Anchoring Boundedly Rational Expectations

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Abstract

How can a central bank avoid losing control over inflation expectations? Under rational expectations, respecting the Taylor principle—increasing rates more than one for one with inflation—prevents self-fulfilling inflation and defines an active monetary policy. We reconsider the issue away from rational expectations, for a large class of boundedly rational expectations featuring both limited foresight and long-term learning. We show three results. (1) When restricting monetary policy to a Taylor rule, the Taylor principle does not prevent self-fulfilling inflation but hyperinflation spirals, unless for unrealistically high degrees of foresight. (2) Against hyperinflation spirals, active monetary policy can be characterized without restricting policy to a Taylor rule, as a sufficient increase of a weighted average of present and future expected policy rates. The weights on future policy rates first increase but then decrease with the horizon, implying that delaying hikes too much requires larger hikes later, with a larger drop in output. (3) Yet, being slow in hiking rates can be optimal provided a large cost on output stabilization, as it spreads the output cost over time.

Keywords: De-anchoring, Taylor Principle, Bounded Rationality.

JEL Classification: E52, E31, E43.

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Introduction

How can a central bank avoid losing control over inflation expectations in the face of a large supply shock? The question has come back front and center following the Covid pandemic, as the world economy experienced supply shocks and inflation levels of magnitude not seen in 40 years. Standard monetary models provide a well-known response to this question: Respecting the Taylor principle—i.e. increasing interest rates more than one-for-one with inflation—prevents inflation from becoming self-fulfilling and defines an active monetary policy (e.g. Woodford 2001).[1] The last worldwide episode of high inflation—the Great Inflation of the 1970s—has been linked to central banks' failure to respect the Taylor principle (Clarida, Galí, and Gertler 2000).

In standard models, the need to respect the Taylor principle is derived under the assumption of rational expectations. In this paper, we revisit the question of how to keep control over inflation expectations when expectations are instead boundedly rational.

What motivates us to move away from the particular case of rational expectations is a recent important qualification to the need to respect the Taylor Principle. The Taylor principle is necessary to prevent self-fulfilling inflation in baseline monetary models, but it no longer is in monetary models developed to solve the shortcomings of the baseline models. Baseline New-Keynesian models find forward guidance (FG) to have unrealistically large effects on inflation and output—the FG puzzle (Del Negro, Giannoni, and Patterson 2012, Carlstrom, Fuerst, and Paustian 2015). Several solutions to the FG puzzle have been proposed. Some, such as cognitive discounting, consist in departures from rational expectations, while others, such as household heterogeneity, do not.[2] But virtually all work by adding discounting to the baseline model. Yet, when adding enough discounting to the baseline model to solve the FG puzzle, a unique equilibrium with no self-fulfilling inflation obtains even under an interest-rate peg—a fully passive monetary policy. This would suggest that the need to adopt a sufficiently active monetary policy—to increase rates sufficiently in response to an inflationary shock—is an artifact of unrealistically forward-looking models. Yet, recent evidence confirms that too passive a monetary policy makes the central bank prone to losing control over inflation (for instance the recent unfortunate experience of Turkey—see Gurkaynak, Kisacikoglu, and Lee 2022).

We consider how to keep control over inflation expectations away from rational expectations by considering instead a large set of boundedly rational expectations. Specifically, we rely on Woodford (2019)'s model of bounded rationality, a generalization of the baseline New-Keynesian model under rational expectations, which obtains as a particular case (section 1). Woodford's model combines two forms of bounded rationality: finite planning horizons and long-term learning. Finite planning horizons make expectations less forward-looking than rational expectations, providing a solution to the FG puzzle. As such, they capture a larger

[1] See e.g. chapter 2 of Woodford (2003) and section 2 of Castillo-Martinez and Reis (2022) for excellent introductions to the Taylor principle and equilibrium determinacy under rational expectations.

[2] On household heterogeneity, see e.g. McKay, Nakamura, and Steinsson (2016), Bilbiie (2020, 2018), Werning (2015) and Acharya and Dogra (2020). On bounded rationality, see e.g. Woodford (2019), Farhi and Werning (2019), Gabaix (2020), Angeletos and Lian (2018), Dupraz, Le Bihan, and Matheron (forthcoming).

class of models of bounded rationality introduced as solutions to the FG puzzle, such as cognitive discounting (Gabaix 2020) and k-level thinking (Farhi and Werning 2019). Long-term learning makes expectations about the long run backward-looking, a feature introduced by Woodford to highlight the fragility of neo-Fisherianism predictions. More broadly, long-term learning captures a departure from rational expectations common to other models of learning, such as least squares learning (Evans and Honkapohja 2001). The combined FPH-learning model generalizes both cognitive discounting models and learning models, allowing expectations to be both backward and forward-looking. As Gust, Herbst, and Lopez-Salido (2023) show, it does a very good job at capturing the dynamics of expectations in surveys. In particular it can replicate the under and over-reaction of expectations to shocks that have been recently documented in surveys and used to discriminate between models of expectation formation (Coibion and Gorodnichenko 2015, Kohlhas and Walther 2021, Angeletos, Huo, and Sastry 2020).[3]

We show three results. First, we show that except for unrealistically high degrees of foresight, active monetary policy does not prevents self-fulfilling inflation but hyperinflation spirals. To do so, we restrict monetary policy to a Taylor rule and study under which conditions it delivers a unique bounded solution (section 2). We show that, bar knife-edge calibrations that we argue are not economically relevant, the Taylor principle remains necessary and sufficient for a unique bounded solution. But it prevents self-fulfilling inflation only for very high degrees of foresight under which the model is subject to the FG puzzle. Whenever foresight is low enough to avoid the FG puzzle, the risk that the Taylor principle prevents is instead the absence of bounded solutions: that the economy necessarily goes on an unbounded, hyperinflation path. This generalizes Woodford (2019)'s result of instability under an interest-rate peg. It also fills the gap between results on the Taylor principle obtained under rational expectations (i.e. purely forward-looking expectations) and results on the Taylor principle obtained in models with purely backward-looking expectations such as least squares learning (Bullard and Mitra 2002, Preston 2005).

What an active monetary policy prevents when the model is not subject to the FG puzzle is therefore very different from what it prevents under rational expectations. The cause of the hyperinflation spiral is a de-anchoring of inflation expectations, as long-run inflation expectations gradually diverge away from target.[4] It takes root in what Wicksell (1898) called the *cumulative process*. Under a passive monetary policy, a burst of inflation increases households' and firms' long-run inflation expectations, which decreases real rates, increases aggregate demand and further increases inflation even once the initial shock has dissipated, and so on in a snowballing spiral. This alternative risk of passive monetary policy, arguably closer to the main risk on central bankers' minds, has an intellectual history that predates risks of equilibrium indeterminacy.[5]

[3] For a review of the survey evidence against full-information rational-expectations, see e.g. Born, Enders, and Muller (2023).

[4] We therefore say that a policy *anchors expectations* if it prevents such divergence of inflation expectations by sufficiently increasing interest rates. Note that a situation of *anchored expectations* can alternatively be understood as a situation in which long-run expectations react little or not al all to recent realized inflation, even if the central bank does not increase rates. See e.g. Carvalho et al. (2023) and Gáti (2023). Because the set-up we consider assumes constant-gain learning, such a situation is excluded by assumption. The central bank can only keep inflation expectations close to target—anchor expectations—by actively bringing them back to target through interest rate hikes.

[5] Such explosive dynamics are also the dominant outcome in laboratory experiments by Assenza et al. (2021) when the Taylor principle is not respected. To completely get rid of such explosive paths in their experiments, the response of policy

Beyond Wicksell (1898), it underlies Friedman (1968)'s argument against pegging interest rates. It is also the risk of concern in purely backward-looking models of expectation-formation, such as least squares learning (Bullard and Mitra 2002, Evans and Honkapohja 2003b, Preston 2005).[6] Our first result shows that it is the risk that active monetary policy prevents not just in purely backward-looking models, but as soon as agents' degree of foresight is low enough to avoid the FG puzzle.

Second, we move beyond Taylor rules (section 3). As is well-known, when expectations are rational and the risk of passive monetary policy consists of equilibrium indeterminacy, monetary policy must necessarily be implemented through some Taylor-like feedback rule. Specifying monetary policy as an exogenous path for the policy rate results in equilibrium indeterminacy just like when the Taylor principle is not satisfied. We show that when the degree of foresight is low enough to rule out the FG puzzle and the risk of concern is instead hyperinflation spirals, characterizing active monetary policy no longer requires to specify monetary policy through a feedback rule. We define an active monetary policy as one that delivers a determinate bounded equilibrium where inflation returns to its target in the long run and show that there exists a large class of exogenous interest-rate paths that do so. This is summarized in Table 1. We characterize the class of interest rate paths that deliver active monetary policy. The characterization states that a weighted average of present and future expected policy rates must increase sufficiently. It captures the idea that inflationary shocks must be met by interest rate hikes, either today or at some point in the future. As such, it captures an intuition similar to the one behind the Taylor principle, but without restraining monetary policy to follow a Taylor rule.

We show that this characterization allows to determine how much interest rate hikes at different horizons matter for anchoring expectations. To this effect, we define what we call the relative anchoring effect of an interest rate at a given horizon: how much it matters for bringing inflation down to target in the long run relative to a contemporaneous interest rate hike. We show that the relative anchoring effect is the highest at intermediary horizons, striking a balance between two opposite forces. First, rate hikes at very short horizons matter little for bringing inflation down because short-term interest rates have by themselves little effect on aggregate demand and therefore inflation. This first effect is present under rational expectations as well but not in purely backward-looking models where aggregate demand does not depend on the entire yield curve beyond present short-term rates. Second, rate hikes at very long horizons matter little for bringing inflation down because, although they can always bring down inflation, it takes much larger future hikes to re-anchor expectations once expectations have had time to drift away. This second effect exists only when hyperinflation spirals are possible and is therefore absent under rational expectations. The horizon of interest rate hikes that matters most for bringing inflation back to target is intermediary, long enough to

rates to inflation need to be significantly above one-for-one. For a response only marginally above one-for-one, many participants extrapolate past trends, leading to explosive dynamics.

[6] In the models of Bullard and Mitra (2002) and Preston (2005) which feature decreasing-gain learning, respecting the Taylor principle actually also guarantees convergence to the rational expectations equilibrium. It therefore wards off both hyperinflation spirals on the way to rational expectations, and self-fulfilling inflation once the economy has converged to rational expectations. In our set-up under constant-gain learning, respecting the Taylor principle does not make the economy converge to rational expectations and only prevent hyperinflation spirals.

Table 1: Outcome Depending on Monetary Policy and the Degree of Foresight

	High Foresight	Low Foresight
Active Taylor Rule	Unique Bounded Inflation Path	Unique Bounded Inflation Path
Passive Taylor Rule	Self-Fulfilling Inflation	Hyperinflation Spirals
Exogenous Interest-Rate Path	Self-Fulfilling Inflation	Depends on the Interest Rate Path

Note: The table sums up when the economy has a unique bounded equilibrium, is subject to self-fulfilling inflation, or diverges into a hyperinflation spiral, depending on agents' degree of foresight and on how monetary policy is set. An active (passive) Taylor rule is one that respects (does no respect) the Taylor principle.

affect long-term rates for several quarters but not so long that it mostly cools the economy once expectations have de-anchored much and there is much to re-anchor. In our main calibration, it is the short-term interest rate 4 quarters ahead that has the highest relative anchoring effect.

This suggests that when the risk of passive monetary policy consists of hyperinflation spirals, delaying interest rate hikes beyond the first few quarters is always ill-advised. Hiking rates today comes at the cost of a lower output today, but delaying hikes to tomorrow will require larger hikes, with a larger output cost. This suggests a trade-off between a recession today and a larger recession tomorrow, a trade-off on which doves and hawks should agree to prefer the former. This however is no longer a question about active and passive monetary policy, but a question about optimal monetary policy.

So third and finally, we derive the optimal way to anchor expectations, i.e. we solve for the optimal monetary policy, both under commitment and under discretion (section 4). Crucially, when the degree of foresight is low and the risk of passive monetary policy consists of hyperinflation spirals, the optimal policy problem captures all that is relevant to determining the best way to anchor expectations. The optimal policy no longer needs to be implemented by combining the optimal interest rate path with an active feedback interest rate rule as it does under rational expectations. The risks of passive monetary policy do exist in the form of hyperinflation spirals, but the solution to the optimal policy problem already selects an interest rate path that increases rates sufficiently to prevent such spirals.

We characterize the optimal policy under commitment through a target criterion which generalizes both the target criterion under rational expectations and the target criterion derived by Molnár and Santoro (2014) in a model of adaptive learning with fully backward-looking expectations. Relative to rational expectations, the target criterion now features the cost of increasing long-run expectations, which can be expressed as the cost of future higher inflation and output gaps. Relative to the case of purely backward-looking expectations the target criterion under commitment still features the need to deliver on past policy promises, since the central bank can still use its ability to commit to affect the forward-looking component of expectations.

We use the characterization of the optimal policy to assess how fast a central bank should increase interest rates in response to a cost-push shock. We show that, contrary to what the relative anchoring effect of policy rates at long horizons could suggest, the response strongly depends on the weight the central bank puts on output stabilization. The more weight it puts on output, the less the central bank should increase policy rates on impact, limiting the fall in output. While the risk of a hyperinflation spiral implies that the lower policy rate on impact must be compensated by higher policy rates in the future, it does not mean that the central bank engineers a recession tomorrow instead of today. Instead, it counters the hyperinflation spiral by keeping rates high for longer, bringing down long-run expectations only slowly. While output is persistently lower in the process, this avoids an outright recession.

This paper is related to several branches of literature. We build crucially on Woodford (2019)'s model of finite planning horizons and long-term learning, following up on a recent literature that has adopted the framework. Gust, Herbst, and Lopez-Salido (2022) show that it fits the dynamics of output and inflation very well, generating macroeconomic persistence without resorting to consumption habits or mechanical indexation. Gust, Herbst, and Lopez-Salido (2023) show that it also fits the dynamics of inflation expectations in the Survey of Professional Forecasters. Woodford and Xie (2022) and Xie (2020) use it to investigate monetary/fiscal interactions. Na and Xie (2022) show it can explain the deviations from the uncovered interest rate parity. We use it to study de-anchoring risks. While Woodford's model is a rare example of a bounded rationality model that combines backward and forward-looking elements, its version without learning connects to other models designed to solve the FG puzzle through bounded rationality, such as Gabaix (2020) and Farhi and Werning (2019), or departure from common knowledge, such as Angeletos and Lian (2018). Because they abstract from long-term learning, these models eliminate the risk of equilibrium indeterminacy without creating a new risk of hyper-inflationary spirals. But this implies passive monetary policy presents no risk in these models.

McKay, Nakamura, and Steinsson (2016) show that household heterogeneity and incomplete markets can provide an alternative solution to the FG puzzle. In these models as well, equilibrium indeterminacy disappears without the new risk of hyper-inflationary spirals appearing. But similarly, this implies that passive monetary policy presents no risk in these models. Besides, Bilbiie (2020), Werning (2015) and Acharya and Dogra (2020) show that household heterogeneity can either attenuate of amplify the FG puzzle, depending on the cyclicality of idiosyncratic income.

The idea that too passive a monetary policy can lead to hyper-inflationary dynamics dates back to Wicksell (1898)'s argument of the cumulative process, later developed by Friedman (1968), first formalized by Howitt (1992), and studied in the least squares learning literature (e.g. Bullard and Mitra 2002, Evans and Honkapohja 2003b, Preston 2005). Relative to these, the present paper analyzes de-anchoring risks in a model where expectations are both forward and backward-looking, spanning all the intermediary cases between fully backward and fully forward (rational) expectations. This allows to show that hyper-inflation is the risk of concern not only in models of fully backward-looking expectations, but as soon as expectations are not so forward-looking as to send the model into the FG puzzle. Allowing for partly forward-looking expectations also allows to derive the interest rate horizons that matter most to bring inflation to target in the long run. Carvalho et al. (2023) and Gáti (2023) consider de-anchoring risks through learning models with endogenous state-dependent learning gains, a dimension the FPH-learning model abstracts from.

Our results on optimal policy connect in particular to the part of the least squares learning literature that considers optimal policy, in particular Molnár and Santoro (2014)—see also Eusepi and Preston (2018), Gaspar, Smets, and Vestin (2010) and the references therein. This literature has emphasized that optimal policy reacts more strongly to inflation than under rational expectations when the risk of hyperinflation spirals exists—a result that still holds in our model. We emphasize instead that whether to increase policy rates strongly and quickly or through a smaller but more persistent increase depends strongly on the weight put on output stabilization. Gáti (2023) shows that when the sensitivity of inflation expectations to realized inflation is state-dependent—a feature from which our paper abstracts—the central bank reacts aggressively when expectations de-anchor, allowing it to react less aggressively when they are anchored.

While the FPH-learning framework we consider embeds cognitive discounting and learning, there are other departures from rational expectations that it does not embed. One is diagnosis expectations (Bordalo, Gennaioli, and Shleifer 2018, Bordalo et al. 2019, 2020, L'Huillier, Singh, and Yoo 2023). Existing versions of diagnosis expectations models have the property that long-term expectations coincide with rational expectations ones and as such are not prone to generating diverging hyper-inflationary spirals under passive monetary policy.[7] Beaudry, Carter, and Lahiri (2023) consider a setup where agents form expectations through level-k thinking, which they show constitutes similarly a midpoint between fully rational and fully backward-looking expectations. They show that in their setup it can be desirable for the central bank to discontinuously pivot from looking through a supply shock to responding aggressively once inflation expectations cross a threshold. The optimal policy that we derive in the FPH-learning model does not display brusque reversals resembling a pivot.[8]

1 Woodford's Boundedly-Rational New-Keynesian Model

To study de-anchoring risks, we build on Woodford (2019)'s model of bounded rationality. We start by laying out Woodford's model and putting it in a tractable form. Woodford's model is an generalization to bounded rationality of the canonical three-equation New-Keynesian model under rational expectations

$$y_t = -\sigma(i_t - E_t(\pi_{t+1})) + E_t(y_{t+1}) + \nu_t^y, \tag{1}$$

$$\pi_t = \kappa(y_t - y_t^e) + \beta E_t(\pi_{t+1}) + \nu_t^p, \tag{2}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t. \tag{3}$$

[7] See e.g. Bordalo et al. (2020), p.2765.

[8] It is not straightforward to define a pivot in our setup in the same way Beaudry, Carter, and Lahiri (2023) do, because we do not restrict monetary policy to consist of a response to current inflation only. But the optimal policy we derive does not display discontinuous shifts in policy.

where π_t is inflation, y_t is output, y_t^e is efficient output which is a function of productivity, i_t is the short-term policy rate, ν_t^y and ν_t^p are demand and cost-push shocks, and equations (1)-(2)-(3) are the Euler equation, New-Keynesian Phillips curve, and the Taylor policy interest-rate rule.[9] Denote it in matrix form

$$Y_t = AE_t(Y_{t+1}) + b\nu_t, \tag{4}$$

where $Y_t = (y_t, \pi_t)'$, $\nu_t = (\nu_t^y, \nu_t^p - \kappa y_t^e)$, and A and b are matrices given in appendix A.

1.1 Finite Planning Horizons

Woodford's model embeds two forms of departures from rational expectations. First, firms and household are assumed to have finite planning horizons. They are assumed to perceive future shocks and to reason through their consequences on endogenous economic variables only until h periods ahead. To evaluate the consequences of their choices beyond their planning horizon h however, they rely on an approximate value function under which all variables that they take as exogenous, such as output and inflation, are equal to their long-run values. When finite planning horizons are the only form of bounded rationality, their long-term value functions is taken to be constant. Woodford shows that an agent with planning horizon h perceives the economy to satisfy the recursion (4) up to its planning horizon h, and imposes the terminal condition $Y_{t+h+1} = 0$ at the end of its planning horizon. As a consequence, its perception Y_t^h of output and inflation at t solves

$$Y_t^h = E_t \sum_{j=0}^{h} A^j b\nu_{t+j}. \tag{5}$$

Under the assumption that planning horizons are distributed according to a geometric distribution, with a share $(1-\rho)\rho^h$ of households and firms having planning horizon h, the model under FPH can be aggregated into

$$Y_t = \rho AE_t(Y_{t+1}) + b\nu_t, \tag{6}$$

where $\rho \in [0, 1]$ increases with the average planning horizon $N = \rho/(1 - \rho)$ in the population and parameterizes the degree of foresight of firms and households. Rational expectations correspond to the limiting case where planning horizons are infinite $\rho = 1$. Relative to rational expectations, finite planning horizons make firms and household discount the future further, providing a solution to the FG puzzle. As such, finite planning horizons capture a departure from rational expectations shared with other solutions to the FG puzzle based on bounded rationality or departures from common knowledge, such as Gabaix (2020), Farhi and Werning (2019), Angeletos and Lian (2018).

[9] Contrary to Woodford (2019), we specify preference demand shocks as discount factor shocks so that they appear as a time-t shock only in the Euler equation. This is simply to avoid having to write the model in bloc form with $t + 1$ shocks on top of t shocks, a straightforward but cumbersome extension.

1.2 Long-Term Learning

Finite planning horizons can be combined with a second, distinct departure from rational expectations: long-term learning. With long-term learning, households and firms do not take their long-term value function to be constant, but adjust it to their evolving expectations of long-run inflation π^*_{t-1} and output y^*_{t-1}, which they build from past realized inflation and output, according to the constant-gain updating rules[10]

$$y^*_t = \mu y^*_{t-1} + (1-\mu)y_t, \tag{7}$$

$$\pi^*_t = \mu \pi^*_{t-1} + (1-\mu)\pi_t, \tag{8}$$

where $\mu \in [0,1]$.[11] Denote it in matrix form

$$Y^*_t = \mu Y^*_{t-1} + (1-\mu)Y_t, \tag{9}$$

where $Y^*_t = (y^*_t, \pi^*_t)'$.

Long-term learning captures a departure from rational expectations shared with other models of learning, such as least squares learning (Bullard and Mitra 2002, Evans and Honkapohja 2003b, Preston 2005). It adds a backward component to expectations that will be essential to capture de-anchoring risks. However, the combined FPH-learning model differs from usual learning models in that firms and households only form their *long-run* expectations in a backward-looking manner. Up to their planing horizons, their expectations remain forward-looking—though of course affected by their expectations for the long run. Ultimately, expectations in the FPH-learning model are partly forward and partly backward. This implies that policy announcements about the future have an effect on expectations right away.

A low value of μ (a high gain $1-\mu$) implies that long-run expectations react quickly and strongly to recent realizations of inflation and output. A high value of μ in contrast implies that long-run expectations drift only slowly and only as a consequence of persistent changes in realized inflation and output.[12] At the limit $\mu = 1$, long-run expectations are constant so they cannot de-anchor. In this case, the model reverts to

[10]Note that we assume that all households have the same long-run expectations regardless of their planning horizons. This is in contrast to the presentation in Woodford (2019), which assumes that a household with planning horizon h bases its expectations of the long run on its past nowcasts of inflation and output, which depend on its planning horizon. We make this change for two reasons. First the assumption is equally meaningful: once output and inflation are realized, households can use these realizations to form long-run expectations instead of their past nowcasts. Second, it simplifies the derivation. As we show in Appendix B, we nevertheless fall exactly on the same aggregated model as in Woodford (2019).

[11]We index long-term expectations of inflation and output by $t-1$ so underline that they are predetermined at t. Indeed, long-run expectations π^*_{t-1} and y^{*h}_{t-1} use only past and not present realizations of inflation and output. Similarly, we index the backward-looking component Y^b below with the time index $t-1$ to underline that it is a state variable. This is only a change in notations however: the variables are defined in exactly the same way as in Woodford (2019). Another notation change is that we use the AR(1) notation μ in equations (7) and (8) instead of the gain notation $\gamma = 1-\mu$.

[12]A higher value of μ can be interpreted as expectations that are *better anchored*, in the sense that the sensitivity of long-run expectations to recent realizations of inflation is low. We refrain from using this terminology however, in order not to bring confusing with the other meaning of *anchoring expectations*, which is the one we use in this paper. We will say the central bank *anchors expectations* when it reacts enough to guarantee that long-run expectations are brought back to target at least at some point in the future, even under a low μ. In Woodford (2019)'s set-up with an exogenous constant-gain parameter, the central bank cannot *anchor expectations* in the alternative sense of making them less responsive to recent inflation even without increasing rates ex post.

the model under finite planning horizons only (6).[13] The model assumes that μ is constant, i.e. abstracts from time variation and state dependence in the learning gain. See e.g. Carvalho et al. (2023) and Gáti (2023) for examples of models with endogenous state-dependent learning gains.

Woodford (2019) shows how the terminal value function of firms and households depend on long-run inflation and output expectations in the FPH-learning model. In Appendix B we show that the FPH-learning model can be rewritten in the following way. A firm or household with planning horizon h still perceives the economy to satisfy the recursion (4) up to its planning horizon h, but now imposes the terminal condition $Y_{t+h+1} = Y^*_{t-1}$ at the end of its planning horizon. As a consequence, its perception Y^h_t of output and inflation at t now solves

$$Y_t^h = E_t \sum_{j=0}^{h} A^j b\nu_{t+j} + A^{h+1} Y^*_{t-1}. \tag{10}$$

Under the assumption that planning horizons are distributed according to a geometric distribution, the FPH-learning model aggregates into

$$Y_t = E_t \sum_{j=0}^{\infty} (\rho A)^j b\nu_{t+j} + (I - \rho A)^{-1}(1-\rho) A Y^*_{t-1}. \tag{11}$$

Define

$$Y^b_{t-1} = (I - \rho A)^{-1}(1-\rho) A Y^*_{t-1}, \tag{12}$$

$$Y^f_t = Y_t - Y^b_{t-1}, \tag{13}$$

the backward-looking and forward-looking components of Y_t.[14] The FPH-learning model (11)-(9) can be written as

$$Y^f_t = (\rho A) E_t(Y^f_{t+1}) + b\nu_t, \tag{14}$$

$$Y^b_t = \Big(\mu I + (1-\mu)(I-\rho A)^{-1}(1-\rho)A\Big) Y^b_{t-1} + \Big((1-\mu)(I-\rho A)^{-1}(1-\rho)A\Big) Y^f_t. \tag{15}$$

2 The Taylor Principle in the Face of Two Risks

In this section, we show that active monetary policy is necessary to retain control over inflation expectations, but that, as soon as the model is not subject to the FG puzzle, the risk it prevents is no longer self-fulfilling inflation but hyperinflation spirals. To do so, we assume that the central bank follows the Taylor rule (3) and derive under which conditions it delivers a unique bounded solution.

[13] Rigorously, at the limit $\mu = 1$, inflation expectations are constant but not necessarily at the assumed stead-state. We assume they are fixed at the assumed steady-state in this case, so that the model under FPH only corresponds to the limit of the FPH-learning model at $\mu = 1$.

[14] For readability, we use the notations Y^f_t and Y^b_{t-1} instead of $\tilde{Y}_t$ and $\bar{Y}_{t-1}$ in Woodford (2019).

Table 2: Default Calibration

Parameter	Value	Interpretation	Note
β	0.995	Discount Factor	
σ	0.5	Intertemporal Elasticity of Substitution	
κ	0.04	Phillips Curve Slope	
ρ	0.5	Forward-lookingness of Planning	Gust et al. (2022)
μ	0.9	Persistence of Learning (one minus learning gain)	Gust et al. (2022)

Note: The table gives the default calibration used in the paper whenever not stated otherwise. The calibration is quarterly.

2.1 Stability with Finite Planning Horizons but no Long-Term Learning

Before getting to this result, consider the stability of the economy under finite planning horizons alone, abstracting temporarily from long-term learning. This corresponds to the particular case $\mu = 1$ where long-run expectations always remain on target. Appendix C shows that the FPH-only economy (6) has a unique bounded equilibrium if and only if the smaller eigenvalue $\lambda_2^*(\phi)$ of A^{-1} is greater than ρ,[15]

$$|\lambda_2^*(\phi)| > \rho. \tag{16}$$

If the economy does not have a unique bounded solution, there is an infinity: inflation is self-fulfilling.

Under rational expectations $\rho = 1$, the determinacy condition (16) is equivalent to the Taylor principle

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1. \tag{17}$$

As the average planning horizons of agents decreases however, ρ decreases below 1 and the determinacy condition (16) becomes easier and easier to satisfy. It holds for lower and lower values of ϕ_π and ϕ_y, putting a less demanding requirement on the responsiveness of monetary policy than the Taylor condition under rational expectations (17). Appendix C shows that for ρ below the threshold

$$\rho^* = \lambda_2^*(0) = \frac{1 + \sigma\kappa + \beta - \sqrt{(1 + \sigma\kappa + \beta)^2 - 4\beta}}{2\beta}, \tag{18}$$

the equilibrium is determinate even under a fully unresponsive monetary policy $\phi_\pi = \phi_y = 0$—an interest rate peg. For a standard calibration of the structural parameters $\beta = 0.995$, $\sigma = 0.5$, $\kappa = 0.04$ (Table 2), the threshold value is $\rho^* = 0.87$.

That not even a weakened version of the Taylor principle is necessary to ensure determinacy when $\rho < \rho^*$

[15]We spell out the dependence of the eigenvalues λ_2^* in $\phi = (\phi_\pi, \phi_y)$ in order to highlight that it depends on the responsiveness of monetary policy.

questions the practical relevance of the Taylor principle in keeping control over inflation expectations. All the more so that for higher values of $\rho > \rho^*$ for which some responsiveness of monetary policy remains necessary, the New-Keynesian model yields widely unrealistic predictions on the effect of interest rate changes—the forward guidance (FG) puzzle. Under rational expectations, and for degrees of foresight $\rho > \rho^*$, announcing an interest rate cut n periods ahead has an effect on output and inflation that increases with the horizon n of the announcement, and becomes infinitely strong as the horizon of the announcement increases. The threshold on the degree of discounting ρ necessary to get rid of the FG puzzle turns out to be exactly the same as the threshold (18) that guarantees determinacy under a peg (see e.g. Dupraz, Le Bihan, and Matheron (forthcoming) for a proof). Finite planning horizons—as well as related models of cognitive discounting such as Gabaix (2020) and Farhi and Werning (2019)—solve the FG puzzle, but taken alone they suggest monetary policy does not need to worry about losing control over inflation expectations.

2.2 Stability with Finite Planning Horizons and Long-Term Learning

We now show that the risk of losing control over inflation expectations resurfaces when adding back long-term learning. Crucially though, it resurfaces under a different form: hyperinflation spirals. This risk, which is arguably more present on central bankers' mind, is distinct from the risk of indeterminate inflation. With an hyperinflation spiral, the risk is instead that inflation become very determinate in a very undesirable direction.

We map the risk of hyperinflation spirals into Blanchard and Kahn (1980) arithmetic, like has long been done for the risk of self-fulfilling inflation. While self-fulfilling inflation occurs when an economy has more roots within the unit circle than it has state variables, hyperinflation spirals occur when an economy has more roots outside the unit circle than it has jump variables. In such a situation, the economy has no bounded solution: all equilibria are explosive.[16]

Because long-run expectations are state variables, hyperinflation spirals can occur. To determine when they do, Appendix D derive the roots of the FPH-learning economy.

Lemma 1. *The roots of the FPH-learning New-Keynesian economy* (14)-(15) *are the following direct function of the roots* $\lambda_i^*(\phi)$ *of the economy under rational expectations*

$$\lambda_i^f(\phi) = \frac{1}{\rho}\lambda_i^*(\phi), \tag{19}$$

$$\lambda_i^b(\phi) = \mu + (1-\mu)(1-\rho)\left(\frac{1}{\lambda_i^*(\phi) - \rho}\right), \tag{20}$$

for $i = 1, 2$.

[16] While the absence of non-explosive path captures the idea of an hyperinflation spiral, the infinitely of explosive paths can also suggest a form of indeterminacy among hyperinflation paths. Section 3 will however show that one particular hyperinflation path stands out, on which *temporarily* explosive hyperinflation dynamics can occur even when inflation eventually returns to target.

With these closed-from expressions for the roots of the FPH-learning economy, it is possible to determine its stability. Appendix E shows the following result.

Proposition 1. *Consider the case with long-term learning $\mu < 1$.*

1. *If the Taylor principle (17) is satisfied, then the FPH-learning economy has a unique bounded solution.*
2. *If the Taylor principle (17) is not satisfied, let $0 < \lambda_2^*(\phi) < 1$ be the smaller root of the economy under rational expectation. Unless (ρ, μ) satisfy:*

$$\rho \in \left[\lambda_2^*(\phi), \frac{1}{2}\left(1 + \lambda_2^*(\phi)\right)\right], \tag{21}$$

$$\mu \in \left[0, \frac{\lambda_2^*(\phi) + 1 - 2\rho}{1 - \lambda_2^*(\phi)}\right]. \tag{22}$$

the FPH-learning economy does not have a unique bounded solution. In this case

   (a) If $\rho > \lambda_2^*(\phi)$, then the equilibrium is indeterminate.

   (b) If $\rho < \lambda_2^*(\phi)$, then there is no bounded equilibrium.

Proposition 1 first states that, up to the narrow exception (21)-(22), the Taylor principle remains necessary and sufficient for a unique bounded solution to exist under a Taylor rule. We argue that the exception is an unappealing knife-edge case, which can be treated as economically irrelevant. It is unappealing because one of the roots of the system is then negative. It is a knife-edge case because it concerns only a very narrow set of values for ρ and μ. For instance, in our baseline calibration $\beta = 0.995$, $\kappa = 0.04$, $\sigma = 0.5$, for $\phi_\pi = \phi_y = 0$, which is the case most conducive to condition (21)-(22), the condition is only satisfied for $\rho \in [0.87, 0.94]$, each time for only a subset of values of μ. One way to get rid of this economically irrelevant case is to consider that the FPH-learning economy under a given value of ρ is stable if and only if it has a unique bounded solution for *any* value of $\mu \in [0, 1)$. Under this definition, Proposition 1 states that the economy is stable if and only if the Taylor principle is satisfied.

Corollary 1. *For a given value of ρ, the FPH-learning economy has a unique bounded solution for all $0 \leq \mu < 1$ if and only if the Taylor principle (17) is satisfied.*

But second, Proposition 1 states that when the Taylor principle is not satisfied, what the FPH-learning economy runs into is not, in most cases, equilibrium indeterminacy but hyper-inflationary spirals. For ρ close enough to 1, $\rho > \lambda_2^*(\phi)$, equilibrium indeterminacy still obtains, as it does under rational expectations. But such high values of ρ are also ones for which the model is subject to the FG puzzle, which occurs whenever $\rho > \lambda_2^*(0) = \rho^*$. When ρ is low enough to solve the FG puzzle, the model is no longer subject to equilibrium indeterminacy. Both $\lambda_1^f(\phi)$ and $\lambda_2^f(\phi)$ are then outside the unit circle, like in the FPH model without learning. Yet crucially, $\lambda_2^b(\phi)$ is then necessarily outside the unit circle as well, so that the economy has 3 roots outside the unit circle, and only one inside, the condition for an absence of bounded solution.

The economy necessarily falls into an explosive inflationary of deflationary spiral as expectations spiral out of control. Pushing the risk of self-fulfilling inflation out the door, it comes back through the window in the form of hyper-inflationary spirals.

Figure 1 gives a graphical illustration of Proposition 1. The diagram represents the 3 possible stability cases as a function of the strength of the feedback in the Taylor rule $\phi_\pi + \frac{1-\beta}{\kappa}\phi_y$ on the x-axis, and of the degree of cognitive discounting ρ on the y-axis. On the diagram, we abstract from the economically irrelevant border-line case associated to condition (21)-(22). Whether the economy has a unique bounded equilibrium or not depends only on the strength of the feedback in the Taylor rule on the x-axis. But respecting the Taylor principle cures very different ailments depending on the value of ρ. Except under very high $\rho > \rho^*$ for which the model is subject to the FG puzzle, too passive a monetary policy does not lead to indeterminacy but to the absence of bounded solution.

What makes the economy inherently unstable under passive monetary policy when $\rho < \lambda_2^*(\phi)$? The economic mechanism is the one underlying Wicksell's *cumulative process* argument against interest rate pegs, later developed by Friedman (1968), first formalized by Howitt (1992), and studied in the least squares learning literature (e.g. Bullard and Mitra 2002, Evans and Honkapohja 2003b, Preston 2005).[17] When monetary policy is not responsive enough, not responding strongly enough to inflation allows transitory shocks to send the economy on a hyper-inflationary (or hyper-deflationary) spiral. Suppose a transitory demand shock (say) hits the economy. Higher demand pushes inflation up, which increases long-term inflation expectations and therefore all inflation expectations. Although the shock then dissipates, households' inflation expectations are now higher, which—absent a strong enough reaction from monetary policy—decreases ex ante perceived real rates. This increases aggregate demand, which increases inflation further, and so on, in a self-feeding spiral. As a consequence, there exists no bounded solution, the exact opposite problem of the infinity of bounded solutions that arises under rational expectations, and more generally when $\rho > \lambda_2^*(\phi)$.

Proposition 1 captures the logic of the cumulative process within a model that has both a forward-looking component, and a backward-looking, learning component. By bridging the gap between the two polar cases of fully rational expectations $\rho = 1$, and purely backward-learning expectations $\rho = 0$, it shows that the generic risk of a passive monetary policy is hyper-inflationary spirals, not indeterminacy, unless for high values of ρ that send the model into the FG puzzle.

3 Beyond Taylor Rules

Proposition 1 states that when restricting monetary policy to the Taylor rule (3), keeping control over inflation expectations requires to respect the Taylor principle (17). But is following a Taylor rule necessary to keep control over inflation expectations? Under rational expectations, it is: Interest-rate policy must

[17] In the least squares literature, when learning is done with a decreasing gain the Taylor principle not only prevents the cumulative process but also guarantees convergence to the rational expectations equilibrium, a stronger stability result. When learning is done with a constant gain, there is no convergence to rational expectations. On the stronger requirements for stability brought by a constant gain under adaptive learning, see e.g. Evans and Honkapohja (2009).

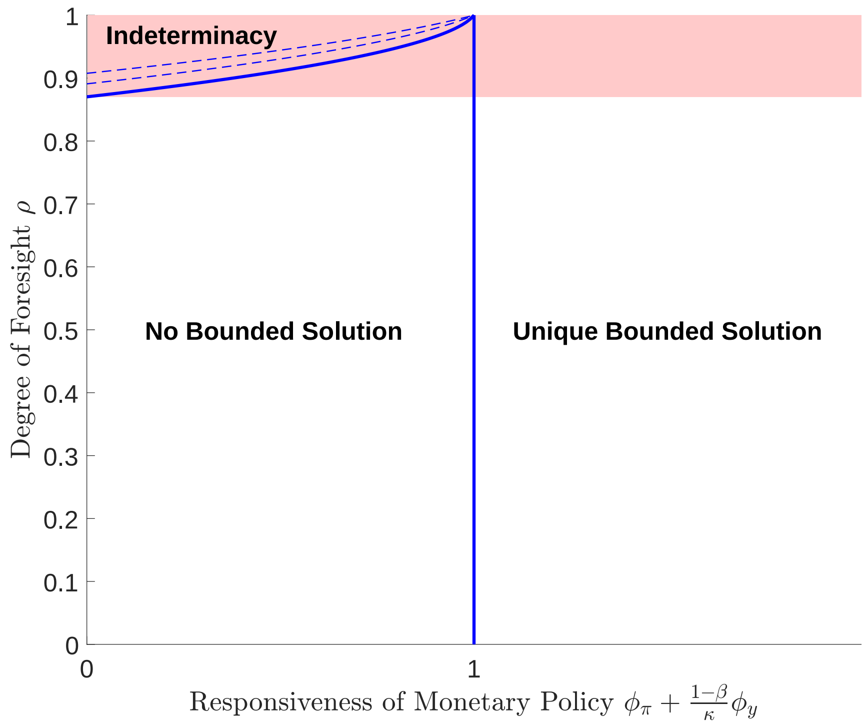

Figure 1: Existence of Solutions in the FPH-Learning New-Keynesian Economy

Note: The diagram represents the 3 possible situations that can arise as a function of the degree of bounded-rationality discounting ρ and the strength of the response in the Taylor rule $\phi_\pi + \frac{1-\beta}{\kappa}\phi_y$. The frontier between the indeterminacy region and the no-solution region plotted in bold is the one for $\phi_y = 0$, which is the one for which the indeterminacy region is the largest. In dashed lines are the frontiers for higher values of $\phi_y = 0.1, 0.2$. The shaded region represents the calibrations under which the model is subject to the FG puzzle: for $\rho > \lambda_2^*(0) = \rho^*$.

necessarily be specified as a feedback rule—a Taylor rule of some form (McCallum 1981). Specifying monetary policy as an exogenous interest rate path results in indeterminacy, just like it does under an interest rate peg. The result extends to the FPH-learning economy when the degree of foresight is high enough that the risk of passive monetary policy lies in equilibrium indeterminacy.

In this section, we show that when the degree of foresight is low enough that the risk of concern is hyperinflation spirals, following a feedback rule is no longer necessary to keep control over inflation expectations. To be sure, many interest-rate paths fail to keep inflation expectations in check. For instance, a constant interest rate—an interest rate peg—results in a hyperinflation spiral since it is a particular case of Proposition 1 for $\phi_\pi = \phi_y = 0$. But for a large class of exogenous paths for the policy rate, the equilibrium is determinate and bounded, and inflation returns to its target in the long run. We characterize all the interest rate paths that bring inflation back to target in the long run. We then show that this characterization allows to determine how powerful interest rate hikes of different horizons are at bringing inflation expectations back to target.

3.1 All the Anchoring Interest Rate Paths

We assume that monetary policy is no longer set as the Taylor feedback rule (3), but as an exogenous interest rate path $(i_{t+n}(\nu))_{n\geq 0}$. An interest-rate path is exogenous if is not specified as a function of endogenous variables such as inflation and output today or tomorrow $(y_{t+n}, \pi_{t+n})_{n\geq 0}$. But it can be a function of the exogenous process ν, something we highlight through the notation $i_t(\nu)$.[18] It can also depend on the lagged endogenous state (y^b_{t-1}, π^b_{t-1}) at t, since it is predetermined at t. What we exclude is a dependence on current and future inflation and output, i.e. the one dependence that is critical to ensure determinacy under rational expectations, by stipulating a response of monetary policy off the equilibrium path.[19]

The FPH-learning economy is then described by (14)-(15) for $\phi_\pi = \phi_y = 0$, only adding the new non-homogeneous term in $i_t(\nu)$

$$Y^f_t = (\rho A_0)E_t(Y^f_{t+1}) + b_0\nu_t - b^i_0 i_t(\nu), \tag{23}$$

$$Y^b_t = \Big(\mu I + (1-\mu)(I-\rho A_0)^{-1}(1-\rho)A_0\Big)Y^b_{t-1} + \Big((1-\mu)(I-\rho A_0)^{-1}(1-\rho)A_0\Big)Y^f_t, \tag{24}$$

where A_0 and b_0 denote the matrices A and b for $\phi_\pi = \phi_y = 0$ and the expression of matrix b^i_0 is given in appendix A.

We consider the case where the model is not subject to the FG puzzle $\rho < \rho^*$. From Proposition 1, the root $\lambda^b_2(0)$ is then outside the unit circle $\lambda^b_2(0) > 1$ and there is no bounded solution if the nominal interest rate is kept constant. Yet, Appendix F shows that a large class of non-constant interest rate paths can deliver a bounded determinate solution, characterized in the following proposition.

Proposition 2.
Assume that the exogenous shocks converge back to steady-state in expectations $\lim_{k\to\infty} E_t(\nu_{t+k}) = 0$.
The FPH-learning economy (23)-(24) has an unique equilibrium where inflation and output remain bounded and the economy returns to steady-state in the long run if and only if:

(i) The interest rate path converges back to steady-state in expectations $\lim_{n\to\infty} E_t(i_{t+n}) = 0$.

(ii) The following condition is satisfied

$$z^b_{2,t-1} + \left(1 - \frac{\mu}{\lambda^b_2(0)}\right)\left(\sum_{n=0}^{\infty}\gamma(n)E_t(v_{2,t+n} - c^i_2 i_{t+n}(\nu))\right) = 0,$$

[18] Our specification of monetary policy is similar to the one advocated by Beaudry, Portier, and Preston (2023) as an alternative to Taylor rules. Relative to them, we allow monetary policy to adjust its entire future expected path for the policy rate in response to changes in fundamentals.

[19] What we restrict to is therefore, in the terminology of Evans and Honkapohja (2003b, 2006, 2003a), fundamentals-based policy rules. Our point is not to dismiss feedback rules—e.g. Evans and honkapohja's expectations-based policy rules—which can have the benefit of simplicity and robustness. Instead, the point is that implementability is no longer an issue away from rational expectations, so that a comparison of various policies within a given model can be done without the need to restrict to a class of policies.

where $z^b_{2,t} = e_2' Y^b_{t-1}$ *for* e_2' *the left eigenvector associated to the root* $\lambda^*_2(0) < 1$ *of* A_0^{-1},

$$\gamma(n) = \frac{\left(\frac{1}{\lambda^f_2(0)}\right)^{n+1} - \left(\frac{1}{\lambda^b_2(0)}\right)^{n+1}}{\frac{1}{\lambda^f_2(0)} - \frac{1}{\lambda^b_2(0)}}, \tag{25}$$

and v_t *and* c^i_2 *are a function of the shocks* ν_t *and a constant given in appendix F.*

Proposition 2 captures the following intuition. Appendix F shows that the economy returns to steady state in the long run if and only if the variable $z^b_{2,t}$ returns to steady-state in the long run. In turn, z^b_2 at $t+k$ is given by

$$E_t(z^b_{2,t+k}) = \lambda^b_2(0)^{k+1}\left(z^b_{2,t-1} + \left(1 - \frac{\mu}{\lambda^b_2(0)}\right)\left(\sum_{n=0}^{\infty}\gamma(n)E_t(v_{2,t+n} - c^i_2 i_{t+n}(\nu))\right)\right) + o_{k\to\infty}(1). \tag{26}$$

Because the root $\lambda^b_2(0)$ is larger than 1, the FPH-learning economy (23)-(24) has a natural tendency to make inflationary shocks snowball into hyperinflation (or hyperdeflation). This is Wicksell's cumulative process. Whenever long-run expectations are not exactly equal to their steady-state values $z^b_{2,t-1} \neq 0$—for instance because an inflationary shock has recently caused them to increase—inflation (and output) is expected to diverge exponentially even absent any future shock. Under an interest-rate peg, this necessarily results in inflation diverging to infinity.

Yet an interest rate path that sufficiently increases interest rates can counter the inflationary shock and bring inflation back to target. Given current values of long-term expectations and given expectations on future shocks, condition (25) assesses whether a given expected path for the policy rate does enough to counter the default tendency of inflation to spiral into hyperinflation. The countering effect of monetary policy can come from interest rate rises either now or in the future, but the size of the necessary increase in rates is different depending on when it occurs. It is captured by the weights $\gamma(n)$ on the interest rate at horizon n.

The idea that sufficiently strong interest rates hikes are necessary to stabilize inflation expectations captures an intuition similar the Taylor principle when monetary policy is restricted to a Taylor rule. As such, it characterizes the paths for the policy rate that constitute active monetary policy. Yet Proposition 2 characterizes what interest rate hikes are necessary without first restraining monetary policy to a given class of interest rate rules.

We now show that the weights $\gamma(n)$ in condition (25) allow to capture the horizon of interest rate hikes that matter most for bringing inflation back to target. We first argue that Taylor rules and the Taylor principle provide little guidance on this issue.

3.2 What Horizons of Rates Matter? The Limited Guidance of Taylor Rules

Under rational expectations, a Taylor rule that respects the Taylor principle wards off self-fulfilling inflation and allows to keep control over inflation expectations. Following such a Taylor rule appears therefore to be a sound advice for central banks, and a natural benchmark to assess whether a central bank is doing enough to counter an inflationary shock. In effect, when in 2021-2022 inflation in the US and many other countries started reaching levels not seen in 40 years, several central bankers and outside observers use Taylor rule benchmarks to assess whether the Federal Reserve and other central banks were lagging behind the curve in their response to inflation.[20]

As far as ensuring equilibrium determinacy is concerned however, Taylor rules provide little practical guidance on how fast to increase interest rates. While in the previous section we considered the simple Taylor rule (3) that responds to current inflation and output only, the interest-rate feedback rules that guarantee determinacy is considerably larger. In particular, the typical Taylor rule considered in applied work generalizes the rule (3) to include interest-rate inertia

$$i_t = \rho_T i_{t-1} + (1 - \rho_T)(i^* + \phi_\pi(\pi_t - \pi^*) + \phi_x x_t). \tag{27}$$

The rule (27) has the policy rate respond to deviations of year-on-year inflation π_t from the inflation target π^* and to the output gap x_t, with inertia captured by the coefficient ρ_T on the lagged policy rate i_{t-1}. Large enough values of the coefficients ϕ_π and ϕ_x still guarantees determinacy in standard New-Keynesian models, but crucially do so for all values of ρ_T between 0 and 1. For example, in the baseline New-Keynesian model (1)-(2), the condition for determinacy is the Taylor principle (17) for all value of ρ between 0 and 1. As a result, a central bank seeking to ensure equilibrium determinacy has an infinitely of Taylor rules to choose from, with widely different recommendations as to the size and timing of interest-rate hikes

To illustrate this, Figure 2 plots the policy rate paths that the rule (27) would have recommended the Fed to follow in February 2022, right before the Fed's first interest-rate hike of March 2022, setting the coefficients ϕ_π and ϕ_x to standard values that guarantee equilibrium determinacy, but varying the inertia coefficient ρ_T to 0, 0.85 and 0.99. While all three rules prevent equilibrium indeterminacy, they offer widely different recommendations as to how fast to raise interest rates.[21]

Whether using Taylor rule benchmarks or not, the question of whether a central bank is lagging behind the curve is typically assessed by looking at whether interest rates have increased enough. But in doing so, beyond the issue of assessing what level of interest rates is sufficiently restrictive, a key issue is what maturity of interest rates is the relevant one to look at.[22] On the one hand, many consumption and investment

[20] Examples of policy-makers and commentators using Taylor rules to make the case that the Fed was lagging behind include for instance Bullard (2022) and Buiter and Sibert (2022). Both use a Taylor rule with no inertia. Bullard notes that the Fed did not look as far behind when using the 2-year Treasury yield instead of the short-term policy rate.

[21] Of course, the recommendations of different Taylor rules can be evaluated and compared through a loss function. We will precisely turn to evaluating policies using the central bank's loss function in Section 4, but without restricting policy to belong to a particular class.

[22] This was again debated in Spring 2022 concerning the Federal Reserve in the US. For instance, on April 5, 2022, Larry

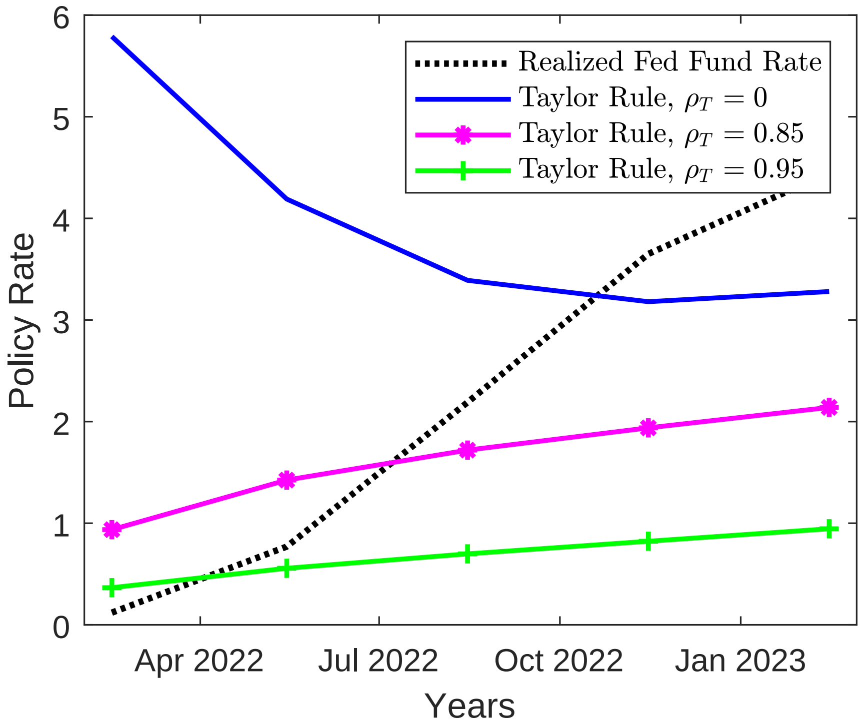

Figure 2: Interest Rate Paths Recommended by Various Taylor Rules in February 2022

Note: The three interest rate paths correspond to the paths that the Taylor rule (27) would have recommended for the Fed to follow in February 2022, for $\phi_\pi = 1.5$, $\phi_x = 0.5$ and various calibrations of the inertia parameter ρ_T. The forecasted paths for the nominal interest rate are calculated using the SPF median expectation for the path of inflation and unemployment in February 2022. The steady-state nominal rate i^* is taken to be 1.8% (a real rate of -0.2%), its average from 2000 to 2019. Inflation π_t is taken to be year-on-year PCE core inflation, and the inflation target $\pi^* = 2\%$. The output gap is proxied with (minus) twice the unemployment gap, in accordance with an Okun's coefficient of 2. The unemployment gap is calculated as the difference between the SPF median expectation of the unemployment rate and the CBO expectation of the natural rate of unemployment.

decisions depend on rather long-term rates. On the other hand, tightening only through a commitment to raising short-term rates in the future runs the risk of letting expectations de-anchor, requiring higher hikes in the future.

What horizon of interest rates matters most to bringing inflation down then? We argue that the rational-expectations set-up is ill-suited to answering this question. Because it assimilates the risks of losing the inflation anchor to self-fulfilling inflation and not to hyperinflation spirals, it does not capture the idea that waiting too long to increase interest rates will give time for inflation expectations to de-anchor. But the FPH-learning model allows to make sense of the trade-off between these two opposing forces.

Summers argued against Paul Krugman's argument that *"only future rates are relevant to spending"*, reminding the *"decades-long tradition of using real Treasury bill or Fed Fund rates to index monetary policy"* (Summers 2022).

3.3 The Relative Anchoring Effect of Interest Rates of Different Horizons

From Proposition 2, there are infinitely many interest rate paths that guarantee inflation returns to target in the long run. The tightening (or loosening) effect that they deliver is however the same, measured by

$$E_t\left(\sum_{n=0}^{\infty}\gamma(n)i_{t+n}(\nu)\right). \tag{28}$$

The weights $\gamma(n)$ therefore play an essential role in assessing how much each horizon of the yield curve matters is delivering a given amount of tightening. The weight $\gamma(n)$ captures how powerful the policy rate i_{t+n} at horizon n is at bringing inflation down. We call it the relative anchoring effect of i_{t+n}.

Definition 1. *We call the weight $\gamma(n)$ the relative anchoring effect of interest rate i_{t+n} at horizon n,*

$$\gamma(n) = \frac{\left(\frac{1}{\lambda_2^f(0)}\right)^{n+1} - \left(\frac{1}{\lambda_2^b(0)}\right)^{n+1}}{\frac{1}{\lambda_2^f(0)} - \frac{1}{\lambda_2^b(0)}}. \tag{29}$$

It is equal to the effect i_{t+n} on $E_t(z_{2,t+k})$ relative to the effect of the current interest rate i_t, for all $k \geq n$,

$$\gamma(n) = \frac{\left(\frac{\partial E_t(z_{2,t+k})}{\partial i_{t+n}}\right)}{\left(\frac{\partial E_t(z_{2,t+k})}{\partial i_t}\right)}. \tag{30}$$

We call $\gamma(n)$ the *relative* anchoring effect of i_{t+n} because it is defined relative to the anchoring effect of the current policy rate i_t. Accordingly and by construction, $\gamma(0) = 1$. Key to seeing $\gamma(n)$ as defined by (30) is the property that the relative effect of i_{t+n} on $z_{2,t+k}^b$ does not depend on the horizon k for $k \geq n$. This is because past $t+n$, the effect of i_{t+n} only manifests itself through the effect it left on the state at $t+n$, $z_{2,t+n}^b$. Beyond that date, any departure of $z_{2,t+n}^b$ from zero diverges exponentially with k at the rate of the explosive root $\lambda_2^b(0) > 1$. Because this rate of divergence is the same for the effect of the contemporary policy rate i_t, the divergence at rate $\lambda_2^b(0)$ cancels out in the relative anchoring effect (30). Since $\gamma(n)$ captures the relative anchoring effect of i_{t+n} for all $k \geq n$, it also captures the relative anchoring effect of i_{t+n} at the limit when k tends to infinity. As a result, it is the weight with which i_{t+n} enters condition (25).

Figure 3 plots the relative anchoring effect $\gamma(n)$ as a function of the horizon n, for various values of the cognitive discounting parameter ρ and the expectations persistence parameter μ. Crucially, the relative anchoring effect $\gamma(n)$ is a single-peaked function of the horizon and peaks at an intermediary horizon. This intermediary peak horizon strikes a balance between two opposite forces.

First, short-horizon rates do not affect aggregate demand for many period, so they have a moderate power to cool down the economy. As the horizon n increases, an increase in i_{t+n} affects aggregate demand at all periods between t and $t+n$ and is therefore better able to cool down the economy. This first effect is present under rational expectations as well but not in purely backward-looking models where aggregate demand

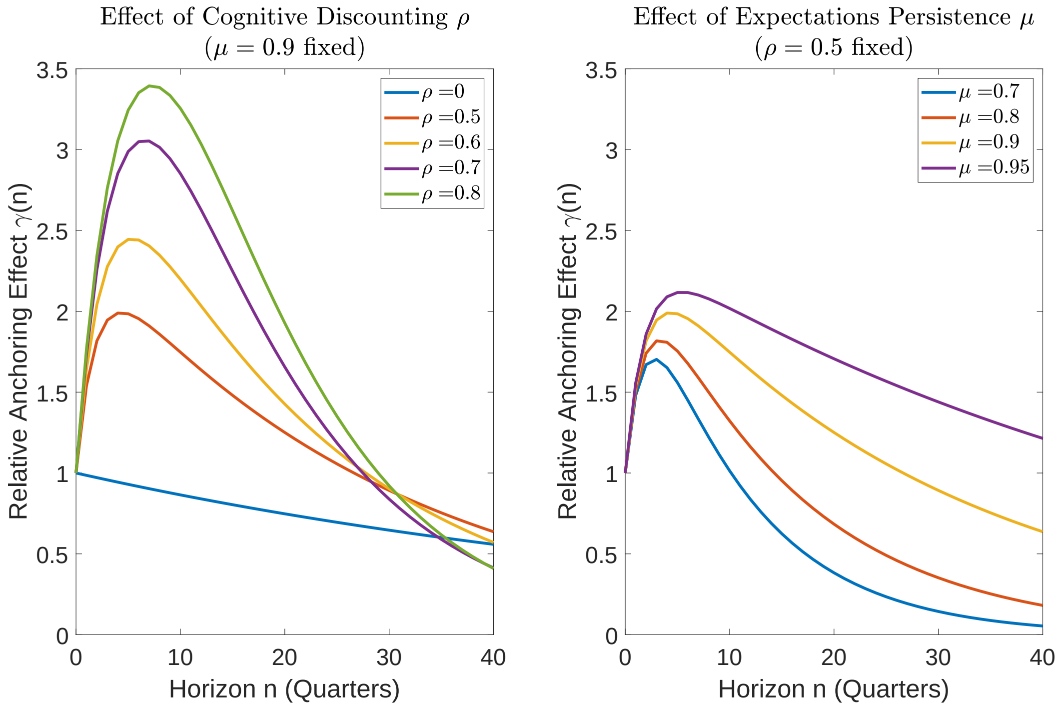

Figure 3: The Relative Anchoring Effect of Interest Rates of Different Horizons

Note: Both panels plot the relative anchoring effect $\gamma(n)$ of the interest rate at horizon n as a function of the horizon n. On the left panel, each curve plots the relationship for a different value of the cognitive discounting parameter ρ, keeping the expectations persistence parameter fixed to $\mu = 0.9$. On the right panel, each curve plots the relationship for a different value of the expectations persistence parameter μ, keeping the cognitive parameter fixed to $\rho = 0.5$. The calibration for the structural parameters is $\beta = 0.995$, $\sigma = 0.5$, $\kappa = 0.04$.

does not depend on the entire yield curve beyond present short-term rates. Accordingly, if in the model expectations are purely backward-looking $\rho = 0$, the relative anchoring effect $\gamma(n)$ is a strictly decreasing function of γ. Current policy rates are then the ones that matter most. But when $\rho > 0$, the relative anchoring function γ is at first increasing in the horizon n.

Yet, as n increases, a second force becomes dominant and makes γ decreasing. The more the increase in rates is delayed, the more expectations de-anchor, requiring a stronger rate hike in the future to re-anchor expectations. This second effect exists only when hyperinflation spirals are possible and is therefore absent under rational expectations. It is also absent at the limit where long-run expectations are fixed $\mu = 1$, as there is then no risk of de-anchoring.

The horizon of interest rate hikes that matters most for re-anchoring expectations is intermediary, long enough to affect long-term rates but not so long that it mostly cools the economy once expectations have de-anchored much and there is much to re-anchor. It increases when expectations are more forward-looking (higher ρ) and when long-run expectations react less to recent inflation (higher μ). In our main calibration in Table 2, which assumes $\rho = 0.5$ and $\mu = 0.9$ as estimated by Gust, Herbst, and Lopez-Salido (2022), the horizon of short-term interest rates with the highest relative anchoring effect is 4 quarters ahead, which matters around twice as much as current short-term rates. For the values of ρ and μ plotted on the figure, the horizon with the highest relative anchoring effect varies between 4 and 8 quarters ahead.

Figure 4 illustrates the relative anchoring effect of interest rates at different horizons through the following thought experiment. We assume that the economy is hit in period 0 by a transitory cost-push shock, and consider the response of the economy under various responses of monetary policy. The thick black line corresponds to the case where monetary policy is fully passive: it does not increase interest rates at any horizon. As a result, while the inflationary shock quickly fades away and inflation falls back strongly in period 1, the shock has shifted long-term inflation expectations, however slightly. Because monetary policy is fully passive, the cumulative process sets in and inflation slowly but inexorably drifts up, at a rate determined by the root $\lambda_2^b(0)$. The thin blue curves correspond to different cases where monetary policy reacts sufficiently to bring the economy back to its stable trajectory. On each curve, the central bank increases the policy rate in a single quarter. The different curves correspond to different choices of the horizon at which it does so, each spaced by two years. The required size of the increase in the policy rate is smaller when done two years after the shock than when done on impact, but increases steadily afterwards. As a consequence, beyond the first two years, the more delayed the hike is, the larger the fall in output that it causes.

4 Optimal Anchoring Policy

That delaying rate hikes beyond the first few quarters increases the size of the required hikes suggests that delaying hikes beyond these first quarters is always ill-advised. Hiking rates today comes at the cost of a lower output today, but hiking tomorrow instead will require larger hikes, with a larger output cost. This suggests a trade-off between a recession today and a larger recession tomorrow, on which doves and hawks should agree to prefer the former. This however is no longer a question about active and passive monetary policy, but a question about optimal monetary policy. In this section, we therefore solve the central bank's optimal policy problem to find the best path of interest rates among those that bring inflation back to target in the long run.

4.1 Characterization: A Generalized Target Criterion

We solve for the optimal monetary policy under commitment in the FPH-learning model (14)-(15). We assume the central bank has a dual mandate to stabilize inflation and the output gap, with a given weight

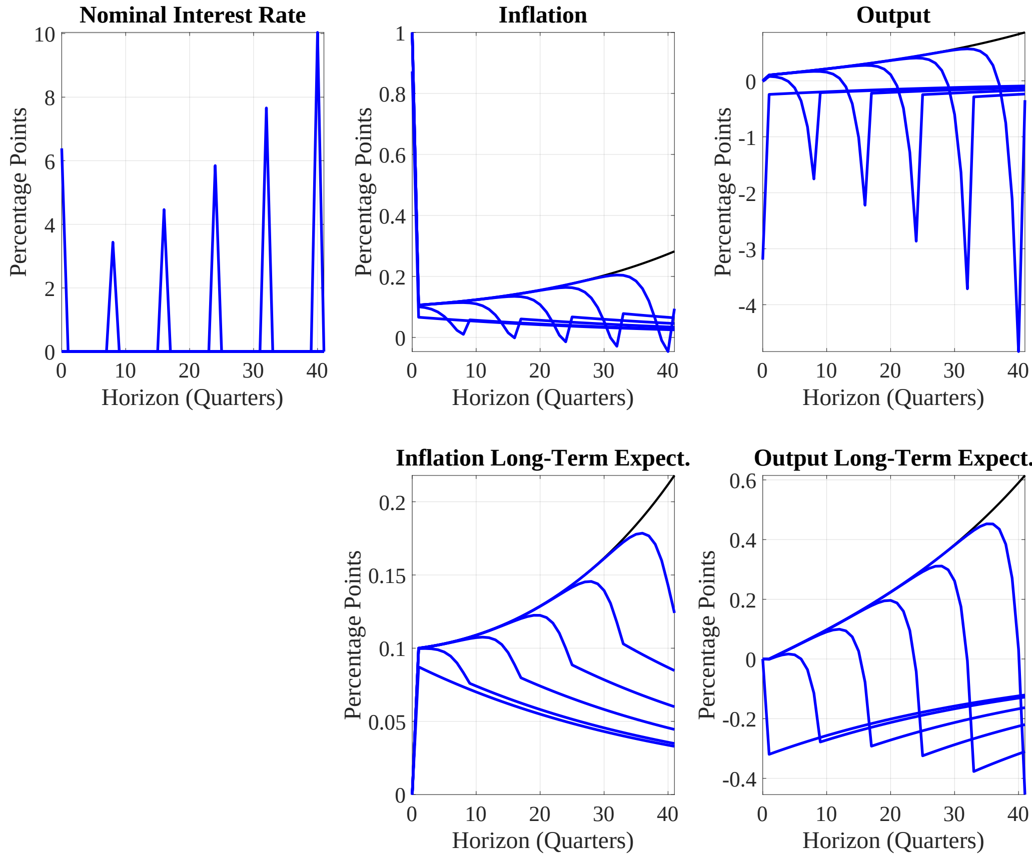

Figure 4: Hiking Now vs. Hiking Tomorrow

Note: The figure gives the response of the economy to a transitory cost-push shock in period 0 under various responses of monetary policy. On the thick black line the central bank does not react at any horizon. On the thin blue curves the central bank responds by increasing its policy rate in a single period, by the amount just necessary to bring the economy back to its stable trajectory. The calibration is the one given in Table 2.

ω on the output gap so that its loss function is[23]

$$E_0 \sum_{t=0}^{\infty} \frac{1}{2} \left(\pi_t^2 + \omega x_t^2 \right). \tag{31}$$

where $x_t = y_t - y_t^e$ is the output gap. We assume that the central bank has rational expectations, i.e. that it fully understands the way in which firms and households form expectations, and therefore the associated de-anchoring risks of failing to react to the shocks that hit the economy. The program of the central bank is

[23] We take this objective as given by the central bank's mandate. It can be shown that in the model under rational expectations, under Calvo price-setting the loss function derived from the representative household's preferences are of the form (31) with $\omega = \kappa/\theta$, where θ is the elasticity of substitution across goods (Woodford 2003, , chapter 6). For a small slope of the Phillips curve κ this puts little weight on output relative to inflation stabilization. The welfare costs under Calvo price-setting are however known to be suspiciously large (Nakamura et al. 2018), so we treat ω as an exogenous parameter. In the version of the FPH-learning model that we consider where planning horizons are heterogeneous among the population, the loss function derived from households' preferences under Calvo pricing is no longer of the form (31) as it is under rational expectations—see Woodford and Xie (2022).

therefore to minimize the loss (31) subject to the constraint of the dynamics of the FPH-learning economy (14)-(15).

Appendix G derives the following characterization of the optimal policies under commitment and under discretion in the FPH-learning model.

Proposition 3. *Let* $M_0 = (I - \rho A_0)^{-1}(1-\rho)A_0$ *and* $\Omega = [\omega, 0; 0, 1]$.

The optimal policy under commitment is characterized by the following target criterion

$$\zeta_t^{\pi} + \frac{1}{\kappa}(\zeta_t^{y} - \rho\zeta_{t-1}^{y}) = 0, \tag{32}$$

while the optimal policy under discretion is characterized by

$$\zeta_t^{\pi} + \frac{1}{\kappa}\zeta_t^{y} = 0, \tag{33}$$

where

$$\zeta_t^{y} = \omega x_t + (1-\mu)\gamma_t^{y^*} \text{ is the marginal cost of higher output at } t, \tag{34}$$

$$\zeta_t^{\pi} = \pi_t + (1-\mu)\gamma_t^{\pi^*} \text{ is the marginal cost of higher inflation at } t, \tag{35}$$

$$(\gamma_t^{y^*}, \gamma_t^{\pi^*})' = E_t \sum_{k=0}^{\infty} (\beta(\mu I + (1-\mu)M_0)')^k \beta M_0' \Omega (Y_{t+k+1} - Y_{t+k+1}^e) \tag{36}$$

is the marginal cost of higher long-term expectations at t.

Together with the dynamics of the economy (14)-(15), both (32) and (33) define a unique equilibrium path for inflation, output and the policy rate.

To understand the target criterion (32), it is useful to see how it generalizes the target criterion under rational expectations. When expectations are rational $\rho = 1$, $M_0 = 0$ and so $\gamma_t^* = 0$. The target criterion under commitment (32) reduces to the classic target criterion

$$\pi_t + \frac{\omega}{\kappa}(x_t - x_{t-1}) = 0. \tag{37}$$

The rational expectations target criterion under commitment sets to zero the sum of the marginal cost of higher inflation, plus $\frac{\omega}{\kappa}$ times the change in the marginal cost of a higher output gap. The marginal cost of higher inflation is simply π_t and the marginal cost of a higher output gap is simply x_t. The fact that the marginal cost of a higher output gap yesterday is subtracted to the criterion is the mark of optimal policy under commitment. It is absent under discretion, with monetary policy under discretion characterized as

$$\pi_t + \frac{\omega}{\kappa}x_t = 0. \tag{38}$$

This history-dependence captures the fact that the optimal monetary policy must deliver on the past promises that it made yesterday in order to move forward-looking expectations yesterday.

The generalized target criterion (32) modifies the one under rational expectations in two ways. First, the cost ζ_t^π of higher inflation at t now includes the cost $\gamma_t^{\pi*}$ of causing higher long-term inflation expectations from tomorrow on—and similarly for the cost of higher output.[24] The cost of higher long-term expectations corresponds itself to the higher inflation and higher output that higher long-term expectations will cause in the future, all else being equal. They can therefore be expressed as a discounted sum of future inflation and future output gaps—equation (36). Because the central bank's ability to commit makes no difference to its ability to affect the backward-looking component of expectations, the expression of (36) is the same under discretion as it is under commitment.

The second modification relative to rational expectations is specific to the policy under commitment. Because expectations are less forward-looking, the term ζ_t^y that captures the commitment of the central bank to past promises is now discounted at the rate ρ. The less forward-looking expectations are, the less the central bank can use its ability to commit to move the forward-looking component of expectations—and so the less it needs to fulfill past promises. When expectations are purely backward-looking $\rho = 0$, there is no role for commitment and so no constraint from past promises. The policy under commitment does not differ from the policy under discretion in this case. Whenever expectations are partly forward-looking $\rho > 0$, a role for commitment and past promises is restored, although it is weaker than under rational expectations.

If the target criterion (32) generalizes the one under rational expectations, il also generalizes the one derived in the adaptive learning model of Molnár and Santoro (2014) where expectations are purely backward-looking. Indeed, Appendix H shows that in the case of purely backward-looking expectations $\rho = 0$, the target criterion (32) reduces to

$$\pi_t + \frac{\omega}{\kappa}\left(x_t - (1-\mu)E_t\left(\sum_{k=0}^{\infty}(\beta\mu)^k\beta^2 x_{t+k+1}\right)\right) = 0, \tag{39}$$

which is the target criterion in the model of Molnár and Santoro (2014).[25]

4.2 Self-Implementing Policy

The characterization of the optimal policy in Proposition 3 holds for any value of $\rho \in [0, 1]$. There is however a key difference depending on whether the degree of foresight ρ is above or below the threshold ρ^*. When foresight ρ is greater than ρ^* and passive monetary policy leads to equilibrium indeterminacy, conducting monetary policy by announcing the interest rates path defined by Proposition 3 together with the dynamics of the economy (14)-(15) does not define a unique equilibrium. To avoid equilibrium indeterminacy, the

[24] Since a one-percentage point increase in inflation at t increases long-term expectations of inflation by $1-\mu$ percentage points, $\gamma_t^{\pi*}$ enters with the coefficient $1-\mu$ in equation (35).

[25] Equation (39) does not appear explicitly in Molnár and Santoro (2014) but can be easily derived from their equations (10) to (16). See also equation (41) in Eusepi and Preston (2018) (written for $\omega = \kappa/\theta$) and equation (24) in Gaspar, Smets, and Vestin (2010) (written for $\beta = 1$).

optimal allocation needs to be implemented through a feedback rule, as it does under rational expectations $\rho = 1$.

But when foresight ρ is lower than ρ^* and passive monetary policy manifests itself as hyperinflation spirals, the issue of equilibrium indeterminacy no longer arises. Announcing the interest rate path defined by Proposition 3 and equations (14)-(15) defines a unique equilibrium. The risk of losing control over inflation expectations still very much exist in the form of hyperinflation spirals. But the optimal policy has already selected an interest rate path that increase rates sufficiently to prevent such spirals. The optimal policy problem captures all that is relevant to determining the best way to anchor expectations.

Corollary 2. *When $\rho < \rho^*$, the optimal policy is self-implementing: Setting the interest rate path defined by Proposition 3 and the dynamics of the economy* (14)-(15) *determines a unique equilibrium.*

4.3 How Fast to Hike?

With this characterization of optimal monetary policy in hands, we return to the question of how fast to hike in response to a supply shock. As Section 3 showed, the relative anchoring effect of policy rates soon decreases with the horizon of the policy rate. Does it imply that the central bank only faces a trade-off between creating a recession today, and creating a worse recession tomorrow? If so, the former may be preferable regardless of a policy-maker's preferences over inflation and output.

Figure 5 plots the response of the economy to a cost-push shock under the optimal policy under commitment for various weights ω on output stabilization.[26] The cost-push shock is assumed to follow an AR(1) shock with persistence $\rho_p = 0.5$. The figures gives the IRF for three different values of the weight ω on output stabilization. As is apparent from the figure, the paths for inflation, output and the policy rate vary considerably with the output weight. When it cares more about output, the central bank increases policy rates much less on impact. As a result, the size of the fall in output is considerably less on impact. To prevent a hyperinflation spiral, the smaller increase in rates early on must be compensated by higher rates later on, in order to bring long-term inflation expectations down. Yet to bring down long-term expectations the central bank does not engineer a recession tomorrow. Instead it keeps the policy rate where it has lifted it for a long time. This leaves output persistently below its steady-state but avoids an outright recession. The persistent tightening does bring long-term expectations back to steady-state in the long run, but it does so slowly.

How does the optimal path for the policy rate compare to the one under rational expectations? The effect of a higher output weight on the optimal way to adjust interest rates is actually qualitatively the same under rational expectations. Figure 6 plots the optimal response under commitment to the same cost-push shock ν_t^p in the rational expectations economy, for the same weights on output stabilization.

[26] We consider the optimal policy under commitment from a timeless perspective, i.e. respecting the commitment to past promises (37) already in the first period (see Woodford 2003, chapter 7). Since we assume that the steady-state level of output is efficient and since we start the IRF in Figure 6 from steady-state, this does matter however.

There are however reasons to be suspicious of this result when it is derived under rational expectations. First, under rational expectations optimal policy under commitment heavily relies on the strong forward-lookingness of rational expectations. In particular, very forward-looking expectations allow the central bank to deliver higher real interest rates not through higher nominal rates but through below-target inflation in the future. Indeed, under rational expectations, the optimal policy (37) is a form of NGDP targeting, which seeks to compensate the inflation on impact with subsequent below-target inflation. This can be seen on Figure 6, where for weights $\omega = 0.1, 0.5$ the nominal policy rate quickly falls below its steady-state value. The increase in real rates is achieved instead through below-target inflation instead, without above steady-state nominal rates.

Second, rational expectations assume away the risk that letting inflation drift away from target will require a costly drop in output tomorrow in order to disinflate the economy. Indeed, under rational expectations, a credible shift in monetary policy brings inflation down at no output cost. Instead, it can easily create an economic boom (Ball 1994). This leaves no room for the argument that hiking early can be beneficial in order to prevent a costly disinflation in the future.

The FPH-learning economy addresses both concerns over rational expectations. First, it makes expectations less foward-looking. Second, it makes disinflation costly. As shown in section (3), in the FPH-learning economy, delaying hikes beyond the first few quarters requires to hike more tomorrow, for a higher cost on output.

Departing from rational expectations does have important consequences for the optimal path of policy rates. For a given weight on output, the central bank increases rates more promptly in the FPH-learning model than under rational expectations. As emphasized in Molnár and Santoro (2014), Eusepi and Preston (2018), Gaspar, Smets, and Vestin (2010), the threat of a de-anchoring of inflation expectations forces the central bank to react more aggressively to a cost-push shock than it does under rational expectations—keeping in mind that the rational expectations benchmark is one in which the increase in rates is extremely gradual for substantial weights on output stabilization.

However, in FPH-learning economy a higher weight on output stabilization still justifies a slower increase in policy rates, as it does under rational expectations. A large weight on output is of course to be judged excessive by policy-makers with a higher weight on inflation—doves are doves. But the recommendation to increase rates strongly and quickly in the face of a large supply shock does not follow mechanically from the possibility of hyperinflation spirals. A relatively lower weight on output stabilization is required for the strong and quick hike to be preferred to a smaller but more persistent one.

5 Conclusion

To stabilize inflation expectations in the face of large supply shocks, a central bank must tighten monetary policy enough. At this broad level the intuition is consistent with the result in rational expectations models that the Taylor principle rules out self-fulfilling inflation. In this paper we have argued that this leaves out important elements of this intuition however. Studying how to stabilize inflation expectations in a bounded-rationality model that captures both forward and backward-looking elements of expectations allows to capture these elements. It captures the risks of passive monetary policy in the form of both self-fulfilling inflation and hyper-inflationary spirals. It allows to identify at what horizons interest rate hikes matter most for anchoring expectations. And it allows to derive policy recommendations directly from the study of optimal policy, since when firms and households' foresight is low enough, a Taylor-type feedback rule is no longer necessary to implement the optimal monetary policy. The optimal monetary policy highlights that while not increasing policy rates today necessarily imposes to increase them later, it does not follow that increasing rates slowly only sets the stage for a worse recession tomorrow. Increasing rates slowly to avoid a recession today can be justified by a large weight on output stabilization, provided policy rates are then kept high for longer.

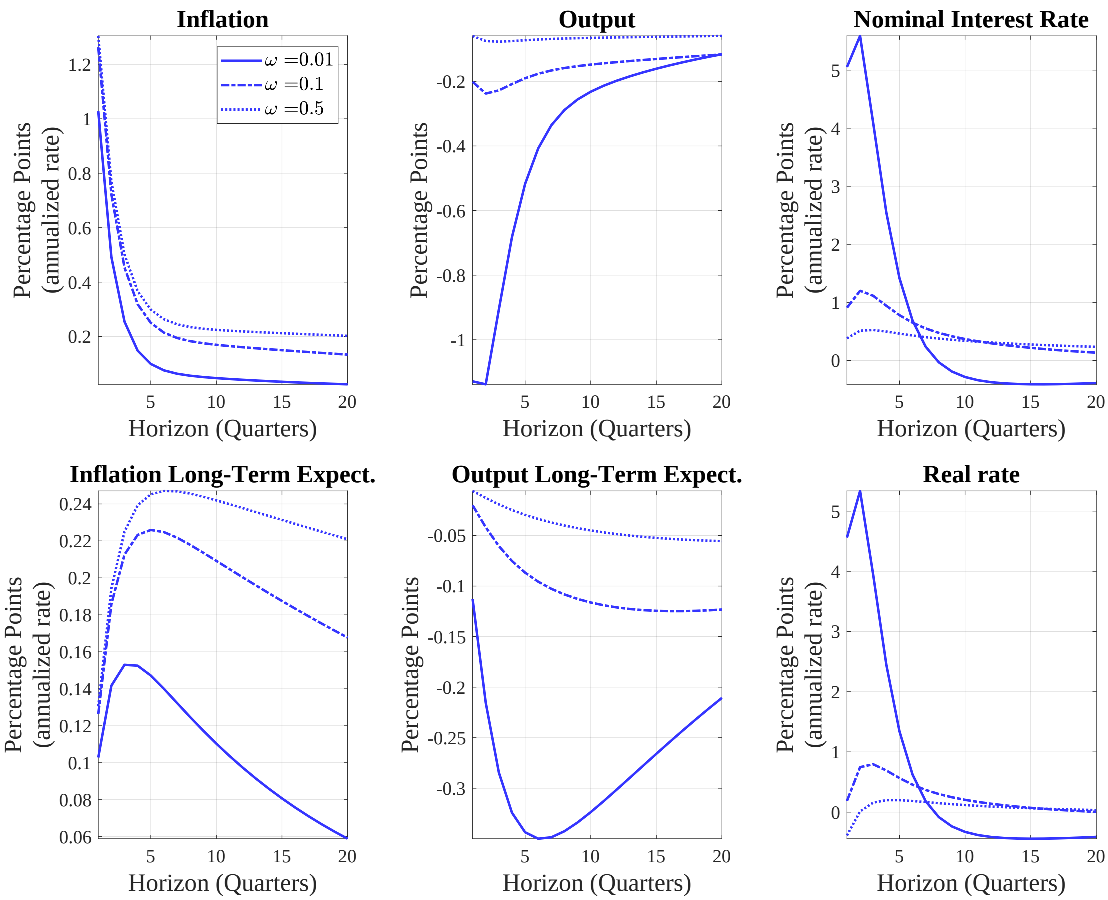

Figure 5: Optimal Response to a Cost-push Shocks in the FPH-Learning Model

Note: The figures gives the impulse response to a cost-push shock in FPH-learning economy (14)-(15)*, when monetary policy is set to minimize the loss function* (31) *under commitment. The persistence of the cost-push shock is* $\rho_p = 0.5$*. The IRF is given for three different values of the weight* ω *on the output gap in the loss function. The calibration of the other parameters is given in Table* 2*. The real interest rate that is plotted is the rationally expected ex-ante real rate* $r_t = i_t - E_t\pi_{t+1}$*.*

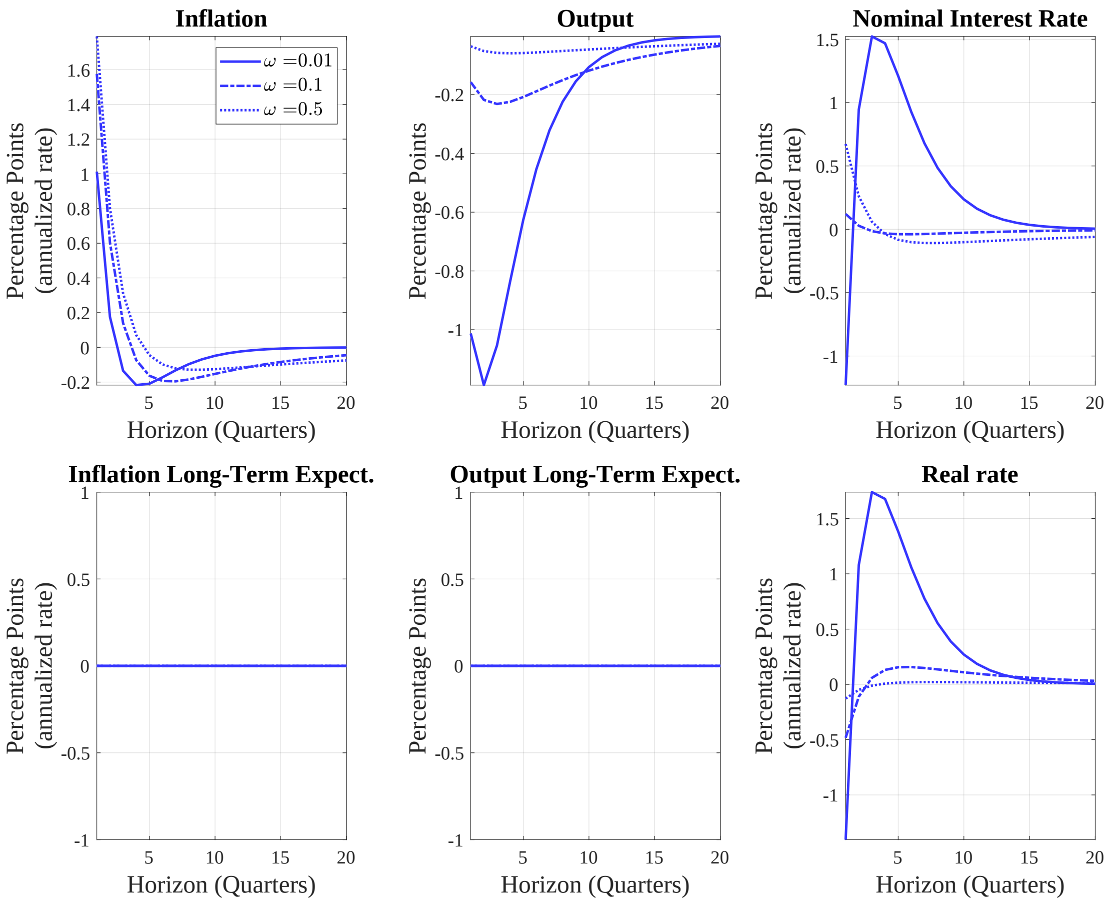

Figure 6: Optimal Response to a Cost-push Shocks under Rational Expectations

Note: The figures gives the impulse response to a cost-push shock in the rational expectations economy (4)*, when monetary policy is set to minimize the loss function* (31) *under commitment. The persistence of the cost-push shock is* $\rho_p = 0.5$*. The IRF is given for three different values of the weight* ω *on the output gap in the loss function. The calibration of the other parameters is given in Table 2.*

A Matrix Expressions

Injecting the Taylor rule into the Euler equation, the New Keynesian model under rational expectations can be written in 2-by-2 matrix form as

$$BY_t = CE_t(Y_{t+1}) + a\nu_t, \tag{A.1}$$

where $Y_t = (y_t, \pi_t)'$, $\nu_t = (\nu_t^y, \nu_t^p - \kappa y_t^e)$, and B, C, a are

$$B = \begin{pmatrix} 1+\sigma\phi_y & \sigma\phi_\pi \\ -\kappa & 1 \end{pmatrix}, \tag{A.2}$$

$$C = \begin{pmatrix} 1 & \sigma \\ 0 & \beta \end{pmatrix}, \tag{A.3}$$

$$a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A.4}$$

It can be rewritten as equation (4) with $A = B^{-1}C$ and $b = B^{-1}a$, i.e.

$$A = \frac{1}{1+\sigma(\phi_y+\kappa\phi_\pi)} \begin{pmatrix} 1 & \sigma(1-\beta\phi_\pi) \\ \kappa & \sigma\kappa+\beta(1+\sigma\phi_y) \end{pmatrix}, \tag{A.5}$$

$$b = \frac{1}{1+\sigma(\phi_y+\kappa\phi_\pi)} \begin{pmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1+\sigma\phi_y \end{pmatrix}. \tag{A.6}$$

For further reference, the inverse of matrix A is:

$$A^{-1} = \begin{pmatrix} 1 + \sigma\phi_y + \frac{\sigma\kappa}{\beta} & \sigma\phi_\pi - \frac{\sigma}{\beta} \\ \\ \frac{-\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}. \tag{A.7}$$

In sections 3 and 4, we use the matrix $b_0^i = B_0^{-1}(\sigma, 0)'$ which is equal to:

$$b_0^i = \begin{pmatrix} \sigma \\ \\ \sigma\kappa \end{pmatrix}. \tag{A.8}$$

B Rewriting of the FPH-Learning Model

We show that the FPH-learning model of Woodford (2019) can be put in the form (11). Note that in contrast to the presentation in Woodford (2019) we assume that all households and firms have the same long-run value functions and long-run expectations regardless of their planning horizons. This simplifies the derivation and is equally meaningful: once output and inflation are realized, households can use these realizations to form long-run expectations instead of their past nowcasts. We show at the end of the Appendix that we nevertheless fall exactly on the same aggregated model as in Woodford (2019).

Throughout this appendix we refer to the equation numbers in Woodford's paper, rewriting them with our notations whenever they slightly differ from Woodford (2019). Woodford (2019) shows that a household with planning horizon h links its expectations at t of output from t to $t+h$ through the standard recursion (equation (2.15))

$$j = 0, ..., h, y_{t+j}^{h-j} = -\sigma(i_{t+j}^{h-j} - E_t(\pi_{t+j+1}^{h-j-1})) + E_t(y_{t+j+1}^{h-j-1}) + \nu_{t+j}^y, \tag{B.1}$$

and at $t+h$ to its long-term value function v_{t-1} through (equation (4.12))

$$y_{t+h}^0 = -\sigma i_{t+h}^0 + v_{t-1} + \nu_{t+h}^y, \tag{B.2}$$

where the superscript $h-j$ takes note of the number of periods until the end of the planning horizon of the agent. The long-term value function v_t satisfies (equation (4.6))

$$v_t = \mu v_{t-1} + (1-\mu)v_t^{est} \tag{B.3}$$

and (equation (4.3))

$$v_t^{est} = y_t + \sigma\pi_t. \tag{B.4}$$

Combining equations (B.2), (B.3) and (B.4) gives

$$y_{t+h}^0 = -\sigma i_{t+h}^0 + (y_{t-1}^* + \sigma\pi_{t-1}^*) + \nu_{t+h}^y. \tag{B.5}$$

On the firm's side, Woodford shows (equation (4.15)) that a firm with planning horizon h that gets to reset its price at t sets it to

$$p_t^{h*} = E_t^h\left(\sum_{j=0}^{h}(\alpha\beta)^j(\pi_{t+j} + (1-\alpha\beta)m_{t+j}) + (\alpha\beta)^{h+1}\tilde{v}_{t-1}\right) \tag{B.6}$$

where $\tilde{v}_{t-1}$ is the firm long-term value function, m denotes marginal cost, and E_t^h denotes firm h's subjective beliefs. The long-term value function $\tilde{v}_t$ satisfies (equation (4.9))

$$\tilde{v}_t = \mu\tilde{v}_{t-1} + (1-\mu)\tilde{v}_t^{est} \tag{B.7}$$

and (equation (4.11))

$$\tilde{v}_t^{est} = \frac{\pi_t}{1-\alpha}. \tag{B.8}$$

Knowing that $\pi_t^h = (1-\alpha)p_t^{h*}$, and $m_t = \zeta(y_t - y_t^e)$, this can be rewritten

$$\pi_t^h = E_t^h\left(\sum_{j=0}^{h}(1-\alpha)(\alpha\beta)^j(\pi_{t+j} + (1-\alpha\beta)\zeta(y_{t+j} - y_{t+j}^e) + (\alpha\beta)^{h+1}\pi_{t-1}^*\right) \tag{B.9}$$

It can be written recursively as

$$j = 0, ..., h-1, \pi_{t+j}^{h-j} = \kappa(y_{t+j}^{h-j} - y_{t+j}^e) + \beta E_t(\pi_{t+j+1}^{h-j-1}), \tag{B.10}$$

$$\pi_{t+h}^0 = \kappa(y_{t+h}^0 - y_{t+h}^e) + \beta\pi_{t-1}^*. \tag{B.11}$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta$ is the slope of the NKPC.

Equations (B.1) and (B.10) write in matrix form

$$j = 0, ..., h-1, Y_{t+j}^{h-j} = AY_{t+j+1}^{h-j-1} + b\nu_{t+j}, \tag{B.12}$$

while equations (B.5) and (B.11) write in matrix form

$$Y_{t+h}^0 = AY_{t-1}^* + b\nu_{t+h}. \tag{B.13}$$

Iterating forward gives equation (10) in the text.

Aggregating across horizons h for the geometric distribution of planning horizons where the fraction of households and firms with planning horizon h is $(1-\rho)\rho^h$ gives equation (11) in the text.

We now show that despite our slight change in assumption that the all households and firms have the same long-run value function and long-run expectations regardless of their planning horizons, the representation of the aggregate economy (9)-(11) (equivalently, the representation (14)-(15)) is exactly equivalent to the representation of the aggregate economy in Woodford (2019).

Equation (14) for the forward-looking component of the economy is simply (4.24) in Woodford (2019), so the forward-looking component is the same. As for the backward-looking component of the economy, Woodford (2019) shows that (equations (4.25) and (4.26)):

$$y_{t-1}^b = \rho y_{t-1}^b - \sigma(i_{t-1}^b - \rho\pi_{t-1}^b) + (1-\rho)(y_{t-1}^* + \sigma\pi_{t-1}^*), \tag{B.14}$$

$$\pi_{t-1}^b = \kappa y_{t-1}^b + \beta\rho\pi_{t-1}^b + \beta(1-\rho)\pi_{t-1}^*. \tag{B.15}$$

This writes in matrix form

$$Y_{t-1}^b = \rho A Y_{t-1}^b + (1-\rho) A Y_{t-1}^*, \tag{B.16}$$

which is exactly equation (12). So the backward-looking components is the same.

C Derivation of the Taylor Principle under FPH only

The FPH-only economy (6) has a unique bounded equilibrium if and only if both eigenvalues of the matrix $(\rho A)^{-1}$ are outside the unit circle. These two eigenvalues are the eigenvalues $\lambda_1^*(\phi)$ and $\lambda_2^*(\phi)$ of the matrix A^{-1}—i.e. of the model under rational expectations—but scaled up by a factor $1/\rho$

$$\lambda_i^f(\phi) = \frac{\lambda_i^*(\phi)}{\rho}. \tag{C.1}$$

The two eigenvalues $\lambda_1^*(\phi)$ and $\lambda_2^*(\phi)$ of A^{-1} are the solutions of the quadratic equation

$$P(\lambda) = \lambda^2 - tr(A^{-1})\lambda + det(A^{-1}). \tag{C.2}$$

where from the expression (A.7) of A^{-1}

$$det(A^{-1}) = \frac{1}{\beta}(1 + \sigma\phi_y + \sigma\kappa\phi_\pi) > 0, \tag{C.3}$$

$$tr(A^{-1}) = \frac{1}{\beta} + \frac{\sigma\kappa}{\beta} + 1 + \sigma\phi_y > 0. \tag{C.4}$$

One of the two roots—λ_1^* without loss of generality—is always outside the unit circle. Indeed, if the roots are real, then $det(A) > 0$, $tr(A) > 0$ implies that both roots are positive, and since $det(A) > 1$ one of them is necessarily greater than one, i.e. outside the unit circle. If the roots are complex, then their common modulus is $\sqrt{det(A)} > 1$ so they are both outside the unit circle.

As a result, $\lambda_1^f(\phi)$ is always outside the unit circle and the condition (C.1) is equivalent to the smaller root $\lambda_2^f(\phi)$ being outside the unit circle. Therefore, there exists a unique bounded solution if and only if $\lambda_2^*(\phi)$ is greater than ρ, which is condition (16) in the text.

Under rational expectations $\rho = 1$, condition (16) is that λ_2^* is outside the unit circle. If the roots are real, this is equivalent to P(1)>0, which is equivalent to the Taylor principle (17). In the case of two complex roots, in which case both roots are outside the unit circle, P(1)>0 so the Taylor principle is also satisfied.

Consider now the case $\phi_\pi = \phi_y = 0$. Denote A_0 the matrix A when $\phi_\pi = \phi_y = 0$. The roots of the matrix A_0^{-1} are then both real, and the smaller root is

$$\lambda_2^*(0) = \frac{tr(A_0^{-1}) - \sqrt{(tr(A_0^{-1}))^2 - 4det(A_0^{-1})}}{2}. \tag{C.5}$$

The condition $\lambda_2^f(0) = \frac{1}{\rho}\lambda_2^*(0) > 1$ is equivalent to $\rho < \rho^* = \lambda_2^*(0)$, which is the expression (18) once replacing the expressions for the trace (C.4) and determinant (C.3).

D Proofs of Lemma 1

The system (11) can be written backward and in matrix form as

$$\begin{pmatrix} E_t(Y_{t+1}^f) \\ Y_t^b \end{pmatrix} = \begin{pmatrix} (\rho A)^{-1} & 0 \\ \Big((1-\mu)(I-\rho A)^{-1}(1-\rho)A\Big) & \Big(\mu I + (1-\mu)(1-\rho)(I-\rho A)^{-1}A\Big) \end{pmatrix} \begin{pmatrix} Y_t^f \\ Y_{t-1}^b \end{pmatrix} - \begin{pmatrix} (\rho A)^{-1} b \\ 0 \end{pmatrix} \nu_t \tag{D.1}$$

This is a triangular system, whose 4 roots are therefore the eigenvalues of the 2-by-2 matrices $(\rho A)^{-1}$ and $(\mu I + (1-\mu)(I-\rho A)^{-1}(1-\rho)A)$. We already encountered the eigenvalues of $(\rho A)^{-1}$, which we denoted λ_i^f, $i = 1, 2$, and whose expression as function of the eigenvalues λ_i^* of A^{-1} is given in (C.1). We denote λ_i^b, $i = 1, 2$ the eigenvalues of the matrix $(\mu I + (1-\mu)(I-\rho A)^{-1}(1-\rho)A)$.

Diagonalize A as:

$$A = Q\Lambda Q^{-1}, \tag{D.2}$$

where $\Lambda = diag(1/\lambda_i^*)$. We have that:

$$(I-\rho A)^{-1}A = (Q(I-\rho\Lambda)Q^{-1})^{-1}Q\Lambda Q^{-1} \tag{D.3}$$

$$= Q(I-\rho\Lambda)^{-1}\Lambda Q^{-1} \tag{D.4}$$

$$= Q diag\left(\frac{1}{\lambda_i^*-\rho}\right)Q^{-1}. \tag{D.5}$$

So that:

$$(\mu I + (1-\mu)(1-\rho)(I-\rho A)^{-1}A) = Q diag\left(\mu + (1-\mu)(1-\rho)\left(\frac{1}{\lambda_i^*-\rho}\right)\right)Q^{-1}. \tag{D.6}$$

So $(\mu I + (1-\mu)(1-\rho)(I-\rho A)^{-1}A)$ is diagonalizable in the same basis as A and its eigenvalues are the ones given in (20).

E Proof of Proposition 1

We first show the following lemma.

Lemma 2. *For a given $i = 1, 2$,*

- *If $\lambda_i^*(\phi)$ is outside the unit circle, then λ_i^f and $\lambda_i^b(\phi)$ are on opposite sides of the unit circle: $\lambda_i^f(\phi)$ outside and $\lambda_i^b(\phi)$ inside.*
- *If $\lambda_i^*(\phi)$ is inside the unit circle—in which case it is necessarily real—distinguish two cases. If*

$$\rho \in \left[\lambda_i^*(\phi), \frac{1}{2}\left(1+\lambda_i^*(\phi)\right)\right], \tag{E.1}$$

$$\mu \in \left[0, \frac{\lambda_i^*(\phi)+1-2\rho}{1-\lambda_i^*(\phi)}\right]. \tag{E.2}$$

then $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are on opposite sides of the unit circle: $\lambda_i^f(\phi)$ inside and $\lambda_i^b(\phi)$ outside. Both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are then real and $\lambda_i^b(\phi)$ is negative.

If condition (E.1)-(E.2) is not satisfied, then $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are on the same side of the unit circle.

1. *If $\rho > \lambda_i^*(\phi)$, then both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are inside the unit circle.*
2. *If $\rho < \lambda_i^*(\phi)$, then both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are outside the unit circle.*

Proof. Assume first that $\lambda_i^*(\phi)$ is outside the unit circle, $|\lambda_i^*(\phi)| > 1$. Then $\lambda_i^f(\phi) = 1/\rho\lambda^*(\phi)$ is outside the unit circle. Besides, since from equation (20)

$$\lambda_i^b(\phi) - \mu = \frac{(1-\rho)(1-\mu)}{\lambda_i^*(\phi)-\rho} \tag{E.3}$$

and

$$|\lambda_i^\star(\phi)-\rho|\geq|\lambda_i^\star(\phi)|-\rho\geq 1-\rho, \tag{E.4}$$

we have that

$$|\lambda_i^b(\phi)|\leq|\lambda_i^b(\phi)-\mu|+\mu<\frac{(1-\rho)(1-\mu)}{|\lambda_i^\star(\phi)-\rho|}+\mu\leq 1.$$

Assume now that $\lambda_i^*(\phi)$ is inside the unit circle $|\lambda_i^*(\phi)|<1$. Because when the $\lambda_i^\star(\phi)$ are complex they have modulus $\sqrt{det(A^{-1})}\geq 1/\sqrt{\beta}$ and are therefore necessarily outside the unit circle, $\lambda_i^*(\phi)$ inside the unit circle is necessarily real. Furthermore, if the $\lambda_i^*(\phi)$ are real, since $det(A^{-1})>0$ and $tr(A^{-1})>0$ they are necessarily positive, so $0<\lambda_i^*(\phi)<1$. Equation (20) implies that

$$sign(\lambda_i^b(\phi)-1)=-sign(\lambda_i^*(\phi)-1)\times sign(\lambda_i^f(\phi)-1), \tag{E.5}$$

so $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are on the same side of 1. It remains possible for them to be on opposite signs of the unit circle however, if $\lambda_i^b(\phi)<-1$ so that $\lambda_i^f(\phi)$ is inside the unit circle and $\lambda_i^b(\phi)$ outside it. This is the case if and only if condition (E.1)-(E.2) is satisfied.

If condition (E.1)-(E.2) is not satisfied, then $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are on the same side of the unit circle, and the side of the unit circle they are on is the side of 1 they are on. If $\rho>\lambda_i^*(\phi)$, then both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are inside the unit circle. If $\rho<\lambda_i^*(\phi)$, then both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are outside the unit circle. □

We now use this lemma to show Proposition 1. We know that one of the two roots $\lambda_1^*(\phi)$ is necessarily outside the unit circle, and only the other root $\lambda_2^*(\phi)$ may be inside or outside the unit circle. From the lemma, it follows that $\lambda_1^f(\phi)=\lambda_1^*(\phi)\rho$ is always outside the unit circle and $\lambda_1^b(\phi)$ is always inside the unit circle. We now distinguish cases for the roots $\lambda_2^f(\phi)$ and $\lambda_2^b(\phi)$.

Assume first that the Taylor principle (17) is satisfied, i.e. $\lambda_2^*(\phi)$ is also outside the unit circle. From the lemma, it follows that $\lambda_2^f(\phi)$ is outside the unit circle and $\lambda_2^b(\phi)$ is inside the unit circle. The system has 2 roots outside the unit circle and two roots inside, so the system has a unique bounded solution.

Assume now that the Taylor principle is not satisfied, i.e. $\lambda_2^*(\phi)$ is inside the unit circle. Consider first the case where condition (21)-(22) is not satisfied. Then from the lemma, $\lambda_2^f(\phi)$ and $\lambda_2^b(\phi)$ are necessarily on the same side of the unit circle, so the system has either 3 roots outside the unit circle and one inside, or 3 roots inside the unit circle and one outside. Either way it has no unique bounded solution. If $\rho>\lambda_2^*(\phi)$, then both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are inside the unit circle, so the system has 3 roots inside the unit circle and the system is indeterminate. If $\rho<\lambda_i^*(\phi)$, then both $\lambda_i^f(\phi)$ and $\lambda_i^b(\phi)$ are outside the unit circle, so the system has no bounded solution.

Consider now the case where condition (21)-(22) is satisfied. The system has then 2 roots outside the unit circle and 2 roots inside, so the system has a unique bounded solution.

F Proof of Proposition 2

We first show the following two lemmas.

Lemma 3. *Let e_2' be the left eigenvector associated to the root $\lambda_2^*(0) < 1$ of A_0^{-1}, and $z_{2,t}^b = e_2' Y_{t-1}^b$.*

The FPH-learning economy (23)-(24) has an unique equilibrium where inflation and output remain bounded and the economy returns to steady-state in the long run if and only if the interest rate path $(i_{t+n}(\nu))_{n\geq 0}$ is such that $E_t(z_{2,t+k}^b) \to 0$ as $k \to \infty$.

The variable $E_t(z_{2,t+k}^b)$ depends on the interest rate path $(i_{t+n}(\nu))_{n\geq 0}$ through:

$$E_t(z_{2,t+k}^b) = \lambda_2^b(0)^{k+1} z_{2,t-1}^b + (\lambda_2^b(0) - \mu)\left(\sum_{n=0}^{k} \phi_k(n) E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu)) + \sum_{n=k+1}^{\infty} \psi_k(n) E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))\right), \tag{F.1}$$

$$\text{where } \phi_k(n) = \lambda_2^b(0)^k \frac{\left(\frac{1}{\lambda_2^f(0)}\right)^{n+1} - \left(\frac{1}{\lambda_2^b(0)}\right)^{n+1}}{\frac{1}{\lambda_2^f(0)} - \frac{1}{\lambda_2^b(0)}}, \tag{F.2}$$

$$\psi_k(n) = \frac{\lambda_2^b(0)^{k+1} - \lambda_2^f(0)^{k+1}}{\lambda_2^b(0) - \lambda_2^f(0)} \left(\frac{1}{\lambda_2^f(0)}\right)^n, \tag{F.3}$$

where, defining, $c = Q^{-1} b_0$, v_t is the function of the shocks $v_t = c\nu_t$ and c^i is a constant vector given by $c^i = Q^{-1} b_0^i$.

Proof. Define

$$Z_t^f = Q^{-1} Y_t^f, \tag{F.4}$$

$$Z_t^b = Q^{-1} Y_t^b. \tag{F.5}$$

The system (23)-(24) in Y_t implies for Z_t:

$$Z_t^f = diag\left(\frac{1}{\lambda_i^f(0)}\right) E_t(Z_{t+1}^f) + c\nu_t - c^i i_t, \tag{F.6}$$

$$Z_t^b = diag\Big(\lambda_i^b(0)\Big) Z_{t-1}^b + diag\Big(\lambda_i^b(0) - \mu\Big) Z_t^f. \tag{F.7}$$

where $c = Q^{-1} b_0$ and $c^i = Q^{-1} b_0^i$.

Denote $z_{1,t}^f$ and $z_{1,t}^b$ the first components of Z_t^f and Z_t^b. They satisfy

$$z_{1,t}^f = \frac{1}{\lambda_1^f(0)} E_t(z_{1,t+1}^f) + v_{1,t} - c_1^i i_t, \tag{F.8}$$

$$z_{1,t}^b = \lambda_1^b(0) z_{1,t-1}^b + (\lambda_1^b(0) - \mu) z_{1,t}^f, \tag{F.9}$$

where $v_t = c\nu_t$ and $v_{1,t}$ is the first component of v_t. Denote $z_{2,t}^f$ and $z_{2,t}^b$ the second components of Z_t^f and

Z_t^b.

$$z_{2,t}^f = \frac{1}{\lambda_2^f(0)} E_t(z_{2,t+1}^f) + v_{2,t} - c_2^i i_t, \tag{F.10}$$

$$z_{2,t}^b = \lambda_2^b(0) z_{2,t-1}^b + (\lambda_2^b(0) - \mu) z_{2,t}^f, \tag{F.11}$$

where $v_{2,t}$ is the second component of v_t.

We first show that the FPH-learning economy (23)-(24) goes back to steady-state at infinite horizon if and only if $E_t(z_{2,t+n}^b)$ to converge to 0. If the FPH-learning economy (23)-(24) is such that $(E_t Y_{t+n}^f, E_t Y_{t+n}^b)$ tends to 0 as n tends to infinity, then necessarily so does $E_t(z_{2,t+n}^b)$. Conversely, we show that a failure of $E_t(z_{2,t+n}^b)$ to converge to 0 is the only thing that can prevent $(E_t Y_{t+n}^f, E_t Y_{t+n}^b)$ from converging to zero, as $E_t(z_{1,t+n}^f)$, $E_t(z_{1,t+n}^b)$ and $E_t(z_{2,t+n}^f)$ necessarily converge to 0.

From equation (F.8), a solution to (23)-(24) that converges to 0 as t tends to infinity is necessarily such that:

$$z_{1,t}^f = E_t \sum_{n=0}^{\infty} \left(\frac{1}{\lambda_1^f(0)} \right)^n (v_{1,t+n} - c_1^i i_{t+n}), \tag{F.12}$$

which is well defined since $\lambda_1^f(0) > 1$. It further implies that $E_t(z_{1,t+n}^f)$ tends to 0 as n tends to infinity. In addition, equation (F.9) implies that $E_t(z_{1,t+n}^b)$ tends to 0 as n tends to infinity, because $\lambda_1^b(0) < 1$.

Just like for $z_{1,t}^f$, from equation (F.8), a solution to (23)-(24) that converges to 0 as t tends to infinity is necessarily such that:

$$z_{2,t}^f = E_t \sum_{n=0}^{\infty} \left(\frac{1}{\lambda_2^f(0)} \right)^n (v_{2,t+n} - c_2^i i_{t+n}), \tag{F.13}$$

which is well defined since $\lambda_2^f(0) > 1$.

We now derive the expression (F.1) for $E_t(z_{2,t+n}^b)$. Iterating equation (F.11) backward gives, injecting (F.13)

$$E_t(z_{2,t+k}^b) = (\lambda_2^b(0) - \mu) \sum_{j=0}^{k} \lambda_2^b(0)^{k-j} \sum_{n=j}^{\infty} \left(\frac{1}{\lambda_2^f(0)} \right)^{n-j} E_t(v_{t+n} - c_2^i i_{t+n}) + \lambda_2^b(0)^{k+1} z_{2,t-1}^b. \tag{F.14}$$

for all $k \geq 0$. Permuting the double summation (distinguishing the cases $n \leq k$ and $n > k$) this can be rewritten as equation (F.1). □

Lemma 4. *Assume that the exogenous shocks converge back to steady-state in expectations* $\lim_{k\to\infty} E_t(\nu_{t+k}) = 0$*, and that (i) holds. Then* $E_t(z_{2,t+k})$ *tends to zero as* $k \to \infty$ *if and only if condition (ii) is satisfied.*

Proof. Let us rewrite the expression (F.1) of $E_t(z_{2,t+k})$ in Lemma (3) as

$$
\begin{aligned}
E_t(z_{2,t+k}) &= a_k + b_k, \text{ where} \\
a_k &= \lambda_2^b(0)^{k+1} z_{2,t-1}^b + (\lambda_2^b(0) - \mu)\left(\sum_{n=0}^{\infty} \phi_k(n) E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))\right) \\
b_k &= (\lambda_2^b(0) - \mu)\left(\sum_{n=k+1}^{\infty} (\psi_k(n) - \phi_k(n)) E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))\right).
\end{aligned}
$$

We show that if (i) is satisfied, then $(b_k)_k$ converges to zero. Let $\varepsilon > 0$. Since $E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))$ tends to zero when n tends to infinity, there exists an integer K such that

$$\forall n \geq K,\ |E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))| \leq \varepsilon.$$

We therefore have that:

$$
\begin{aligned}
\forall k \geq K,\ |b_k| &\leq (\lambda_2^b(0) - \mu) \sum_{n=k+1}^{\infty} |\psi_k(n) - \phi_k(n)||E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))| \\
&\leq (\lambda_2^b(0) - \mu) \sum_{n=k+1}^{\infty} |\psi_k(n) - \phi_k(n)| \times \varepsilon
\end{aligned}
$$

We can calculate

$$|\psi_k(n) - \phi_k(n)| = \frac{-\lambda_2^f(0)}{\lambda_2^b(0) - \lambda_2^f(0)} (\lambda_2^b(0)^{k-n} - \lambda_2^f(0)^{k-n})$$

so that

$$
\begin{aligned}
\sum_{n=k+1}^{\infty} |\psi_k(n) - \phi_k(n)| &= \frac{-\lambda_2^f(0)}{\lambda_2^b(0) - \lambda_2^f(0)} \sum_{m=1}^{\infty} (\lambda_2^b(0)^{-m} - \lambda_2^f(0)^{-m}) \\
&= \frac{\lambda_2^f(0)}{(\lambda_2^b(0) - 1)(\lambda_2^f(0) - 1)}
\end{aligned}
$$

So:

$$\forall k \geq K,\ |b_k| \leq (\lambda_2^b(0) - \mu) \frac{\lambda_2^f(0)}{(\lambda_2^b(0) - 1)(\lambda_2^f(0) - 1)} \times \varepsilon,$$

which proves that $(b_k)_k$ tends to zero. It follows that $E_t(z_{2,t+k}^b)$ tends to zero when k tends to infinity if and only if a_k does. It can be written as

$$a_k = \lambda_2^b(0)^{k+1}\left(z_{2,t-1}^b + \left(1 - \frac{\mu}{\lambda_2^b(0)}\right)\left(\sum_{n=0}^{\infty}\gamma(n)E_t(v_{2,t+n} - c_2^i i_{t+n}(\nu))\right)\right)$$

The term in brackets does not depend on k, so for a_k to tend to zero when k tends to infinity, it is necessary that the therm in brackets is equal to zero, i.e. that condition (ii) holds. □

With these two lemmas in hand, we now show Proposition 2. Accordingly, we maintain the assumption that the exogenous shocks converge back to steady-state in expectations $\lim_{k\to\infty} E_t(\nu_{t+k}) = 0$.

Necessary Condition: Assume that the economy Y_{t+k} converges back to steady-state (i.e. zero) in expectations as k tends to infinity. Then it must be that i_{t+k} converges back to steady-state (i.e. zero) in expectations as well, i.e. condition (i). Indeed, from equation (9), $Y = 0$ in steady-state implies $Y^* = 0$ and so $Y^b = Y^f = 0$ (from equations (12) and (13)). So from equation (23) implies $b_0\nu - b_0 i = 0$, and under the assumption that ν_t returns to steady-state implies that i_t returns to steady-state.

Let us now show condition (ii). From Lemma 3, if the economy returns to steady-state in expectations then $E_t(z_{2,t+k})$ tends to zero. From Lemma 4, it implies that (ii) is satisfied.

Sufficient Condition: Assume now that condition (i) and (ii) are satisfied. From Lemma 3, we need to show that $E_t(z_{2,t+k})$ tends to zero. Since condition (i) is satisfied, Lemma 4 states that this is equivalent to condition (ii).

Finally, note that equation (30) is a corollary of Lemma 3. For $k \geq n$, equation (F.1) implies that

$$\frac{\partial E_t(z_{2,t+k})}{\partial i_{t+n}} = -c_2^i(\lambda_2^b(0) - \mu)\phi_k(n). \tag{F.15}$$

Equation (30) follows.

G Proof of Proposition 3

We first derive the characterization of the optimal policy under commitment, then under discretion.

Optimal Policy under Commitment

The policy rate i_t shows up only in the first row of the forward-looking component of the economy (14) so we can drop i_t and the first row of (14) from the optimization program. (It will give i_t residually.) The second row of (14) writes

$$\kappa y_t^f - \pi_t^f + \beta\rho E_t \pi_{t+1}^f + \nu_t^p = 0. \tag{G.1}$$

For a more economically meaningful interpretation of the Lagrange multipliers, we use the representation of the system in (Y_t^f, Y_{t-1}^*) instead of (Y_t^f, Y_{t-1}^b). The variables Y_t^b is simply the linear transformation of Y_t^*:

$$Y_{t-1}^b = M_0 Y_{t-1}^*. \tag{G.2}$$

where $M_0 = (I - \rho A)^{-1}(1-\rho)A$. The backward-looking component of the economy (15) is then replaced by the following recursion on Y_t^*:

$$Y_t^* = D_0 Y_{t-1}^* + (1-\mu)Y_t^f. \tag{G.3}$$

where $D_0 = \mu I + (1-\mu)M_0$.

The program of the central bank can therefore be written as

$$\min_{Y_t^f, Y_t^*} E_0 \sum_{t=0}^{\infty} \frac{1}{2}(Y_t^f + M_0 Y_{t-1}^* - Y_t^e)'\Omega(Y_t^f + M_0 Y_{t-1}^* - Y_t^e), \tag{G.4}$$

$$\text{s.t. } (\kappa, -1)Y_t^f + \beta(0,\rho)E_t Y_{t+1}^f + (0,1)\nu_t = 0, \tag{G.5}$$

$$\text{s.t. } Y_t^* = D_0 Y_{t-1}^* + (1-\mu)Y_t^f, \tag{G.6}$$

where $\Omega = [\omega, 0; 0, 1]$.

Denote by $\tilde{\gamma}_t$ the univariate Lagrange multiplier associated to the forward constraint (G.1). Denote $\gamma_t^* = (\gamma_t^{y*}, \gamma_t^{\pi*})'$ the 2-dimensional multiplier associated to constraint (G.3). The first-order conditions of the central bank's optimality program are

$$\Omega(Y_t - Y_t^e) + \begin{bmatrix} \kappa \\ -1 \end{bmatrix} \tilde{\gamma}_t + \begin{bmatrix} 0 \\ \rho \end{bmatrix} \tilde{\gamma}_{t-1} + (1-\mu)\gamma_t^* = 0, \tag{G.7}$$

$$\gamma_t^* = \beta E_t\Big(M_0'\Omega(Y_{t+1} - Y_{t+1}^e)\Big) + \beta E_t\Big(D_0'\gamma_{t+1}^*\Big). \tag{G.8}$$

Iterating (G.8) forward gives equation (36) in the text. The first 2-dimensional equations (G.7) can be rewritten to eliminate $\tilde{\gamma}_t$ as

$$\zeta_t^\pi + \frac{1}{\kappa}(\zeta_t^y - \rho\zeta_{t-1}^y) = 0, \tag{G.9}$$

where

$$\zeta_t^\pi = \pi_t + (1-\mu)\gamma_t^{\pi^*}, \tag{G.10}$$

$$\zeta_t^y = \omega(y_t - y_t^e) + (1-\mu)\gamma_t^{y^*}. \tag{G.11}$$

Optimal Policy under Discretion

The optimal monetary policy under discretion can be obtained following the same steps as under commitment. The only difference is that the central bank now takes as given the forward-looking component of expectations $E_t Y_{t+1}^f$ in equation (G.5). (It however still takes into account the effect of its policy on the backward-looking long-run component of expectations Y^*.) The first-order conditions (G.7)-(G.8) under commitment therefore become under discretion

$$\Omega(Y_t - Y_t^e) + \begin{bmatrix} \kappa \\ -1 \end{bmatrix} \tilde{\gamma}_t + (1-\mu)\gamma_t^* = 0, \tag{G.12}$$

$$\gamma_t^* = \beta E_t\Big(M_0'\Omega(Y_{t+1} - Y_{t+1}^e)\Big) + \beta E_t\Big(D_0'\gamma_{t+1}^*\Big). \tag{G.13}$$

Following the same steps as under commitment, equation (G.12) can be rewritten as equation (33).

H Target Criterion in the Purely Backward-Looking Case $\rho = 0$

This appendix shows that in the case of purely backward-looking expectations $\rho = 0$, the target criterion (32) can be written as (39), like in the model of Molnár and Santoro (2014). To show this, we rewrite the optimization program of the central bank this time in the variables (Y_t, Y_t^*). We denote by λ_t and γ_t^* the associated Lagrange multipliers. We get the following relations

$$\Omega(Y_t - Y_t^e) + \begin{bmatrix} \kappa \\ -1 \end{bmatrix} \lambda_t + (1-\mu)\gamma_t^\star = 0$$

$$\beta M_0' \begin{bmatrix} \kappa \\ -1 \end{bmatrix} \lambda_{t+1} - \gamma_t^\star + \beta\mu\gamma_{t+1}^\star = 0$$

Thus, we obtain

$$\gamma_t^\star = \sum_{k=0}^{\infty} (\beta\mu)^k \beta M_0' \begin{bmatrix} \kappa \\ -1 \end{bmatrix} \lambda_{t+k+1}$$

Noticing that

$$M_0' \begin{bmatrix} \kappa \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\beta \end{bmatrix}$$

We finally obtain

$$\gamma_t^{y,*} = 0,$$
$$\kappa\lambda_t + \omega x_t = 0$$

and

$$\pi_t + \frac{\omega}{\kappa} x_t + (1-\mu)\gamma_t^{\pi,*} = 0$$
$$\gamma_t^{\pi,*} = -\beta^2 \sum_{k=0}^{\infty} (\beta\mu)^k \frac{\omega}{\kappa} x_{t+k+1}$$

Combining the two equations gives the target criterion (39).

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