detailnonres_inv2.xlsx`. Total fixed assets is computed as the sum of all non-residential fixed assets, including equipment, machinery, land, buildings, and intellectual property. The stock-based measure is our baseline measure of real-estate intensity.

Monetary shocks. We use three alternative measures of monetary policy changes. First, the high-frequency monetary shock from Jarociński and Karadi (2020) obtained with median rotation sign restrictions, mp_med_t, computed by aggregating daily shocks during each year. [24] This is our baseline measure. Second, the high-frequency based monetary shock from Jarociński and Karadi (2020) obtained with poor man's sign restrictions, mp_pm_t, also computed by aggregating daily shocks during each year. Both of these shocks extract the first principal component of surprises in interest rate futures with maturities from one month to one year, within a short window of 30 minutes around the times of the Federal Reserve's monetary policy

[24] The original dataset called jarocinski-karadi.zip is available from the AEJ Macro website and provides data for the period 1990 to 2016. We are grateful to Peter Karadi for providing us with an updated version of the dataset that extends the monetary shocks until 2019. The dataset provides daily monetary shocks. We transform these daily shocks into annual shocks by aggregating the daily shocks for each year.

announcements. They extend the approach in Nakamura and Steinsson (2018) by stripping out information shocks from policy shocks using a VAR model with sign restrictions. Both of these measures are true shocks in the sense that they are exogenous to current economic activity. Third, the annual change in the real effective Federal Funds rate, Δr_t, obtained from the Federal Reserve Bank of St. Louis' FRED database. This variable proxies for the change in real short-term interest rates. We construct this variable by taking the difference between the annual change in the nominal effective Federal Funds rate, available at: `https://fred.stlouisfed.org/series/FEDFUNDS`, and the annual CPI inflation rate, available from `https://fred.stlouisfed.org/series/FPCPITOTLZGUSA`.

Land supply elasticity. Local real-estate supply elasticities at the MSA level, denoted H_j, for a total of 95 MSAs are obtained from Table VI in Saiz (2010), which is available from `https://academic.oup.com/qje/article/125/3/1253/1903664?login=true`. These elasticities capture the amount of local land that can be developed and are estimated using satellite-generated images of the terrain. We transform this dataset into the latest available MSA codes, as of March 2020, as defined by the United States Office of Management and Budget (OMB), by averaging elasticities across merged MSAs in those few cases where MSA definitions have changed because of the merging of several regions.

Financial leverage. To capture balance-sheet effects related to financial leverage, we compute a sector-level measure of the leverage ratio, $Leverage_{i,t}$, using firm-level data from COMPUSTAT (downloaded from WRDS). We define financial leverage as the ratio of short-term and long-term debt (DLC+DLTT) divided by the book value of total assets (AT). We construct the sector-level measure of financial leverage by taking the median of financial leverage across firms by sector. We merge this variable into the database at the sector level by matching the COMPUSTAT NAICS codes to the BEA sector codes using the concordance table available from the BEA website.

Table 3 reports the descriptive statistics of our main variables. In our sample, real estate is a sizeable fraction of the fixed assets that corporations hold on their balance sheet. For the median sector in the sample, real estate represents 62 percent of total fixed assets. Real-estate supply is quite elastic, with an elasticity of 2 for the average MSA. The interest rate (shocks) data indicate that our sample period includes both periods of monetary expansions and contractions. Annual changes in short-term interest rates ranged from -4.1 to $+2$ percentage points, with a median of 0.

B.2 Robustness checks

Table 4 presents several robustness checks on our main results. Column 1 shows results when replacing the interest rate variable with the cumulative monetary shock from Jarociński and Karadi (2020), obtained with poor man sign restrictions. Results are broadly unchanged. We now obtain a statistically negative coefficient on the interaction variable of interest of -0.030. This implies a semi-elasticity of 0.03 units for a one standard deviation increase in both housing inelasticity and real-estate intensity.

Column 2 presents results when using the change in the Federal Funds rate as measure of the change in monetary policy. We have reversed the sign on this variable such that higher values

Table 3: **Summary statistics**

VARIABLES	Mean	Median	St. Dev.	Min	Max	N. Obs.
$\Delta y_{i,j,t}$	.018	.021	.139	-2.314	2.927	17,816
mp_med_t	.019	.022	.109	-.159	.181	18
mp_pm_t	.043	.055	.128	-.321	.198	18
Δr_t	-.000	.000	.013	-.041	.020	18
H_j	2.011	1.795	1.015	.670	5.450	74
$RE_{i,t}$	.598	.621	.202	.258	.928	324
$RE\_inv_{i,t}$	.265	.206	.205	.049	.898	324
$Leverage_{i,t}$	.241	.229	.099	.025	.562	306

Notes: Δy_{ijt} is the real GDP growth rate at the sector-MSA level for a given year from the US Bureau of Economic Analysis (BEA)'s Regional Accounts. GDP is expressed in chained 2012 US dollars and growth rates are constructed using log changes. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over each year. H_j is the elasticity of real-estate supply at the MSA level from Saiz (2010). mp_pm_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with poor man's sign restrictions, aggregated over each year. δr_t is the change in the Federal Funds rate over the year. H_j is the elasticity of housing supply at the MSA level from Saiz (2010). RE_{it} is the ratio of real-estate assets to total fixed assets of the sector in a given year from the BEA fixed assets tables. RE_inv_{it} is the ratio of real-estate investment to total fixed investment of the sector in a given year from the BEA fixed assets tables. $Leverage_{it}$ is the leverage ratio of the sector, computed at the sector-year level as the median across firms of the ratio of short and long term debt over total assets.

Table 4: **Monetary transmission, real-estate supply, and real-estate intensity: robustness**

VARIABLES	(1) $\Delta y_{i,j,t}$	(2) $\Delta y_{i,j,t}$	(3) $\Delta y_{i,j,t}$	(4) $\Delta y_{i,j,t}$
$mp\_pm_t * H_j * RE_{i,t-1}$	-0.030**			
	(0.009)			
$\Delta r_t * H_j * RE_{i,t-1}$		-0.173*		
		(0.062)		
$mp\_med_t * H_j * REinv_{i,t-1}$			-0.034**	
			(0.010)	
$mp\_med_t * H_j * RE_{i,t-1}$				-0.039***
				(0.007)
$mp\_med_t * H_j * Leverage_{i,t-1}$				0.0025
				(0.012)
$y_{i,j,t-1}$	-0.237***	-0.237***	-0.237***	-0.239***
	(0.027)	(0.027)	(0.027)	(0.027)
Sector-Year FE	Yes	Yes	Yes	Yes
Sector-Region FE	Yes	Yes	Yes	Yes
Region-Year FE	Yes	Yes	Yes	Yes
Observations	17752	17752	17752	17096
R-squared	0.345	0.344	0.345	0.347
p>F	0.000	0.000	0.000	0.000
Clustering	Yes	Yes	Yes	Yes

Notes: Regression results from estimating the following specification: $\Delta y_{ijt} = \alpha_{ij} + \alpha_{it} + \alpha_{jt} + \delta \cdot y_{ijt-1} + \beta_1 \cdot r_t \cdot H_j + \beta_2 \cdot r_t \cdot H_j \cdot RE_{it-1} + \gamma L_{ijt-1} + \varepsilon_{it}$. Δy_{ijt} is real GDP growth rate at the sector-MSA level, and y_{ijt-1} is the one-period lag of the log of real GDP. mp_pm_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with poor man's sign restrictions, aggregated over the year. Δr_t is the annual change in the Federal Funds rate. mp_med_t is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over the year. H_j is the elasticity of housing supply at the MSA level. We invert the sign of the monetary shock and H_j variables to ease interpretation, so that higher values of the monetary shock denote an easing of monetary policy and higher values of H_j denote a more inelastic housing supply. RE_{it} is the ratio of real-estate assets to total fixed assets of the sector. $REinv_{it}$ is the ratio of real-estate investments to total fixed investments of the sector. All explanatory variables except the monetary shock are standardized. $Leverage_{it}$ is the median across firms in the sector of the ratio of short and long term debt over total assets. All variables are as defined in Table 2. Standard errors are clustered two-ways by sector-region and year.

denote easing. Results are qualitatively similar. The regression coefficient of interest increases in size but is estimated less precisely compared to our baseline estimate.

Column 3 shows that our main results are robust to using a measure of real estate intensity based on investments in real estate instead of the stock of real estate.

Previous research has provided strong evidence of the existence of a balance sheet effect, whereby the transmission of monetary shocks to firm output is affected by financial constraints. In Column 4, we control for balance sheet effects through the inclusion of financial leverage. We compute financial leverage using balance sheet data from COMPUSTAT. We do not find evidence of a balance sheet effect related to financial leverage. Our main result is not affected when controlling for financial leverage.

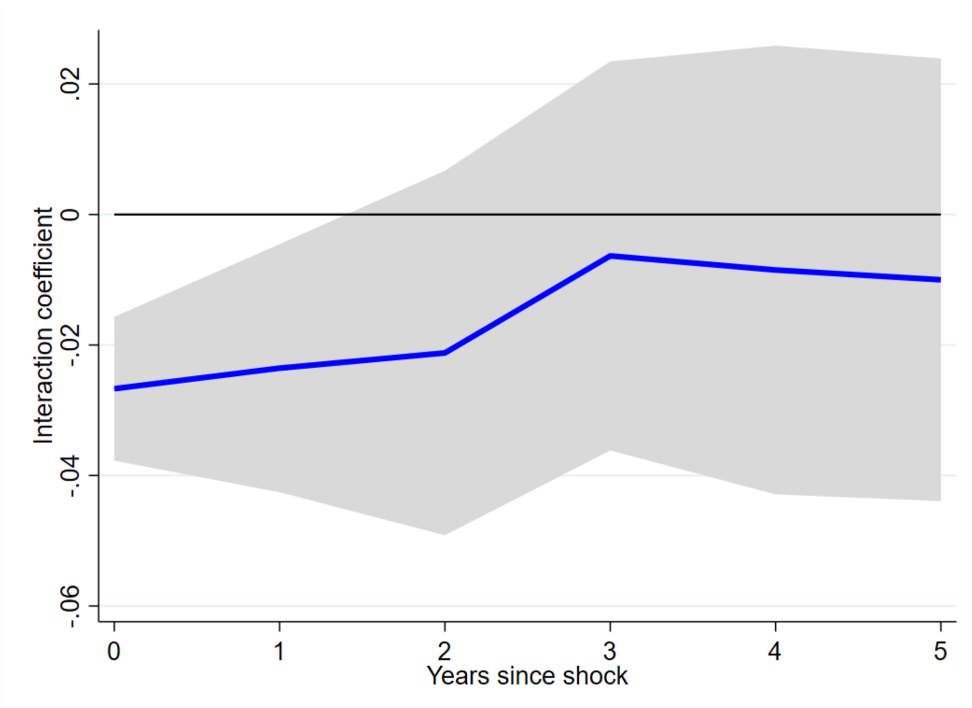
To analyze the dynamic impact of interest rate changes over time, we also estimate a Jordà (2005) type of local projection of Equation (37) as follows:

$$y_{ij,t+h} - y_{ij,t-1} = \alpha_{ijh} + \alpha_{jth} + \alpha_{ith} + \delta_h \cdot y_{ijt-1} + \beta_h \cdot r_t \cdot H_j \cdot RE_{i,t-1} + \varepsilon_{ijth} \tag{52}$$

for $h \geq 0$. The local projection shows the dynamics of the response of GDP to monetary shocks differentiated by housing elasticity H_j and real estate intensity $RE_{i,t-1}$. Specifically, the regression coefficients β_h indicate how the cumulative response of GDP over the period t to $t+h$ to a monetary shock in year t depends on the interaction between H_j and $RE_{i,t-1}$ in year $t-1$.

As before, all explanatory variables other than the monetary shock are standardized, and the signs of the monetary shock and housing elasticity are reversed to ease interpretation. The results of the local projection exercise are depicted in Figure 7. We use our baseline measure, mp_med_t, for the monetary shock. The results indicate that the result reported in Table 2 is persistent. Namely, the output of sectors that are more real-estate intensive and are located in regions where housing supply is more inelastic shows a consistently weaker response to expansionary monetary policy shocks for up to 5 years, even if this effect is estimated with large standard errors as shown by the 90% confidence intervals.

Figure 7: **Dynamics of differential response of output to monetary shocks**



Notes: The figure displays the regression coefficients β_h (the blue line) obtained when estimating the following specification: $y_{ij,t+h} - y_{ij,t-1} = \alpha_{ijh} + \alpha_{jth} + \alpha_{ith} + \delta_h \cdot y_{ijt-1} + \beta_h mp\_med_t \cdot H_j \cdot RE_{i,t-1} + \varepsilon_{ijth}$ for $h = 0, 1, 2, 3, 4, 5$. The grey area depicts the 90% confidence interval. All variables are as defined in Table 2. Standard errors are clustered twoways by sector-region and year.

C Online Appendix

C.1 Unconstrained firms

Consider a variation of our baseline economy where, in addition to entrepreneurs, there is a mass of firms that do not face financial constraints, i.e., they are unconstrained. For concreteness, we assume that these firms are owned and operated by the capitalists. In particular, assume that these firms have a production technology that can convert k units of capital in $t = 0$ into $f(k)$ units of the consumption good in $t = 1$, where f is twice differentiable with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) < 0$.

For simplicity, suppose that the capital supply is inelastic and entrepreneurs cannot pledge their output to creditors, i.e., $\lambda = 0$. Lastly, to ensure that some entrepreneurs operate side-by-side with these firms, we assume that $f'(\bar{K}) < 1$; that is, the marginal product of capital in these firms – if they were to employ the entire stock of capital – is lower than that of the most productive entrepreneur.

In this economy, there are two types of equilibria depending on whether the interest rate, R, is above or below the threshold $\widetilde{R}$.[25] If $R > \tilde{R}$, then the unconstrained firms are inactive, $R \cdot q > f'(0)$, and the analysis of the previous section applies. If $R < \tilde{R}$, the unconstrained firms are active and $R \cdot q = f'(K^T(q, R)))$, where $K^T(q, R) > 0$ denotes the aggregate units of capital employed by these firms; it only depends on (and is decreasing in) $R \cdot q$. In this case, market clearing requires that:

$$\bar{K} = K^T(q, R) + \int_{q \cdot R}^{1} \frac{w}{q} \cdot dG\left(A\right), \tag{53}$$

It follows that in equilibrium a fall in R raises the price of capital q (and weakly reduces $R \cdot q$). The response of aggregate output to changes in the interest rate is given by:

$$\frac{dY}{dR} = \int_{R \cdot q}^{1} (A - R \cdot q) \cdot \frac{d}{dR}\left(\frac{w}{q}\right) \cdot dG(A), \tag{54}$$

which note only features the capital-reallocation effect (as capital supply is fixed). Moreover, since w/q decreases in response to a fall in R, we have that $dY/dR > 0$.

What is going on? When the unconstrained firms are active, the price of capital is determined by their marginal product of capital, since $R \cdot q = f'(K^T)$. Thus, any decline in the interest rate must be compensated by an increase in the price of capital, which implies that the demand of capital by supra-marginal entrepreneurs must fall. Ultimately, a decline in the interest rate simply redistributes capital from supra-marginal entrepreneurs to the less productive unconstrained firms, thereby reducing output. Although this example with fixed capital stock and no credit whatsoever is stark, it is straightforward to prove the analogue of Proposition 1 for

[25] If $\lim_{k \to 0} f'(k) = \infty$, then $\widetilde{R} = \infty$. Otherwise, $\tilde{R}$ is implicitly defined by:

$$\int_{\widetilde{R} \cdot q}^{\bar{A}} \frac{w}{q} \cdot dG\left(A\right) \equiv \bar{K},$$

with $q = f'(0) \cdot \widetilde{R}^{-1}$. At this interest rate, the entrepreneurs are just able to absorb the entire capital stock $\bar{K}$.

this setting: namely, as the capital supply becomes less elastic, dY/dR increases and becomes positive provided that λ is below a threshold.

C.2 Diminishing returns at entrepreneur level

Consider a variation of our baseline economy in which entrepreneurial technology has diminishing returns. In particular, suppose that an entrepreneur with productivity A can convert k units of capital at $t = 0$ into $A \cdot h(k)$ units of the consumption good at $t = 1$, where $h(\cdot)$ satisfies $h'(\cdot) > 0$, $h''(\cdot) < 0$ and $\lim_{k \to 0} h'(k) \to \infty$. As before, A is distributed according to the cdf G on interval $[0, 1]$ that has an associated density g. For simplicity, assume that the capital supply is inelastic and entrepreneurs cannot pledge their output to creditors, i.e., $\lambda = 0$.

Optimal investment of entrepreneur with productivity A is given by:

$$k_A(q, R) = \min \left\{ h'^{-1}\left(\frac{R \cdot q}{A}\right), \frac{w}{q} \right\}. \tag{55}$$

Therefore, it follows that there is a cut-off productivity given by:

$$\widetilde{A} = \frac{R \cdot q}{h'\left(\frac{w}{q}\right)},$$

such that all entrepreneurs with productivity $A > \widetilde{A}$ are constrained by their resources while the rest are unconstrained.

In this economy, the market clearing condition for capital is given by:

$$\bar{K} = \int_0^{\widetilde{A}} h'^{-1}\left(\frac{R \cdot q}{A}\right) \cdot g(A) \cdot dA + \frac{w}{q} \cdot (1 - G(\widetilde{A})), \tag{56}$$

and aggregate output is:

$$Y = \int_0^{\widetilde{A}} A \cdot h\left(h'^{-1}\left(\frac{R \cdot q}{A}\right)\right) \cdot g(A) \cdot dA + \int_{\widetilde{A}}^1 A \cdot h\left(\frac{w}{q}\right) \cdot g(A) \cdot dA. \tag{57}$$

Combining Equations (56) and (57), we have:

$$\frac{dY}{dR} = \int_{\widetilde{A}}^1 \left(A \cdot h'\left(\frac{w}{q}\right) - q \cdot R\right) \cdot \frac{d}{dR}\left(\frac{w}{q}\right) \cdot g(A) \cdot dA. \tag{58}$$

Since $A \cdot h'(k_A) > q \cdot R$ for all $A > \widetilde{A}$ and $\frac{d}{dR}\left(\frac{w}{q}\right) > 0$, it follows that dY/dR is positive.

What is the intuition? In partial equilibrium, as R falls, the investment of unconstrained entrepreneurs expands while the investment of constrained entrepreneurs is unchanged. This implies that, in general equilibrium, the price of capital must rise. Ultimately, exactly as in our baseline economy with linear technology, the decline in R simply reallocates capital from entrepreneurs with $A > \widetilde{A}$ who are constrained to entrepreneurs with $A < \widetilde{A}$ who are unconstrained, thereby reducing output. Thus, in a world with diminishing returns at entrepreneurial level, it is the capital reallocation from high- to low-marginal product of capital entrepreneurs that depresses aggregate TFP and output.

Again, although this example with fixed capital stock and no credit is stark, it is straightforward to prove the analogue of Proposition 1 for this setting: namely, as the capital supply becomes less elastic, dY/dR increases and becomes positive provided that λ is below a threshold.

C.3 Default risk and credit spreads

In this appendix, we extend our analysis to incorporate heterogeneity in entrepreneurial funding costs. To do so, we incorporate idiosyncratic risk into the entrepreneurial production technology. If the entrepreneur invests k at $t = 0$, then she succeeds with probability p at $t = 1$, in which case she receives output $A \cdot k$; otherwise, she fails with probability $1 - p$ and receives output 0. We assume that entrepreneurs differ both in the probability of success p and in the productivity A in case of success. As before, we assume an entrepreneur can pledge at most a fraction λ of her output to outsiders.

Since it is without loss of generality to assume that the entrepreneur does not default upon success, it follows that $R \cdot (1 + \frac{1-p}{p})$ is the interest rate at which an entrepreneur of type (A, p) can raise funds from lenders, where $R \cdot \frac{1-p}{p}$ is the spread over the risk-free rate due to the possibility of default. Let $e(A, p) = p \cdot A$ denote the expected productivity of an entrepreneur of type (A, p). The distributions over p and A induce a distribution $\widetilde{G}(\cdot)$ over the expected productivity e, which we will assume has full support on $[0, 1]$. Observe that the model can accommodate either positive or negative correlation between p and A.

Entrepreneurial demand for capital can be shown to take the form:

$$k_{(A,p)}(q, R) = k_e(q, R) \begin{cases} = 0 & \text{if } \frac{e}{R} < q \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot e}{R}} \cdot w\right] & \text{if } \frac{e}{R} = q \\ = \frac{1}{q - \frac{\lambda \cdot e}{R}} \cdot w & \text{if } \frac{\lambda \cdot e}{R} < q < \frac{e}{R} \\ \infty & \text{if } q \leq \frac{\lambda \cdot e}{R} \end{cases}; \tag{59}$$

that is, the demand of entrepreneur (A, p) depends only on her expected productivity $e = p \cdot A$, and not on p and A separately. As a result, q must be such that:

$$K^S(q) = K = K^D(q, R) = \int_0^1 k_e(q, R) \cdot d\widetilde{G}(e). \tag{60}$$

Aggregate output is in turn given by:

$$Y = \int_0^1 e \cdot k_e(q, R) \cdot d\widetilde{G}(e). \tag{61}$$

By comparing Equations (59)-(61) with their counterparts in the baseline model, it follows immediately that Proposition 1 holds in this extended setting as well (we only need to relabel A with e). Importantly, note that for a given distribution over expected productivity $\widetilde{G}$, the correlation of p and A is irrelevant for the equilibrium q, K, and Y.

C.4 Closed economy: endogenous interest rates and savings gluts

Throughout our main analysis, we considered a small open economy that experienced an exogenous fall in the world interest rate. In this Appendix, we show that none of our main insights would change if the economy were closed and the fall in the interest rate were the result of a savings glut, i.e., an increase in the economy's desired savings.

Suppose now that the economy is closed, that the agents preferences are given by:

$$U = E_0\{c_0 + \beta \cdot c_1\} \tag{62}$$

for some $\beta \in (0,1)$, and that the capitalists have an endowment $w^C > 0$ of the consumption good at $t = 0$. Given these adjustments, we next show that the main results from our baseline setting can be obtained by raising the desired savings in this economy.

Proposition 3 *The effects of a fall in the interest rate, R, as described in Proposition 1 are isomorphic to those of an increase in w^C and/or β.*

In what follows, we illustrate the proof of this result. First, note that the equilibrium interest rate, R, must be weakly greater than β^{-1}. Otherwise, there would be a positive credit demand but no savings, as all agents who do not invest in capital would want to consume; hence, the credit market would not clear.

Second, observe that, given prices $\{q, R\}$, the aggregate savings of the savers (i.e., the capitalists and entrepreneurs with productivity $A < q \cdot R$) are given by:

$$S(q,R) \begin{cases} = w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R) & \text{if } R > \beta^{-1}, \\ \in [0,\ w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R)] & \text{if } R = \beta^{-1} \end{cases}. \tag{63}$$

Equation (63) states that if $R > \beta^{-1}$, then the savers save all their resources, which are given by their endowments of the consumption good plus the profits of the capitalists. If $R = \beta^{-1}$, then the savers are indifferent between saving and consuming these resources. As a result, the credit market clearing condition is given by:

$$S(q,R) = \int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A), \tag{64}$$

which together with Equations (5), (6), (8), (9) and (10), characterizes the equilibrium.

Lastly, observe that the aggregate credit demand can be expressed as:

$$\int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A) = q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{65}$$

since the entrepreneurs who invest in capital use all of their endowment plus borrowing to finance purchases of capital.

Therefore, we can immediately see that there are two possibilities in equilibrium.

Case 1. Consider a candidate equilibrium where the interest rate, R, is equal to β^{-1}. For this to be an equilibrium, it must be that:

$$w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot \beta^{-1}) \geq q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{66}$$

which holds if and only if:

$$w^C + w \geq \chi(K^S(q)), \tag{67}$$

where the equilibrium price of capital, q, clears the capital market:

$$K^S(q) = \int_{q \cdot R}^{1} k_A(q, \beta^{-1}) \cdot dG(A). \tag{68}$$

It is therefore immediate that in this case the effects of an increase β on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3. Moreover, observe that this candidate is an equilibrium if w^C and/or β are large enough.

Case 2. Consider a candidate equilibrium where the interest rate R is above β^{-1}. This candidate is an equilibrium if at $R = \beta^{-1}$ the inequality (67) is violated, i.e., if w^C and/or β are small. Hence, in this case, the equilibrium prices $\{q, R\}$ are such that:

$$w^C + w = \chi(K^S(q)), \tag{69}$$

and

$$K^S(q) = \int_{q \cdot R}^{1} k_A(q, R) \cdot dG(A). \tag{70}$$

Here, a rise in w^C raises the capital price (as $\chi(K^S(q))$ is increasing in q) and lowers the interest rate (to offset the effect of a higher q that depresses capital demand). Hence, the effects of an increase in w^C on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3.

Lastly, note that if the equilibrium is initially in Case 2, then an increase in w^C eventually moves the equilibrium into Case 1.

C.5 Our mechanism in the Kiyotaki-Moore model

In this Appendix, we show that the capital-reallocation effects induced by falling interest rates that we emphasized through the main text are also present in the class macro-finance model of Kiyotaki and Moore (1997).

Time is now infinite, $t = 0, 1, ...$ Assume, for simplicity, that all entrepreneurs in the modern sector have the same productivity $A \in (0, 1)$, and that the capital stock is fixed at $\bar{K} > 0$. Thus, aggregate output in any period t depends solely on the allocation of capital between the modern and traditional sectors:

$$Y_t = A \cdot K_t + a \cdot f(\bar{K} - K_t), \tag{71}$$

where K_t denotes the aggregate stock of capital employed in the modern sector at time t.

We focus on equilibria in which the traditional sector is active in all periods and, hence, its demand for capital is given by:

$$\frac{a \cdot f'\left(\bar{K} - K_{t+1}\right) + q_{t+1}}{q_t} = R, \tag{72}$$

i.e., the return to capital within the traditional sector must equal the interest rate.

As in the static model, we introduce a financial friction by assuming that – in any period – an entrepreneur can walk away with a fraction $1 - \lambda$ of her resources, which now include her output and the market value of her capital. It thus follows that entrepreneurs face the following borrowing constraint:

$$R \cdot B_t \leq \lambda \cdot (A + q_{t+1}) \cdot K_{t+1}, \tag{73}$$

where B_t and K_{t+1} respectively denote entrepreneurial borrowing and investment in period t.[26] Note that, since all entrepreneurs are identical, B_t and K_{t+1} also represent aggregate borrowing and investment in the modern sector.

In any period t, the net worth of entrepreneurs equals the sum of their output and the market value of their capital minus repayments to creditors: $A \cdot K_t + q_t \cdot K_t - R \cdot B_{t-1}$. We assume that entrepreneurs consume a fraction $1 - \rho$ of this net worth in every period, where $\rho \cdot R < 1$.[27] This ensures, in the spirit of Kiyotaki and Moore (1997), that the financial constraint holds with equality in all periods. As a result, the modern-sector demand for capital is given by:

$$K_{t+1} = \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot (1 - \lambda) \cdot (A + q_t) \cdot K_t, \tag{74}$$

where we make parametric assumptions to ensure that both sectors are active in a neighborhood of the steady state.[28]

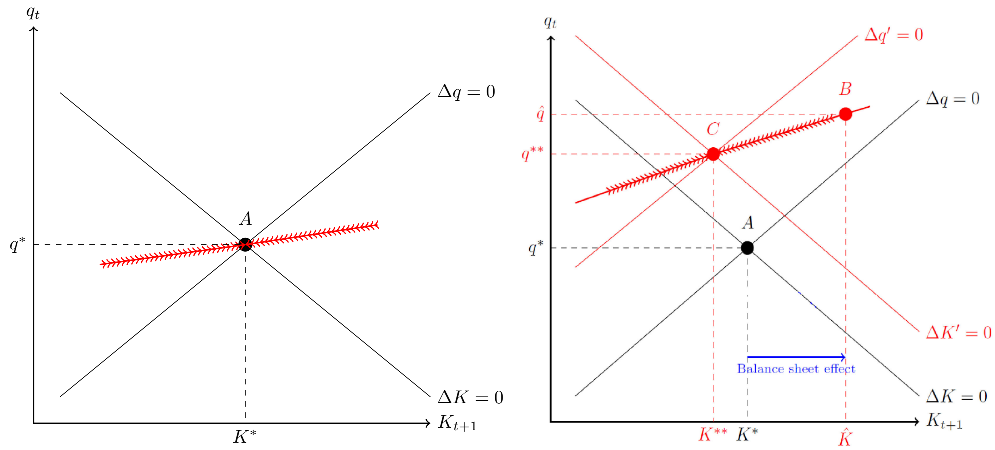
Thus, given an initial value for $K_0 > 0$ and a no bubbles condition on the price of capital, Equations (72) and (74) fully characterize the equilibrium of this economy. Panel (a) of Figure 8 portrays the equilibrium dynamics with the help of a phase diagram in the (K_{t+1}, q_t)-space. The $\Delta q = 0$ locus depicts all the combinations of K_{t+1} and q_t for which Equation (72) is satisfied with $q_t = q_{t+1}$. The locus is upward sloping because a higher level of modern-sector investment, K_{t+1}, is associated with a higher productivity of capital in the traditional sector and – since capital is priced by this sector – with a higher level of q_t. The $\Delta K = 0$ locus depicts instead all the combinations of K_{t+1} and q_t for which Equation (72) is satisfied with $K_t = K_{t+1}$. The locus is downward sloping because a higher level of modern-sector investment,

[26] In Kiyotaki and Moore (1997), the output of investment is not pledgeable but the resale value of capital is fully pledgeable. Although our results would also go through under that specification, we have chosen the current specification in order to preserve symmetry with the baseline model of Section 2.

[27] E.g., it is sufficient to assume that entrepreneurs have log-preferences, i.e., $U^E = \sum_{t=0}^{\infty} \rho^t \cdot log(c_t)$. Note that the preferences of other agents (i.e., capitalists and traditional investors) are irrelevant for the evolution of q_t, K_t and Y_t.

[28] In particular, if K_0 is close to steady state, this requires that:

$$\frac{a \cdot f'(0)}{R - 1} > \frac{R \cdot \rho \cdot (1 - \lambda) + \lambda}{R - R \cdot \rho \cdot (1 - \lambda) - \lambda} \cdot A > \frac{a \cdot f'(\bar{K})}{R - 1}.$$



(a) Convergence to steady-state (b) Effects of unexpected fall in R

Figure 8: **Equilibrium dynamics and balance sheet effects.** The figure illustrates a phase diagram for the joint evolution of the price of capital and the stock of capital in the modern sector. The saddle path of the system is depicted by a red curve with arrows pointing to the steady state: the left panel depicts the dynamics before the unexpected decline in the interest rate, whereas the right panel depicts the dynamics after it.

K_{t+1}, is only affordable to constrained entrepreneurs if the equilibrium price of capital, q_t, is lower. As the figure shows, the system displays saddle-path dynamics. From an initial condition $K_0 < K^*$, both K and q increase monotonically as the economy transitions to the steady state and modern-sector entrepreneurs accumulate net worth. The opposite dynamics follow from an initial condition $K_0 > K^*$.

The right-hand panel of Figure 8 portrays the response to a permanent and unanticipated decline in R in a given period t_0. In response to a lower R, both loci shift upwards. The $\Delta q = 0$ locus shifts up because the traditional sector's willingness to pay for capital increases alongside the net present value of dividends; the $\Delta K = 0$ also shifts up because entrepreneurs' ability to pay for capital increases as lower interest rates relax their borrowing constraint. The presence of financial frictions, however, mitigates the shift in the $\Delta K = 0$ locus. Thus, as the figure shows, a decline in R triggers an increase in the steady-state price of capital to q^{**}, and a reduction in the capital employed in the modern sector to K^{**}. Hence, a reduction in the interest rate leads to a fall in the steady-state level of output despite the presence of dynamics.

This does not mean, however, that balance sheet effects do not play a role. Indeed, on impact, in response to a decline in the interest rate, the value of capital increases from $q^* \cdot K^*$ to $q_{t_0} \cdot K^*$ while entrepreneurial debt payments - which are pre-determined - remain unaffected and equal to $R \cdot B^*$.[29] Therefore:

$$K_{t+1} = \begin{cases} \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot ((1-\lambda) \cdot (A + q^*) + q_t - q^*) \cdot K^* & \text{if } t = t_0 \\ \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot (1-\lambda) \cdot (A + q_t) \cdot K_t & \text{if } t > t_0 \end{cases}. \tag{75}$$

The evolution of q_t is still given by Equation (72). This means that the adjustment of K to

[29] As in Kiyotaki and Moore (1997), these balance sheet effects require that entrepreneurs' debt payments are not indexed to the price of capital.

the new steady-state is not monotonic. As the right-hand panel of Figure 8 shows, K_{t+1} rises to $\widehat{K}$ on impact: this, as stated in the figure, is the balance sheet effect. The expansion of the modern sector is short-lived, though, since from that period onwards the economy evolves along the saddle-path towards its new steady state, which features a higher price of capital but a lower capital stock in the modern sector and thus a lower level of output. This decline from $\widehat{K}$ to K^{**} is, as stated in the figure, due to the reallocation effect: the higher demand of capital by the traditional sector keeps capital prices high, and these slowly erode the net worth of modern-sector entrepreneurs. As a result, the dynamic behavior of aggregate output in this economy resembles closely that of the dynamic economy in Section 4, illustrated in Figure 6.

The key takeaway is that the same reallocation forces that we analyzed in our baseline model of Section 2 are also at work in a dynamic environment. Moreover, these forces are persistent in response to a permanent decline in the interest rate, while the balance-sheet effects that are often highlighted in the literature are transitory. To be sure, an unexpected decline in the interest rate does have an initial balance-sheet effect that benefits productive entrepreneurs and reallocates capital towards them, raising average productivity and output. But this effect is by nature temporary: the reason is that it represents a one-time shock to the level of entrepreneurial net worth, but it does not affect the dynamic evolution of net worth thereafter.

C.6 Closed-form solution to the steady state of the dynamic model

In this Appendix, we derive the steady state of the dynamic model in closed form, for the case in which productivity is i.i.d. over time and uniformly distributed on the unit interval.

As Equation (31) in the text shows, in steady state:

$$W_A = \begin{cases} \frac{\Theta}{\Theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)} \cdot g(A) \cdot W & \text{if } A \geq r \cdot q \\ \frac{\Theta}{\Theta+\rho-r} \cdot g(A) \cdot W & \text{otherwise} \end{cases}, \tag{76}$$

where $g(A) = 1$ in this case because of the uniform distribution. Let $x \equiv A/(r \cdot q)$, then:

$$\frac{W_{r\cdot q\cdot x}}{W} = \begin{cases} \frac{\Theta}{\Theta+\rho-\frac{x-\lambda}{1-\lambda}\cdot r} & \text{if } x \in \left[1, \frac{1}{r\cdot q}\right] \\ \frac{\Theta}{\Theta+\rho-r} & \text{if } x \in [0,1] \end{cases}, \tag{77}$$

with:

$$\int_0^{\frac{1}{r\cdot q}} \frac{W_{r\cdot q\cdot x}}{W} dx = 1\ . \tag{78}$$

From substituting (77) into (78), it follows that:

$$\frac{1}{r\cdot q} = \lambda + \frac{1-\lambda}{r} \cdot \left[\Theta + \rho - \frac{\Theta+\rho-r}{\exp\left\{\frac{1}{\Theta}\cdot\frac{r}{1-\lambda}\cdot\frac{\rho-r}{\Theta+\rho-r}\right\}}\right]. \tag{79}$$

Equation (79) allows us to express price q as a function of model parameters.

Aggregate output is given by:

$$Y = \frac{1}{\int_1^{\frac{1}{r \cdot q}} W_{r \cdot q \cdot x} \cdot dx} \cdot \left[\int_1^{\frac{1}{r \cdot q}} r \cdot q \cdot x \cdot W_{r \cdot q \cdot x} \cdot dx \right] \cdot \bar{K} \ . \tag{80}$$

Thus, we have that:

$$Y = r \cdot q \cdot \frac{\Theta + \rho - r}{\rho - r} \cdot \Theta \cdot \frac{\left[\left(\Theta + \rho + \frac{\lambda}{1-\lambda} \cdot r \right) \ln \left(\Theta + \rho + \frac{\lambda}{1-\lambda} \cdot r - \frac{r}{1-\lambda} \cdot x \right) + \frac{r}{1-\lambda} \cdot x \right] \Big|_1^{\frac{1}{r \cdot q}}}{\left(\frac{r}{1-\lambda} \right)^2} \cdot \bar{K} \ , \tag{81}$$

which together with Equation (79) allow us to express Y as a function of parameters.