The welfare cost of ignoring the beta*

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Abstract

Because of risk aversion, any sensible investment valuation system should value less projects that contribute more to the aggregate risk, i.e., that have a larger income-elasticity of net benefits. In theory, this is done by adjusting discount rates to consumption betas. But in reality, most public and private institutions and people use a discount rate that is rather insensitive to the risk profile of their investment projects. I show in this paper that the economic consequences of the implied misallocation of capital are dire. To do this, I calibrate a Lucas model in which the investment opportunity set contains a myriad of projects with different expected returns and risk profiles. The welfare loss of using a single discount rate is equivalent to a permanent reduction in consumption that lies somewhere between 15% and 45%, either at the level of the irrational agents, or at equilibrium if all agents make the same mistake. Economists should devote more energy to support a reform of public discounting systems in favor of what has been advocated by the normative interpretation of modern asset pricing theories over the last four decades.

Keywords: Discounting, investment theory, asset pricing, carbon pricing, Arrow-Lind theorem, WACC fallacy, rare disasters, capital budgeting.

JEL codes: G12, H43, Q54.

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1 Introduction

It is an enduring common practice in most western countries to value public investments and policies by measuring the present value of their flow of expected social benefits using a single discount rate. As already noticed by Bazelon and Smetters (1999) for example, this means that no insurance or hedging value is recognized to policies that reduce ex ante the consequences of natural catastrophes or of a pandemic for example. Symmetrically, no penalty is imposed to policies involving benefits materializing mostly in good states of nature, such as expanding the capacity of energy and transportation infrastructures. It is never too late to change this inefficient practice. In this paper, I estimate its social cost. It is large.

An obvious candidate to evaluate the impact of an investment on the risk borne by its stakeholders is its "consumption beta", hereafter called beta. The beta of a project is defined as the elasticity of its future benefit to changes in future aggregate consumption. The larger the project's beta, the larger its impact on the aggregate risk in the economy. Any decision criterion that recognizes risk aversion should value less projects with a larger beta, everything else unchanged. Modern asset pricing and investment theories translated this simple idea into practice by recommending that discount rates be adjusted for the projects' beta. The Consumption-based Capital Asset Pricing Model (CCAPM) pioneered by Rubinstein (1976), Lucas (1978) and Breeden (1979), and its extensions (Bansal and Yaron, 2004; Barro, 2006) to solve the asset pricing puzzles (Mehra and Prescott, 1985; Weil, 1989), provide a normative framework to justify this methodology. In these models, there is a linear relationship between the socially desirable discount rate for a project and its consumption beta. This discounting system is thus characterized by two key variables: the risk-free discount rate and the aggregate risk premium. The risk-free discount rate describes our willingness to care about the future in general, whereas the risk premium characterizes our collective distaste for acts that raises the aggregate risk. The large market risk premium relative to the risk-free market rate observed over the last century suggests that the risk-adjustment embedded in this efficient discounting system should play a crucial role in the investment evaluation process.

The practice of investment evaluation and selection is often distant from these recommendations universally supported by normative economic theory. This is particularly the case in the public sector. Indeed, most countries and international organizations have established guidelines for policy and investment evaluation in which the recommended discount rate is unique and not sensitive to the risk profile of the decision under scrutiny.[1] This dogma of a single discount rate for the public sector has long been supported by the influential Arrow-Lind theorem (Arrow and Lind, 1970), which claims that "the government invests in a greater number of diverse projects and is able to pool risks to a much greater extent than private investors", thereby washing out risk completely. Most people interpreted this result as meaning that all public investment projects should be discounted at the risk-free interest rate. But, as stated by Sandmo (1972), Lucas (2014), Baumstark and Gollier (2014) and the CCAPM theory, this result is valid only for projects with a zero CCAPM beta. Notice that Arrow and Lind mentioned this point in their paper: "The results ... depend on returns from a public investment being independent of other components of national income" (p. 373). As stressed by Bazelon and Smetters (1999) and Cherbonnier and Gollier (2019), the use of a single discount rate tends to overvalue positive-beta policies such as building new

[1] See Section 2 for more details on this. France is the only exception.

transportation infrastructures, and to undervalue policies that hedge the macroeconomic risk such as improving earthquake-resistant construction norms, increasing pandemic-treatment capacities, or building a strategic petroleum reserve. Because a vast majority of projects have a positive beta, the use of the risk-free rate as the discount rate implies an excess of positive NPV projects compared to the capacity of public funding, thereby often forcing governments to impose a capital rationing scheme on top of the valuation process. The fallacious interpretation of the Arrow-Lind also prevailed in the debate about public spending in the wake of the financial crisis of 2008 or of the COVID-19 crisis of 2020 when many experts recommended using the low cost of public capital to implement ambitious recovery plans in the United States and in Europe.[2]

The absence of consensus on the Social Cost of Carbon (SCC) in our profession illustrates the mess in which economists and practitioners have to survive under this inefficient discounting system. In climate economics since the publication of the Stern Review (Stern (2007)), most proponents to the debate used the Ramsey rule (Ramsey, 1928) to evaluate the rate at which future climate damages should be discounted.[3] The problem is that the Ramsey rule and its extension to uncertainty (Hansen and Singleton, 1983) characterize the rate at which *safe* benefits should be discounted. Weitzman (2001) made things more confusing by suggesting that when the long-term risk-free rate is uncertain, its harmonic mean should be used to discount long-term climate damages. The first reference to the necessity to adjust the climate discount rate to the risk profile of the climate damages emerged when the Obama administration convened a commission aimed at making recommendation on the SCC. The Technical Support Document (Interagency Working Group on Social Cost of Carbon, 2010) used three discount rates: 2.5%, 3% and 5%, this latter rate reflecting "the possibility that climate damages are positively correlated with market returns." Dietz et al. (2018) showed that in the DICE model of Nordhaus (2008), the CCAPM beta of climate damages is close to unity: In the business-as-usual scenario, future climate damages will be larger if the future will be more prosperous. This implies that the entire debate on the SCC has long been misleading by ignoring the crucial risk-adjustment of the climate discount risk.

Inefficient discount systems generate a myriad of other issues. For example, at which price should governments sell specific infrastructures, such as highways, railroads, or hospitals? What is the value of public investments in defense, schools, or research institutions? How should public funds from covid recovery funds and green deals be allocated? How should students value their different educational options? All these questions are certain to receive bad answers when using an inefficient discounting system, in particular when contemplating long-lasting investments.

The bottom line is that the practice of capital budgeting and investment evaluation is still far from what would be compatible with social welfare maximization. In this paper, I measure the welfare loss associated with using a single discount rate when performing the benefit-cost analysis to determine the optimal allocation of capital, either at the individual level, or in the economy as a whole. Contrary to the standard endowment economy that is

[2]See for example Paul Krugman's op-ed in the NYT entitled "Ideology and investment" (October 26, 2014): "The federal government can borrow incredibly cheaply... So borrowing to build roads, repair sewers and more seems like a no-brainer." Although a reduction in the interest rate implies a uniform reduction in risk-adjusted discount rate, it does not mean that the public cost of capital should be used to uniformly discount all public projects. Boyer (2018, 2022) provides other illustrations of this fallacy in the public sector.

[3]See for example Arrow (2007), Nordhaus (2007), Dasgupta (2008) and Weitzman (2010).

used in the CCAPM (Lucas (1978), Martin (2013)), I examine a dynamic model in which investments are endogenously selected in an opportunity set with heterogeneous risk profiles and expected benefits. At the beginning of each period, identical infinitely-lived agents must determine what share of their wealth should be consumed, and which investments should be implemented. The first-best investment rule entails a CCAPM discounting system in which the project-specific discount rate is a linear function of the project's beta. I calibrate this model by assuming that the common risk factor is affected by extreme events as in Barro (2006) in order to solve the standard asset pricing puzzles. I then compare this dynamic equilibrium to another equilibrium in which the representative agent uses a single discount rate to determine her investment strategy. I show that the absence of risk-adjustment in this procedure has catastrophic effects on intertemporal welfare. This is a reminder of the importance of the allocation of capital in our economy for its functioning and for our collective prosperity.

2 Public discounting in practice

France is currently the only country in the world in which public investment projects must be evaluated using a discount rate that is sensitive to the project's risk profile (Quinet (2013)). The French discounting system is based on the CCAPM with a risk-free discount rate of 2.5% and a systematic risk premium of 2%.[4] The evaluators are thus required to estimate the consumption beta of their project, which is defined as the elasticity of the project's net benefit to changes in aggregate consumption. Personal anecdotes suggest that lobbies from high-beta sectors have periodically attempted to go back to a single discount rate, or to reduce the level of the aggregate risk premium by referring to the equity premium puzzle.[5]

Between 1997 and 2012, Norway used a simplified version of the CCAPM to evaluate large public investment projects, with project-specific discount rates ranging from 3.5% to 8% depending upon the project's beta. But a report published in 2012 (Hagen et al. (2012)) claimed that "considerable room for discretionary assessments with regard to estimates as to project-specific risk ... may offer incentives to choose assumptions that may influence the outcome of the analysis in the direction favoured by various interest parties... These circumstances suggest that it may be preferable to recommend simple and transparent rules that capture the most important aspects of the matter, without being too complex to understand or to apply" (page 77). Consequently, the report recommends the use of a single discount rate of 4%. It has been determined by combining a risk-free rate of 2.5% and an average risk premium of 1.5%.

More recently, the Netherlands has adopted three public discount rates (Rijksoverheid (2020)): An all-purpose discount rate of 2.25%, with two exceptions. A lower discount rate of 1.6% should be used for "costs that are largely or wholly independent of usage (i.e. fixed costs)". A larger discount rate of 2.9% should be used for "benefits that are highly non-linear

[4] All discount rates discussed in this paper are real discount rates. I limite this description to short-term discount rates. The French system also imposes a smaller risk-free discount rate and a larger risk premium for long maturities.

[5] A pernicious strategy yielding the same outcome consists in proposing to discount the certainty equivalent benefits at the risk-free discount rate. This methodology is supported by the theory as explained for example by Bazelon and Smetters (1999), but it fails to fit observed investment decisions because of the standard asset pricing puzzles.

relative to usage, where usage, moreover, depends on the state of the economy." This could be interpreted as a simplified version of the CCAPM discounting system, with the partition of the investment opportunity set into three beta segments.

All other countries that have published a discounting guideline have been using since a long time – and are still using – a single discount rate. In the United Kingdom, the official discount rate has been 3.5% since 2003, using the Ramsey rule (Treasury (2018)). In the European Union, it is equal to 5.5% for the "Cohesion countries" (basically the more recent member states) and 3.5% for the others (Florio (2008)). The most confusing discounting system can be found in the United States. Since the publication of Circular A-4 by OMB (2003), regulatory analysis should "provide estimates of net benefits using both 3 percent and 7 percent" discount rates. This official document justifies these two rates as respectively the "real rate of return on long-term government debt" and the "average before-tax rate of return to private capital in the U.S. economy". Nordhaus (2013) claims that in Circular A-4, "the OMB discussion is completely confused... because the difference is not the difference between investment and consumption" but instead "the risk premium on leveraged corporate capital" (quoted by Sunstein (2014)). This confusion and the absence of guideline about which of these two discount rates should be used in practice represents a procedural failure that has been used by the Trump administration to arbitrarily increase the discount rate for carbon pricing to 7 percent, yielding a carbon price of 1 USD/tCO_2 (Environmental Protection Agency (2018)), from around 50 USD/tCO_2 at the end of the Obama administration (Interagency Working Group on Social Cost of Carbon (2015)).[6]

Whether the private sector uses more efficient investment decision rules remains an open question. On one side, standard textbooks in finance strongly recommend the CCAPM rule to evaluate investment projects (Bodie and Merton, 2000; Brealey et al., 2017), and most CFOs claim to use it (Graham and Harvey, 2001; Jacobs and Shivdasani, 2012). On the other side, there is ample evidence in observed asset prices that the CCAPM pricing rule is only partially able to explain them. The Security Market Line – which links expected returns to betas – is too flat (Fama and French, 1992). This generates a problem similar to the one observed in the public sector, with low-beta projects being undervalued, and large-beta projects being overvalued. Dessaint et al. (2019) confirm this finding by examining a large sample of mergers and acquisitions. Another standard misunderstanding in this field is what Krueger et al. (2015) have termed the "WACC fallacy". It consists in using the Weighted Average Cost of Capital (WACC) of an institution as the single discount rate used by this institution to evaluate its investment opportunities. In a sense, this is the private sector version of the fallacious interpretation of the Arrow-Lind theorem.

3 The model

The model is an adaptation of the CCAPM in which the dynamics of heterogenous capital allocation is endogenous. There is a single consumption good that can be either consumed or invested. An investment project is characterized by a pair $(\theta, \beta) \in \mathbb{R}^2$, and the investment opportunity set in the economy is described by a distribution function F over this pair. This distribution is stationary. For simplicity, capital is short-lived. One unit of capital invested

[6] The other ingredient used by the Trump administration to reduce the social cost of carbon is the limitation of the benefits to those accruing to U.S. citizens.

in project (θ, β) at date $t-1$ generates a single benefit $x_t(\theta, \beta)$ that materializes at date t, with

$$x_t(\theta, \beta) = \theta + \beta y_t + \tilde{\varepsilon}_t(\theta, \beta), \tag{1}$$

with $E_{t-1}\tilde{\varepsilon}_t = 0$. We assume that risks $\tilde{\varepsilon}(\theta, \beta)$ are idiosyncratic, in the sense that $\tilde{\varepsilon}(\theta, \beta)$ and $\tilde{\varepsilon}(\theta', \beta')$ are statistically independent for all $(\theta, \theta', \beta, \beta')$. The project-specific benefit $x_t(\theta, \beta)$ is sensitive to the realization of a common factor whose realization y_t at date t is unknown at date $t-1$, with $E_{t-1}y_t = 0$. We assume that $(y_0, y_1, y_2, ...)$ is a vector of independent and identically distributed random variables that are independent of the idiosyncratic risks $\tilde{\varepsilon}$. To sum up, a project is characterized by its expected gross return on investment (ROI) θ and by its sensitivity β to the common factor y. Without loss of generality, we assume that the average β among all projects belonging to the investment opportunity set is equal to unity:

$$\iint \beta dF(\theta, \beta) = 1. \tag{2}$$

The decision variable $\alpha_t(\theta, \beta)$ represents the capital invested in projects (θ, β) at date t. If the investment strategy $\alpha_t : \mathbb{R}^2 \to \mathbb{R}$ is chosen at date t, it generates total wealth z_{t+1} at date $t+1$, which is equal to

$$z_{t+1} = \iint \alpha_t(\theta, \beta) x_{t+1}(\theta, \beta) dF(\theta, \beta) = \overline{\theta}_t + \overline{\beta}_t y_{t+1}, \tag{3}$$

with

$$\overline{\theta}_t = \iint \alpha_t(\theta, \beta)\theta dF(\theta, \beta) \tag{4}$$

and

$$\overline{\beta}_t = \iint \alpha_t(\theta, \beta)\beta dF(\theta, \beta). \tag{5}$$

Observe from equation (3) that idiosyncratic risks $\tilde{\varepsilon}_t(\theta, \beta)$ associated to investing in the family of projects (θ, β) are washed out by diversification. Consumption at date t equals $c_t = z_t - \overline{\alpha}_t$, where $\overline{\alpha}_t$ is total investment expenditure at date t, with

$$\overline{\alpha}_t = \iint \alpha_t(\theta, \beta) dF(\theta, \beta) \tag{6}$$

I assume that the capital which can be invested at date t in any project (θ, β) in the economy is constrained to be non-negative and smaller than capacity z_t/η, with $\eta \in [0, 1]$. So for example, a feasible investment strategy is to implement a fraction η of the investment projects to full capital capacity z_t/η, and to invest nothing in the other projects. A capacity constraint is necessary in this model to discard an investment strategy in which the entire capital of the economy would be invested in projects with the best risk-return profile.

There is a continuum of infinitely-lived agents in the economy. They are endowed with the same initial wealth and they all face the same opportunity set of investment projects. They maximize the discounted expected utility of their flow of consumption. Their preferences are characterized by their common utility discount factor δ and by their increasing and concave utility function u over consumption. I assume a CRRA utility function with $u(c) = c^{1-\gamma}/(1-\gamma)$, with $\gamma > 0$. In the calibration section of this paper, I will solve the asset pricing puzzles by assuming rare disasters in the distribution of y.

4 The rational equilibrium

4.1 Characterization

A rational equilibrium is an allocation in which all agents follow the investment strategy that maximizes their intertemporal welfare. Because all agents have the same preferences and the same initial endowment, autarky is an equilibrium. I first characterize the optimal investment strategy in this economy. It solves the following recursive problem:

$$V(z_t) = \max_{\alpha_t:\mathbb{R}^2\to[0,z_t/\eta]} u(z_t - \overline{\alpha}_t) + \delta E V(\overline{\theta}_t + \overline{\beta}_t y). \tag{7}$$

The first-order condition for the investment decision in project (θ, β) can be written as follows: For all (θ, β) such that $dF(\theta, \beta) > 0$,

$$u'(z_t - \overline{\alpha}_t^*) = \delta E\left[(\theta + \beta y)V'(\overline{\theta}_t^* + \overline{\beta}_t^* y)\right] + \psi_t(\alpha, \beta) \tag{8}$$

with

$$\psi_t(\theta, \beta)\begin{cases} \geq 0 & \text{if} \quad \alpha_t^*(\theta, \beta) = 0, \\ = 0 & \text{if} \quad \alpha_t^*(\theta, \beta) \in \ ]0, z_t/\eta[, \\ \leq 0 & \text{if} \quad \alpha_t^*(\theta, \beta) = z_t/\eta. \end{cases} \tag{9}$$

I consider the following guess solution:

$$V^*(z) = h^* \frac{z^{1-\gamma}}{1-\gamma} \tag{10}$$

Proposition 1 describes the solution of this problem, which is based on this guess solution. It is easy to check that this solution satisfies the equilibrium conditions (7)-(9).

Proposition 1. *If it exists, the rational equilibrium investment strategy $\alpha_t^*(\theta, \beta) = \alpha^*(\theta, \beta) z_t/\eta$ is such that*

$$\alpha^*(\theta, \beta)\begin{cases} = 0 & \text{if} \quad \theta \leq R^* + \beta\pi^*, \\ \in \ [0, 1] & \text{if} \quad \theta = R^* + \beta\pi^*, \\ = 1 & \text{if} \quad \theta \geq R^* + \beta\pi^*, \end{cases} \tag{11}$$

The risk-free rate R^* and the aggregate risk premium π^* are defined respectively as

$$R^* = \frac{\eta^{1-\gamma} - \delta E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{1-\gamma}}{\delta(\eta - \overline{\alpha}^*) E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{-\gamma}}, \tag{12}$$

and

$$\pi^* = -\frac{Ey\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{-\gamma}}{E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{-\gamma}}, \tag{13}$$

where the triplet $(\overline{\alpha}^*, \overline{\theta}^*, \overline{\beta}^*) \in \mathbb{R}^3$ is such that $\overline{\alpha}_t^* = \overline{\alpha}^* z_t/\eta$, $\overline{\theta}_t^* = \overline{\theta}^* z_t/\eta$ and $\overline{\beta}_t^* = \overline{\beta}^* z_t/\eta$. The welfare measure h^* at equilibrium equals

$$h^* = \frac{(\eta - \overline{\alpha}^*)^{1-\gamma}}{\eta^{1-\gamma} - \delta E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{1-\gamma}}. \tag{14}$$

This rational equilibrium exists if and only if $\eta^{1-\gamma}$ is larger than $\delta E\left(\overline{\theta}^* + \overline{\beta}^* y\right)^{1-\gamma}$.

Function $\alpha^*(\theta, \beta)$ describes the optimal investment strategy. Projects (θ, β) are implemented at full capacity if and only if their expected rate of return θ is larger than the project-specific discount rate $R^* + \beta\pi^*$. Variable $\overline{\alpha}^*$ can thus be interpreted as the proportion of projects in the investment opportunity set that are implemented. It implies a constant consumption/wealth ratio equaling $1 - \overline{\alpha}^*/\eta$.

Along this optimal stationary investment strategy, the growth process entails serially independent shocks:

$$\frac{z_t}{z_{t-1}} = \frac{c_t}{c_{t-1}} = \frac{\overline{\theta}^*}{\eta} + \frac{\overline{\beta}^*}{\eta} y_t. \tag{15}$$

In equation (1), we expressed the return of any project (θ, β) as a linear function of the artificial common factor y. Using the above equation, we can rewrite this return as a linear function of the growth rate of aggregate consumption:

$$x_t(\theta, \beta) = \left(\theta - \beta\frac{\overline{\theta}^*}{\overline{\beta}^*}\right) + \beta\frac{\eta}{\overline{\beta}^*}\frac{c_t}{c_{t-1}} + \tilde{\varepsilon}_t(\theta, \beta). \tag{16}$$

Notice that this equation is the classical CCAPM regression in which the return of a project (θ, β) is regressed on the growth rate of consumption. It implies that the CCAPM beta of this project is defined as follows:

$$\beta^{CCAPM}(\theta, \beta) = \beta\frac{\eta}{\overline{\beta}^*}. \tag{17}$$

The optimal intertemporal welfare is measured by $V^*(z_0)$. Normalizing z_0 to unity, it can be more intuitively measured by the permanent equivalent consumption level c^{pe} that generates the same intertemporal utility, yielding

$$c^{pe*} = ((1-\delta)h^*)^{\frac{1}{1-\gamma}}. \tag{18}$$

This variable is a convenient measure of optimal intertemporal welfare. A similar policy evaluation instrument has been used by Epstein et al. (2014) in another context.

4.2 Calibration

The parameters of the benchmark calibration are summarized in Table 1. We assume a constant relative risk aversion of $\gamma = 3$ and a utility discount factor of $\delta = 0.99$. We also assume that $\eta = 0.5$, which means that the entire wealth in the economy would be able to finance 50% of all possible investment projects.

We assume that the mean payoff θ and the sensitivity β to the common factor are independently distributed in the investment opportunity set.[7] We also assume that they are normally distributed, with $\theta \sim N(\mu_\theta, \sigma_\theta^2)$ and $\beta \sim N(\mu_\beta, \sigma_\beta^2)$. The expected return of feasible projects in the investment opportunity set has a mean of 3%, and a standard deviation of 2%. Parameter σ_β measures the heterogeneity of the investment risk profiles in the economy. If all projects would have the same beta, they should all be optimally evaluated with a single discount rate. There would be no inefficiency associated with the WACC and Arrow-Lind

[7] Because of the optimal selection process, they will be positively correlated within the family of implemented projects.

parameter	value	description
γ	3	relative risk aversion
δ	0.99	utility discount factor
$1/\eta$	2	investment capacity per project
μ_θ	1.03	mean expected payoff per unit of capital
σ_θ	0.02	standard deviation of expected payoff per unit of capital
μ_β	1	mean payoff sensitivity to the common factor
σ_β	0.5	standard deviation of payoff sensitivity to the common factor
p	1.7%	annual probability of a macroeconomic catastrophe
μ_{bau}	0	technical parameter of the common factor
σ_{bau}	0.04	technical parameter of the common factor
μ_{cat}	-0.40	technical parameter of the common factor
σ_{cat}	0.40	technical parameter of the common factor

Table 1: Benchmark calibration of the model.

fallacies in that case. We conjecture that the welfare loss of using a single discount rate is increasing in the standard deviation of β. We take $\sigma_\beta = 0.5$, and we will check ex post that this degree of heterogeneity is aligned with the distribution of sectoral CCAPM betas in the U.S. economy.

In order to solve the classical asset pricing puzzles that prevail in the standard CCAPM, we use the Barro's approach based on the possibility of macroeconomic catastrophes (Barro (2006), Martin (2013)). We assume that the common factor y is distributed as random variable $\exp(Y) - E\exp(Y)$ with

$$Y \sim \Big(N(\mu_{bau}, \sigma_{bau}^2), 1-p; N(\mu_{cat}, \sigma_{cat}^2), p\Big). \tag{19}$$

With probability $1-p$, the state is business-as-usual (bau) and the distribution of Y conditional to that state is normal with mean μ_{bau} and volatility σ_{bau}. But with a small probability p, the catastrophic state occurs, and the distribution of Y conditional to that state is normal with mean $\mu_{cat} << 0$ and volatility σ_{cat}.

I estimate the rational equilibrium described in Proposition 1 numerically. Because R^* and π^* depend upon the triplet $(\overline{\alpha}^*, \overline{\theta}^*, \overline{\beta}^*)$ that is determined by the optimal investment strategy $\alpha^*(.,.)$, this proposition describes the optimal solution only implicitly. I solve this problem by observing that this optimal strategy is a function of pair (R^*, π^*), so are $\overline{\theta}^*(R^*, \pi^*)$, $\overline{\beta}^*(R^*, \pi^*)$ and $\overline{\alpha}^*(R^*, \pi^*)$, using respectively equations (4), (5) and (6). Thus, equations (12) and (13) can be interpreted as a system of two equations with two unknowns, R^* and π^* that I solve numerically.

The rational investment strategy and its implication in terms of risk, return and intertemporal welfare are described in Table 2. The risk-free discount rate and the aggregate risk premium are respectively equal to 0.86% and 2.22%, which are close to their historical averages over the last century in the United States.[8] Wealth and consumption grow at a trend of 1.51%, with a volatility of 2.76%, in line with the observation. In case of a macroeconomic

[8] The aggregate risk premium, which is defined as the risk premium of a claim on aggregate consumption, differs from the equity premium because equity has a consumption beta which typically assumed between 2

variable	value	description
$r^* = R^* - 1$	0.86%	risk-free discount rate
π^*	2.22%	risk premium
$\overline{\alpha}^*$	48.60%	proportion of projects implemented
$1 - (\overline{\alpha}^*/\eta)$	2.80%	consumption/wealth ratio
$(\overline{\theta}^*/\overline{\alpha}^*) - 1$	4.43%	average expected return of implemented projects
$\overline{\beta}^*/\overline{\alpha}^*$	0.80	average sensitivity to the common factor of the implemented projects
$(\overline{\theta}^*/\eta) - 1$	1.51%	expected growth of consumption
$\overline{\beta}^*\sigma_y/\eta$	2.76%	volatility of consumption growth
c^{pe*}	0.0464	permanent equivalent consumption

Table 2: Description of the rational equilibrium under the benchmark calibration.

catastrophe, consumption drops in expectation by almost 20%, which is representative of the macro catastrophes documented in Barro (2006). Given the optimal discounting system, 48.60% of the investment projects pass the test of a positive NPV. Because each implemented project requires two units of wealth ($\eta = 1/2$), 97.20% of total wealth is reinvested every period, yielding a consumption-wealth ratio of 2.80%.

The rational selection of projects allows for both an increase in the mean expected return and a reduction in the mean sensitivity of the selected projects compared to their distribution in the opportunity set. The mean sensitivity is 1 in the opportunity set, and is only 0.80 among implemented projects. The mean expected return is 3% in the opportunity set, and it increases to 4.43% among implemented projects. It yields a price-earning ratio of 22.57. The intertemporal welfare obtained from following this optimal investment strategy is equivalent to consuming a constant flow of 4.64% of initial wealth z_0.

I now come back to the degree of heterogeneity in the risk profile of implemented projects in the economy. The key parameter is σ_β which is equal to 0.5 in this calibration. It yields a distribution of sensitivity β to the common factor y of the implemented projects that is determined by the optimality condition $\theta \geq R^* + \beta\pi^*$. One can translate this into a distribution for the CCAPM betas of these projects by using equation (17). This distribution is described by the smooth curve in Figure 1. These CCAPM betas have a mean of 1.03 and and a standard deviation of 0.59. We compare this distribution to the empirical distribution of CCAPM betas in the US stock market. To do this, we use the Fama-French dataset of annual value-weighted returns of 49 industries over the period 1930-2018. The list of estimated CCAPM betas for these 49 industries is given in Table 6. The distribution of these betas is summarized in Figure 1. Its support goes from -0.25 (precious metals) to 2.35 (printing and publishing), with a mean of 1.02 and a standard deviation of 0.63. The relative concordance of the distribution of the CCAPM betas predicted by the model with this empirical distribution provides an additional support to this calibration exercise.

and 3 (see for example Bansal and Yaron (2004)). This suggests an equity premium around 4-6%, which is compatible with the recent estimation by Jorda et al. (2019).

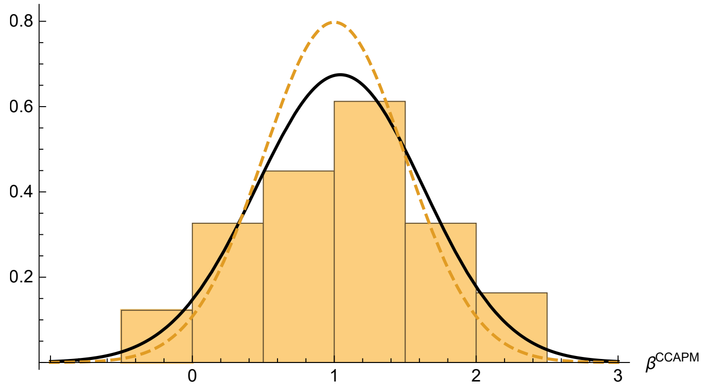

Figure 1: Histogram of the OLS estimators of the CCAPM betas of the 49 Fama-French industries of the US economy, based on industry-specific value-weighted equity returns (Table 6). Its standard deviation equals 0.63. The plain curve describes the density function of the distribution of the CCAPM betas of the implemented project predicted by the model. The standard deviation of these CCAPM betas predicted by the model is 0.59. The dashed curve is the density function $N(\mu_\beta, \sigma_\beta^2)$ of the betas of the projects in the opportunity set.

5 The individual welfare cost of the WACC and Arrow-Lind fallacies in the rational equilibrium

In this section, I consider the case of an irrational agent who uses a single discount rate to determine his dynamic investment strategy. All other agents behave optimally as described in the previous section. Therefore, the existence of this marginal agent has no effect on the equilibrium. The dynamics of the economy, and therefore on equilibrium asset prices, are the ones that have been examined in the previous section. The irrational agent uses the following decision rule based on his single discount rate ρ:

$$\alpha^*(\theta,\beta) \begin{cases} = 0 & \text{if} \quad \theta \leq \rho, \\ \in \ [0,1] & \text{if} \quad \theta = \rho, \\ = 1 & \text{if} \quad \theta \geq \rho. \end{cases} \tag{20}$$

This rule has the advantage of not requiring the irrational agent to estimate the beta of the projects under scrutiny, but it implies an inefficient portfolio allocation. Given the ρ selected by the agent, one can characterize his investment portfolio and his wealth and consumption dynamics. It yields a triplet $(\overline{\alpha}_\rho, \overline{\theta}_\rho, \overline{\beta}_\rho)$ similar to what has been described earlier, expect that this triplet is now a function of ρ. Since it is assumed that θ and β are independently distributed, the investment decision rule (20) implies that the mean beta of the implemented projects will be equal to unity, so that $\overline{\beta}_\rho$ equals $\overline{\alpha}_\rho$ in this model. The intertemporal welfare of this agent with initial wealth z_0 and using the single discount rate ρ is denoted $V_\rho(z_0)$. It is defined recursively as follows:

$$V_\rho(z) = u\left(z\left(1 - \frac{\overline{\alpha}_\rho}{\eta}\right)\right) + \delta E V_\rho\left(\frac{z}{\eta}(\overline{\theta}_\rho + \overline{\beta}_\rho y)\right). \tag{21}$$

	Optimal strategy	WACC strategy	Arrow-Lind strategy
discount rate	$0.86\% + \beta \times 2.22\%$	$0.86\%+ 2.22\% = 3.08\%$	0.86%
risk-free rate	0.86%	0.86%	0.86%
risk premium	2.22%	2.22%	2.22%
% projects implemented	48.60%	48.40%	85.76%
consumption/wealth ratio	2.80%	3.20%	
E[return]	4.43%	4.65%	
E[sensitivity]	0.80	1.00	1.00
E[growth]	1.51%	1.30%	
growth volatility	2.76%	3.44%	
c^{pe}	0.0464	0.0339	

Table 3: Comparisons of outcomes in an economy in which all agents use the optimal investment strategy, except one isolated agent who uses a single discount rate. In the "WACC strategy" column, this discount rate ρ is selected to be the WACC $R^* + \pi^*$ of the portfolio of investments undertaken by this agent. In the "Arrow-Lind strategy" column, the discount rate is R^*, yielding an infeasible solution. The "Optimal strategy" column is copy-pasted from Table 2.

Using this decision rule implies that the intertemporal welfare of the isolated agent is equal to $V_\rho(z_0) = h_\rho z_0^{1-\gamma}/(1-\gamma)$ where h_ρ satisfies the following condition:

$$h_\rho = \frac{(\eta - \overline{\alpha}_\rho)^{1-\gamma}}{\eta_\rho^{1-\gamma} - \delta E\left(\overline{\theta}_\rho + \overline{\beta}_\rho y\right)^{1-\gamma}}. \tag{22}$$

This solution is a function of the single discount rate ρ that is used by the irrational agent As a benchmark, let us first examine the "WACC strategy" which consists in using the average cost of capital in the economy as the all-purpose discount rate used by the irrational agent. The agent knows that the average beta of the projects that he will implement is equal to 1. Because all other agents behave rationally, the equilibrium asset returns are as described in the previous section, with $r^* = 0.86\%$ and $\pi^* = 2.22\%$. The WACC of the irrational agent will thus be equal to $r^*+\pi^* = 3.08\%$. He selects this rate as the single discount rate for investment evaluation. In Table 3, I describe the outcome of this investment strategy and I compare it to the optimal strategy already described in the previous section. The two investment strategies are described in Figure 2. The irrational agent invests in approximately the same number of projects (48.4%) than the rational agents (48.6%). However, the compositions of the portfolio are quite different. The irrational agent undertakes too many risky projects (those in the north-east red quadrant in Figure 2 should not be implemented), and too few safe projects (those in the south-west red quadrant should be implemented). This yields more uncertainty about future consumption, with a volatility of wealth and consumption growth going up from 2.76% to 3.44% for rational investors. This is only partially compensated by a larger expected portfolio return (4.65% up from 4.43%). The bottom line is a massive

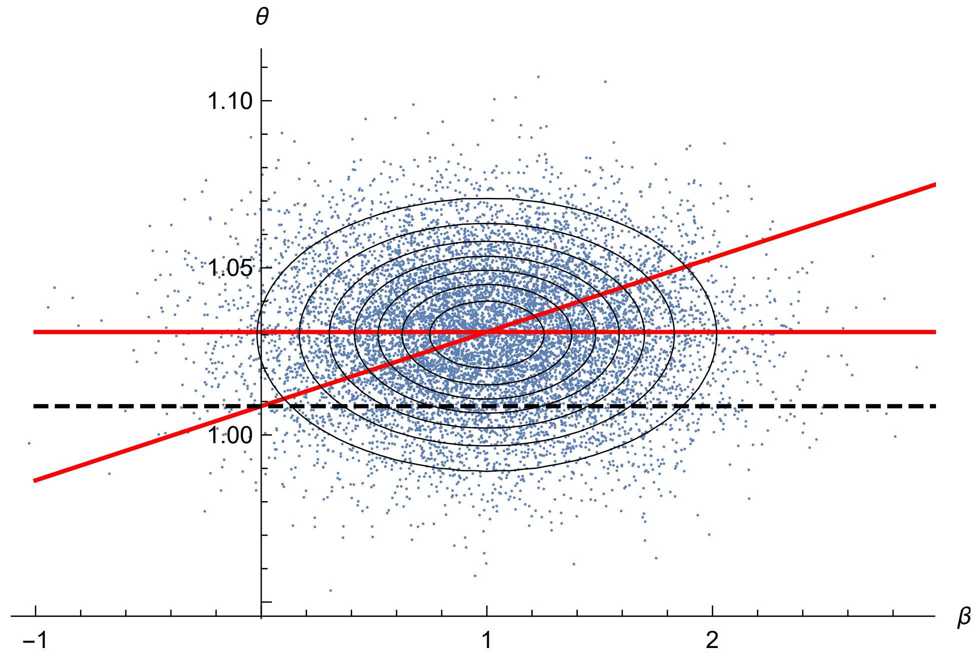

Figure 2: Comparison of the "optimal", the "WACC" and the "Arrow-Lind" investment strategies. We draw a sample of 10.000 projects from the joint normal distribution of (β, θ), using the benchmark calibration described in Table 1. The ellipses are iso-density curves of this joint distribution. The oblique and horizontal plain lines describe respectively the optimal and WACC frontiers, with the set of implemented projects above these lines. The dashed line corresponds to the Arrow-Lind strategy.

27% reduction in the measure of intertemporal welfare. Indeed, the permanent equivalent consumption level c^{pe} goes down from 0.0464 to 0.0339.

One could alternatively examine the "Arrow-Lind strategy" which would consist in using the risk-free interest rate $r^* = 0.86\%$ as the single discount rate. However, this discount rate is too small to yield a feasible solution. Indeed, implementing this investment evaluation procedure would imply that 86% of the investment projects would yield a positive NPV, implying that the irrational agent should spend every period 172% of his wealth to invest.

In fact, the irrational agent has a very narrow interval of possible single discount rates to choose from in order to generate a positive intertemporal welfare. More specifically, if he chooses a single discount rate smaller than 3%, consumption would be negative, as in the Arrow-Lind strategy. If he chooses a discount rate larger than 3.09%, early consumption is too large and capital accumulation too small to support a positive permanent consumption equivalent. In Figure 3, I show how the intertemporal welfare of the irrational agent is related to the choice of the single discount rate.

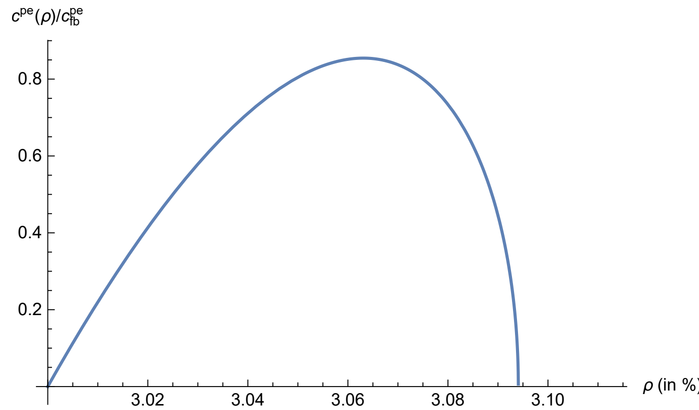

Figure 3: Ratio of the permanent equivalent consumption under a single discount rate ρ to the first-best permanent equivalent consumption c_{fb}^{pe}. Among these second-best strategies, the optimal single discount rate is $\rho_1 = 3.06\%$.

6 The WACC equilibrium

In this section, I assume that *all* agents in the economy use the same single-DR strategy (20). Contrary to the previous section, the fact that all agents follow the same inefficient investment strategy means that the dynamics of growth and thus the equilibrium asset prices are affected by the irrationality of the agents. The WACC equilibrium is defined as a dynamic allocation of capital is which all individuals select their portfolio based on decision rule (20), where $\rho = \rho_1$ is the average cost of capital in the economy, i.e., ρ_1 equals $R_1 + \pi_1$. In that economy, the risk-free interest rate and the risk premium must satisfy the following equilibrium conditions:

$$R_1 = \frac{(\eta - \overline{\alpha}_{\rho_1})^{-\gamma}}{\delta h_{\rho_1} E\left(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y\right)^{-\gamma}}, \tag{23}$$

and

$$\pi_1 = \frac{-Ey\left(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y\right)^{-\gamma}}{E\left(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y\right)^{-\gamma}}. \tag{24}$$

By replacing h_{ρ_1} by its expression in (22), we can rewrite the WACC condition $\rho_1 = R_1 + \pi_1$ as follows:

$$\rho_1 = \frac{\eta^{1-\gamma} - \delta E(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{1-\gamma}}{\delta(\eta - \overline{\alpha}_{\rho_1}) E(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{-\gamma}} + \frac{-Ey(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{-\gamma}}{E(\overline{\theta}_{\rho_1} + \overline{\beta}_{\rho_1} y)^{-\gamma}}. \tag{25}$$

This equation, which is solved numerically, characterizes the WACC equilibrium. It is described in Table 4. The global WACC is equal to $\rho_1 = 3.06\%$. Notice that this single discount rate combines a risk-free interest rate and a risk premium that are very different from the individual WACC strategy described in the previous section in the isolated case. Indeed, under this equilibrium in which all agents behave irrationally, the equilibrium interest rate goes down to -1.48% because of precautionary savings, and the equilibrium risk premium goes up to 4.54% because of the larger macroeconomic uncertainty. Again, under this single discount

	Rational equilibrium	WACC equilibrium
discount rate	$0.86\% + \beta \times 2.22\%$	-1.48%+ 4.54% = 3.06%
risk-free rate	0.86%	-1.48%
risk premium	2.22%	4.54%
% projects implemented	48.60%	48.70%
consumption/wealth ratio	2.80%	2.52%
E[return]	4.43%	4.64%
E[sensitivity]	0.80	1.00
E[growth]	1.51%	2.00%
growth volatility	2.76%	3.46%
c^{pe}	0.0464	0.0396

Table 4: Comparisons of outcomes of the rational equilibrium and the WACC equilibrium.

rate rule, the decision-maker overinvests in risky projects and underinvests in relatively safer ones. This absence of selectivity on the risk dimension implies that the average beta is equal to 1.00, to be compared to only 0.80 under the equilibrium with rational agents. The good news is that the average expected return equals 4.64% under the second-best strategy, to be compared to only 4.43% under the first best. Also, people invest a larger fraction of their wealth in projects, so that the consumption-wealth ratio is reduced from 2.80% to 2.52%. This is due to a precautionary effect, since the volatility of consumption growth is markedly increased from 2.76% to 3.46%. The bottom line is again an important deterioration of intertemporal welfare. The permanent equivalent consumption drops from 0.0464 to 0.0396, a permanent 15% reduction in consumption. Notice that the small difference between the single discount rate used in the individual WACC solution and in the WACC equilibrium implies a sizeable effect on welfare. This is because the marginally smaller discount rate in the isolated case marginally increases the saving rate. But because the consumption-wealth ratio is small, this has a sizeable effect to reduce the consumption rate, yielding an important impact on intertemporal welfare.

It is useful to search for the single discount rate that maximizes the intertemporal welfare of irrational agents that use a single-DR strategy. In other words, what is the ρ that maximizes $V_\rho(z_0) = h_\rho u(z_0)$? The answer to this question is obtained by using equation (22). It is easy to check that the first-order condition to this problem is given by equation (25). In short, the equilibrium WACC $\rho_1 = 3.06\%$ is the single discount rate that corresponds to the maximum in Figure 3. This result is summarized in the following proposition.

Proposition 2. *Suppose that all agents in the economy use the same single-discount-rate rule to determine their investment strategy. The single discount rate that minimizes the welfare cost of this irrational behavior is the equilibrium WACC $\rho_1 = R_1 + \pi_1$, in which R_1 and π_1 are respectively the equilibrium interest rate and the equilibrium risk premium in this economy.*

Of course, the WACC equilibrium is dominated by the rational equilibrium, but if all agents in the economy apply the same single discount rate rule, using the equilibrium WACC as the all-purpose discount rate is the rule that maximizes intertemporal welfare in the set of single-discount-rate allocations.

7 The rationed Arrow-Lind equilibrium

In this section, I examine an economy in which all agents believe in the fallacious interpretation of the Arrow-Lind theorem consisting in using the equilibrium interest rate as the all-purpose discount rate to evaluate investment projects. As we know from the previous two sections, the equilibrium interest rate is typically too small to be use as a single discount rate, so that the capital necessary to finance all investment projects that pass the test of a positive NPV is larger than aggregate wealth. No equilibrium exists under this approach. In practice, experts who have been using the Ramsey rule to estimate the public discount rate often addressed the excess demand for public funds that this solution generated by proposing a capital rationing scheme.[9] In practice, the inability to fund all positive-NPV projects under a too low public discount rate has offered discretion to politicians and high-ranked public servants to prioritize public investments.

I hereafter characterize a family of rationed Arrow-Lind equilibria. Such equilibria are parametrized by a scalar q which denotes the probability for a project with a positive NPV to be implemented. The fact that q is less than 1 means that capital is rationed in the economy. So, an Arrow-Lind equilibrium with rationing q is defined by the fact that all agents use the equilibrium interest rate in the economy as a all-purpose discount rate, but only a proportion q of non-negative-NPV projects are actually implemented. The equilibrium Arrow-Lind discount rate $\rho_{AL}(q)$ must thus satisfy the following equilibrium condition:

$$\rho_{AL}(q) = \frac{\left(\frac{\eta}{q}\right)^{1-\gamma} - \delta E(\overline{\theta} + \overline{\beta} y)^{1-\gamma}}{\delta(\frac{\eta}{q} - \overline{\alpha}) E(\overline{\theta} + \overline{\beta} y)^{-\gamma}} = R. \tag{26}$$

I describe in Table 5 two rationed AL equilibria, respectively with rationing ratio $q = 0.6$ and $q = 0.8$. Of course, it is inefficient to randomize the access to capital for good projects to compensate for a single discount rate that is too small. This implies for example that when we allow only $q = 60\%$ of the non-negative-NPV projects to be implemented, the permanent equivalent consumption level is limited to 0.0253, a catastrophic 45% permanent reduction in consumption compared to the rational strategy. The risk-free interest rate in this economy (and thus the single discount rate) is equal to 1.18%. The demand for capital is 63.6% larger than total wealth in the economy, but only 60% of the demand is satisfied, which leaves 1.84% of wealth for consumption. Financial risk and economic growth are highly volatile in this economy.

[9] This is illustrated for example by the last report in France that recommended a single discount rate, where the Ramsey rule was used, combined with a public capital rationing scheme (Lebègue (2005), pp. 72-76).

	Rational equilibrium	Rationed Arrow-Lind equilibrium	
		$q = 0.8$	$q = 0.6$
discount rate	$0.86\% + \beta \times 2.22\%$	2.44%	1.18%
risk-free rate	0.86%	2.44%	1.18%
risk premium	2.22%	4.61%	4.73%
% projects implemented	48.60%	48.88%	49.08%
consumption/wealth ratio	2.80%	2.24%	1.84%
E[return]	4.43%	4.25%	3.65%
E[sensitivity]	0.80	1.00	1.00
E[growth]	1.51%	2.74%	6.96%
growth volatility	2.76%	4.34%	5.81%
c^{pe}	0.0464	0.0339	0.0253

Table 5: Description of two rationed Arrow-Lind equilibria. Parameter q is the proportion of non-negative-NPV projects that are implemented.

$$\begin{aligned}
\text{equity premium} &: \quad \mu - r = 0.06; \\
\text{standard deviation of stock returns} &: \quad \sigma = 0.165; \\
\text{concavity parameter} &: \quad \gamma = 2.2; \\
\text{rate of impatience} &: \quad \rho = 0.037; \\
\text{risk-free rate} &: \quad r = 0.01.
\end{aligned}$$

8 Concluding remarks

One of the most puzzling feature of the experts' debate on the public discount rate is its reliance on its misleading cornerstone, the Ramsey rule. This rule, adjusted for the uncertainty affecting economic growth, provides the right basis to estimate the rate at which risk-free benefits and costs should be discounted. Using that rule to recommend an all-purpose discount rate in the economy does not only represent a very dangerous interpretation of the theory, as explained in this paper. It also makes it impossible to initiate a constructive debate about how to value the future. As long as one ignores the necessity to adjust discount rates to risk characteristics, all sorts of difficulties materialize, from the WACC fallacy to the rationing of public investments with a positive NPV. Over the last ten years, the remarkable stalemate prevailing in the Stern/Nordhaus debate on the social cost of carbon is another vivid illustration of our collective inability to transform our consensual asset pricing theory into practical evaluation rules. The impossibility for the U.S. administration to revise its deeply flawed discounting system is a puzzle, in particular given the effort of some prominent experts to change that system (Arrow et al. (2013), Lucas (2014), Sunstein (2014)). The social cost of this failure is huge, and the credibility of our profession is at stake given the ability of lobbies and politicians to play with the current inefficient rules. In the U.S., this means updating Circular A-4.

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Table 6 of the consumption betas using sectoral equity returns

CCAPM beta	Fama-French Industry
0.36	Agriculture
0.44	Food Products
0.43	Candy & Soda
0.74	Beer & Liquor
-0.09	Tobacco Products
1.27	Recreation
1.91	Entertainment
2.35	Printing and Publishing
0.97	Consumer Goods
0.68	Apparel
0.02	Healthcare
1.15	Medical Equipment
0.26	Pharmaceutical Products
0.63	Chemicals
1.19	Rubber and Plastic Products
1.15	Textiles
0.65	Construction Materials
1.70	Construction
1.41	Steel Works Etc
0.42	Fabricated Products
1.61	Machinery
1.68	Electrical Equipment
1.22	Automobiles and Trucks
1.01	Aircraft
1.10	Shipbuilding, Railroad Equipment
0.21	Defense
-0.25	Precious Metals
1.01	Non-Metallic and Industrial Metal Mining
1.04	Coal
1.07	Petroleum and Natural Gas
0.64	Utilities
0.84	Communication
2.23	Personal Services
1.03	Business Services
1.00	Computers
-0.10	Software

2.20	Electronic Equipment
0.97	Measuring and Control Equipment
1.55	Business Supplies
0.36	Shipping Containers
1.44	Transportation
1.74	Wholesale
0.72	Retail
1.57	Restaurants, Hotels, Motels
0.90	Banking
1.21	Insurance
2.15	Real Estate
1.63	Trading
0.58	Almost Nothing

Table 6: Estimation of the CCAPM betas of the 49 Fama-French industries of the US economy. The CCAPM beta of an industry is the OLS estimator of the regression of the industry-specific value-weighted return on the growth rate of real GDP/cap, using annual data from 1930 to 2018. Source: Own computations using the Fama-French database.