H-DERIVATIVES OF SOME FUNCTIONALS FROM THE FRACTIONAL BROWNIAN MOTION

TOMASZ ROLSKI

We consider a family of sup-functionals of (drifted) fractional Brownian motion with Hurst parameter $H \in (0,1)$. This family includes, but is not limited to: expected value of the supremum

$$\mathcal{M}_H(T,a) := \mathbf{E}\Big(\sup_{t\in[0,T]} B_H(t) - at\Big),$$

the Wills functional, and the Piterbarg-Pickands constant. Explicit formulas for the derivatives of these functionals as functions of Hurst parameter evaluated at $H = \frac{1}{2}$ are established. For functional $\mathcal{M}_H(T,a)$ the H-derivative is also calculated for $H = 1$ (from the left side).

In order to derive these formulas, it was developed the concept of derivatives of fractional α-stable fields introduced by Stoev & Taqqu (2004) and propose a Paley-Wiener-Zygmund representation of the fractional Brownian motion.

The research was prompted by a result of Delorme *et al* (2017) wherein, rather informally, it was shown that the derivative of the Pickands constant at $1/2$ equals to $-2\gamma_E$, where γ_{E} is the Euler-Mascheroni constant.

The presented results are the outcome of a joint project with Krzysztof Bisewski and Krzysztof Dębicki.

References

[1] Derivative of the expected supremum of fractional Brownian motion at H=1. *Queueing Syst.* **102** (2022), no. 1-2, pp. 53–68. DOI = 10.1007/s11134-022-09859-3, with Bisewski, Krzysztof and Debicki, Krzysztof.

[2] Derivatives of sup-functionals of fractional Brownian motion evaluated at $H = 1/2$. *Electron. J. Probab.* **27** (2022), Paper No. 129, 35 pp. DOI = 10.1214/22-ejp848 with Bisewski, Krzysztof and Debicki, Krzysztof.