Parental educational styles with externalities*

Antonio Cabrales[†] Esther Hauk[‡]

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Abstract

In this paper we develop a model that allows to understand the circumstances under which a society, or groups within a society, may decide to pursue a collaborative education model or an individualist one. One important aspect of our model is that there are externalities and it can feature multiple equilibria. We can thus explain why one observes different local educational cultures even within relatively homogeneous countries. In addition, both features generate important and subtle insights for public policies. Depending on the parameters, the policymakers may need to operate "only" on beliefs, or they may need to change parental and teachers' abilities. We study the incentives and possible policy responses to a desire for segregation.

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[†]Department of Economics, Universidad Carlos III de Madrid; e-mail: antonio.cabrales@uc3m.es

[‡]Instituto de Anàliisis Econòmico (IAE-CSIC), Move and Barcelona School of Economics, Campus UAB, 08193 Bellaterra (Barcelona), email: esther.hauk@iae.csic.es

1 Introduction

The technology of education combines elements that require no interaction (doing homework alone) with elements where collaboration between pupils is important (exchange of ideas about problem solving). There are different cultures about which elements of the technology to emphasize. Some pedagogies (Montessori, Waldorf, Reggio Emilia, see e.g. Lopata, Wallace, and Finn 2005, Edwards 2002, or Frierson 2021) stress the importance of students working in groups.[1] More traditional pedagogies are skeptical about collaboration, and prioritize work by the student on her own. The skepticism can be explained partly because group work can easily turn into simple recreation, with doubtful pedagogical value, so some parents and educators tend to eschew it.

The research (Bietenbeck 2014, Foldnes 2016, Pagcaliwagan 2016, Cecchini et al. 2021) shows that both approaches can be fruitful, but there is a lot of individual variation. One difficulty for the interpretation of results is that collaborative work is more effective if others take a more conscientious role in the collective effort. Thus, it may not be effective in a very individualistic group. Also educators' guidance and parental collaboration are important for the collaborative model to provide good results. While the return of individual work is well understood, parents may perceive more uncertainty about the return of the collective type of work.

The aim of this paper is to develop a model that allows to understand the circumstances under which a society, or groups within a society, may decide to pursue one model or the other. This will allow us to understand some phenomena that are of empirical and policy interest. For example, the externalities that some parents impose on others when different educational cultures coexist within a country, or a smaller local area. These externalities, in turn, can lead to self-segregation efforts, to avoid the spillover from local interaction with other educational cultures.

We assume that for learning to sink in, children have to make a minimal individual effort. After that happens, they can choose to split their remaining unit of

[1] "The third feature of Montessori's moral theory shatters this individualism by highlighting deep forms of shared agency beyond mutual respect or even mere cooperation. This 'third thing' is 'harmony between people who work together' and 'work in a group" Frierson (2021).

time between individual and collective effort. The return of this extra individual effort is a fixed rate per unit of time. The return to collective effort, on the other hand, also depends on the average level of time other members of the students' peer group devote to it. The child's preferences can be influenced by parental education effort, which consists in influencing the relative rates of return of collective versus individualistic effort, within some limits. Parents can lower the innate relative rates of return to some extent, thereby inducing either a collaborative or individualistic effort choice by their children. In the former case, we will call their education style collaborative. In the latter, we call their education style individualistic. While parents and children agree on the return from individual effort, parents may have a different judgement on the child's true utility from collective effort. This happens due to their parental skepticism as well as their perceived uncertainty about this return, but also because they abstract from the pure enjoyment value of collective activities. Therefore parental and child utility differ in the relative value they place on collective effort, which is smaller for parents than children. This leads to a potential tension. The parent can always induce collective effort in her child if this is her preferred alternative. However, she might not be able to induce her child to do individual effort even if the latter is the parental preferred alternative.

Our model features two types of peer effect. On the one hand, we model the standard peer effect among children. On the other hand, there is the peer effect derived from parental educational style. That is, a child's educational outcome can be affected because of the behavior of the parents of the child's peers. This peer effect is often overlooked in the literature. One paper that does look at this channel of influence is Chen, Chung and Wang (2023). They ask how the presence of children of parents with power (in this case high officials of the Chinese Communist Party) affects the school performance of these children's peers. They use between cohort variability (as in Hoxby 2000) to show that one more son of a senior official in the class increases the performance of his peers by 2%. The effect of these children is practically the same if controlled by their personal characteristics, in particular the official's child own school results. This suggests that the most likely reason for the effect is through the official.

In other words, in our paper parental educational styles have externalities. Therefore our model can feature multiple pure strategy symmetric equilibria even in a homogenous population depending on which education style parents coordinate. We can thus explain why one observes different local educational cultures even within relatively homogeneous countries. We start our equilibrium analysis precisely with homogenous parents where children are randomly matched to their peers. Then, we split the parents with homogenous preferences into two separated groups. The children - the students - mainly interact with members of their own group, but are also sometimes exposed to members of the other group. We then study a model where there is "geography" and students interact with others that are "locally close" to them.

All these variations of the model can feature equilibria where some parents are dissatisfied with the environment in which they live. Therefore, we study the possibility for parents to self-segregate into environments that are potentially better for them and allow for parental heterogeneity. We examine three different sources of heterogeneity. The first one is parent's perceived return from collaborative effort. Secondly, we explore what happens when different parents have different abilities to control the utility of their children. Finally, we assess what happens when parents can provide different returns from individualistic learning.

For all the versions of the model, we characterize the pure strategy symmetric equilibria of the games featured. Because of the externalities, there are often multiple equilibria. The elements that are crucial for the equilibria are the productivity of the individual effort, the productivity of the standalone and synergistic components of the collaborative effort, and the uncertainty parameter of the parents about the return of collaborative effort. In the case of separated societies and local interaction, the degree of separation, and the network structure are also crucial. For the segregation results, the parents that value collaboration the most are usually the ones that have more incentives to segregate and they do it in equilibrium more often.

Our model generates important and subtle insights for public policies to achieve coordination on the more efficient equilibrium in case of multiplicity. A key parameter for policy is the parameter that is inversely related to parental skepticism about collaboration. If that parameter is high (low skepticism), a welfare improving policy "only" needs to shift parental expectations about what other parents are going to do to coordinate their behaviors on collaboration. If, on the other hand, the parameter is low (high skepticism) then the problem goes beyond coordinating expectations. The issue now is about the ability of parents to control the activities of the children. Parents in that context cannot persuade the children to undertake the individual effort activities that they consider more productive. Achieving a higher persuasive power might require parental training and pedagogy. That could be more costly and difficult. Alternatively, the policymakers might need to increase the (perception of) returns on collaborative effort. This might entail (re)training the parents and teachers about the collaborative technology, which is intrinsically more complicated.

In this context, we study a situation where differences in socioeconomic status lead to some parents having a lower persuasive ability for their children to exert high individual effort. Would an intervention to improve parental persuasion ability be universally beneficial? The answer to this question is mostly positive, except for some range of parameters, and for particular beliefs about equilibria, something that could be addressed by additional policies.

Not all policies are public, of course. A "bad" equilibrium can entice individuals to search for better educational arrangements for their children. If there is heterogeneity in the parental styles, there can be incentives for some parents (e.g. those with less skepticism about collaboration) to create their own schools. That is, they would want to "secede" from the standard schools and create more collaborative schools. This interacts with public policy. If a social planner has optimistic expectations about equilibrium coordination, secession is always to be encouraged. The seceding parents believe their utility will improve under secession, whereas the ones that stay in the standard school will remain at the high individualistic effort equilibrium and their utility is unaltered.

We argue that optimistic expectations is the reasonable case in the game under consideration. Secession is likely to entail some costs, and those costs can focus expectations on the optimistic side, if players are rational enough to avoid dominated strategies. Formally we show that result holds if one uses forward induction as a solution concept in the game of secession. Note, in that context, that the cost of secession plays a role in the "signaling" value towards equilibrium selection. This implies that a high subsidy for secession is not always a good idea.

An alternative to building new schools is to move children between schools. This can be done by the parents themselves, but also with busing policies. Angrist and Lang (2004) show that the effects of these policies are small. Perhaps this is due to an expectations problem given the low cost for the "public" movers.

Continuing with the topic of heterogeneous groups, we also consider a society that is split into two groups where parents differ in the return they can provide to their children due to individual learning efforts. One of those groups, perhaps a more elite one, derives a higher return from individual effort. This also creates incentives for segregation for the parents with lower returns to individualistic education. They would rather have their children surrounded by only collaborative learners to benefit maximally from the externalities of collaborative learning. In this case, the policy could stimulate secession. It may also want to equalize the returns. However, equilibrium selection can be an issue sometimes.

All these possibilities for policy (private or public) to make a positive change in children's and parent's lives could be a reason why many leading educational organizations are increasingly interested in collaborative learning. For example, after reviewing the literature, the Education Endowment Foundation (2021) concludes that "The impact of collaborative approaches on learning is consistently positive, with pupils making an additional 5 months' progress, on average, over the course of an academic year". According to the Cornell's Center for Teaching innovation "Research shows that educational experiences that are active, social, contextual, engaging, and student-owned lead to deeper learning."

There are some connections between what we call "individualistic" and "collaborative" educational styles and the "authoritative" and "liberal" parenting styles in Doepke and Zilibotti (2017). Our parents mirror Doepke and Zilibotti's (2017) "authoritative" parents since our parents influence their children by modifying their children's utility function. In particular, parents can lower the benefits that children received from collaborative activities which they might want to do due to their skepticism towards collaborative education outcomes. Parents might refrain from doing so, either because they prefer the equilibrium outcome with the "collaborative" educational style or they cannot incentivize their children to the individualistic educational style which the parents, but not the children would prefer. In these cases the equilibrium parenting style might seem "liberal" since parents refrain from influencing their children. Doepke and Zilibotti's (2017) liberal style often involves the children doing activities together, although in that context these activities tend to be non-productive. In contrast, our collaborative style centers on productive activities, but it can also be combined with more social elements. We also consider an extension that mirrors Doepke and Zilibotti's (2017) authoritarian parents who force children to choose the learning effort the parents want. We show that the authoritarian educational style is more attractive to parents who are very skeptical about collective learning.

Since parental skepticism towards collaborative learning is a key parameter in our model, the question arises how it could be identified in the data. We include an empirical exercise where we argue that parents with scientific and technical professions are more likely to understand the returns from collaborative learning than those in other high-level occupations (e.g. managers) and show that in the region of Madrid private schools with collaborative pedagogies attract disproportionately the children of technical and scientific parents.

The paper is organized as follows. In section 2 we present the main model with homogeneous parents and children, characterize its equilibria and welfare. Continuing with homogeneous preferences Section 3 studies the possibility of coexistence of two different groups adopting different educational styles under global interaction while Section 4 considers different local interaction structures. Section 5 analyzes a society with parents that have heterogeneous preferences and who interact globally and studies secession. Section 6 analyzes some further extensions. Section 7 explains our empirical exercise. Section 8 concludes. Some extensions and proofs are relegated to the Appendix.

2 The main model

We study the decision problem of single-parent single-child families where children choose how to split one unit of time between collaborative and individual learning. Let x be the proportion of time dedicated to individual learning which yields a marginal return of R. The choice of x refers to additional units after the student has made minimal individual effort needed for knowledge to sink in. We do not model this minimal individual effort explicitly.

Children have a preference parameter a on collaboration that can be shaped by their parent. The marginal return per unit of time to collaborative learning depends on this parameter a, on a constant term K_1 independent of peer activity and a term which depends on the average commitment of peers to collaborative learning $K_2(1-\overline{x})$. Therefore the child's utility is given by

$$U^C(x) = a\left(K_1 + K_2\left(1-\overline{x}\right)\right)(1-x) + Rx \tag{1}$$

Parents can shape their child's preference parameter a on collaboration which can be chosen from an interval $a \in [\underline{a}, \overline{a}]$. We assume that $a = \overline{a}$ is the default value if parents do not intervene and that by intervening they can reduce a. While parents evaluate the return from individualistic learning in the same way as their child, their perception of the return from collaborative learning might differ. Parents assign a weight $\mu \in (0,1)$ to the return from the collaborative activities of their child which might reflect different considerations. On the one hand, there could be a doubt about the effectiveness of collaborative effort. For example, it may have a strong dependence on the peer or teacher quality, or their understanding of the technology (Qureshi et al. 2023). On the other hand, collaboration may have an effect on the enjoyment of the activity that is not reflected in an increase in human capital, and some parents may downweight momentary enjoyment with respect to future human capital.[2] The parental utility is therefore given by

$$U^P(a) = \mu a \left(K_1 + (1-\overline{x}) K_2\right)(1-x_1) + R x_1 \tag{2}$$

We assume that if parents cannot change the action of the children by changing their choice of a, they choose to leave $a = \overline{a}$. This can be motivated if changing a has a very small cost. The timing of the model is as follows: Parents first choose a to maximize (2). Once this choice has been realized, children choose x to maximize (1). The game is solved by backward induction.

2.1 The child's choice

The child's best reply is given by

$$x_1 = \begin{cases} 1 \text{ if } R > a\left(K_1 + K_2(1-\overline{x})\right) \\ 0 \text{ if } R \leq a\left(K_1 + K_2(1-\overline{x})\right) \end{cases} \tag{3}$$

resulting in the following symmetric pure strategy equilibria:

1. All children individually learning full-time ($x = \overline{x} = 1$) is the unique equilibrium if $R > a(K_1 + K_2)$.

2. All children collaboratively learning full-time ($x = \overline{x} = 0$) is the equilibrium if $R \leq aK_1$.

3. $x = 0$ and $x = 1$ are both equilibria if $aK_1 < R \leq a(K_1 + K_2)$.

In the last case multiple equilibria arise in a homogeneous population. This happens because the time spent on collaborative learning positively impacts other collaborative learnings, leading to strategic complementarities. The homogenous population is playing a coordination game.

[2]This creates a problem to evaluate social welfare, as there is not an obvious superior standard in the evaluation of that tradeoff. Policymakers with different convictions about the value of enjoyment versus human capital, or who are differently informed about the value of collaborative work might want to push different policies.

2.2 Parental choice of a

Using the child's best response (3) parental utility becomes

$$U^P(a) = \begin{cases} R \text{ if } \frac{R}{K_1+(1-\overline{x})K_2} \geq a \\ \mu a\left(K_1+(1-\overline{x})K_2\right) \text{ if } \frac{R}{K_1+(1-\overline{x})K_2} < a \end{cases} \tag{4}$$

If $\frac{R}{K_1+(1-\overline{x})K_2} > \overline{a}$ children will always choose not to collaborate for all possible values of a. If $\frac{R}{K_1+(1-\overline{x})K_2} < \underline{a}$ children will always want to collaborate. Hence, in both cases, the parent's best response is to choose $\overline{a}$. For $\underline{a} < \frac{R}{K_1+(1-\overline{x})K_2} < \overline{a}$ the choice of a determines whether or not children make individual effort. Parents want their child to make individual effort if $\mu\overline{a}\left(K_1+(1-\overline{x})K_2\right) \leq R$, or equivalently, when $\overline{a} \leq \frac{R}{\mu(K_1+(1-\overline{x})K_2)}$. Thus, the combined condition for making individual effort $x=1$ in this interval is $\frac{R}{K_1+(1-\overline{x})K_2} \leq \overline{a} \leq \frac{R}{\mu(K_1+(1-\overline{x})K_2)}$, which yields a utility of $U^P = U^C = R$. Otherwise, if in this interval $\overline{a} > \frac{R}{\mu(K_1+(1-\overline{x})K_2)}$,the parents want their children to collaborate.

We assume that parents choose the minimal reduction of a that allows them to induce their desired effort choice by the child. This could be justified by having a tiny cost $\varepsilon \to 0$ per unit of reduction in a. With this assumption the parental best replies taking into account the child's best reply can be summarized as follows:

$$\begin{aligned} a &= \min\left[\overline{a}, \frac{R}{K_1+(1-\overline{x})K_2}\right] \text{ for} & (5)\\ R &> \max\left\{\underline{a}\left(K_1+(1-\overline{x})K_2\right), \overline{a}\mu\left(K_1+(1-\overline{x})K_2\right)\right\} \\ a &= \overline{a} \text{ for } R < \max\left\{\underline{a}\left(K_1+(1-\overline{x})K_2\right), \overline{a}\mu\left(K_1+(1-\overline{x})K_2\right)\right\} & (6)\end{aligned}$$

Notice that (5) is an individualistic parenting style to which children best respond by choosing $x=1$ while (6) captures the collaborative parenting style that makes children choose $x=0$.

The max operator in (5) and (6) reflects the potential conflict between parental and child's incentives.

Let

$$\max\left\{\underline{a}\left(K_1+\left(1-\overline{x}\right)K_2\right),\overline{a}\mu\left(K_1+\left(1-\overline{x}\right)K_2\right)\right\}=\underline{a}\left(K_1+\left(1-\overline{x}\right)K_2\right)$$

which is the cutoff level for R for which parents can induce individualistic effort. Parents would like to induce individualistic effort whenever

$$\overline{a}\mu\left(K_1+\left(1-\overline{x}\right)K_2\right)<R$$

but for

$$\overline{a}\mu\left(K_1+\left(1-\overline{x}\right)K_2\right)<R<\underline{a}\left(K_1+\left(1-\overline{x}\right)K_2\right)$$

they cannot, since the child's best response to any level of a that can be implemented is collaborative effort. On the other hand, let

$$\max\left\{\underline{a}\left(K_1+\left(1-\overline{x}\right)K_2\right),\overline{a}\mu\left(K_1+\left(1-\overline{x}\right)K_2\right)\right\}=\overline{a}\mu\left(K_1+\left(1-\overline{x}\right)K_2\right)$$

which is the cutoff level for R for which parents want to induce individualistic effort in their child. In this case they can always implement their preferred option. In other words when $\underline{a}>\overline{a}\mu$ or equivalently $\mu\leq\frac{\underline{a}}{\overline{a}}$ parents might not be able to induce individualistic effort even if they wanted to.

Lemma 1

1. The symmetric pure strategy equilibria for $\mu\leq\frac{\underline{a}}{\overline{a}}$ are:

$$a=\overline{a}\text{ and }x=0\text{ for }R\leq K_1\underline{a}$$

$$a=\min\left[\overline{a},\frac{R}{K_1}\right]\text{ and }x=1\text{ for }R>\underline{a}\left(K_1+K_2\right)$$

2. The symmetric pure strategy equilibria for $\mu>\frac{\underline{a}}{\overline{a}}$ are:

$$a=\overline{a}\text{ and }x=0\text{ for }R<\overline{a}\mu K_1$$

$$\left.\begin{array}{c} a=\overline{a} \text{ and } x=0 \\ a=\min\left[\overline{a},\frac{R}{K_1}\right] \text{ and } x=1 \end{array}\right\} \text{ for } \overline{a}\mu K_1 \leq R \leq \overline{a}\mu\left(K_1+K_2\right)$$

$$a=\min\left[\overline{a},\frac{R}{K_1}\right] \text{ and } x=1 \text{ for } R>\overline{a}\mu\left(K_1+K_2\right)$$

Proof. See Appendix. ■

Notice that there is a unique equilibrium where a collaborative parenting style prevails as long as parents do not have an incentive to deviate to implementing high individual effort even if all other parents do. Similarly, there is a unique equilibrium with individualistic parenting style as long as parents do not have an incentive to deviate to implementing collective effort even if all other parents do. Multiplicity occurs in between these two bounds (the lower bound for the individualistic style equilibrium to exist and the upper bound for the collective style equilibrium to exist) and captures the parental peer effect where the choice of parental educational styles creates an externality.

2.3 Welfare and policy

Suppose a social planner took the point of view of maximizing parental utility. Then she would like all children to choose $x=1$ when $R>\overline{a}\mu\left(K_1+K_2\right)$ and all children to choose $x=0$ when $R<\overline{a}\mu\left(K_1+K_2\right)$.With this in mind, we can review the equilibrium structure of Lemma 1 in terms of efficiency.

1. For $\mu>\frac{\underline{a}}{\overline{a}}$ parents can always induce their child to choose their preferred options. In this case when the equilibrium is unique, it is always efficient. When multiple equilibria are possible, the high individual effort equilibrium is inefficient, since $R\leq\overline{a}\mu\left(K_1+K_2\right)$. The multiplicity of equilibria results from a strategic complementarity, but the externalities of collaborative effort are lost when everybody chooses the high individual effort equilibrium.

2. For $\frac{\underline{a}}{\overline{a}}\frac{K_1}{K_1+K_2}<\mu<\frac{\underline{a}}{\overline{a}}$, when the equilibrium is unique, it is always efficient since the condition guarantees that $\overline{a}\mu\left(K_1+K_2\right)>K_1\underline{a}$. When multiple equilibria are possible, the low individual effort equilibrium is efficient for

$K_1\underline{a} < R < \overline{a}\mu\,(K_1 + K_2)$, while the high individual effort equilibrium is efficient for $\overline{a}\mu\,(K_1 + K_2) < R < \underline{a}\,(K_1 + K_2)$. When R is sufficiently high the externalities of collaborative effort fall short compared to the returns of individual effort from the parental point of view, even if everybody else collaborates. However, parents cannot induce their child to choose individual effort, because their child's lowest return from collaborative effort is $\underline{a}\,(K_1 + K_2) > R$.

3. For $\mu < \frac{\underline{a}}{\overline{a}}\frac{K_1}{K_1+K_2}$, implying $\overline{a}\mu\,(K_1 + K_2) < K_1\underline{a}$, the low individual effort equilibrium becomes inefficient even when it is unique for values of R such that $\overline{a}\mu\,(K_1 + K_2) < R \leq K_1\underline{a}$. The incentives of parents and children to choose the high individual effort equilibrium lie very far apart. The children prefer to choose collective effort even if nobody else does, and even when parents set a to its lowest possible value $\underline{a}$. When multiple equilibria exist, the high individual effort equilibrium is efficient, and it is also efficient when it is the unique equilibrium.

From these results, we can generate some policy insights.

1. If μ is high (Case 2 of Lemma 1), then the only possible inefficiency arises when there are multiple equilibria. This means that "all" the policy needs to do is to shift parenetal expectations about what other parents are going to do to coordinate their behaviors on a cooperative education style making children choose $x = 0$. This could be done in principle with temporary measures and activities that need not be very costly.

2. If μ is low (Case 1 of Lemma 1), then there are circumstances where the issue is not just about expectations, but about the ability of parents to control the activities of their children. That is, $\underline{a}$ is not low enough, so parents cannot persuade the children to undertake the individual effort activities that parents consider to be more productive even if they chose a individualistic education style which they do not in equilibrium since it is not effective. Achieving a lower $\underline{a}$ might require parental training on persuasiveness and pedagogy that could be more costly and difficult. When the tension between parental and

child's perspective is very strong, namely $\mu < \frac{\underline{a}}{\bar{a}} \frac{K_1}{K_1+K_2}$ an alternative could be to influence the returns on collaborative effort (raising K_2) such that the inequality flips and inefficiency only arises in the area of multiple equilibria when parents coordinate on the wrong equilibrium. Raising K_2 may require (re)training the parents and teachers about the collaborative technology, which is intrinsically more complicated.

3 Multiple groups

Consider a situation where parameters are such that multiple equilibria can arise in a homogeneous population and imagine we have two isolated population groups that coordinate on different equilibria for the effort decision of their children. We want to examine under which conditions these population groups can coexist when they interact with each other. Let $\lambda > 1/2$ be the fraction of the time that each population group interacts with their own group. We will refer to the group that chooses $x_1 = x_L = 0$ as the low individual effort group and the group that chooses $x_2 = x_H = 1$ as the high individual effort group.

3.1 No parental choice

For the time being we shut down the parental choice channel and study whether two population groups can coordinate on different effort decisions for their children in a homogenous society where all children have the same preference parameter a.

Lemma 2 *In an a priori homogenous group of children an equilibrium with two population groups where one group of children coordinates on $x_L = 0$ and the other one on $x_H = 1$ is indeed possible if*

$$a\left(K_1 + \lambda K_2\right) > R > a\left(K_1 + (1-\lambda) K_2\right) \tag{7}$$

Proof. The choice problem of a child in group i is to choose individual effort x_i

maximizing

$$a\left(K_1+\left(1-\left(\lambda\overline{x_i}+(1-\lambda)\,\overline{x_{-i}}\right)\right)K_2\right)(1-x_i)+Rx_i$$

where $\overline{x_i}$ refers to the average choice of x in group i and $\overline{x_{-i}}$ refers to the average choice of x in the other group $-i$. The above expression is equivalent to

$$\begin{aligned}&a\left(K_1+\left(1-\left(\lambda\overline{x_i}+(1-\lambda)\,\overline{x_{-i}}\right)\right)K_2\right)\\&+\left(R-a\left(K_1+\left(1-\left(\lambda\overline{x_i}+(1-\lambda)\,\overline{x_{-i}}\right)\right)K_2\right)\right)x_i\end{aligned}$$

Given that everybody else in group 1 chooses $x_L=0$ and everybody in group 2 chooses $x_H=1$, choosing $x_L=0$ is indeed a best response for a member of group 1 if $R-a\left(K_1+\left(1-(1-\lambda)\right)K_2\right)<0$ or equivalently if

$$R<a\left(K_1+\lambda K_2\right) \tag{8}$$

Similarly, choosing $x_2=x_H$ is indeed a best response for a member of group 2 if $R-a\left(K_1+(1-\lambda)K_2\right)>0$ or equivalently

$$R>a\left(K+(1-\lambda)K_2\right) \tag{9}$$

Condition (7) results from joining (8) and (9). ■

The lower the interaction among the groups (the closer λ to 1), the bigger the range of parameters for which this condition (7) can hold.

3.2 Parental choice

We now turn to the full problem with parental choice and analyze the same question: can we have a group where children choose $x=x_L=0$ and another group were children choose $x=x_H=1$ when each group interacts with a fraction $(1-\lambda)$ of the other group? The following Lemma states when this is possible.

Lemma 3 *An equilibrium with two separate subgroups where parents of the high individual effort children choose the individualistic education style* $a=\min\left[\overline{a},\frac{R}{K_1+(1-\lambda)K_2}\right]$

and parents of the low individual effort children choose the collaborative education style $a = \overline{a}$ *exists when*

$$\underline{a}\left(K_1 + (1-\lambda)K_2\right) \ < R < \underline{a}(K_1 + \lambda K_2) \ \textit{for} \ \mu < \frac{\underline{a}}{\overline{a}} \tag{10}$$

or when

$$\mu\overline{a}\left(K_1 + (1-\lambda)K_2\right) < R < \mu\overline{a}(K_1 + \lambda K_2) \ \textit{for} \ \mu > \frac{\underline{a}}{\overline{a}} \tag{11}$$

Proof. We first formulate the utility function of the parents in the different groups and then look for their best response functions and the corresponding equilibrium.

Children who live in the low individual effort group where they choose $x = x_L = 0$ and interact a fraction λ of their time with the children in their own group and $(1-\lambda)$ with children in the other group which choose $x = x_H = 1$ will face an average individual effort time of their fellow students given by $\overline{x} = 1 - \lambda$.Therefore the utility of their parents becomes

$$U^P_{x_L=0}(a) = \begin{cases} R \text{ if } \frac{R}{K_1+\lambda K_2} > a \\ \mu a\left(K_1 + \lambda K_2\right) \text{ if } \frac{R}{K_1+\lambda K_2} \leq a \end{cases}$$

Children who live in the high individual effort group where they choose $x = x_H = 1$ and interact a fraction λ of their time with the children in their own group and $(1-\lambda)$ with children in the other group which choose $x = x_L = 0$ face an average individual study time of their fellow students given by $\overline{x} = \lambda$.Therefore the utility of their parents becomes

$$U^P_{x_H=1}(a) = \begin{cases} R \text{ if } \frac{R}{K_1+(1-\lambda)K_2} > a \\ \mu a\left(K_1 + (1-\lambda)K_2\right) \text{ if } \frac{R}{K_1+(1-\lambda)K_2} \leq a \end{cases}$$

If children in the low individual effort group indeed best respond by choosing $x = x_L = 0$, and if children in the high individual effort group best respond by choosing $x = x_H = 1$,the low individual effort group parents have to be in an equilibrium where they induce $x_L = 0$ and high individual effort group parents where they induce $x_H = 1$. Using (5) and (6) the best response function of parents of the high individual effort group and parents of the low individual effort group become

$$
\begin{aligned}
a &= \min\left[\overline{a}, \frac{R}{K_1 + (1-\lambda) K_2}\right] && (12)\\
&\text{for } \max\left\{\underline{a}\left(K_1 + (1-\lambda) K_2\right), \overline{a}\mu\left(K_1 + (1-\lambda) K_2\right)\right\} \leq R \\
a &= \ \overline{a} \text{ for } R < \max\left\{\underline{a}\left(K_1 + \lambda K_2\right), \overline{a}\mu\left(K_1 + \lambda K_2\right)\right\} && (13)
\end{aligned}
$$

respectively. Conditions (12) and (13) combine to

$$
\begin{aligned}
&\max\left\{\underline{a}\left(K_1 + (1-\lambda) K_2\right)\ , \mu\overline{a}\left(K_1 + (1-\lambda) K_2\right)\right\} < R \\
&< \max\left\{\underline{a}\left(K_1 + \lambda K_2\right), \overline{a}\mu\left(K_1 + \lambda K_2\right)\right\}
\end{aligned}
$$

which can be rewritten as conditions (10) and (11) when replacing the max by its corresponding value. ■

3.3 Welfare and policy Implications

Observe that the conditions for coexistence of two separate subgroups who interact with each other given in Lemma 3 lie strictly inside the bounds for the existence of multiple equilibria in pure strategies given in Lemma 1.[3] This observation has important policy implications. Whenever multiple equilibria exist, one of those equilibria is inefficient. This could happen for example when parents coordinate on the high individual effort equilibrium at one school and on the low individual effort equilibrium at the other school. One policy tool that is sometimes used (or proposed) in reality is to move (say, by busing) children from the inefficient school to the efficient school (see Agostinelli et al. 2020). If parents can restrict the interaction of their children so λ of the time they are "with their own kind", this policy might backfire. This will happen if the children can coordinate on a "bad" multiple group equilibrium: one where the collaborative learners are worse off than in separate subpopulations because fewer of their classmates make collaborative ef-

[3] $\overline{a}\mu K_1 < \mu\overline{a}\left(K_1 + (1-\lambda) K_2\right) < R < \mu\overline{a}(K_1 + \lambda K_2) \leq \overline{a}\mu\left(K_1 + K_2\right)$ for $\mu > \frac{\underline{a}}{\overline{a}}$ while for $\mu < \frac{\underline{a}}{\overline{a}}$ we have $\underline{a}K_1 < \underline{a}\left(K_1 + (1-\lambda) K_2\right) < R < \underline{a}(K_1 + \lambda K_2) \leq \underline{a}\left(K_1 + K_2\right)$

fort. Even if the groups cannot coexist together, conditions exist where the whole society converges to an inefficient outcome. The whole society will converge to a high individual effort equilibrium if $R > \max\left[\mu\overline{a}(K_1 + \lambda K_2), \underline{a}(K_1 + \lambda K_2)\right]$ and to a low individual effort equilibrium $R < \max\left[\mu\overline{a}(K_1 + (1-\lambda)K_2), \underline{a}(K_1 + \lambda K_2)\right]$. But the high individual effort equilibrium is only efficient if $R > \mu\overline{a}(K_1 + K_2)$ so for $\mu\overline{a}(K_1 + K_2) > R > \max\left[\mu\overline{a}(K_1 + \lambda K_2), \underline{a}(K_1 + \lambda K_2)\right]$ the policy of moving children between the schools would move both groups to the high individual effort equilibrium when it is inefficient. When $\mu > \frac{\underline{a}}{\overline{a}}$ and $R < \mu\overline{a}(K_1 + (1-\lambda)K_2)$ this policy will always achieve the efficient low individual effort equilibrium, but for $\mu < \frac{\underline{a}}{\overline{a}}$ the low individual effort equilibrium might be inefficient for a value of μ low enough that $\mu\overline{a}(K_1 + K_2) < R < \underline{a}(K_1 + (1-\lambda)K_2)$. Hence moving children from an inefficient school to an efficient school does not necessarily improve welfare, even if parents are a priori identical.

We will show in the next sections that similar situations can arise under local interaction structures.

4 Local interaction

In this section we will examine if different equilibrium clusters can coexist if children only interact locally and examine different particular local interaction structures. In order to do so, we study the incentives of parents living on the boundary of low and high individual effort regions to switch to the other equilibrium. Coexistence of different equilibrium clusters is possible only if none of these parents in the boundary has an incentive to switch. In the entire section we will only consider situations where initially a low and a high individual effort region form and parents best respond to this situation.

4.1 Linear interaction on a circle

If interaction happens on the circle, each child interacts with the k closest neighbors to each side. If different clusters arise, there will be a parent of a low individual effort child and a parent of a high individual effort child located on each side of the boundary and their child will live in an environment where half of the other children choose high individual effort and half of the other children choose low individual effort. Therefore a necessary condition for the survival of the two clusters is that two different parental subgroups can exist with $\lambda = \frac{1}{2}$. But for $\lambda = \frac{1}{2}$ the conditions (10) and (11) of Lemma 3 collapse to $\underline{a}\,(K_1 + 0.5K_2) \; < R < \underline{a}(K_1 + 0.5K_2)$ if $\mu < \frac{\underline{a}}{\overline{a}}$ or $\mu\overline{a}\,(K_1 + 0.5K_2) < R < \mu\overline{a}(K_1 + 0.5K_2)$ if $\mu > \frac{\underline{a}}{\overline{a}}$ which can only happen non generically. Therefore, local interaction on a circle will always lead to a unique equilibrium. However, as in Section 3.3 this unique equilibrium is not always efficient. In particular, convergence to $x = 1$ is inefficient when $\max[\underline{a}\,(K_1 + 0.5K_2)\,, \mu\overline{a}\,(K_1 + 0.5K_2)] \; < R < \mu\overline{a}\,(K_1 + K_2)$ which can only happen for $\mu > \frac{\underline{a}}{\overline{a}}\frac{K_1+0.5K_2}{K_1+K_2}$. Convergence to $x = 0$ is inefficient when $\mu\overline{a}\,(K_1 + K_2) < R < \underline{a}\,(K_1 + 0.5K_2)$ when $\mu < \frac{\underline{a}}{\overline{a}}\frac{K_1+0.5K_2}{K_1+K_2}$.

When these inefficiencies arise, one could have a policy to avoid local interaction, so everyone connects to everyone else, to allow for the possibility for convergence to an efficient equilibrium.

In particular, local interaction on a circle leads to inefficient convergence on $x = 1$ for $\mu > \frac{\underline{a}}{\overline{a}}\frac{K_1+0.5K_2}{K_1+K_2}$ when $\max[\underline{a}\,(K_1 + 0.5K_2)\,, \mu\overline{a}\,(K_1 + 0.5K_2)] \; < R < \mu\overline{a}\,(K_1 + K_2)$. Local interaction on a circle leads to inefficient convergence on $x = 0$ for $\mu < \frac{\underline{a}}{\overline{a}}\frac{K_1+0.5K_2}{K_1+K_2}$ when $\mu\overline{a}\,(K_1 + K_2) < R < \underline{a}\,(K_1 + 0.5K_2)$. In both cases, for this parameter range under global interaction multiple equilibria exist, so allowing for global interaction could open the door that parents coordinate on the efficient equilibrium.

4.2 Further examples with local interaction structures

In this subsection, we will look at further examples of local interaction structures. Unlike the circle, in these local interaction structures coexistence of different equilibrium clusters are possible. In an equilibrium where two subgroups coexist either one group or the other will be worse off than in the alternative group since preferences are homogeneous. But that does not mean that moving to a society without local interaction will necessarily yield a Pareto improvement. Indeed, we will show that the conditions that allow for the coexistence of two different subgroups under local interaction are a subset of the conditions for multiple equilibria under global interaction. Whenever there are multiple equilibria under global interaction one of them is inefficient, and it might be the case that under global interaction society ends up coordinating on the inefficient equilibrium.

When the conditions for coexistence are violated, the system will converge to a homogeneously behaved population but the outcome might be inefficient.[4]

The next subsections explain the local interaction structures and derive the conditions of coexistence of different equilibrium clusters.[5]

4.2.1 Nearest neighbor interaction in $m-$dimensions.

Players interact on an m-dimensional lattice with the nearest neighbors in the m-dimensions. Hence there are $2m$ neighbors, and in an advancing $m-1$ boundary, only one of them will be affected by the other side so that $1/(2m)$ fraction of neighbors will be of the other type[6] and $(2m-1)/2m$ of the own type. Recall that a high individual effort parent always gets R irrespective of the behavior of the other parents and children. Hence coexistence of different equilibrium clusters is possible for

$$\begin{aligned} \underline{a}\left(K_1+\frac{1}{2m}K_2\right) &< R<\underline{a}\left(K_1+\frac{2m-1}{2m}K_2\right) \text{ if } \mu<\frac{\underline{a}}{\overline{a}} \text{ or} \\ \mu\overline{a}\left(K_1+\frac{1}{2m}K_2\right) &< R<\mu\overline{a}\left(K_1+\frac{2m-1}{2m}K_2\right) \text{ if } \mu>\frac{\underline{a}}{\overline{a}} \end{aligned} \tag{14}$$

These conditions for coexistence lie strictly inside the range of parameters for which multiple equilibria exist by Lemma 1 and they converge to the boundaries of Lemma 1 when $m\to\infty$ in which case this local interaction structure coincides with our original model.

[4]We will not state the exact conditions for inefficiency. They follow the same logic as the conditions for multiple subgroups.

[5]For more general interaction structures, see Morris (2000).

[6]This fraction of neighbors of the other type is referred to by Morris (2000) as the contagion threshold.

4.2.2 n-max distance interaction in m dimensions.

Each player interacts with all players who are less than n steps away in each of the m dimensions. Hence each player has have $(2n+1)^m - 1$ neighbors.

Example 4 *As an example, with $m = 2$ and $n = 1$, parents are on a two-dimensional lattice and we had two subgroups at the linear boundary each person would interact with 5 people of their own type and 3 people of the different type and hence the linear boundary can serve as place to stop change since conditions (10) and (11) become*

$$\begin{aligned} \underline{a}\left(K_1 + \frac{3}{8}K_2\right) &< R < \underline{a}\left(K_1 + \frac{5}{8}K_2\right) \text{ if } \mu < \frac{\underline{a}}{\overline{a}} \text{ or} \\ \mu\overline{a}\left(K_1 + \frac{3}{8}K_2\right) &< R < \mu\overline{a}\left(K_1 + \frac{5}{8}K_2\right) \text{ if } \mu > \frac{\underline{a}}{\overline{a}} \end{aligned}$$

More generally, the number of neighbors of the other type at the boundary is given by $\frac{n(2n+1)^{m-1}}{(2n+1)^m - 1}$. Hence two different equilibrium clusters can coexist if

$$\begin{aligned} & \underline{a}\left(K_1 + \frac{n(2n+1)^{m-1}}{(2n+1)^m - 1}K_2\right) < R \\ < \; & \underline{a}\left(K_1 + \left(1 - \frac{n(2n+1)^{m-1}}{(2n+1)^m - 1}\right)K_2\right) \text{ if } \mu < \frac{\underline{a}}{\overline{a}} \text{ or} \\ & \mu\overline{a}\left(K_1 + \frac{n(2n+1)^{m-1}}{(2n+1)^m - 1}K_2\right) < R \\ < \; & \mu\overline{a}\left(K_1 + \left(1 - \frac{n(2n+1)^{m-1}}{(2n+1)^m - 1}\right)K_2\right) \text{ if } \mu > \frac{\underline{a}}{\overline{a}} \end{aligned}$$

In the limit when $n \to \infty$ the conditions cannot be satisfied generically as the thresholds are

$$\lim_{n\to\infty} \frac{n(2n+1)^{m-1}}{(2n+1)^m - 1} = \frac{1}{2}, \lim_{n\to\infty}\left(1 - \frac{n(2n+1)^{m-1}}{(2n+1)^m - 1}\right) = \frac{1}{2}$$

In the limit when $m \to \infty$ the conditions can be satisfied, for finite n, generically as the thresholds are

$$\lim_{m\to\infty}\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}=\frac{n}{2n+1},\ \lim_{m\to\infty}\left(1-\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}\right)=\frac{n+1}{2n+1}$$

Again the conditions for coexistence of multiple groups lie strictly inside the range of parameters for which multiple equilibria exist by Lemma 1.

4.2.3 Regions

The population is divided into an infinite number of "regions" of m players each. Each player in a region interacts with every other player in that region. The regions are arranged in a line and each player also interacts with one player in each neighboring region. In this case, the number of neighbors is $m+1$ and at the boundary of two different clusters just 1 neighbor is of the other type so conditions (10) and (11) become

$$\begin{aligned}\underline{a}\left(K_1+\frac{1}{m+1}K_2\right) &< R<\underline{a}\left(K_1+\frac{m}{m+1}K_2\right)\text{ if }\mu<\frac{\underline{a}}{\overline{a}}\text{ or}\\ \mu\overline{a}\left(K_1+\frac{1}{m+1}K_2\right) &< R<\mu\overline{a}\left(K_1+\frac{m}{m+1}K_2\right)\text{ if }\mu>\frac{\underline{a}}{\overline{a}}\end{aligned}$$

These conditions for coexistence lie strictly inside the range of parameters for which multiple equilibria exist by Lemma 1 and reassuringly converge the the boundaries of Lemma 1 when $m\to\infty$ in which case this local interaction structure coincides with our original model.

4.2.4 Hierarchies

The population is arranged in a hierarchy. Each player has m subordinates. Each player, except the root player, has a single superior. In this case the number of neighbors is $m+1$ and at the boundary between different behavioral clusters there is again just one neighbor of the different type. Therefore the conditions of coexistence of different equilibrium clusters for the hierarchy coincide with the one of regions.

5 Parental heterogeneity and parental secession

In this section we will depart from a homogeneous population and introduce parental heterogeneity. We will study a situation where we have two different types of parents that either differ in their evaluation for the returns from collaborative effort capture by the μ_i or in their resources to control their children activities reflected by $\underline{a_i}$ or in the returns they can provide from individualistic effort R_i. While different μ_i merely correspond to different evaluations of the education technologies, differences in $\underline{a_i}$ or R_i can be interpreted differences in socioeconomic status where "upper class" parents have more resources either leading either to a lower $\underline{a_i}$ or to a higher R_i. Our main aim is to analyse when these different groups have a desire to avoid interaction with the other group which we label as a secession desire and when secession actually occurs and its welfare consequences. We will set up the general preferences which reflect all three different possible cases of parental heterogeneity and then use the differences in the evaluation for the returns from collaborative effort as our workhorse example in the sense of providing a full equilibrium analysis while only focusing on secession in the other two cases.

5.1 Preferences and best responses

We study a situation where we have two population groups H and L where the H group has size $\gamma \in (0,1)$ and the L group $1-\gamma$. These groups either differ in μ_i where $\mu_H > \mu_L$ or in $\underline{a_i}$ where $\underline{a_H} > \underline{a_L}$ or in R_i where $R_H > R_L$. When both groups interact, the choice problem of a child in group i is to choose individual effort x_i maximizing

$$U_i^c(x_i) = a_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\, \overline{x_L}\right)\right) K_2\right)(1-x_i) + R_i x_i \tag{15}$$

where $\overline{x_H}$ is the average choice of x of the children from the H group and $\overline{x_L}$ is the average choice of x of the children from the L group. The child's best response is therefore

$$x_i = \begin{cases} 1 \text{ if } R_i > a_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right) \\ 0 \text{ if } R_i \le a_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right) \end{cases} \tag{16}$$

Parental utility is given by

$$U_i^P(a_i) = \mu_i a_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right)(1 - x_i) + R_i x_i \tag{17}$$

which after incorporating the child's best response function becomes

$$U_i^P(a_i) = \begin{cases} R_i \text{ if } \frac{R_i}{K_1 + (1 - (\gamma \overline{x_H} + (1-\gamma)\overline{x_L}))K_2} \ge a_i \\ \mu_i a_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right) \\ \text{if } \frac{R_i}{K_1 + (1 - (\gamma \overline{x_H} + (1-\gamma)\overline{x_L}))K_2} < a_i \end{cases} \tag{18}$$

The parental optimal choice is characterized as follows

- for

$$\max\left\{\underline{a}_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right), \right.$$
$$\left. \overline{a}\mu_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right)\right\} < R_i,$$

  the parental optimal choice is $a_i = \min\left[\overline{a}, \frac{R_i}{K_1 + (1 - (\gamma \overline{x_H} + (1-\gamma)\overline{x_L}))K_2}\right]$ implying $x_i = 1$.

- for

$$R_i < \max\left\{\underline{a_i} \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right), \right.$$
$$\left. \overline{a}\mu_i \left(K_1 + \left(1 - \left(\gamma \overline{x_H} + (1-\gamma)\,\overline{x_L}\right)\right) K_2\right)\right\},$$

  the parental optimal choice is $a_i = \overline{a}$ implying $x_i = 0$.

Two types of equilibria can arise, (i) homogeneous equilibria where all children make the same choices and (ii) heterogeneous equilibria where children from different groups choose different individualistic effort levels. These equilibria might coexist under certain parameters. In the next section we illustrate the possible equilibrium outcomes when the source of heterogeneity is in the parental evaluation of the return to collaborative effort μ_i.

5.2 Differences in the parental evaluation of the return to collaborative effort

Let $\mu_H > \mu_L$ be the only difference between parent groups, i.e. $\underline{a}_i = \underline{a}$ and $R_i = R$.

5.2.1 Equilibrium outcomes

The equilibrium outcomes are derived in the Appendix. Proposition 6 in the Appendix describes in detail all equilibria constellations that arise for different values of μ_H and μ_L. Cases 1 and 2 of the Proposition describe the equilibria for $\mu_L < \frac{\underline{a}}{\overline{a}}$ and Case 3 for $\frac{\underline{a}}{\overline{a}} < \mu_L$. We will discuss what happens when μ_H grows from $\mu_H = \mu_L + \varepsilon$ to 1. This is represented in Figure 1 and 2 where for high and low values of γ respectively for $\mu_L < \frac{\underline{a}}{\overline{a}}$ (hence capturing Case 1 and Case 2 of Proposition 6).

As can be seen from the Figures for low μ_H no mixed population equilibrium is possible. Observe that at $\mu_H = \frac{\underline{a}}{\overline{a}}$ the lower bound for $x_i = 1$ for all i switches from $\underline{a}K_1$ to $\overline{a}\mu_H K_1$ and the mixed population equilibrium starts to appear with lower bound $\underline{a}\,(K_1 + \gamma K_2)$ (which initially lies above the now moving lower bound for $x_i = 1$ for all i). Observe that the lower bound of the mixed population equilibrium coincides with its upper bound $\overline{a}\mu_H\,(K_1 + \gamma K_2)$ at $\mu_H = \frac{\underline{a}}{\overline{a}}$. Also, the upper bound of the mixed population equilibrium initially lies below the fixed upper bound of $\underline{a}\,(K_1 + K_2)$ of the equilibrium where $x_i = 0$ for all i.

When μ_H grows further the lower bound for $x_i = 1$ for all i increases, and so does the upper bound for the mixed population equilibrium which increases even faster. Hence when μ_H increases, we go from a situation with no mixed population equilibrium to a situation where a mixed population equilibrium appears between the lower bound for $x_i = 1$ for all i equilibrium and the upper bound of the $x_i = 0$ for all i equilibrium. The more μ_H increases, the bigger the area of R where the mixed population equilibrium exist and the smaller the area of coexistence of $x_i = 0$

for all i and $x_i = 1$ for all i which finally disappears for μ_H sufficiently high, so that the lower bound for the $x_i = 1$ for all i equilibrium has overtaken the upper bound for the $x_i = 0$ for all i equilibrium.

Whether the mixed equilibrium first becomes possible before the high individual effort equilibrium or when the low individual effort equilibrium is no longer possible depends on γ, the proportion of μ_H types in the population. For sufficiently low γ (Case 2 b(i)) the mixed equilibrium first becomes possible before the high individual effort equilibrium while for sufficiently high γ (Case 2 b(ii)) the mixed equilibrium first continues to be possible when the low individual effort equilibrium is no longer possible. In other words, when μ_H increases we go from Case 1 to Case 2a, to Case 2b (either (i) or (ii) depending on the size of γ), then to Case 2c and finally to Case 2d.[7]

5.2.2 Parental secession

In the present model only parents with μ_H might want to secede, since parents with μ_L have stronger incentives to implement individual effort where the return is independent of what other parents do. To understand the incentives for secession we first have to specify expectations of the seceding group in case there are multiple equilibria under secession.

We will start our analysis with what we define as *pessimistic expectations*, namely that if there are multiple equilibria after secession the least favorable equilibrium from the perspective of the seceding parents will occur. With pessimistic expectations the μ_H parents want to secede if after secession in the unique equilibrium they choose $\overline{a}$ and that induces low individual effort. It also requires that before secession they lived in an environment where other parents did not induce low individual effort. This situation can happen for a sufficiently high μ_H and values of R such that either multiple equilibria exist but there is no high individual effort, or there is a unique

[7] Increasing μ_H in Case 3 where $\frac{\underline{a}}{\overline{a}} < \mu_L < \mu_H$ leads to a similar picture. The lowest μ_H possible is in case 3a. Then we move to case 3b(i) or 3b(ii) depending on whether γ is low or high respectively. Then we move to Case 3c and finally to Case 3d. Again it is the lower bound of the high individual effort equilibrium that grows with μ_H and the upper bound of the mixed population equilibrium that also grows and at a faster rate with μ_H. The same logic than before applies.

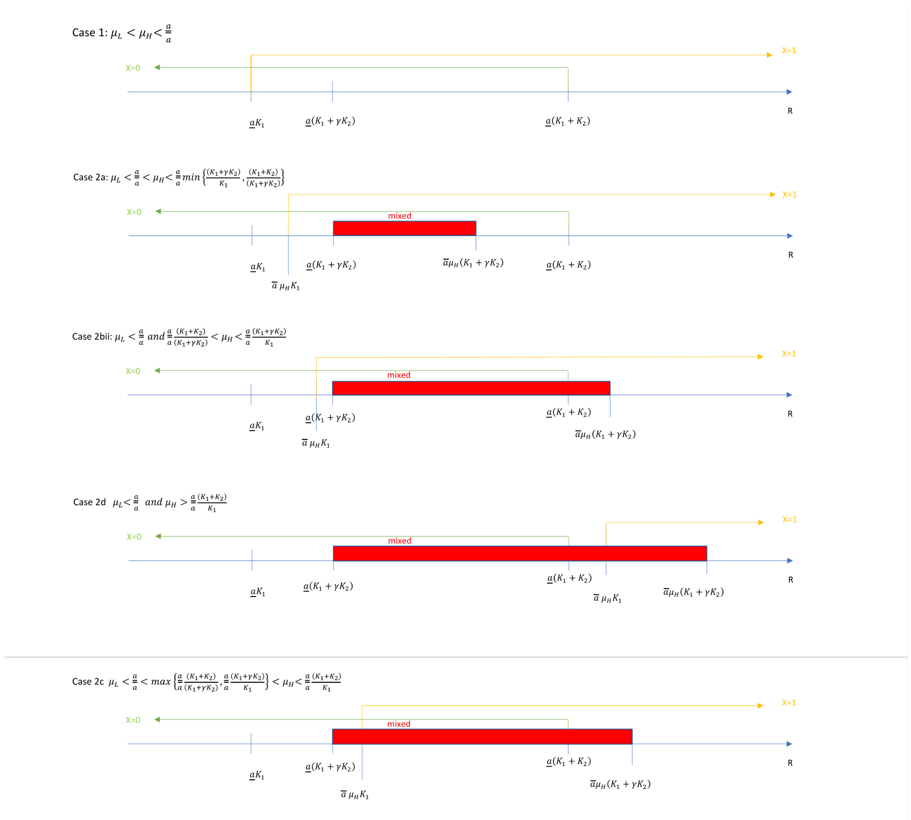


Figure 1 Equilibrium cases when $\mu_m < \frac{\underline{a}}{\overline{a}}$ and γ is high

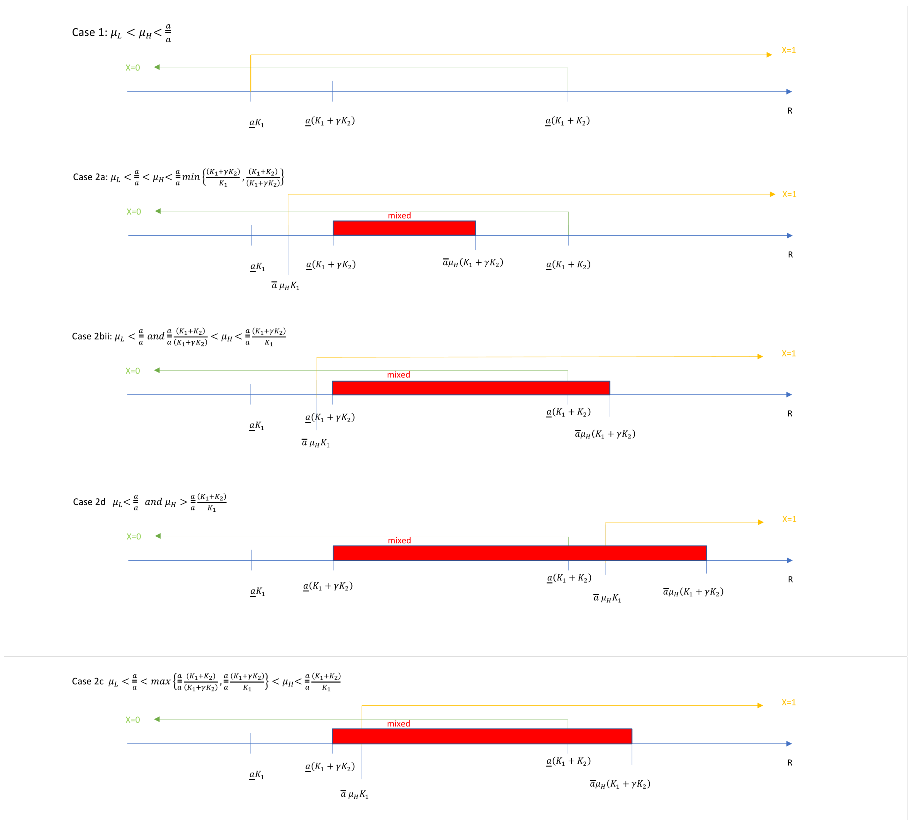


Figure 2 Equilibrium cases when $\mu_m < \frac{\underline{a}}{\overline{a}}$ and γ is low

mixed population equilibrium.[8] In these cases, μ_H parents can avoid the mixed population equilibrium and coordinate on the low individual effort equilibrium if $R < \overline{a}\mu_H K_1$.[9]

Is secession optimal for a social planner who only takes into consideration the parental preferences under pessimistic expectations?. Secession happens under 2 scenarios:

1. *The first is under a unique mixed equilibrium.* In this case, after secession, the μ_L parents who were inducing high individual effort retain their preferred option, and the μ_H parents now coordinate on low individual effort so their utility improves. *In this first case, secession always increases social welfare, and it can be facilitated.*

2. *The second case* in which secession happens under pessimistic expectations is if, after secession, parents end up in a *mixed equilibrium when an equilibrium where all students make low individual effort also exists.* If secession happens in this case with pessimistic expectations, the μ_H parents now coordinate on low individual effort so their utility improves. However, it is theoretically possible that after secession the μ_L parents and their children change their beliefs to thinking that in their group low individual effort will prevail and then switch to the low individual effort equilibrium, which is worse than the original situation from the point of view of μ_L parents. *In this second case, secession can potentially decrease social welfare for that group, and thus, it is unclear what the social planner stance should be.* Notice, though, that this requires a big change in beliefs for μ_L parents and children.

[8] This can never happen in case 1 of Proposition 6 where $\mu_M < \frac{\underline{a}}{\overline{a}}$. A mixed equilibrium does not even exist in this case.

[9] To be precise secession happens in the following cases of Proposition 6

- in the area of coexistence of $x_i = 0$ for all i and the mixed equilibrium in Case 2b (i), 2c and 2d, as well as in Case 3b(i), 3c and 3d
- in the area of a unique mixed equilibrium in Cases 2d and 3d

A different situation arises if seceding parents have optimistic expectations. This means that when multiple equilibria after secession are possible, the seceding parents believe that they will coordinate on their preferred equilibrium. Then, the μ_H parents can reach the low individual effort equilibrium when $R \leq \underline{a}(K_1 + K_2)$ if $\mu_H \leq \frac{\underline{a}}{\overline{a}}$ or for $R \leq \overline{a}\mu_H (K_1 + K_2)$ if $\mu_H > \frac{\underline{a}}{\overline{a}}$. This means that when $\mu_H \leq \frac{\underline{a}}{\overline{a}}$ (Case 1) secession will occur in the area of coexistence of the high and low individual effort equilibrium. In Cases 2 and 3 where $\mu_H > \frac{\underline{a}}{\overline{a}}$ the relevant cutoff for the upper bound of secession is now $R \leq \overline{a}\mu_H (K_1 + K_2)$ while the lower bound is defined by the maximum value of R for which the low individual effort equilibrium in both groups is a unique equilibrium. Observe that the upper bound for secession $\overline{a}\mu_H (K_1 + K_2)$ is bigger than the lower bound for which there is a unique high individual effort equilibrium in the entire population, hence there is even an area with a unique high individual effort equilibrium where μ_H parents prefer to secede.

If a social planner has optimistic expectations about equilibrium coordination, secession is always to be encouraged. The seceding μ_H parents do it because they believe utility will improve, whereas μ_L parents will remain at the high effort equilibrium.

5.2.3 Forward induction and equilibrium selection

We argue that optimistic expectations is the reasonable case in a game like the one we are considering. Secession is likely to entail some costs, and those costs can focus expectations on the optimistic side, if players are rational enough to avoid dominated strategies. Formally we are going use forward induction as a solution concept in the game of secession.

Note that given that the cost of secession plays a role in the "signaling" value towards equilibrium selection, this implies that a very high subsidy for secession is not always a good idea.

Suppose the game of secession works in the following way. First, the μ_H parents decide whether to sign a contract that states that if X of them sign it, each pays an amount d towards the construction of a new school, where the signatories can take their children. If at least X of them sign, the signatories secede, and then they play our standard schooling game.

Let $U^*_{B\mu_H}$ be the equilibrium payoffs for μ_H parents in the original schooling game.

Proposition 5 *Assume* $U^*_{B\mu_H} \leq \overline{a}\mu_H (K_1 + K_2) - d$ *and*

$$\max \{\overline{a}\mu_H (K_1), \underline{a}(K_1)\} < R < \max \{\overline{a}\mu_H (K_1 + K_2), \underline{a}(K_1 + K_2)\}.$$

Then, in any equilibrium that survives forward induction, the μ_H *parents decides to sign the contract to secede, and then their children choose* $x = 0$ *in the new school.*

Proof. The

$$\max \{\overline{a}\mu_H (K_1), \underline{a}(K_1)\} < R < \max \{\overline{a}\mu_H (K_1 + K_2), \underline{a}(K_1 + K_2)\}$$

defines the areas where the parents with optimistic expectations want to secede. Then, all we need to show is that it is a weakly dominated strategy to sign the contract and take actions that lead children to choose $x = 1$ after secession. The reason is that by not signing the contract, the parents expect to get $U^*_{B\mu_H}$.Then signing the contract and choosing actions that lead the child to choose $x = 1$,leads to a payoff of $R - d$, and since actions without secession were part of an equilibrium, this means that $U^*_{B\mu_H} \geq R > R - d$. This in turn implies that signing the contract and then choosing actions that lead to $x = 1$ is a dominated strategy. Also, after secession, since all the parents expect others in the new school to take actions that lead to $x = 0$,the payoff for taking that action are $\overline{a}\mu_H (K_1 + K_2) - d$,and we assumed $\overline{a}\mu_H (K_1 + K_2) - d \geq U^*_{B\mu_H}$. This means that choosing actions that lead to $x = 0$ after secession is optimal. ■

5.2.4 Secession by μ_L parents

Note that in our model the only people with an interest in seceding are those characterized by μ_H. The reason is that they are the only ones benefitting from an externality from other children, because the externality only happens for collaborative effort. If individual effort also created an externality (say because of emulation or contagion), there would be a reason for μ_L parents to secede as well, for symmetric reasons as those uncovered for μ_H parents. However, it is reasonable to assume that collaborative effort generates higher externalities.

Another possibility is that collaborative effort has a negative externality on individualistic effort. Think of a classroom with a lot of noise when some children want to concentrate, or even that individualistic children are pressured to abandon their activity. In that case we can write the child's utility as

$$U_i^c(x_i) = a_i\left(K_1 + K_2\left(1 - \overline{x}\right)\right)\left(1 - x_i\right) + Rx_i\left(1 - \delta\left(1 - \overline{x}\right)\right)$$

where the $\delta \in (0, 1)$ coefficient reflects the negative externality which the fraction of collaborative effort students generate on the individualistic ones. The parental utility with $a_i \in [\underline{a}, \overline{a}]$ is now given by

$$U_i^P\left(a_i\right) = \mu a_i\left(K_1 + \left(1 - \overline{x}\right)K_2\right)\left(1 - x_i\right) + Rx_i\left(1 - \delta\left(1 - \overline{x}\right)\right)$$

It is clear (proof in supplementary material) that the main change this induces is that the ranges of parameters for different kinds of equilibria are different. In particular, the range of parameter values where individualistic effort is a unique equilibrium is reduced. Also, this creates an incentive for μ_L parents to segregate, when there are mixed equilibria in the population, as in an isolated population they can get the full return R, rather than $R\left(1 - \delta\gamma\right)$.

5.3 Differences in socioeconomic status

Consider a society in which parents have the same μ and the same R but are split into two groups that differ in $(\overline{a}, \underline{a}_i)$ where $i \epsilon \{L, H\}$ with $\underline{a}_H > \underline{a}_L$. This difference could reflect differences in socioeconomic status (SES). Parents with $\underline{a}_L$ can influence their children's preferences more than parents with $\underline{a}_H$ and we therefore refer to the former as having a higher socioeconomic status. Given the parental best responses derived after the parental utility function (18) it is easy to show that the mixed equilibrium where $\underline{a}_H$ parents do not influence their children and choose $\overline{a}$ leading to no individualistic effort by the child $x_H = 0$ and $\underline{a}_L$ parents influence their children leading to full individualistic effort $x_L = 1$ exists for

$$\begin{aligned} &\max\left\{\underline{a}_L\left(K_1+\gamma K_2\right), \overline{a}\mu\left(K_1+\gamma K_2\right)\right\} < R \\ &< \max\left\{\underline{a}_H\left(K_1+\gamma K_2\right), \overline{a}\mu\left(K_1+\gamma K_2\right)\right\} \end{aligned} \tag{19}$$

Obviously, when the mixed equilibrium occurs, this would create reasons for segregation as in the section with parental secession. Again it will be the parents who implement no individualistic effort that would like segregation, since their children benefit most if they are only interacting with other children who choose full collaborative effort.

In the mixed equilibrium, L parents have utility R and H parents have utility $\overline{a}\mu\left(K_1+\gamma K_2\right)$. Since

$$\max\left\{\underline{a}_L\left(K_1+\gamma K_2\right), \overline{a}\mu\left(K_1+\gamma K_2\right)\right\} < R,$$

this means that $\overline{a}\mu\left(K_1+\gamma K_2\right) < R$ and L parents are always better off. H parents induce high collaborative effort only because they cannot induce their children to choose high individual effort, but their preferred unconstrained option would be individual effort. *Would we always reach a better equilibrium if a policy intervention allowed H type parents to also have $\underline{a}_L$?*

Suppose the L parents who were previously getting their children have $x = 1$, stay that way. Now, since $\underline{a}_L\left(K_1+\gamma K_2\right) < R$,then even if the children of the former H parents would all stay with $x = 0$, every parent would want to deviate since $\underline{a}_L\left(K_1+\gamma K_2\right) < R$ and the children of those parents want to shift even alone, which makes the mixed equilibrium no longer feasible.

The only question is whether with this change to everybody having access to $\underline{a}_L$ there will be another equilibrium with all using $x = 0$.This can happen for example if $\mu > \frac{\underline{a}_L}{\overline{a}}$ and $R \leq \overline{a}\mu\left(K_1+K_2\right)$ where there is an equilibrium with $x = 0$ and everyone is better off.

Before this policy intervention total parental welfare was $\gamma\bar{a}\mu\left(K_1+\gamma K_2\right)+(1-\gamma)R<R$. If the intervention leads to everybody implementing individual effort the new welfare is R whereas if the intervention leads to everybody implementing collective effort the new welfare is $\bar{a}\mu\left(K_1+K_2\right)$. By Lemma 1 this latter equilibrium can only be reached for $R\leq\bar{a}\mu\left(K_1+K_2\right)$ if $\mu>\frac{\underline{a_L}}{\bar{a}}$ in which case it is the efficient equilibrium. *Thus if $\mu>\frac{\underline{a_L}}{\bar{a}}$ no matter which equilibrium is reached the policy intervention improves welfare* even when the $x=0$ equilibrium is reached in the case of multiplicity when it is inefficient.

If on the other hand $\mu<\frac{\underline{a_L}}{\bar{a}}$ then the collaborative equilibrium can be reached for $R\leq\underline{a_L}\left(K_1+K_2\right)$ but it is no longer necessarily true that $R\leq\bar{a}\mu\left(K_1+K_2\right)$ since $\underline{a_L}>\bar{a}\mu$. If $\bar{a}\mu\left(K_1+K_2\right)<R\leq\underline{a_L}\left(K_1+K_2\right)$ then in the area of multiplicity the high individual effort equilibrium is efficient. If for these parameters players coordinate on the collaborative effort equilibrium the former H parents increase their welfare from $\bar{a}\mu\left(K_1+\gamma K_2\right)$ to $\bar{a}\mu\left(K_1+K_2\right)$ but the former L parents reduce their welfare to $\bar{a}\mu\left(K_1+K_2\right)<R$. *Thus. if $\mu<\frac{\underline{a_L}}{\bar{a}}$ the overall welfare effect is unclear.* In this case the policy to improve the technology to $\underline{a_L}$ to everybody opens the door to harming the former L parents even to the extent that the overall societal welfare decreases if $\gamma\bar{a}\mu\left(K_1+\gamma K_2\right)+(1-\gamma)R>\bar{a}\mu\left(K_1+K_2\right)$

5.3.1 Differences in returns to individualistic effort

Consider now that the society is split into two groups where parents have the same μ and the same $\underline{a}$ but differ in the return they can provide to their children due to individualistic learning efforts R_i for $i\in\{L,H\}$, with $R_L<R_H$ where an upper class child has R_H and therefore a higher return from individual effort. The proportion of upper class children is given by γ. Since the upper class children have a higher return from individualistic education in a mixed equilibrium their parents will induce individualistic effort while lower class children will choose collaborative effort, hence the proportion of collaborators is $1-\gamma$. It is easy to show that a necessary condition

for such a mixed equilibrium to exist is given by

$$R_L < \max\left\{\underline{a}\left(K_1 + (1-\gamma)K_2\right), \overline{a}\mu\left(K_1 + (1-\gamma)K_2\right)\right\} < R_H \tag{20}$$

The parents with lower returns to individualistic education have an incentive for segregation. They would rather have their children being surrounded by only collaborative learners to benefit maximally from the externalities of collaborative learning.

6 Extensions

We now analyze some further extensions to test the robustness of our results to some different assumptions.

6.1 Authoritarian parents

The parents in our model mirror Doepke and Zilibotti's (2017) "authoritative" parents since parents influence their children by modify their children's utility function. Parents cannot always induce their preferred effort option for their child since they are restrained by their child's best response function. When this constraint binds parents would choose Doepke and Zilibotti's (2017) authoritarian parenting style if this technology was available. Authoritarian parents force their child to have a specific x. Parental and child's interest are in conflict when $\underline{a}K_1 + (1-\overline{x})K_2 > R > \mu\underline{a}\left(K_1 + (1-\overline{x})K_2\right)$. Obviously, this is more likely to hold for μ_L parents. Specifically, both parents want to manipulate x if

$$\underline{a}\left(K_1 + (1-\overline{x})K_2\right) > R > \mu_H\underline{a}\left(K_1 + (1-\overline{x})K_2\right),$$

and only the μ_L parents if

$$\mu_H\underline{a}\left(K_1 + (1-\overline{x})K_2\right) > R > \mu_L\underline{a}\left(K_1 + (1-\overline{x})K_2\right),$$

One way this authoritarian control can easily be achieved is to send the child to a school where teaching emphasizes only individual work on non-cooperative tasks, and which have a very competitive environment, so the children do not want to help each other.

The authoritative parents would be happy that these schools exist, because they are the ones that, when mixed equilibria exist, have an incentive for secession, so their children are in an environment with more cooperative/collaborative classmates.

6.2 Spillovers from total collaborative effort

Up to now only the average collaborative effort mattered for the returns to collaborative learning. We now examine how our setup changes when the total collaborative effort affects the externality. The child's best response function in this case becomes

$$x_i = \begin{cases} 1 \text{ if } R > a\left(K_1 + NK_2\left(1 - \overline{x}\right)\right) \\ 0 \text{ if } R \leq a\left(K_1 + NK_2\left(1 - \overline{x}\right)\right) \end{cases}$$

where N is the size of the population. It is easy to see that most of our results would not be affected qualitatively. Note, though, that for secession the size of the seceding group matters. If a fraction γ of parents prefers collaborative effort, they have to take into account that under secession their children will only interact with γN other children. This lowers the interest of secession. With **pessimistic expectations**, and under the same process we discussed before, there are no incentives for secession. This is because the secessionists are not integrating themselves into a new group, but they are constructing a new school with part of the people that they were already interacting with in the old place. This means that the $N\left(1 - \overline{x}\right)$ people from which they already got the externalities before, are the same ones from which they will get the externalities in the new place, so no new positive externalities can be obtained. It would be different if the new school gathered groups of collaborative minded parents arriving from several different schools. In that case, one could have $\sum_{j=i}^{n} N_j\left(1 - \overline{x}\right)$ with j being the index from the different schools of origin of the students in the new school.

Under **optimistic expectations**, there could still be incentives for secession. In that case, the new school might gather students from a school that was locked into an individualistic equilibrium, in a situation in which multiple equilibria are possible. In the new school, and for the equilibrium selection reasons explained in section 5.2.3, a different equilibrium can be reached.

7 Empirical illustration

We end with an empirical exercise.[10] It illustrates how our approach allows to make sense of available data productively. One prediction of our model is that with parental heterogeneity comes a potential for secession into schools where parents anticipate others will have similar parental preferences (measured, for example, by different μ_i's). As we argue in the introduction "alternative" pedagogies (Montesssori, Waldorf,..) are more supportive of collaborative learning. We also think that in scientific or technical professions are more likely to understand collaboration than those in other high-level occupations, such as managerial ones. Thus we hypothesized that there are more children of scientific and technical parents in alternative schools, than children of managers. Since alternative schools are typically private, we wanted to contrast the parental occupations in those schools, with those other private, but more traditional, schools.

We use administrative data on standardized exams administered to all Grade 3, Grade 6 and Grade 10 students in the region of Madrid between the 2015/2016 and 2018/2019 school years. The tests are low-stakes, have no academic consequences, and come along with several socio-demographic questionnaires for students, parents, teachers and school principals. To identify parental occupation, we use data from the parent questionnaire, in particular, a specific question where both mother and father report their main occupation.[11]

[10]We are extremely grateful to José Montalbán for the high-quality analysis we report here, which uses data he collected for another project. He is of course not resposible for the mistakes we may have done in the interpretation of his analysis. https://sites.google.com/site/josemontalbancastilla/

[11]For a more detailed description of the data see Montalban and Sevilla (2023).

To find out which are the "alternative" schools, we used the website of an organization called Ludus that promotes that kind of school.[12] We complemented their school locator with manual searches on Google to check the comprehensiveness of Ludus and found no differences of note.[13] Our control group are students from all private schools that are not "alternative". The private schools are labelled as such in our database.

The database has parental occupation for both fathers and mothers of students. We selected as our categories to compare, Technicians and Scientific Professionals, on the one hand, and Directors and Managers on the other.

Our variable of interest is the difference in the school share of parents with technical and scientific profession vs. directors and managers. Figure 3 shows the histogram of that variable for traditional private schools (white bars) vs. non-traditional ones (grey bars) for both fathers, mothers, and both of them combined. One can easily see that, for fathers, the distribution is quite evenly spread across traditional private schools, but it is very heavily concentrated on the positive side for non-traditional ones. Basically, non-traditional schools attract disproportionately the children of technical and scientific fathers. The pattern is similar, but less clear for mothers, because there are fewer managers and directors among women.

Figure 4 confirms that the observation from Figure 3 is statistically clear and very robust. Figure 4 shows the coefficient of parental occupation in a regression where the difference in parental profession is the dependent variable. We do the regression without controls, with year and grade fixed effects, and with a comprehensive list of controls. The results are very consistent for all specifications. For fathers, they are very clearly significant, and of similar in magnitude in all cases. They are also similar for mothers, but significance is only achieved when comprehensive controls are included.

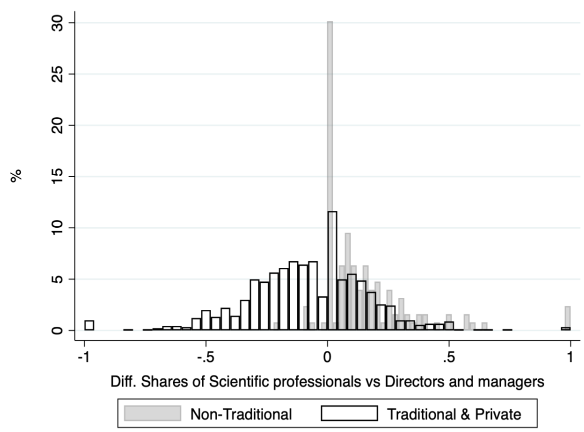
Overall, the hypothesis is well supported in the data. Obviously we are not making causal statements, and other explanations for the association could be proposed. But we hope the exercise illustrates the empirical possibilities of our approach.

[12] https://ludus.org.es/

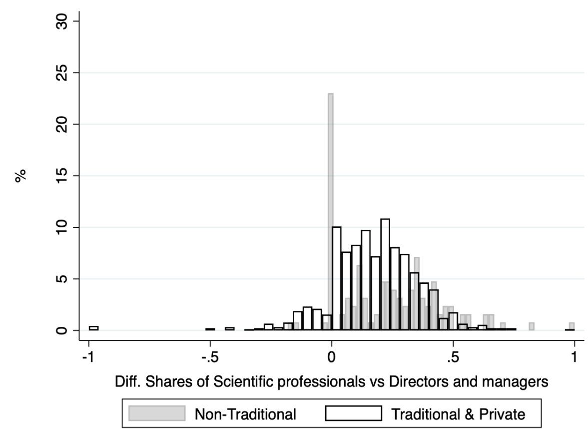
[13] We include the full list of "alternative" schools in the Appendix.

Figure 3 Histogram of School Differences on Share Parents with Technicians and Scientific Professionals vs Directors and Managers.

(a) Father Occupation



(b) Mother Occupation



(c) At least one parent in Occupation

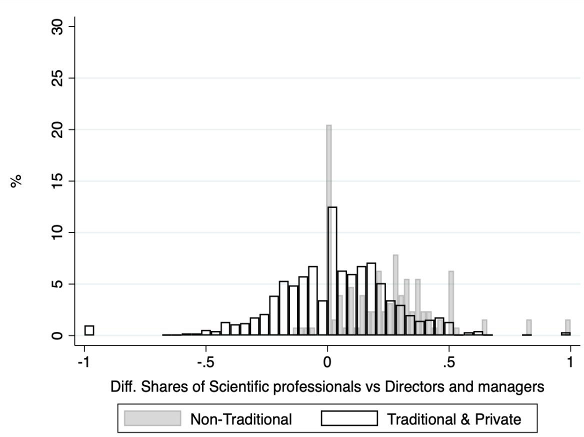
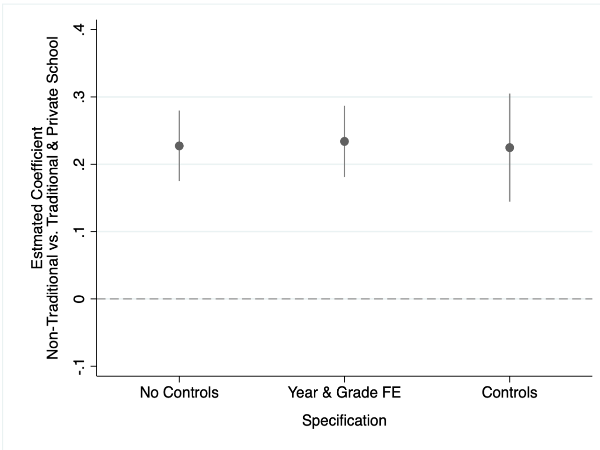
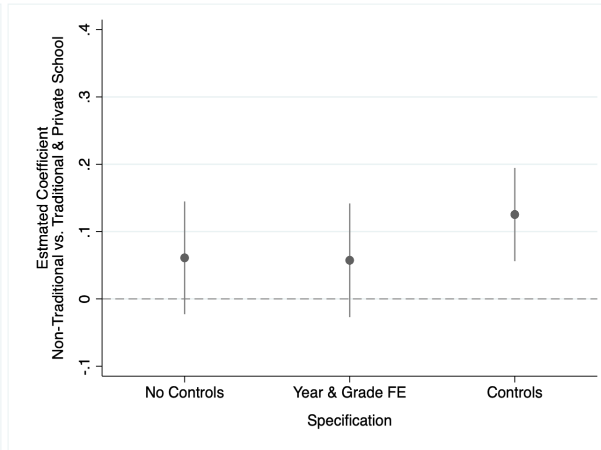


Figure 4 Estimated Coefficient of Non-Traditional vs. Traditional & Private school

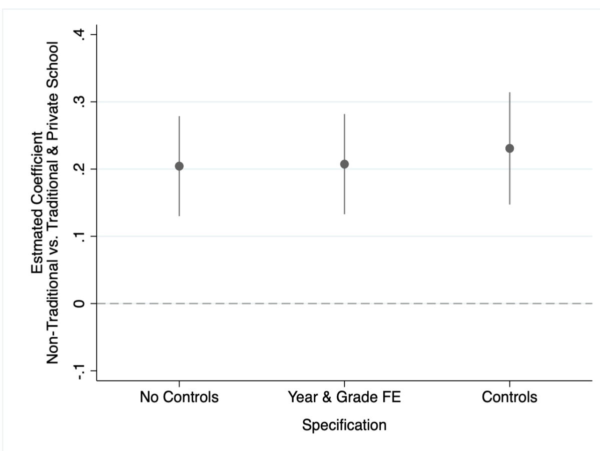
(a) Father Occupation



(b) Mother Occupation



(c) At least one parent in Occupation



Notes: This figure plots the estimated coefficient and 95% confidence intervals where the outcome variable is the difference between the school share of parents with Technicians and Scientific Professionals and Directors and Managers. Each coefficient comes from three separate regressions: (i) Regressing the outcome variable on an indicator for Non-Traditional school; (ii) Specification (i) adding year and grade fixed-effects; (iii) Specification (ii) adding school control variables. The school control variables used in the analysis are the following school shares: female, Spanish, Spanish Father, Spanish Mother, School start before two years old, Number of digital devices at home, Less than 50 books at home, Mother years of education, Father years of education, Mother full-time worker, Mother Unemployed, Father full-time worker, Father unemployed, Mother occupation Elementary occupation or lower, father occupation Elementary occupation or lower, mother occupation Administrative staff, father occupation Administrative staff, Dummy for missing occupation. Robust standard errors are clustered at the school level .

8 Concluding remarks

In this paper we developed a model that allows to understand the circumstances under which a society, or groups within a society, may decide to pursue a collaborative education model or an individualistic one. Externalities in collaborative learning can induce multiple equilibria even in a homogeneous society and lead to inefficiencies. We proposed policies to achieve the more efficient equilibrium. A key parameter for policy is parental perceived return from collaborative learning μ. If that parameter is high, policies only need to shift parental expectations about the other parents' behavior to coordinate the behavior on collaboration. However, if this parameter is low policy measures that go beyond coordinating parental behavior are required.

Collaborative learning is a newer technology than individualistic learning. Therefore the benefits from collaborative learning are less well understood.[14] This justifies the presence of the parental skepticism parameter concerning the returns from collaborative learning in our model. Early adopters of the collaborative technology tend to be parents who have invested in collecting the information about collaborative learning (the high μ parents in our model). This research, and the realization that their children welfare could be superior if surrounded by other collaborative students, may induce those parents to segregate their children into "alternative" schools. This is a costly process, and it justifies our forward induction criterion whereby pioneer parents coordinate their expectations on an optimistic outcome. Pioneers anticipate their children's classmates after segregation are likely to choose collaboration. Choosing the costly path to segregate and then adopting an individualistic strategy is not very sensible option.

More research on the benefits and shortcomings of collaborative learning becomes available over time. The technology itself is improved by incorporating the new knowledge about when collaborative learning works best. Thus, the parental skepticism parameter is likely to diminish, resulting in a higher μ as time goes by,

[14] Dell et al. (1989) study the perception of parents towards cooperative learning in elementary classrooms via questioneers. Interestingly they checked whether parents needed or wanted more information about cooperative learning. As a group parents were undecided whether they knew sufficiently about cooperative learning but most parents wanted to learn more about it.

leading to a higher adoption of the technology. Indeed, there has been an increase in the types of education that parents choose for their children around the world (Plank and Skype 2003).[15] Starting in the Western context, alternative education with emphasis on collaboration gradually has gained increasing popularity in some Asian countries such as Japan, Korea, Thailand, and Sri Lanka (Nagata 2007) and even in China (Xu and Spruyt 2022) and Hong-Kong (Chan and Yeung 2020).

Nevertheless, differences in the parental skepticism parameter are likely to persist even with better knowledge about the returns from cooperative learning. Indeed, these differences might reflect preferences about cultural education styles which are hard to shift. With this interpretation, a fruitful avenue for future research is to study the intergenerational transmission of preferences about the parental skepticism parameter in the spirit of Bisin and Verdier (2000, 2001). Another interesting future extension of our main model is to allow parents to influence the return to collaborative effort K_2. On the one hand, parental involvement can increase the returns from collaborative learning in general. On the other hand, parents might also be interested in manipulating the externality in a way that collaborating with classmates with a specific characteristic is much more rewarding to their children than with classmates that lack this specific characteristic to avoid their children to interact with a certain subgroup of the population. It will be interesting to analyze the effects on overall efficiency in this context.

Appendix

8.1 Proof of Lemma 1

Proof. Using the best responses (5) and (6) of parents we find the symmetric pure strategy equilibrium by setting (i) $x = 1$ and $a = \min\left[\overline{a}, \frac{R}{K_1}\right]$ implying $\overline{x} = 1$ and

[15] To give a few examples: In Spain the demand for project based learning has surged (Martínez-Celorrio 2016), Montessori education in the US has made its way into the public sector (National Center for Montessori in the Public Sector 2014). According to the report by Stehlik and Stehlik (2019) Waldorf school had experienced and exponential growth and by 2019 there were 1100 Waldorf Schools and 2000 Waldorf Kindergartens in more than 80 countries.

(ii) $a = \overline{a}$ and $x = 0$ implying $\overline{x} = 0$. Hence parental best responses become

$$\begin{aligned} a &= \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ for } \max\{\underline{a}K_1, \overline{a}\mu K_1\} < R \text{ implying } x = 1 \\ a &= \overline{a} \text{ for } R < \max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\} \text{ implying } x = 0 \end{aligned}$$

leading to the following the pure symmetric strategy equilibria

- $a = \overline{a}$ and $x = 0$ for

$$\begin{aligned} & R < \min\{\max\{K_1\underline{a}, \overline{a}\mu K_1\}, \max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\}\} \\ & = \max\{K_1\underline{a}, \overline{a}\mu K_1\} \end{aligned}$$

- $\left.\begin{array}{c} a = \overline{a} \text{ and } x = 0 \\ a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \end{array}\right\}$ for

$$\max\{K_1\underline{a}, \overline{a}\mu K_1\} \leq R \leq \max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\}$$

- $a = \min\left[\overline{a}, \frac{R}{K_1}\right]$ and $x = 1$ for $R > \max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\}$. Observe $\max\{K_1\underline{a}, \overline{a}\mu K_1\} = K_1\underline{a}$ iff $\mu < \frac{\underline{a}}{\overline{a}}$.

Observe $\max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\} = \underline{a}(K_1+K_2)$ iff $\mu < \frac{\underline{a}}{\overline{a}}$ which allows us to restate the symmetric pure strategy equilibria as in Lemma 1. ■

8.2 Equilibria with heterogeneous μ

8.2.1 Homogeneous equilibria

We first examine equilibria where all children/parents make the same choices:

- All children choose $x_i = 1$.

$$a_i = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ for } \max\{\underline{a}(K_1), \overline{a}\mu_i K_1\} < R \text{ implying } x_i = 1$$

This has to be true for both types, so it is binding for the μ_H

$$a_i = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ for } \max\left\{\underline{a}\left(K_1\right), \overline{a}\mu_H K_1\right\} < R \text{ implying } x_i = 1$$

or equivalently

$$a_i = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ implying } x_i = 1 \text{ for } \begin{cases} \underline{a}K_1 < R \text{ if } \mu_H < \frac{\underline{a}}{\overline{a}} \\ \overline{a}\mu_H K_1 < R \text{ if } \mu_H > \frac{\underline{a}}{\overline{a}} \end{cases}$$

- All children choose $x_i = 0$ if

$$a_i = \overline{a} \text{ for } R < \max\left\{\underline{a}\left(K_1 + K_2\right), \overline{a}\mu_i\left(K_1 + K_2\right)\right\} \text{ implying } x_i = 0$$

The binding one is for the μ_L

$$a_i = \overline{a} \text{ for } R < \max\left\{\underline{a}\left(K_1 + K_2\right), \overline{a}\mu_L\left(K_1 + K_2\right)\right\} \text{ implying } x_i = 0$$

or equivalently

$$a_i = \overline{a} \text{ implying } x_i = 0 \text{ for} \begin{cases} R < \underline{a}\left(K_1 + K_2\right), \text{ if } \mu_L < \frac{\underline{a}}{\overline{a}} \\ R < \overline{a}\mu_L\left(K_1 + K_2\right) \text{ if } \mu_L > \frac{\underline{a}}{\overline{a}} \end{cases}$$

8.2.2 Heterogeneous equilibria

The mixed equilibrium where each type of parent makes a different choice, and thus the children as well. This means $x_H = 0$ and $x_L = 1$ requires for the μ_L

$$a_L = \min\left[\overline{a}, \frac{R}{K_1 + \gamma K_2}\right] \text{ for}$$
$$\max\left\{\underline{a}\left(K_1 + \gamma K_2\right), \overline{a}\mu_L\left(K_1 + \gamma K_2\right)\right\} < R \text{ implying } x_i = 1$$

and for the μ_H

$$a_H = \overline{a} \text{ for } R < \max\left\{\underline{a}\left(K_1+\gamma K_2\right), \overline{a}\mu_H\left(K_1+\gamma K_2\right)\right\} \text{ implying } x_i = 0$$

so combining the two requires

$$\begin{aligned} &\max\left\{\underline{a}\left(K_1+\gamma K_2\right), \overline{a}\mu_L\left(K_1+\gamma K_2\right)\right\} < R \\ < \ &\max\left\{\underline{a}\left(K_1+\gamma K_2\right), \overline{a}\mu_H\left(K_1+\gamma K_2\right)\right\} \end{aligned}$$

This is generically not possible for $\mu_H < \frac{\underline{a}}{\overline{a}}$. The condition is possible when $\mu_H > \frac{\underline{a}}{\overline{a}}$ and can be rewritten as

$$\begin{aligned} \overline{a}\mu_L\left(K_1+\gamma K_2\right) &< R < \overline{a}\mu_H\left(K_1+\gamma K_2\right) \text{ if } \mu_L > \frac{\underline{a}}{\overline{a}} \text{ and } \mu_H > \frac{\underline{a}}{\overline{a}} \text{ or} \\ \underline{a}\left(K_1+\gamma K_2\right) &< R < \overline{a}\mu_H\left(K_1+\gamma K_2\right) \text{ if } \mu_H > \frac{\underline{a}}{\overline{a}} > \mu_L \end{aligned}$$

8.2.3 Full equilibrium analysis

Proposition 6 *1. for $\mu_H < \frac{\underline{a}}{\overline{a}}$ the equilibria are*

- *uniqueness in $x_i = 0$ for all i when $R < \underline{a}K_1$.*
- *coexistence of $x_i = 0$ for all i and $x_i = 1$ for all i when $\underline{a}K_1 \leq R \leq \underline{a}\left(K_1+K_2\right)$*
- *uniqueness in $x_i = 1$ for all i when $R > \underline{a}\left(K_1+K_2\right)$*

2. For $\mu_H > \frac{\underline{a}}{\overline{a}} > \mu_L$ and

(a) For $\frac{\underline{a}}{\overline{a}}\frac{\left(K_1+\gamma K_2\right)}{K_1} > \mu_H > \frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2} > \frac{\underline{a}}{\overline{a}} > \mu_L$ and $\mu_H < \frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}$ or equivalently $\min\left\{\frac{\underline{a}}{\overline{a}}\frac{\left(K_1+\gamma K_2\right)}{K_1}, \frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}\right\} > \mu_H > \frac{\underline{a}}{\overline{a}} > \mu_L$ the equilibria are

- *uniqueness in $x_i = 0$ for all i when $R < \overline{a}\mu_H K_1$*
- *coexistence of $x_i = 0$ for all i and $x_i = 1$ for all i when $\overline{a}\mu_H K_1 < R < \underline{a}\left(K_1+\gamma K_2\right)$*

- *coexistence of all three when* $\underline{a}\left(K_1+\gamma K_2\right)<R<\overline{a}\mu_H\left(K_1+\gamma K_2\right)$
- *coexistence of* $x_i=0$ *for all* i *and* $x_i=1$ *for all* i *when* $\overline{a}\mu_H\left(K_1+\gamma K_2\right)<R<\underline{a}\left(K_1+K_2\right)$
- *uniqueness in* $x_i=1$ *for all* i *when* $R>\underline{a}\left(K_1+K_2\right)$

(b) *For* $\min\left\{\frac{\underline{a}}{\overline{a}}\frac{(K_1+\gamma K_2)}{K_1},\frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}\right\}<\mu_H<\max\left\{\frac{\underline{a}}{\overline{a}}\frac{(K_1+\gamma K_2)}{K_1},\frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}\right\}$ *and* $\mu_L<\frac{\underline{a}}{\overline{a}}$

i. *for* $\frac{\underline{a}}{\overline{a}}\frac{(K_1+\gamma K_2)}{K_1}<\mu_H<\frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}\left(<\frac{\underline{a}}{\overline{a}}\frac{(K_1+K_2)}{K_1}\right)$ *and* $\mu_L<\frac{\underline{a}}{\overline{a}}$ *the equilibria are:*

$$\textit{uniqueness in } x_i=0 \textit{ for all } i \textit{ when } R<\underline{a}\left(K_1+\gamma K_2\right)$$

$$\begin{aligned}\textit{coexistence of } x_i &= 0 \textit{ for all } i \textit{ and mixed when}\\ \underline{a}\left(K_1+\gamma K_2\right) &< R<\overline{a}\mu_H K_1\end{aligned}$$

$$\textit{coexistence of all three when } \overline{a}\mu_H K_1<R<\overline{a}\mu_H\left(K_1+\gamma K_2\right)$$

$$\begin{aligned}\textit{coexistence of } \begin{cases} x_i=0 \textit{ for all } i\\ x_i=1 \textit{ for all } i\end{cases} \textit{for all } i \textit{ when}\\ \overline{a}\mu_H\left(K_1+\gamma K_2\right) < R<\underline{a}\left(K_1+K_2\right)\end{aligned}$$

$$\textit{uniqueness in } x_i=1 \textit{ for all } i \textit{ when } R>\underline{a}\left(K_1+K_2\right)$$

ii. *For* $\frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}<\mu_H<\frac{\underline{a}}{\overline{a}}\frac{(K_1+\gamma K_2)}{K_1}\left(<\frac{\underline{a}}{\overline{a}}\frac{(K_1+K_2)}{K_1}\right)$ *and* $\mu_L<\frac{\underline{a}}{\overline{a}}$ *the equilibria are:*

$$\textit{uniqueness in } x_i=0 \textit{ for all } i \textit{ when } R<\overline{a}\mu_H K_1$$

$$\textit{coexistence of } \begin{cases} x_i=0 \textit{ for all } i\\ x_i=1 \textit{ for all } i\end{cases} \textit{when } \overline{a}\mu_H K_1<R<\underline{a}\left(K_1+\gamma K_2\right)$$

$$\textit{coexistence of all three when } \underline{a}\left(K_1+\gamma K_2\right)<R<\underline{a}\left(K_1+K_2\right)$$

$$\begin{aligned} \textit{coexistence of } x_i &= 1 \textit{ for all } i \textit{ and mixed when} \\ \underline{a}\left(K_1+K_2\right) &< R<\overline{a}\mu_H\left(K_1+\gamma K_2\right) \end{aligned}$$

uniqueness in $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_H\left(K_1+\gamma K_2\right)$

(c) *For* $\mu_L < \frac{\underline{a}}{\overline{a}} < \max\left\{\frac{\underline{a}}{\overline{a}}\frac{(K_1+\gamma K_2)}{K_1}, \frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}\right\} < \mu_H < \frac{\underline{a}}{\overline{a}}\frac{(K_1+K_2)}{K_1}$ *the equilibria are:*

- *uniqueness in* $x_i = 0$ *for all* i *when* $R < \underline{a}\left(K_1+\gamma K_2\right)$
- *coexistence of* $x_i = 0$ *for all* i *and mixed when* $\underline{a}\left(K_1+\gamma K_2\right) < R < \overline{a}\mu_H K_1$
- *coexistence of all three when* $\overline{a}\mu_H K_1 < R < \underline{a}\left(K_1+K_2\right)$
- *coexistence of* $x_i = 1$ *for all* i *and mixed when* $\underline{a}\left(K_1+K_2\right) < R < \overline{a}\mu_H\left(K_1+\gamma K_2\right)$
- *uniqueness in* $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_H\left(K_1+\gamma K_2\right)$

(d) *for* $\mu_H > \frac{\underline{a}}{\overline{a}}\frac{(K_1+K_2)}{K_1}$, $\frac{\underline{a}}{\overline{a}} > \mu_L$ *the equilibria are:*

- *uniqueness in* $x_i = 0$ *for all* i *when* $R < \underline{a}\left(K_1+\gamma K_2\right)$
- *coexistence of* $x_i = 0$ *for all* i *and mixed when* $\underline{a}\left(K_1+\gamma K_2\right) < R < \underline{a}\left(K_1+K_2\right)$
- *mixed when* $\underline{a}\left(K_1+K_2\right) < R < \overline{a}\mu_H K_1$
- *coexistence of* $x_i = 1$ *for all* i *and mixed when* $\overline{a}\mu_H K_1 < R < \overline{a}\mu_H\left(K_1+\gamma K_2\right)$
- *uniqueness in* $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_H\left(K_1+\gamma K_2\right)$

3. *Let* $\mu_H > \mu_L > \frac{\underline{a}}{\overline{a}}$

(a) *If* $\mu_L > \frac{\underline{a}}{\overline{a}}$ *and* $\mu_H < \min\left\{\mu_L\frac{(K_1+\gamma K_2)}{K_1}, \mu_L\frac{(K_1+K_2)}{(K_1+\gamma K_2)}\right\}$ *the equilibria are:*

- *uniqueness in* $x_i = 0$ *for all* i *when* $R < \overline{a}\mu_H K_1$
- *coexistence of* $x_i = 0$ *for all* i *and* $x_i = 1$ *for all* i *when* $\overline{a}\mu_H K_1 < R < \overline{a}\mu_L\left(K_1+\gamma K_2\right)$
- *coexistence of all three when* $\overline{a}\mu_L\left(K_1+\gamma K_2\right) < R < \overline{a}\mu_H\left(K_1+\gamma K_2\right)$

- *coexistence of* $x_i = 0$ *for all* i *and* $x_i = 1$ *for all* i *when* $\overline{a}\mu_H (K_1 + \gamma K_2) < R < \overline{a}\mu_L (K_1 + K_2)$
- *uniqueness in* $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_L (K_1 + K_2)$

(b) *Let* $\mu_L > \frac{\underline{\underline{a}}}{\overline{a}}$ *and*

$$\min\left\{\mu_L \frac{(K_1 + \gamma K_2)}{K_1}, \mu_L \frac{(K_1 + K_2)}{(K_1 + \gamma K_2)}\right\} < \mu_H$$
$$< \max\left\{\mu_L \frac{(K_1 + \gamma K_2)}{K_1}, \mu_L \frac{(K_1 + K_2)}{(K_1 + \gamma K_2)}\right\}$$

i. $\mu_L \frac{(K_1+\gamma K_2)}{K_1} < \mu_H < \mu_L \frac{(K_1+K_2)}{(K_1+\gamma K_2)}$ *the equilibria are*

uniqueness in $x_i = 0$ *for all* i *when* $R < \overline{a}\mu_L (K_1 + \gamma K_2)$

$$\begin{aligned} \textit{coexistence of } x_i &= 0 \textit{ for all } i \textit{ and mixed when} \\ \overline{a}\mu_L (K_1 + \gamma K_2) &< R < \overline{a}\mu_H K_1 \end{aligned}$$

$$\begin{aligned} &\textit{coexistence of all three equilibria when} \\ \overline{a}\mu_H K_1 &< R < \overline{a}\mu_H (K_1 + \gamma K_2) \end{aligned}$$

$$\begin{aligned} &\textit{coexistence of} \begin{cases} x_i = 0 \textit{ for all } i \\ x_i = 1 \textit{ for all } i \end{cases} \textit{when} \\ \overline{a}\mu_H (K_1 + \gamma K_2) &< R < \overline{a}\mu_L (K_1 + K_2) \end{aligned}$$

uniqueness in $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_L (K_1 + K_2)$

ii. $\mu_L \frac{(K_1+K_2)}{(K_1+\gamma K_2)} < \mu_H < \mu_L \frac{(K_1+\gamma K_2)}{K_1}$ *the equilibria are:*

uniqueness in $x_i = 0$ *for all* i *when* $R < \overline{a}\mu_H K_1$

$$\text{coexistence of} \begin{cases} x_i = 0 \text{ for all } i \\ x_i = 1 \text{ for all } i \end{cases} \text{ when}$$
$$\overline{a}\mu_H K_1 < R < \overline{a}\mu_L (K_1 + \gamma K_2)$$

$$\text{coexistence of all three when}$$
$$\overline{a}\mu_L (K_1 + \gamma K_2) < R < \overline{a}\mu_L (K_1 + K_2)$$

$$\text{coexistence of} \begin{cases} x_i = 0 \text{ for all } i \\ \text{mixed} \end{cases} \text{ when}$$
$$\overline{a}\mu_L (K_1 + K_2) < R < \overline{a}\mu_H (K_1 + \gamma K_2)$$

uniqueness in $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_H (K_1 + \gamma K_2)$

(c) *Let* $\mu_L > \frac{\underline{a}}{\overline{a}}$ *and* $\max\left\{\mu_L \frac{(K_1+\gamma K_2)}{K_1}, \mu_L \frac{(K_1+K_2)}{(K_1+\gamma K_2)}\right\} < \mu_H < \mu_L \frac{(K_1+K_2)}{K_1}$ *the equilibria are:*

- *uniqueness in* $x_i = 0$ *for all* i *when* $R < \overline{a}\mu_L (K_1 + \gamma K_2)$
- *coexistence of* $x_i = 0$ *for all* i *and mixed when* $\overline{a}\mu_L (K_1 + \gamma K_2) < R < \overline{a}\mu_H K_1$
- *coexistence of all three equilibria when* $\overline{a}\mu_H K_1 < R < \overline{a}\mu_L (K_1 + K_2)$
- *coexistence of* $x_i = 1$ *for all* i *mixed when* $\overline{a}\mu_L (K_1 + K_2) < R < \overline{a}\mu_H (K_1 + \gamma K_2)$
- *uniqueness in* $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_H (K_1 + \gamma K_2)$

(d) *Let* $\mu_L > \frac{\underline{a}}{\overline{a}}$ *and* $\mu_H > \mu_L \frac{(K_1+K_2)}{K_1}$ *the equilibria are:*

- *uniqueness in* $x_i = 0$ *for all* i *when* $R < \overline{a}\mu_L (K_1 + \gamma K_2)$
- *coexistence of* $x_i = 0$ *for all* i *and mixed when* $\overline{a}\mu_L (K_1 + \gamma K_2) < R < \overline{a}\mu_L (K_1 + K_2)$
- *mixed when* $\overline{a}\mu_L (K_1 + K_2) < R < \overline{a}\mu_H K_1$
- *coexistence of* $x_i = 1$ *for all* i *and mixed when* $\overline{a}\mu_H K_1 < R < \overline{a}\mu_H (K_1 + \gamma K_2)$

- *uniqueness in* $x_i = 1$ *for all* i *when* $R > \overline{a}\mu_H\left(K_1 + \gamma K_2\right)$

Proof. We need to understand and rank and combine the different conditions for which the equilibria exist. For the different cases the conditions are:

1. Let $\mu_H < \frac{\underline{a}}{\overline{a}}$.
   - Then the high individual effort equilibrium exists for $\underline{a}K_1 < R$
   - The low individual effort equilibrium exists for $R < \underline{a}\left(K_1 + K_2\right)$
   - The mixed equilibrium does not exist
2. Let $\mu_H > \frac{\underline{a}}{\overline{a}} > \mu_L$
   - Then the high individual effort equilibrium exists for $\overline{a}\mu_H K_1 < R$
   - The low individual effort equilibrium exists for $R < \underline{a}\left(K_1 + K_2\right)$
   - The mixed equilibrium exists for $\underline{a}\left(K_1 + \gamma K_2\right) < R < \overline{a}\mu_H\left(K_1 + \gamma K_2\right)$
   - Observe that $\overline{a}\mu_H K_1 < \underline{a}\left(K_1 + K_2\right)$ iff $\mu_H < \frac{\underline{a}}{\overline{a}}\frac{(K_1+K_2)}{K_1}$
   - Also $\underline{a}\left(K_1 + \gamma K_2\right) < \underline{a}\left(K_1 + K_2\right)$ always
   - $\overline{a}\mu_H K_1 < \overline{a}\mu_H\left(K_1 + \gamma K_2\right)$ always
   - $\underline{a}\left(K_1 + \gamma K_2\right) < \overline{a}\mu_H K_1$ whenever $\mu_H > \frac{\underline{a}}{\overline{a}}\frac{(K_1+\gamma K_2)}{K_1}$
   - $\overline{a}\mu_H\left(K_1 + \gamma K_2\right) < \underline{a}\left(K_1 + K_2\right)$ iff $\mu_H < \frac{\underline{a}}{\overline{a}}\frac{K_1+K_2}{K_1+\gamma K_2}$
3. Let $\mu_H > \mu_L > \frac{\underline{a}}{\overline{a}}$
   - Then the high individual effort equilibrium exists for $\overline{a}\mu_H K_1 < R$
   - The low individual effort equilibrium exists for $R < \overline{a}\mu_L\left(K_1 + K_2\right)$
   - The mixed equilibrium exists for $\overline{a}\mu_L\left(K_1 + \gamma K_2\right) < R < \overline{a}\mu_H\left(K_1 + \gamma K_2\right)$
   - Observe that
     - $\overline{a}\mu_L\left(K_1 + \gamma K_2\right) < \overline{a}\mu_L\left(K_1 + K_2\right)$ always and $\overline{a}\mu_H K_1 < \overline{a}\mu_H\left(K_1 + \gamma K_2\right)$ always

- $\overline{a}\mu_L(K_1+K_2) > \overline{a}\mu_H K_1$ iff $\mu_L > \mu_H \frac{K_1}{(K_1+K_2)}$ or equivalently $\mu_H < \mu_L \frac{(K_1+K_2)}{K_1}$
- $\overline{a}\mu_L(K_1+\gamma K_2) > \overline{a}\mu_H K_1$ iff $\mu_L > \mu_H \frac{K_1}{(K_1+\gamma K_2)}$ or equivalently $\mu_H < \mu_L \frac{(K_1+\gamma K_2)}{K_1}$
- $\overline{a}\mu_H(K_1+\gamma K_2) < \overline{a}\mu_L(K_1+K_2)$ iff $\mu_L > \mu_H \frac{(K_1+\gamma K_2)}{(K_1+K_2)}$ or equivalently $\mu_H < \mu_L \frac{(K_1+K_2)}{(K_1+\gamma K_2)}$

- now $\frac{(K_1+\gamma K_2)}{K_1} < \frac{(K_1+K_2)}{K_1}$ always and $\frac{(K_1+K_2)}{(K_1+\gamma K_2)} < \frac{(K_1+K_2)}{K_1}$ always
- $\frac{(K_1+K_2)}{(K_1+\gamma K_2)} < \frac{(K_1+\gamma K_2)}{K_1}$ if $K_1(K_1+K_2) < (K_1+\gamma K_2)(K_1+\gamma K_2)$

$$K_1 < 2\gamma K_1 + \gamma^2 K_2$$

- So γ has to be big enough for this to be satisfied.

■

Secession by μ_L parents

$$a(K_1+K_2(1-\overline{x}))(1-x) + Rx(1-\delta(1-\overline{x})) \tag{21}$$

where the $\delta \in (0,1)$ coefficient reflects the negative externality which the fraction of collaborative effort students generate on the individualistic ones. This can be rewritten as

$$a(K_1+K_2(1-\overline{x})) + (R(1-\delta(1-\overline{x})) - a(K_1+K_2(1-\overline{x})))\,x$$

Children choose x to maximize (1) yielding the best reply

$$x_1 = \begin{cases} 1 \text{ if } R > \frac{a(K_1+K_2(1-\overline{x}))}{1-\delta(1-\overline{x})} \\ 0 \text{ if } R \leq \frac{a(K_1+K_2(1-\overline{x}))}{1-\delta(1-\overline{x})} \end{cases} \tag{22}$$

The parental utility with $a \in [\underline{a}, \overline{a}]$ is now given by

$$U^P(a) = \mu a(K_1+(1-\overline{x})K_2)(1-x_1) + Rx_1(1-\delta(1-\overline{x})) \tag{23}$$

Using the child's best response (22) parental utility becomes

$$U^P(a) = \begin{cases} R \text{ if } \frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} \geq a \\ \mu a\left(K_1 + (1-\overline{x})K_2\right) \text{ if } \frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} < a \end{cases} \tag{24}$$

If $\frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} > \overline{a}$ children will always choose not to collaborate for all possible values of a. If $\frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} < \underline{a}$ children will always want to collaborate, hence parent's best response is to choose $\overline{a}$. For $\underline{a} < \frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} < \overline{a}$ the choice of a determines whether or not children make individual effort. Parents want their child to make individual effort if $\mu\overline{a}\left(K_1 + (1-\overline{x})K_2\right) \leq R\left(1-\delta\left(1-\overline{x}\right)\right)$, or equivalently when $\overline{a} \leq \frac{R(1-\delta(1-\overline{x}))}{\mu(K_1+(1-\overline{x})K_2)}$, so the combined condition for making individual effort in this interval is $\frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} \leq \overline{a} \leq \frac{R(1-\delta(1-\overline{x}))}{\mu(K_1+(1-\overline{x})K_2)}$ and $x = 1$ and $U^P = U^C = R\left(1-\delta\left(1-\overline{x}\right)\right)$. Otherwise if in this interval $\overline{a} > \frac{R(1-\delta(1-\overline{x}))}{\mu(K_1+(1-\overline{x})K_2)}$ parents want their children to collaborate.

We assume that children are born with $\overline{a}$ and the investment of parents consists in reducing a. If there was a tiny cost $\varepsilon \to 0$ per unit of reduction in a then the parental best reply would be as follows:

1. $a = \overline{a}$ for $\frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} > \overline{a}$ leading to $U^P = U^C = R\left(1-\delta\left(1-\overline{x}\right)\right)$
2. $a = \frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2}$ if $\underline{a} < \frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} < \overline{a} \leq \frac{R(1-\delta(1-\overline{x}))}{\mu(K_1+(1-\overline{x})K_2)}$ leading to $U^P = U^C = R\left(1-\delta\left(1-\overline{x}\right)\right)$
3. $a = \overline{a}$ if $\underline{a} < \frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} < \overline{a}$,and $\frac{R(1-\delta(1-\overline{x}))}{\mu(K_1+(1-\overline{x})K_2)} \leq \overline{a}$ leading to $U^P = \mu\overline{a}\left(K_1 + (1-\overline{x})K_2\right)$ and $U^C = \overline{a}\left(K_1 + (1-\overline{x})K_2\right)$
4. $a = \overline{a}$ if $\frac{R(1-\delta(1-\overline{x}))}{K_1+(1-\overline{x})K_2} < \underline{a}$ leading to $U^P = \mu\overline{a}\left(K_1 + (1-\overline{x})K_2\right)$ and $U^C = \overline{a}\left(K_1 + (1-\overline{x})K_2\right)$

We can join the first and second line and the third and fourth line to obtain the simplified expression for the parental best response taking into account the own child's best response as

$$
\begin{aligned}
& a = \min\left[\overline{a}, \frac{R}{K_1 + (1-\overline{x})K_2}\right] \text{ for} \\
& \max\left\{\underline{a}\left(K_1 + (1-\overline{x})K_2\right), \overline{a}\mu\left(K_1 + (1-\overline{x})K_2\right)\right\} < R\left(1 - \delta(1-\overline{x})\right) \\
& a = \ \overline{a} \text{ for} \\
& R\left(1 - \delta(1-\overline{x})\right) < \max\left\{\underline{a}\left(K_1 + (1-\overline{x})K_2\right), \overline{a}\mu\left(K_1 + (1-\overline{x})K_2\right)\right\}
\end{aligned}
$$

Lemma 7

1. If $\mu \leq \frac{\underline{a}}{\overline{a}}$ we have the following symmetric pure strategy equilibria,

$$
a = \overline{a} \text{ and } x = 0 \text{ for } R \leq \frac{K_1 \underline{a}}{1-\delta}
$$

$$
\left.\begin{array}{c} a = \overline{a} \text{ and } x = 0 \\ a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \end{array}\right\} \text{ for } \frac{K_1 \underline{a}}{1-\delta} \leq R \leq \frac{\underline{a}(K_1 + K_2)}{1-\delta}
$$

$$
a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \text{ for } R > \frac{\underline{a}(K_1 + K_2)}{1-\delta}
$$

2. If $\mu > \frac{\underline{a}}{\overline{a}}$ we have the following symmetric pure strategy equilibria

$$
a = \overline{a} \text{ and } x = 0 \text{ for } R < \frac{\overline{a}\mu K_1}{1-\delta}
$$

$$
\left.\begin{array}{c} a = \overline{a} \text{ and } x = 0 \\ a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \end{array}\right\} \text{ for } \frac{\overline{a}\mu K_1}{1-\delta} \leq R \leq \frac{\overline{a}\mu(K_1 + K_2)}{1-\delta}
$$

$$
a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \text{ for } R > \frac{\overline{a}\mu(K_1 + K_2)}{1-\delta}
$$

Proof. Using the best responses (5) and (6) of parents we find the symmetric pure strategy equilibrium by setting (i) $x = 1$ and $a = \min\left[\overline{a}, \frac{R}{K_1}\right]$ implying $\overline{x} = 1$ and

(ii) $a = \overline{a}$ and $x = 0$ implying $\overline{x} = 0$. Hence parental best responses become

$$a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ for } \frac{\max\{\underline{a}K_1, \overline{a}\mu K_1\}}{1-\delta} < R \text{ implying } x = 1$$
$$a = \overline{a} \text{ for } R < \frac{\max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\}}{1-\delta} \text{ implying } x = 0$$

leading to the following the pure symmetric strategy equilibria

- $a = \overline{a}$ and $x = 0$ for $R < \frac{\max\{K_1\underline{a}, \overline{a}\mu K_1\}}{1-\delta}$
- $\left.\begin{array}{c} a = \overline{a} \text{ and } x = 0 \\ a = \min\left[\overline{a}, \frac{R}{K_1}\right] \text{ and } x = 1 \end{array}\right\}$ for $\frac{\max\{K_1\underline{a}, \overline{a}\mu K_1\}}{1-\delta} \leq R \leq \frac{\max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\}}{1-\delta}$
- $a = \min\left[\overline{a}, \frac{R}{K_1}\right]$ and $x = 1$ for $R > \frac{\max\{\underline{a}(K_1+K_2), \overline{a}\mu(K_1+K_2)\}}{1-\delta}$Observe $\max\{K_1\underline{a}, \overline{a}\mu K_1\} = K_1\underline{a}$ iff $\mu < \frac{\underline{a}}{\overline{a}}$.

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