Distorted Prices and Targeted Taxes in the New Keynesian Network Model

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Abstract

The defining feature of the New Keynesian model is that goods prices are adjusted infrequently. In the one-sector version of the model, goods are intrinsically homogeneous and should trade at the same price. By targeting inflation, monetary policy can achieve the efficient allocation. In the network version of the model, sectoral shocks call for an adjustment of relative prices and give rise to a trade-off between adjusting relative prices across sectors and maintaining price stability within sectors. Monetary policy alone can no longer achieve the first best. Against this background, we study the optimal tax response to sectoral shocks. It features twice as many tax instruments as there are sectors, is budget-neutral, and is not confined to the sector where the shock originates. A simple rule that targets sectoral inflation approximates the optimal policy well. We illustrate the quantitative relevance of our results using a calibrated version of the model.

Keywords: Network, pricing frictions, price distortions, optimal policy, sectoral taxes, subsidies, monetary policy

JEL-Codes: E32, E62

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1 Introduction

Aggregate fluctuations are to a large extent driven by sector-specific shocks because some sectors are particularly large and/or because shocks propagate to other sectors through input-output linkages (Foerster et al., 2011; Gabaix, 2011; De Graeve and Schneider, 2023). Recent examples include but are not limited to the global financial crisis, the Covid-19 recession, and the energy crisis caused by the Russian invasion of Ukraine. In this case, assuming that prices are adjusted infrequently, monetary policy is generally unable to implement the first-best allocation (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). The divine coincidence that characterizes the canonical New Keynesian one-sector model breaks down and monetary policy can no longer ensure price stability by closing the output gap.

Against this background, we turn to fiscal policy because it can be targeted to specific sectors in a way conventional monetary policy can not.[1] Specifically, we ask how sectoral taxes should be adjusted in the face of sectoral shocks and establish conditions under which it is possible to restore the efficient allocation. We take up the question within a variant of the New Keynesian Network (NKN) model due to Rubbo (2023). We extend her model in two ways as we introduce a) sector-specific demand shocks (in addition to supply shocks) and b) two fiscal instruments—each sector-specific and potentially time-varying: production subsidies and sales taxes. As a genuine contribution, we derive the canonical representation of the NKN model in the vector space: two distinct sets of sectoral Phillips curves and dynamic IS curves.

Based on this representation, we derive our main result: If applied jointly, subsidies and sales taxes are sufficient to restore the efficient allocation in the face of sectoral shocks. Importantly, the first-best policy relies on *both* instruments because as only a subset of firms adjusts prices, relative prices are distorted within *and* across sectors. Hence, in the very spirit of Tinbergen (1952), the optimal policy needs to operate on both sides of the distortion to replicate the flexible-price allocation. It does so by subsidizing firms' production and simultaneously taxing their sales. The optimal policy is budget neutral and adjusts taxes and subsidies across *all* sectors in a way that reflects their distance from the sector where the shock originates. We also put forward a simple-rule policy that can approximate the first-best policy arbitrarily well.

[1] We abstract from targeted asset purchases by central banks that may be tailored to address sectoral distortions (e.g. Papoutsi et al., 2022).

Our results may inform the design of actual policies and provide a benchmark for assessing policies that have been deployed in the past in response to sectoral shocks, if only on ad hoc basis. For instance, to mitigate the adverse impact of the Covid-19 pandemic on the German restaurant industry, value-added taxes (VAT) in that sector were lowered from 19 to 7 percent during a 3-year period. In response to the surge in energy prices triggered by the Russian invasion of Ukraine, the French government rolled out a large-scale subsidy of energy prices (Langot et al., 2023). More generally, as a way to provide fiscal stimulus to investment, U.S. firms are at times granted a depreciation bonus during recessions. This policy, too, has a sectoral dimension because it impacts sectors differently depending on whether investment is more short-term or long-term (Zwick and Mahon, 2017).

The NKN model offers a framework to systematically analyze sectoral stabilization policies. It features N sectors which are connected through input-output linkages. Production in each sector employs labor and uses intermediate goods, potentially sourced from all sectors. In each sector, there is a continuum of monopolistically competitive firms that operate identical technologies but adjust prices only infrequently and, importantly, asynchronously. Factor inputs within a sector are adjusted flexibly and instantaneously in order to meet demand at posted prices. Households allocate expenditures within and across sectors in order to minimize expenditures and supply labor which we assume to be imperfectly mobile across sectors. As a result, demand shocks call for an adjustment of relative prices which they wouldn't otherwise. There are also sector-specific shocks to total factor productivity (TFP), or "supply shocks," for short.

In order to evaluate policies we derive a welfare measure based on a second-order approximation to household utility. It features three terms related, in turn, to the aggregate output gap, sectoral output gaps, and the price dispersion within sectors. In this case, assuming certain conditions are satisfied, it is optimal for monetary policy to close the aggregate output gap (Rubbo, 2023). Yet such a policy will imply non-zero output gaps and price dispersion at the sectoral level. At this level, the model features a fundamental tradeoff: The more strongly prices adjust in response to sectoral shocks, the *smaller* is the sectoral output gap; yet at the same time—since not all firms in a sector can adjust prices—sectoral inflation and price dispersion are *larger* within the sector. In this environment, the optimal sectoral tax policy can restore the efficient allocation by using two instruments that operate on different sides of the distortion.

We refer to this as the "$2 \cdot N$ policy": by subsidizing production it incentivizes those firms that may adjust prices to keep them stable; via sales taxes, it steers demand so as to close sectoral output gaps. Importantly, sales taxes apply to consumption and intermediate goods trade alike.

In light of these results, three remarks are in order. First, the optimal fiscal policy is budgetary neutral: The subsidy is fully funded by the sales tax. Second, *seller prices* do not change in equilibrium under the optimal tax policy. Sales taxes effectively take over the job of ensuring allocative efficiency across sectors by altering *buyer prices*. In a sense, our results thus turn the "socialist calculation debate" about the allocative role of prices on its head (Von Mises, 1953; Lerner, 1934; Lange, 1936): Because in the New Keynesian model prices fail to adjust instantaneously, taxes can—in theory—be adjusted in their stead to signal scarcity to buyers while seller prices remain constant. Third, the tax policy which we find to be optimal in the presence of sticky prices would have no real effects if prices were fully flexible. And indeed, earlier work by Poterba et al. (1986) performs a test for nominal rigidities based on budget-neutral shifts from direct to indirect taxes.

The network is central to our analysis because it shapes the response of fiscal instruments across sectors. Under the optimal policy, the sales tax and the production subsidy move in sync in all sectors. How strong the response is, however, depends on the distance of a sector from the sector where the shock originates as well as on the nature of the shock. For supply shocks, the tax response is governed by a measure of "downstream" distance, for demand shocks it is "upstream" distance. The response of taxes and subsidies depends also on whether monetary policy responds to the shock. If it does not, the sign of the tax response is the same in all sectors. If, instead, monetary policy accommodates the shock by stabilizing the output gap, the sign of the tax response in distant sectors is the opposite of that in sectors close to the origin of the shock.

Lastly, we also solve for an optimal simple rule that does not directly respond to shocks, but instead to sectoral inflation rates, using again both subsidies and taxes in all sectors. We refer to this as the "simple $2 \cdot N$ rule." The more aggressive the response to inflation, the more closely the simple $2 \cdot N$ rule approximates the optimal policy, a result that is familiar from the normative analysis of monetary policy in one-sector models (Galí, 2015). Likewise, we derive the optimal response of taxes and subsidies under the constraint that only either of the two instruments may be adjusted.

In the last part of the paper, we calibrate the model to capture key features of the production network of the U.S. economy. Specifically, we set $N = 373$ based on the 6-digit classification of the Bureau of Economic Analysis. A key input for the calibration is the actual input-output table for which we rely on data for the year 2007, but we also allow pricing frictions to differ across sectors. We simulate the calibrated model to illustrate the quantitative importance of the optimal response for specific shock scenarios and further decompose the welfare loss under various suboptimal policies into the contribution of a) the output gap, b) price distortions within sectors, and c) price distortions across sectors. We find that a simple $2 \cdot N$ rule can approximate the optimal policy arbitrarily well, bringing the welfare loss all the way down to zero. Instead, policies that have been used in the past, such as subsidizing buyers' energy prices amplify the welfare loss relative to the laissez-faire case. Finally, we compute the ex-ante welfare loss using the actual distribution of shocks across sectors. We find that as monetary policy closes the output gap, substantial welfare losses remain due to distortions within and across sectors. They can be substantially reduced through the simple rule policy.

The paper is structured as follows. In the remainder of the introduction, we place the paper in the context of the literature. Section 2 presents the model, and derives its canonical representation as well as the welfare loss function. In Section 3 we characterize the optimal policy and state our main results. We calibrate and simulate the model in order to quantify our results in Section 4. A final section concludes.

Related literature. In addition to the studies referenced above, our paper relates to several strands of the literature. First, there is work on how sectoral shocks propagate through networks, typically with a focus on TFP shocks and in models without nominal frictions (Horvath, 1998, 2000; Acemoglu et al., 2012; Caliendo et al., 2017), but also in versions of the NKN model (Pastén et al., 2024). Recently, the effects of sectoral government spending shocks have been also investigated, in models with and without pricing frictions (Proebsting, 2022; Bouakez et al., 2022; Flynn et al., 2022; Devereux et al., 2023; Cox et al., 2024b), as well as the sectoral transmission of the economic impact of the Covid-19 pandemic (Guerrieri et al., 2022; Baqaee and Farhi, 2022).

Second, regarding optimal policy, we note that our results appear to conflict with the classic Diamond-Mirrless result according to which intermediate goods should not be taxed (Diamond and Mirrlees, 1971a,b). Yet while Diamond-Mirrless is about avoiding distortions due to taxation in an otherwise efficient economy, we consider an economy subject to (pricing) frictions, which, as we show, can be undone through the appropriate choice of fiscal instruments. More generally, because in our setup the optimal policy is budgetary neutral, we sidestep a number of issues central to the literature on optimal taxation which, following Ramsey (1927), is concerned with minimizing the distortionary impact of raising revenues, notably on capital formation (Chamley, 1986; Judd, 1985; Straub and Werning, 2020), and the interaction with monetary policy in models price flexibility and without (Lucas and Stokey, 1983; Chari et al., 1991; Schmitt-Grohé and Uribe, 2004).

Third, the analysis of optimal policy which accounts for nominal rigidities has largely been limited to one-sector and/or open-economy models, highlighting possible constraints on monetary policy—either through an exchange-rate peg, a monetary union, or the zero-lower bound (Eggertsson and Woodford, 2004; Adao et al., 2009; Correia et al., 2008, 2013; Schmitt-Grohé and Uribe, 2016).[2] In particular, an influential study by Farhi et al. (2014) finds that "fiscal devaluations" in monetary unions require a simultaneous adjustment of several tax instruments, just like our $2 \cdot N$ policy does. Yet, in an open-economy context, deviations from the law of one price may also require the optimal policy to resort to tax instruments even as monetary policy is unconstrained (Chen et al., 2021; Egorov and Mukhin, 2023). We show that the insights from the open-economy literature carry over to the closed-economy version of the New Keynesian model once we move beyond its one-sector version.

Last, there is work on optimal stabilization policy in versions of the NKN model. Woodford (2022) studies optimal transfers in the face of sectoral demand failures. Cox et al. (2024a) derive the optimal adjustment of government spending at the sectoral level in the face of sectoral supply shocks. Because it is costly to deviate from the optimal level of public goods provision, they find that the jointly optimal monetary-fiscal policy does generally not fully restore the first best.

2 The model

We extend the NKN model of Rubbo (2023) in two ways. First, we allow for demand shocks and assume that labor is imperfectly mobile across sectors. Sec-

[2] And indeed, empirical work suggests that tax policies can and are used to make up for the lack of monetary stabilization under these conditions (D'Acunto et al., 2018; Bachmann et al., 2021).

ond, we allow for time-varying taxes and subsidies. Section 2.1 offers a compact outline of the original model and our extensions. In Section 2.2 we depart from Rubbo and derive the canonical representation of the NKN model familiar from the one-sector version of the New Keynesian model. To do so, we resort to vector notation and establish a system of dynamic IS equations and New Keynesian Phillips curves, one set for each sector. We also solve for the allocation under flexible prices and present a stylized example for the propagation of shocks through the network in Sections 2.3 and 2.4, respectively. Section 2.5 introduces the welfare loss function that we use to assess alternative policies in Section 3 below. In what follows refer to Appendix A for additional details and derivations.

2.1 Outline

A representative household enjoys expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \log(C_t) - \sum_i \frac{(L_{t,i}^I)^{1+\gamma}}{1+\gamma} - \frac{(L_t^M)^{1+\gamma}}{1+\gamma} \right\},$$

where E_0 is the expectation operator, $\delta \in [0, 1)$ is the discount factor, $L_{t,i}^I$ is sector-specific (immobile) labor supplied to sector i, L_t^M is non-sector-specific (mobile) labor supplied to all sectors, such that $L_t^M = \sum_i L_{t,i}^M$; C_t is a consumption basket composed of N sectoral goods $C_{t,i}$:

$$C_t = \prod_i C_{t,i}^{\beta_{t,i}}. \tag{1}$$

Here $\beta_{t,i}$ is a demand shifter which tilts preferences towards sector-i goods. For the steady state we assume $\sum_i \bar{\beta}_i = 1$. The consumer pays the *buyer price* $P_{t,i}$ for sectoral good i. It may differ from the *seller price* because of a sales tax introduced below. The consumer price index is then given by $P_t = \prod_i \left(\frac{P_{t,i}}{\beta_{t,i}} \right)^{\beta_{t,i}}$. The household's flow budget constraint reads as follows:

$$P_t C_t + Q_t B_t = B_{t-1} + W_t L_t^M + \sum_i W_{t,i} L_{t,i}^I + T_t.$$

Here B_t is a riskless discount bond that trades at price Q_t, $W_{t,i}$ is the wage earned by sector-specific labor in sector i, W_t is the wage paid to non-sector-specific labor, and T_t are lump-sum profits and government transfers. We rule out Ponzi schemes.

In each sector, there is a continuum of monopolistically competitive firms. Sectoral output is an aggregate over the output of all $k \in [0, 1]$ firms: $Y_{t,i} = \left(\int_0^1 Y_{t,i,k}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$. The sectoral price index is given by $P_{t,i} = \left(\int_0^1 P_{t,i,k}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$, where $P_{t,i,k}$ is the price of firm k operating in sector i. Firm-specific demand is given by:

$$Y_{t,i,k} = \left(\frac{P_{t,i,k}}{P_{t,i}}\right)^{-\epsilon} Y_{t,i}. \tag{2}$$

The production technology has constant returns to scale and given by:

$$Y_{t,i,k} = A_{t,i} \cdot (L_{t,i,k}^{I})^{\alpha_i^I} \cdot (L_{t,i,k}^{M})^{\alpha_i^M} \cdot \prod_j X_{t,ij,k}^{\omega_{ij}(1-\alpha_i)},$$

where $A_{t,i}$ is sector-specific productivity, $L_{t,i,k}^{I}$ and $L_{t,i,k}^{M}$ is sector-specific and non-sector-specific labor used by firm k in sector i, $X_{t,ij,k}$ is the input sourced from sector j by firm k in sector i. ω_{ij}, in turn, corresponds to the share of input j in the intermediate input costs such that $\sum_j \omega_{ij} = 1$. The shares of immobile and mobile labor in each sector are given by α_i^I and α_i^M such that $\alpha_i = \alpha_i^I + \alpha_i^M$ is the total labor share. In what follows, to simplify the analysis, we assume that mobile and immobile labor is employed in constant proportions in each sector, that is, $\alpha_i^I = \kappa\alpha_i$, where κ is the importance of sector-specific labor. This specification nests the case of full labor mobility for $\kappa = 0$ and the case of fully sector-specific labor for $\kappa = 1$, as well as intermediate cases.

The marginal cost of production in sector i is

$$MC_{t,i} = \frac{1}{(\alpha_i^I)^{\alpha_i^I}(\alpha_i^M)^{\alpha_i^M}\prod_j(\omega_{ij}(1-\alpha_i))^{\omega_{ij}(1-\alpha_i)}} \cdot \frac{1}{A_{t,i}} \cdot W_{t,i}^{\alpha_i^I} W_t^{\alpha_i^M} \prod_j P_{t,j}^{\omega_{ij}(1-\alpha_i)}, \tag{3}$$

that is, a function of sectoral productivity, input prices, and wages.

Firms are subject to a Calvo-type price rigidity such that the share of firms updating their prices in sector i in period t is $1 - \lambda_i$ (λ_i is price stickiness). A generic firm k adjusts the seller price (pre taxes), $P_{t,i,k}^s$, such that $P_{t,i,k} = (1 + \tau_{t,i}^s) \cdot P_{t,i,k}^s$, where $\tau_{t,i}^s$ is a sales tax paid by the buyer of a good (either households or downstream firms). As firms adjust their seller price in period t, they maximize the stream of expected future profits conditional on not resetting their price in the future:

$$\max_{P_{t,i}^{s,\star}} E_t \left\{ \sum_{s=t}^{\infty} Q_{t,s}\lambda_i^{s-t} \left(P_{s,i,k}^s Y_{s,i,k} - (1 - \tilde{s}_{s,i}^p) \cdot MC_{s,i} Y_{s,i,k}\right) \right\},$$

subject to firm-specific demand (2), expressed in terms of seller prices. The stochastic discount factor for nominal payoffs is $Q_{t,s} = \delta^{s-t}\left(\frac{P_t C_t}{P_s C_s}\right)$.

In the expression above the term $\tilde{s}_{t,i} = 1/\epsilon + s_{t,i}$ is a sector-specific production subsidy.[3] It is central to our analysis. The constant component $1/\epsilon$ offsets the distortion from monopolistic competition. Below we analyze how the time-varying component $s_{t,i}$ impacts the equilibrium allocation. Importantly, the subsidy is paid to producers, while taxes are paid by buyers—taxes and subsidies thus affect opposite sides of the market. With flexible prices, this does not matter, but with sticky prices, the side of the market to which the tax or subsidy is applied becomes relevant (Poterba et al., 1986).

We define the markup in sector i as the buyer price relative to marginal costs:

$$\mathcal{M}_{t,i} \equiv \frac{(1+\tau^s_{t,i}) \cdot P^s_{t,i}}{MC_{t,i}}. \tag{4}$$

Given this expression, we may anticipate our main result below: the optimal policy neutralizes the impact of changes in marginal costs on the seller price via subsidies but ensures via taxes that the buyer price adjusts in such a way as to keep the markup constant—thus replicating the flexible-price allocation.

Finally, we specify fiscal and monetary policy. At each point in time and for each sector, fiscal policy sets a tax and a subsidy paid by the buyer and to the seller of the good, $\tau^s_{t,i}$ and $s^p_{t,i}$, respectively. The government budget is balanced in each period via a lump-sum tax:

$$T_t = \sum_i \bar{\tau} MC_{t,i} Y_{t,i} + \sum_i \tau^s_{t,i} P^s_{t,i} Y_{t,i} - \sum_i s^p_{t,i} MC_{t,i} Y_{t,i}.$$

Monetary policy controls the money supply M_t which equals nominal private spending $M_t = P_t C_t$.

In equilibrium, all agents behave optimally and all markets clear. Product market clearing in sector i implies that production of sector i is either consumed by households or used as an intermediate input.

$$Y_{t,i} = C_{t,i} + \sum_j X_{t,ji}. \tag{5}$$

[3] In the Online Appendix we show that the production subsidy nests the case of a labor subsidy or any other input-specific subsidy as special cases.

2.2 Canonical representation

We solve the model using a log-linear approximation around the efficient steady state. In the steady state, all time-varying fiscal instruments are set to zero. To derive our results, we first show how to cast the NKN model into the canonical representation familiar from the one-sector NK model. For this purpose, we resort to vector notation and define several matrices. The input-output matrix Ω, collects the input-specific shares in the total cost of production such that $\Omega_{ij} = (1-\alpha_i)\omega_{ij} = \frac{X_{ij}P_j}{MC_iY_i}$ is the cost share of input j in the production of Sector-i goods.

We also define the corresponding Leontief inverse matrix as $L = (I-\Omega)^{-1}$ and a vector of steady-state sector-specific sales shares $\boldsymbol{\xi}$ such that $\xi_i = \frac{P_iY_i}{PC}$ (Domar weights). We refer to column vectors $[X_1, ..., X_N]'$ with bold letters $\boldsymbol{X}$. The log-deviation of X from its steady state, in turn, is denoted by a lowercase letter so that $x = \log(X) - \log(\bar{X})$, where $\bar{X}$ denotes the steady-state value. Matrix I_X is a diagonal matrix with vector $\boldsymbol{X}$ on the diagonal and $\mathbf{1}$ is a column vector of ones.

Given these definitions, we can cast the equilibrium conditions of the model into the canonical (system of) IS curve(s) and Phillips curve(s). In the first step, to derive the system of sectoral IS curves, we rely on market clearing at the sectoral level as well as for the labor market so as to relate sectoral wages, $w_{t,i}$ and the non-sector-specific wage w_t, to monetary policy and sectoral markups (in logs, $\mu_{t,i} = \log(\mathcal{M}_{t,i})$):

$$\boldsymbol{w_t} = \mathbf{1}\cdot m_t + \frac{\gamma}{1+\gamma} I_\xi^{-1} L' I_\beta \boldsymbol{b_t} - \frac{\gamma}{1+\gamma} I_\xi^{-1} L' I_\xi \boldsymbol{\mu_t}, \tag{6}$$

$$w_t = m_t - \frac{\gamma}{1+\gamma}\boldsymbol{\xi}'\boldsymbol{\mu_t}, \tag{7}$$

where $\boldsymbol{b_t}$ is a vector of demand shifters, capturing log-deviations in sector-specific consumption shares, $b_{t,i} = \log(\beta_{t,i}) - \log(\bar{\beta}_i)$.

Using the definition of markups, we can write seller prices as a function of marginal costs, in turn, linked to wages, productivity, and taxes:

$$\boldsymbol{p_t^s} = L(\boldsymbol{\mu_t} - \boldsymbol{a_t}) + L I_\alpha(\kappa \boldsymbol{w_t} + (1-\kappa)\mathbf{1}\cdot w_t) - \boldsymbol{\tau_t^s}. \tag{8}$$

where $\boldsymbol{\tau_t^s}$ is the vector of sector-specific sales taxes.

We also introduce a vector of sector-specific output gaps $\boldsymbol{\tilde{y}_t}$, capturing the log deviations of sector-specific final output from its efficient level. By combining (6) and (7) with (8) and aggregating, we obtain:

$$\boldsymbol{\tilde{y}_t} = -(L - \hat{L} I_\xi)\cdot \boldsymbol{\mu_t}, \tag{9}$$

where matrix $\hat{L} = \frac{\gamma}{1+\gamma} \cdot [\mathbf{1}\boldsymbol{\beta}' + \kappa(I - \mathbf{1}\boldsymbol{\beta}')] \cdot LI_{\alpha}I_{\xi}^{-1}L'$ and $\boldsymbol{\beta}$ is a vector of steady-state sectoral consumption shares. The aggregate output gap is given by $\tilde{y}_t = \boldsymbol{\beta}'\tilde{\boldsymbol{y}}_t$. By combining equations (6) - (9) we link prices to output gaps:

$$\boldsymbol{p}_t^s = -\tilde{\boldsymbol{y}}_t + [m_t \cdot \mathbf{1} - \boldsymbol{\tau}_t^s] + [\hat{L}I_{\beta}\boldsymbol{b}_t - L\boldsymbol{a}_t]. \tag{10}$$

Evaluating (10) in period $t+1$, taking expectations, and subtracting it from the (10), we finally obtain a *system* of dynamic IS equations:

$$\tilde{\boldsymbol{y}}_t = -(R_t \cdot \mathbf{1} - E_t\boldsymbol{\pi}_{t+1}^s - \boldsymbol{r}_t^n) + E_t\tilde{\boldsymbol{y}}_{t+1} + \boldsymbol{\Delta\tau}_{t+1}^s, \tag{11}$$

where $R_t = E_t[\Delta m_{t+1}]$ is the nominal interest rate, $\boldsymbol{\pi}_t^s$ is a vector of sector-specific seller price inflation ($\pi_{t,i}^s = p_{t,i}^s - p_{t-1,i}^s$), and $\boldsymbol{r}_t^n = -E_t[\hat{L}I_{\beta}\boldsymbol{\Delta b}_{t+1} - L\boldsymbol{\Delta a}_{t+1}]$ is vector of sector-specific natural interest rates. These expressions illustrate that natural rates differ across sectors to the extent that shocks are sector-specific and, hence, monetary policy will generally be unable to adjust the common nominal interest rate accordingly—one size doesn't fit all (sectors). And even if only one sector is experiencing a shock, in the presence of input-output linkages natural rates will move differently across sectors. Note further, however, how (changes in) sales taxes emerge as an additional term in the system—an observation which will be relevant below as we derive the optimal tax response to sectoral shocks.

The sectoral Phillips curves are derived from the price-setting block of the model. Log-linearizing the first-order condition associated with the price-setting problem and rearranging terms, we obtain a link between sectoral seller price inflation and sectoral markups

$$\boldsymbol{\pi}_t^s = \tilde{I}_{\lambda}(-\boldsymbol{\mu}_t + \boldsymbol{\tau}_t^s - \boldsymbol{s}_t^p) + \delta E_t\boldsymbol{\pi}_{t+1}^s. \tag{12}$$

Here $\boldsymbol{s}_t^p$ is a vector of sector-specific production subsidies. Matrix $\tilde{I}_{\lambda} = I_{\lambda}^{-1}(I - I_{\lambda})(I - \delta I_{\lambda})$ and I_{λ} feature sectoral price stickiness on the diagonal. Substituting sectoral output gaps using (9), we obtain a system of New Keynesian Phillips curves linking sectoral inflation to sectoral output gaps:

$$\boldsymbol{\pi}_t^s = \tilde{I}_{\lambda}(L - \hat{L}I_{\xi})^{-1} \cdot \tilde{\boldsymbol{y}}_t + \delta E_t\boldsymbol{\pi}_{t+1}^s + \tilde{I}_{\lambda}(\boldsymbol{\tau}_t^s - \boldsymbol{s}_t^p). \tag{13}$$

Note that taxes and subsidies also show up as additional terms in this Phillips curve system—a sector-specific Phillips curve residual. Also, note that our representation of the equilibrium dynamics differs from Rubbo (2023). Her analysis links sectoral inflation to the *aggregate* output gap as it is concerned with the tradeoffs faced by monetary policy, while our representation is geared towards the role of fiscal instruments at the sectoral level.

Note that equations (11) and (13) pin down the dynamics of sectoral output gaps and inflation rates as a function of shocks and policies only. As such they represent the equilibrium dynamics of the NKN model in the vector space in an analogous way to what is familiar as the canonical representation of the one-sector model (Galí, 2015).

2.3 Natural benchmark

To assess the equilibrium outcome under alternative policies, we first solve the model assuming that prices are flexible and time-varying tax instruments are zero. In this way, we obtain the "natural" allocation which provides a benchmark for policy. In this allocation, markups are constant in all periods and all states: $\boldsymbol{\mu_t} = 0$. Letting $\boldsymbol{p_t^n}$ and $\boldsymbol{y_t^n}$ denote the vectors of sectoral prices and final outputs that obtain in the natural allocation, system (10) combined with sectoral consumption demand yields the following solution:

$$\boldsymbol{p_t^n} = m_t \cdot \mathbf{1} - L\boldsymbol{a_t} + \hat{L}I_\beta \boldsymbol{b_t}, \tag{14}$$

$$\boldsymbol{y_t^n} = (I - \hat{L}I_\beta) \cdot \boldsymbol{b_t} + L\boldsymbol{a_t}. \tag{15}$$

A couple of remarks are in order. First, note that the money supply m_t shifts all flexible prices uniformly, determining the aggregate price level. Next, the expression above shows how sectoral productivity and demand shifts, $\boldsymbol{a_t}$ and $\boldsymbol{b_t}$, move prices and sector outputs depending on the network structure. Specifically, the impact of productivity shocks is governed by the Leontief inverse matrix L the ij-th element of which captures the effect of a shock in sector j on sector i. Recall that the matrix L is constructed from the underlying input-output matrix where element ij is the cost-based share of input j in production of i. As such the Leontief inverse provides a measure of *downstream proximity* (Acemoglu et al., 2016). Accordingly, the negative (positive) effect of productivity shocks on prices (quantities) is stronger in more closely connected downstream sectors.

In addition, expressions (14) and (15) show that the sectoral impact of demand shocks is determined by matrix $\hat{L}I_\beta$. For prices the effect depends on this matrix only, for quantities the overall effect is given by a direct positive effect of increased demand and a negative effect due to increased prices. Formally, as we show in the Online Appendix, we can write the matrix as follows:

$$\hat{L}I_\beta = \kappa\frac{\gamma}{1+\gamma} \cdot L \cdot I_\alpha \underbrace{I_\xi^{-1} L' I_\xi}_{=U} \cdot I_\xi^{-1} I_\beta + (1-\kappa)\frac{\gamma}{1+\gamma}\mathbf{1}\boldsymbol{\beta}', \tag{16}$$

where $I_\xi^{-1}I_\beta$ is a diagonal matrix with elements $\frac{P_iC_i}{P_iY_i}$ (final sales in total sales) on the diagonal. $U = I_\xi^{-1}L'I_\xi = (I - \tilde{\Omega})^{-1}$ is a modified Leontief inverse such that $\tilde{\Omega}_{ij} = \frac{X_{ji}}{Y_i}$ is the share of sector-i sales purchased by sector j in steady state. Hence, U provides a measure of *upstream proximity*. In addition, expression (16) features L, reflecting the fact that demand shocks propagate not only upstream, but also downstream.

Finally, the flex-price solution (14) and (15) is also informative about the role of labor mobility. To see this, consider the limiting case where labor is perfectly mobile across sectors ($\kappa = 0$). In this case, matrix (16) simplifies to:

$$(\hat{L}I_\beta)|_{\kappa=0} = \frac{\gamma}{1+\gamma}\mathbf{1}\boldsymbol{\beta}', \tag{17}$$

meaning that if labor is perfectly mobile across sectors, a shock that shifts sectoral demand, $b_{t,i}$, has a *uniform* effect on natural prices across sectors. The effect on sectoral output is then given by the direct effect of the shock net of the uniform price response. In sum, with perfectly mobile labor the origin of the demand shock does matter for quantities but not for prices.

Note that the uniform effect of demand shocks on relative sectoral prices under fully mobile labor pertains to natural price levels. Actual relative prices may still be affected by demand shocks if sectors differ in the degree of price rigidity. To see this, combine systems (10) and (12) and rearrange terms to obtain the dynamic system of seller prices:

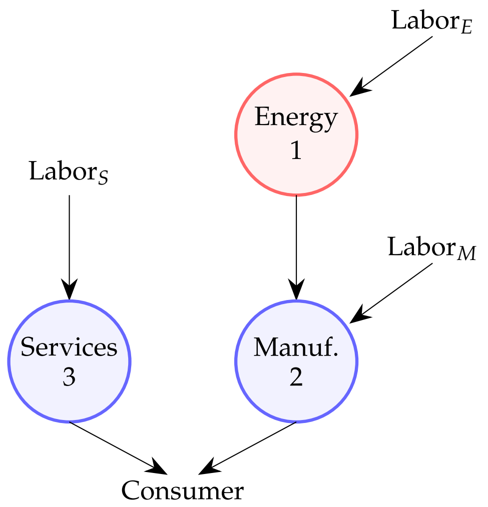
$$\boldsymbol{p_t^s} = Z \cdot \boldsymbol{p_{t-1}^s} + ZX \cdot (\boldsymbol{p_t^n} - \boldsymbol{\tau_t^s}) + Z\tilde{I}_\lambda \cdot (\boldsymbol{\tau_t^s} - \boldsymbol{s_t^p}) + \delta Z \cdot E_t \boldsymbol{p_{t+1}^s}, \tag{18}$$

where $X = \tilde{I}_\lambda \cdot (L - \hat{L}I_\xi)^{-1}$ and $Z = ((1+\delta) \cdot I + X)^{-1}$. The expression shows that the propagation of a shock onto seller prices operates at two levels. First, shocks affect the vector of natural prices through the propagation mechanism described above. Then, this change is passed through onto actual prices according to the weights in the matrix ZX which combines sectoral price stickiness and network parameters.

2.4 Example

To fix ideas, we consider a three-sector economy as an example. It consists of two final-good sectors, services and manufacturing, and one intermediate-

Figure 1: Three-sector economy



Notes: Graphical illustration of 3-sector economy. Service and energy sector use labor as only input. Manufacturing uses labor and intermediate inputs sourced from Energy sector.

good sector, energy. We assume that manufacturing uses energy as input, while services are produced exclusively with labor. Figure 1 provides a graphical illustration of the network structure. As we study the transmission of shocks in this economy, we assume that labor is fully immobile across sectors and that prices are flexible.

As discussed above, the propagation of sectoral productivity shocks onto natural prices is determined by the Leontief matrix which in the present case takes a particularly simple form:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1-\alpha_M & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Its entries determine the effect of a productivity shock originating in either of the three sectors (columns) on the sector itself and the two other sectors (rows). For instance, the first column shows the effect of an energy shock on—from top to bottom—energy, manufacturing, and services, respectively. In this example, services prices do not respond, but prices in the other sectors do:

$$p_E^n = -a_E, \text{ and } p_M^n = -(1-\alpha_M)\cdot a_E.$$

The shock propagates downstream: an adverse productivity shock in the energy sector raises the natural price of energy but also that of manufacturing, exactly proportional to the energy use in manufacturing (reflecting the underlying Cobb-Douglas technology).

Instead, the transmission of sectoral demand shocks is governed by the matrix:

$$\hat{L}I_{\beta} = \frac{\gamma}{1+\gamma}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

As a result, the impact of a demand shock originating in manufacturing (sector 2) leaves services prices unchanged and alters manufacturing and energy prices as follows

$$p_M^n = \frac{\gamma}{1+\gamma} b_M, \text{ and } p_E^n = \frac{\gamma}{1+\gamma} b_M.$$

In other words, a negative demand shock in manufacturing leads to a natural price decrease not only in manufacturing but also in the upstream energy sector. In this example, the upstream propagation is not followed by a subsequent downstream propagation because energy is not used in the production of other downstream goods or for final consumption. However, the demand shocks will also propagate downstream if energy is used as input in the services sector, as we show in the Online Appendix.

2.5 Welfare loss

We assess the outcome of alternative policies in terms of the associated welfare loss based on a second-order approximation to household utility. As we show in the Online Appendix, it can be expressed as follows:

$$\Delta_t \approx \frac{1}{2} \cdot E_0 \sum_{t=0}^{\infty} \delta^t \cdot \{ \underbrace{f_y \cdot \tilde{y}_t^2}_{\text{due to output gap}} + \underbrace{\tilde{\boldsymbol{y}}_t' \cdot F_y \cdot \tilde{\boldsymbol{y}}_t}_{\text{due to cross-sector}} + \underbrace{\boldsymbol{\pi}_t^{s\prime} \cdot F_p \cdot \boldsymbol{\pi}_t^s}_{\text{due to within-sector}} \} + t.i.p. \tag{19}$$

The welfare loss, Δ_t, depends on three terms in addition to terms independent of policy (t.i.p.): the aggregate output gap, sector-specific output gaps, and sector-specific seller price inflation rates.[4] The corresponding weights are

$$f_y = 1 + \gamma,$$
$$F_y = (L' - I_\xi \hat{L}')^{-1}[(I_\xi - \boldsymbol{\xi\xi}') + 2 \cdot N - \frac{\gamma}{1+\gamma} M](L - \hat{L} I_\xi)^{-1},$$
$$F_p = \epsilon \cdot I_\xi I_\lambda (I - I_\lambda)^{-1} (I - \delta I_\lambda)^{-1};$$

with $N_{ij} = \xi_i (\Omega (I - \Omega)^{-1})_{ij}$ and $M = I_\xi (I_\xi^{-1} L' - \mathbf{11}')' I_\xi I_\alpha (I_\xi^{-1} L' - \mathbf{11}') I_\xi$.

The first term in expression (19) captures aggregate misallocation: the aggregate output gap is proportional to the average markup (which is zero in the first best). There is also a welfare loss due to sector-specific output gaps (second term): because prices are sticky, the buyer price measured in relative terms across sectors will generally adjust insufficiently to sectoral shocks. The second term in expression (19) reflects the resulting misallocation of resources *across* sectors; note that sectoral output gaps are related to inefficient sectoral markups by eq. (9). At the same time, price stickiness induces an inefficiency within sectors. Since some firms within a sector adjust prices in response to sector-specific shocks and others do not, there is price dispersion within sectors which would be absent if prices were flexible. The third term in expression (19) represents the resulting misallocation *within* sectors; this within-sector inefficiency due to sticky prices induces a welfare loss that is familiar from the basic one-sector version of the New Keynesian model.

In that version of the model, the second term in the welfare loss function disappears. And indeed, in the baseline version of the one-sector model, monetary policy is able to stabilize inflation by closing the output gap (divine coincidence). In the multi-sector economy, however, monetary policy faces a tradeoff between tolerating inefficiencies within and across sectors. Generally, a higher level of inflation is conducive to a stronger adjustment of relative prices. At the same time, it induces stronger price dispersion within sectors. We revisit this tradeoff below, as we consider the use of additional policy instruments.

[4] Instead, Rubbo (2023) expresses the welfare loss as a function of the aggregate output gap and sectoral inflation rates because, in the model without taxes, there exists a linear homogeneous transformation between sectoral output gaps and inflation rates. La'O and Tahbaz-Salehi (2022) express the welfare loss as a sum of three terms. However, in contrast to the expression above, they use expectation errors to characterize each term.

3 Results

In this section, we present our main theoretical result which characterizes a tax policy that achieves the first-best allocation. To set the stage, we first establish the limitations of monetary policy to stabilize the economy in the face of sector-specific shocks. We delegate the proofs of all results to Appendix B.

3.1 Limitations of monetary policy

Assume for now that time-varying fiscal instruments are kept at their steady-state level of zero. In this case, monetary policy is generally unable to achieve the first best allocation. Nevertheless, monetary policy can still play an important stabilization role. In particular, Rubbo (2023) shows that stabilizing the output gap is the optimal monetary policy in the NKN model once one assumes full discounting (static economy).[5] We spell out this policy formally in what follows.

Proposition 1 (Output gap targeting w/o the use of fiscal instruments)**.** *Consider a static economy ($\delta = 0$), which is at the steady state at time $t-1$ ($\boldsymbol{p^s_{t-1}} = 0$). Assuming that fiscal instruments are not used, closing the output gap, $\tilde{y}_t = 0$, requires for the money supply:*

$$m_t = -\frac{1}{\boldsymbol{\beta'}(X-I)\mathbf{1}} \cdot \boldsymbol{\beta'}(X-I) \cdot [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]. \tag{20}$$

By adjusting m_t to close the output gap in this way, monetary policy improves welfare significantly, as we show in Section 4 below. However, it cannot restore the first best allocation because as already discussed in Section 2.2 above, it cannot simultaneously track the natural rates of interest that generally differ across sectors. An exception is the special case of the one-sector economy in which case targeting the output gap implies $m_t = a_t - \frac{\gamma}{1+\gamma} b_t$. And indeed, this policy removes the residual in the demand curve (see eq. 11) while stabilizing inflation—divine coincidence obtains.

[5] As noted in Rubbo (2023), the output gap targeting remains nearly optimal in the dynamic economy ($\delta > 0$).

3.2 Optimal fiscal policy

Given the limitations of monetary policy, we turn to fiscal instruments, specifically, to sectoral production subsidies $\boldsymbol{s_t^p}$ and sales taxes $\boldsymbol{\tau_t^s}$.[6] We will show that these instruments are sufficient to restore the first best allocation.[7] The following proposition establishes a combination of sector-specific sales taxes and production subsidies that simultaneously stabilize sectoral output gaps and sector-specific seller price inflation. An implication is that the aggregate output gap is also closed.

Proposition 2 ($2 \cdot N$ policy)**.** *The first best sectoral tax policy implies that the output gap, all sectoral markups, and seller price inflation in all sectors are fully stabilized:* $\tilde{y}_t = 0$*,* $\boldsymbol{\mu}_t = 0$*,* $\boldsymbol{\pi^s}_t = 0$*. Let initial seller prices be at their steady-state value* $\boldsymbol{p^s_{-1}} = 0$*. Then, the sectoral production subsidies and sales taxes achieving this outcome are*

$$\boldsymbol{s_t^p} = m_t \cdot \mathbf{1} + [\hat{L} I_\beta \boldsymbol{b_t} - L \boldsymbol{a_t}], \tag{21}$$

$$\boldsymbol{\tau_t^s} = \boldsymbol{s_t^p}. \tag{22}$$

Several remarks are in order. First, the optimal tax policy requires two tax instruments to be adjusted in each sector: the sales tax and the production subsidy. Hence, the optimal policy features $2 \cdot N$ tax instruments in total.[8] Formally, the optimal sales tax insulates output gaps from changes in natural rates in the dynamic IS system (11), while the optimal production subsidy stabilizes sectoral seller price inflation in the Phillips curve system (13) by offsetting the Phillips curve residuals created by sales taxes.

Second, we emphasize that the optimal policy eliminates the *twofold* distortion due to sticky prices, within and across sectors. To the extent that prices are

[6] Recall that the sales taxes apply to consumers and downstream firms alike, in contrast to conventional value-added taxes (VAT) for which producers are typically reimbursed. A distinct property of our instruments is therefore that they allow targeting upstream sectors whose output is not directly used in final consumption.

[7] They are not necessary in the sense that for special cases of the network and/or the incidence of shocks a subset of instruments is sufficient. There are also other instruments that may be used instead. In a complementary paper Cox et al. (2024a) consider the optimal sectoral government spending policy which, however, turns out to be generally not sufficient to restore the first best.

[8] The $2 \cdot N$ instruments are required for a given monetary policy. In case fiscal and monetary policy are coordinated to achieve the first best, $2 \cdot (N-1)$ tax instruments are sufficient since monetary policy can be tailored to stabilize one sector. For instance, one can adjust the money supply to eliminate the residual in the first equation of the demand system (10), while setting the tax and subsidy in this sector to 0. Then elimination of the remaining N-1 residuals would require adjusting taxes (and subsidies) in the remaining N-1 sectors.

adjusted infrequently, sectoral shocks induce a misallocation within sectors. To eliminate this welfare loss, the optimal production subsidy offsets the effect of shocks on marginal costs within a sector. This incentivizes firms that can adjust prices to leave them unchanged. However, as a result, the relative prices across sectors would also remain unchanged, failing to induce a sectoral reallocation of expenditure which is called for in the face of sectoral shocks. This is where the sales tax comes in: it is designed to mimic the efficient pricing mechanism. It ensures that buyer prices continue to fluctuate, thus signaling relative scarcity, even when seller prices remain constant. In other words, while the subsidy eliminates the within-sector welfare loss by stabilizing $\boldsymbol{\pi}^s$, the sales tax takes care of cross-sector welfare loss by stabilizing sectoral output gaps $\tilde{\boldsymbol{y}}$.

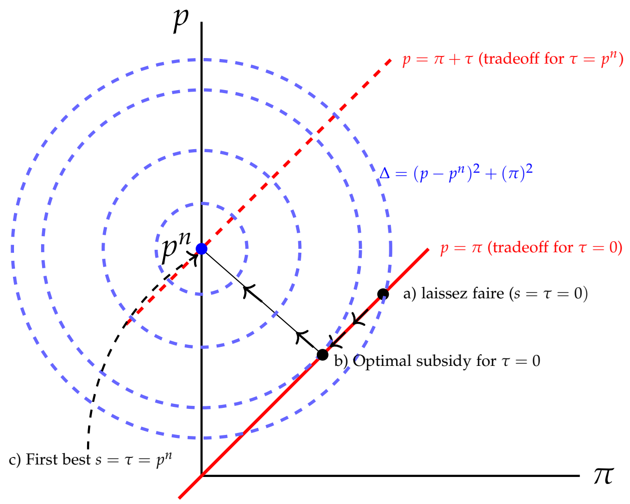
Third, optimal sales taxes move exactly one-for-one with the vector of counterfactual flexible prices. To see this, substitute the optimal policy (21) and (22) in (10) and (12), yielding $\boldsymbol{p}_t^s = 0$, and note that $\boldsymbol{p}_t = \boldsymbol{p}_t^s + \boldsymbol{\tau}_t^s$ to obtain: $\boldsymbol{p}_t = \boldsymbol{p}_t^n = m_t \cdot \mathbf{1} + \hat{L} I_\beta \boldsymbol{b}_t - L\boldsymbol{a}_t$ which is precisely the flexible-price solution in (14). This is the distinct feature of the $2 \cdot N$ policy: it implements the efficient allocation by ensuring that effective prices move as if they were fully flexible—even though firms do not adjust prices at all. In this way, the optimal policy provides a remedy for the infrequent price adjustment that is the defining friction of the New Keynesian model. Given the discussion in Section 2.3 above, this also implies that a productivity shock in an upstream sector calls for an adjustment of taxes and subsidies downstream (as productivity shocks propagate downstream). At the same time, a demand shock might require setting taxes/subsidies both upstream and downstream according to the pattern of its propagation onto flexible prices.

To illustrate the issue graphically, Figure 2 zooms in on a generic sector. Its horizontal and vertical axes measure seller price inflation and the market price of the sectoral good respectively. Consider an adverse, sector-specific TFP shock that raises marginal costs. Absent any policy intervention, only a fraction of firms in the sector will raise their price: price dispersion in the sector increases in sync with (sectoral) inflation. At the same time, there is an insufficient response of relative prices (captured by the gap between p and p^n) resulting in a sectoral output gap. In the figure this laissez-faire scenario is indicated by point a).

Now consider a possible policy intervention and, more specifically, an adjustment of the fiscal instruments under consideration.[9] If fiscal policy relies on

[9]Recall that by adjusting money supply monetary policy affects all sectors simultaneously

Figure 2: The relative price-inflation tradeoff in a generic sector



Notes: Illustration omits sector indices, abstracts from network, $\Omega = 0$, assumes full discounting, $\delta = 0$, and that the economy is initially in steady state ($p_{t-1} = 0$), see Appendix B for details.

subsidies only, it faces a tradeoff, indicated by the red (solid) line. By paying a subsidy the equilibrium outcome is pushed along this line towards the origin: the firms that adjust prices respond less to the shock and price dispersion is reduced—but at the expense of a weaker response of relative prices. To simplify, the figure omits sector indices, assumes $\delta = 0$ (full discounting), $\boldsymbol{p_{t-1}} = 0$ (economy is in steady state at time $t-1$), and abstracts from the input-output network: $\Omega = 0$, ensuring that welfare loss is independent across sectors. In this case, the optimal level of the subsidy (absent taxes) obtains in point b): it minimizes the sectoral welfare loss indicated by the radius of the dashed (blue) circles and given by $\Delta = f_\mu(p - p^n)^2 + f_\pi(\pi^s)^2$

The bliss point is in point c). Here price dispersion is zero and the relative price changes in line with the natural relative price. However, to get there, one needs to resort to taxes in addition to the subsidy. In the special case under consideration, the buyer price is linked to inflation in the following way: $p = \pi^s + \tau^s$ (since $p_{t-1} = 0$). Hence, raising taxes shifts the tradeoff up. In the figure, this is illustrated by the shift from the solid (red) line towards the dashed (red) line. As we show in Appendix B, there is a tax/subsidy pair τ and s which uniquely pins down the pair of seller price inflation and the market

and can thus not be tailored to address the distortions in a specific sector.

price as $\pi^s = \frac{1}{1+b} \cdot [p^n - (1-a) \cdot \tau - a \cdot s]$ and $p = \frac{1}{1+b} \cdot [p^n + (a-b) \cdot \tau - a \cdot s]$; the optimal tax-subsidy pair moves the economy to the point c).

The optimal policy features tax instruments of the same size but of different signs (tax vs subsidy): see Proposition 2. This implies that it is budgetary neural around the efficient steady state. The following Corollary establishes this result.

Corollary 1 (Budget neutrality)**.** *To the first order, the ratio of net government revenue from sales tax and production subsidies to output equals* $T_\tau = \boldsymbol{\xi}'\boldsymbol{\tau}_t^s - \boldsymbol{\xi}'\boldsymbol{s}_t^p$. *The optimal policy is budget-neutral, that is* $T_\tau = 0$.

Finally, consider a situation when only one tax instrument is available: either sales taxes or production subsidies. In this case, as our discussion above makes clear, the first best allocation is infeasible. However, we may still characterize the optimal one-instrument policy. Intuitively, the optimal policy trades off stabilizing the relative prices $\boldsymbol{p_t} - \boldsymbol{p_t^n}$ and stabilizing $\boldsymbol{\pi_t^s}$. The following Proposition establishes the constrained optimal policy.[10]

Proposition 3 (Single tax instrument policies)**.** *Consider a static economy* $\delta = 0$, *which is at the steady state at time* $t-1$ ($\boldsymbol{p_{t-1}^s} = 0$). *The vector of production subsidies maximizing welfare when* $\boldsymbol{\tau_t^s} = 0$ *is*

$$\boldsymbol{s_t^p} = (L - \hat{L}I_\xi)^{-1} \cdot (I - X^{-1}(\hat{F}_\mu + \hat{F}_p)^{-1}\hat{F}_\mu) \cdot \underbrace{(m_t \cdot \mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t})}_{\text{first best prod. subsidy}}$$

The vector of sales taxes maximizing welfare when $\boldsymbol{s_t^p} = 0$ *is*

$$\boldsymbol{\tau_t^s} = [X(L - \hat{L}I_\xi - I) + A]^{-1} \cdot (A - X) \cdot \underbrace{(m_t \cdot \mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t})}_{\text{first best sales tax}}$$

where $\hat{F}_\mu = \tilde{F}_\mu + \tilde{F}_\mu'$ *and* $\tilde{F}_\mu = (L' - I_\xi \hat{L}')^{-1} \cdot (\frac{f_y}{(1+\gamma)^2}\boldsymbol{\xi}\boldsymbol{\xi}' + F_\mu) \cdot (L - \hat{L}I_\xi)^{-1}$ *and* $\tilde{F}_p = F_p$, $A = (((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu + (L - \hat{L}I_\xi - I)'X'\hat{F}_p)^{-1} \cdot ((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu$.

Note that the optimal single-instrument policies are linear combinations of the unconstrained policy response. However, in contrast to the $2 \cdot N$ policy, the constrained policies depend on the details of the welfare loss function, that is the weights put on the output gaps and inflation in the welfare loss. These weights, in turn, depend on the details of sector-specific price stickiness.

[10]The proof is available in the Online Appendix.

3.3 A simple rule

While the optimal policy described above can achieve the first best allocation, it relies on observing the underlying shocks. Hence, we consider, as an alternative, a *simple* fiscal rule that is constrained to respond to observable variables only, as in the analysis of optimal monetary policy (for instance, Galí, 2015). Specifically, we posit a rule that adjusts fiscal instruments in response to sectoral seller price inflation only. The following proposition establishes that a simple rule may approximate the first-best policy in the limit.

Proposition 4 (Simple rule)**.** *Consider the following rule for adjusting subsidies and taxes:*

$$\boldsymbol{s}_t^p = \phi \cdot \boldsymbol{\pi}_t^s,$$
$$\boldsymbol{\tau}_t^s = \boldsymbol{s}_t^p,$$

where ϕ governs the strength of adjustment to sectoral producer price inflation. Then, the resulting allocation becomes first-best for $\phi \to \infty$.

For the static economy, $\delta = 0$ and $\boldsymbol{p}_{t-1}^s = 0$, the simple rule implies the solution:

$$\boldsymbol{s}_t^p = (I + \phi X)^{-1} \cdot \phi X \cdot \underbrace{(m_t \cdot \mathbf{1} + \hat{L} I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t})}_{\text{optimal subsidy}}.$$

The strength of the fiscal response is governed by the parameter ϕ. It regulates by how much the outcome under this simple-rule policy deviates from the first best. To see this, note that in the static case, the simple-rule subsidy is the "discounted" linear transformation of the subsidy under the $2 \cdot N$ rule: The larger ϕ, the more closely the simple rule resembles the optimal policy. Note that the simple rule with large ϕ represents a strong commitment to stabilize sectoral seller price inflation by adjusting sectoral production subsidies (and sales taxes). Our model simulation below shows that in equilibrium, this commitment results in muted seller price inflation rather than in large subsidies/taxes.[11]

4 Quantitative analysis

In this section, we calibrate the model to U.S. data and illustrate the quantitative relevance of our results. We do so by first studying the optimal policy response

[11]Also for $\delta = 0$, we obtain: $\boldsymbol{\pi}_t^s = \frac{\boldsymbol{s}_t^p}{\phi} = (I + \phi X)^{-1} \cdot X \cdot (m_t \cdot \mathbf{1} + \hat{L} I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t})$ which is decreasing in ϕ.

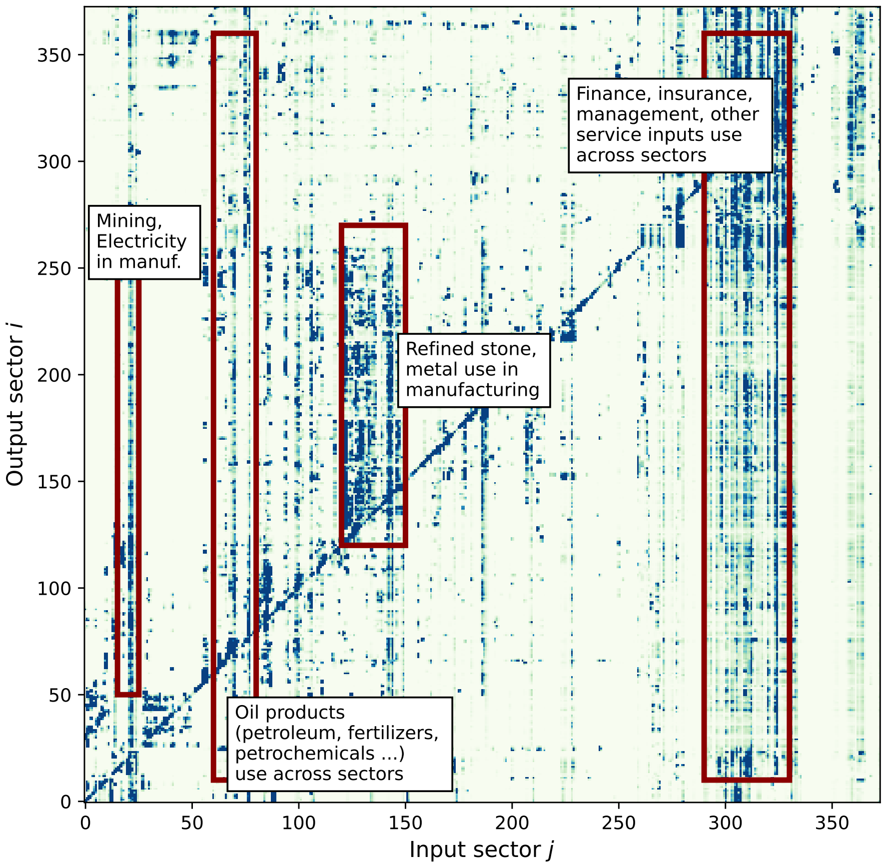
to specific shock scenarios. Second, we compute the welfare loss for various alternative policies, both conditional on the shock scenarios and unconditionally based on the actual cross-sectional distribution of shocks.

4.1 Model calibration and solution

In our calibration, we distinguish three sets of parameters: those that characterize the input-output network, the other structural parameters, and the parameters that govern the shock processes. We postpone the specification of the shock distribution until subsection 4.4 where we compute the expected welfare loss under various sub-optimal policies. In this subsection, we focus on the calibration of the first two sets of parameters. We discuss them in turn. First, we use the input-output account data for the year 2007 as reported in the "Use table" of the Bureau of Economic Analysis (BEA). This table contains information on intermediate input costs, labor costs, and consumption expenditures for 402 industries, defined based on a 6-digit classification. Following Baqaee and Farhi (2020), and in line with our model analysis above, we drop the government, scrap, and non-comparable import sectors from the data. Then we drop the industries for which no "use data" is available (for both intermediate input and final consumption). This leaves us with 373 sectors in total. We then use the table to determine for each sector i the intermediate input share ω_{ij} as the share of input-j costs in the total intermediate input costs, the labor share α_i as the ratio of employee compensation to total cost, and finally β_i as the ratio of final consumption expenditure to total final consumption expenditure across sectors.

In Figure 3 we use a heatmap to represent the input-output network. An element in the matrix indicates the extent to which a specific sector positioned along the vertical axis sources from the sectors positioned along the horizontal axis. The sectoral indices run from the bottom (left) to the top (right), ranging from the upstream sectors such as agriculture and natural resource mining to the downstream sectors such as manufacturing and services. The diagonal elements are mostly dark, indicating that many firms source a lot from their own sector, even at the very granular 6-digit level of disaggregation. As an example, we observe that energy inputs produced in "mining" and "electricity generation" are widely used in manufacturing sectors. Likewise, a broader set of oil products, including petroleum, fertilizers, and chemicals, is used across many sectors of the economy, ranging from agriculture to services. Additionally, we see that the products of more upstream manufacturing sectors, such as stone

Figure 3: Input-output network in 373-sector economy



Notes: Visualization of input-output table as provided by BEA for the year 2007 (6-digit industry classification). Sector i (vertical axis) uses input from sector j (horizontal axis), measured in terms of cost shares. Darker entries correspond to larger values. The maximum (darkest) value corresponds to 94.6 percent; the maximum value on the diagonal is 60.4 percent.

and metals, are used by more downstream manufacturing industries. Finally, a broad range of services such as insurance, finance, and management is again widely used across sectors.

Second, we calibrate the remaining structural parameters, assuming a period in the model represents a month. We account for the sectoral heterogeneity in price rigidity and set $1 - \lambda_i$ to match the estimates for the sector-specific price flexibility reported by Antonova (2024). According to her estimates, price flexibility varies substantially across sectors, ranging from 0.052 to 0.989, with a median of 0.277.[12] The remaining structural parameters are common across

[12] This implies that 27.7% of firms reset their price within a month in the median sector, which is consistent with a median price duration of 4.3 months reported in Bils and Klenow (2004)

sectors. Specifically, we set the inverse Frisch elasticity to $\gamma = 2$. We set the discount factor $\delta = 0.997$ and the elasticity of substitution between varieties within sectors to $\epsilon = 8$. The degree of labor mobility is $\kappa = 0.5$ in the baseline calibration and we report results for alternative values in the Online Appendix.

Solving our dynamic rational expectation model requires specifying stationary dynamic processes governing sectoral productivity and demand. We assume that both of them follow an AR(1) process in each sector with a common persistence parameter $\rho = 0.97$.[13] Given our calibration, solving the model amounts to finding the solution to the system of equations describing seller price dynamics, see eq. (18). We find a solution to this system using the method of undetermined coefficients. The solution algorithm is described in the Online Appendix. The solution is unique under an exogenous monetary policy because we assume a money supply policy rather than an interest rate rule (Galí, 2015).

4.2 The optimal response to sectoral shocks: An illustration

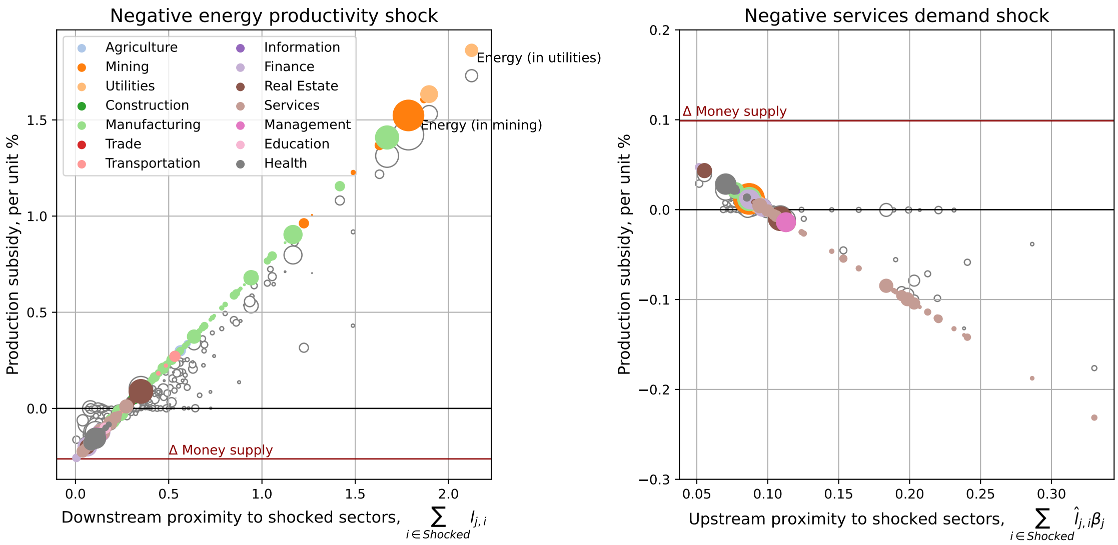
We are finally in a position to trace out the quantitative implications of the optimal policy response to sectoral shocks. For illustrative purposes, we consider two types of shocks that originate simultaneously in a number of generic sectors, as either supply or demand shocks. We define an adverse "energy productivity shock" first. It lowers productivity, $\boldsymbol{a}_t$, simultaneously by 1 percent in key energy-related industries as defined by the 2-digit sectoral classification: 21 - "Mining" and 22 - "Utilities". Energy-related sectors in "Mining" include oil and gas extraction, while those in "Utilities" encompass electric power generation. Overall there are 10 6-digit sectors in these categories, accounting for about 10% of total sales in the economy. Productivity in the remaining sectors is unchanged.

Next, we define a "services demand shock" as a simultaneous and exogenous decline in demand, $\boldsymbol{b}_t$, by 1 percent in service-related industries using the 2-digit sector groups: 71 - "Arts, Entertainment, and Recreation", 72 - "Accommodation and Food Services", and 81 - "Other Services". In total, there are 22 6-digit sectors in these categories, accounting for about 7% of total sales in the economy. Demand in the remaining sectors does not shift exogenously in this shock scenario.

We compute the optimal policy response to both shock scenarios based on

[13] Specifically, we assume that $\boldsymbol{a}_t = \rho \boldsymbol{a}_{t-1} + \boldsymbol{\epsilon}_t^a$ and $\boldsymbol{b}_t = \rho \boldsymbol{b}_{t-1} + \boldsymbol{\epsilon}_t^b$ where the vectors for sectoral productivity and demand shock innovations are given by $\boldsymbol{\epsilon}_t^a$ and $\boldsymbol{\epsilon}_t^b$, respectively.

Figure 4: Subsidy distribution under optimal policy



Notes: Solid dots show optimal policy response in each sector to negative productivity shock in energy sectors (left panel) and adverse demand shock in services sectors (right panel). Circles show response under simple rule policy ($\phi = 50$). The size of dots/circles represents the size of sectors measured by sales share.

the result of Proposition 2 and show it in Figure 4. In the left panel, we consider the adverse productivity shock in "energy," and measure—for each sector—the optimal policy response in terms of the subsidy along the vertical axis against an increasing (average) network proximity to "energy" along the horizontal axis. The right panel of the same figure is organized in the same way but shows results for the demand shock, in which case network proximity differs, as discussed in Section 2.3 above.

We make three observations. First, the optimal policy response varies systematically across sectors. In response to the productivity shock, the optimal policy implies a production subsidy in most sectors as well as higher sales taxes (not shown).[14] In response to the demand shock, there is a reduction of the subsidy in most sectors as well as reduced taxes (again not shown). In both instances, the strength of the response increases linearly in the proximity to the shocked sectors where it is, in fact, strongest. Over time, the effect of the shocks declines almost monotonically, as we illustrate through impulse responses, shown in the Online Appendix.

[14]Recall from Proposition 2 that taxes move one-for-one with subsidies under the $2 \cdot N$ policy. Hence, we do not show it to economize on space.

Second, we compute the optimal tax policy under the assumption that monetary policy adjusts the money supply according to eq. (20). This means that the money supply is lowered in response to an adverse productivity shock but raised in response to an adverse demand shock. In the panels, we indicate the monetary policy response by the red line. It is a horizontal line since it applies uniformly across sectors and, hence, does not depend on network proximity. It matters, however, for the optimal tax policy. While the subsidy is raised (lowered) in most sectors in response to the productivity (demand) shock, its sign flips very distant sectors because here the extent of monetary accommodation can be "excessive".

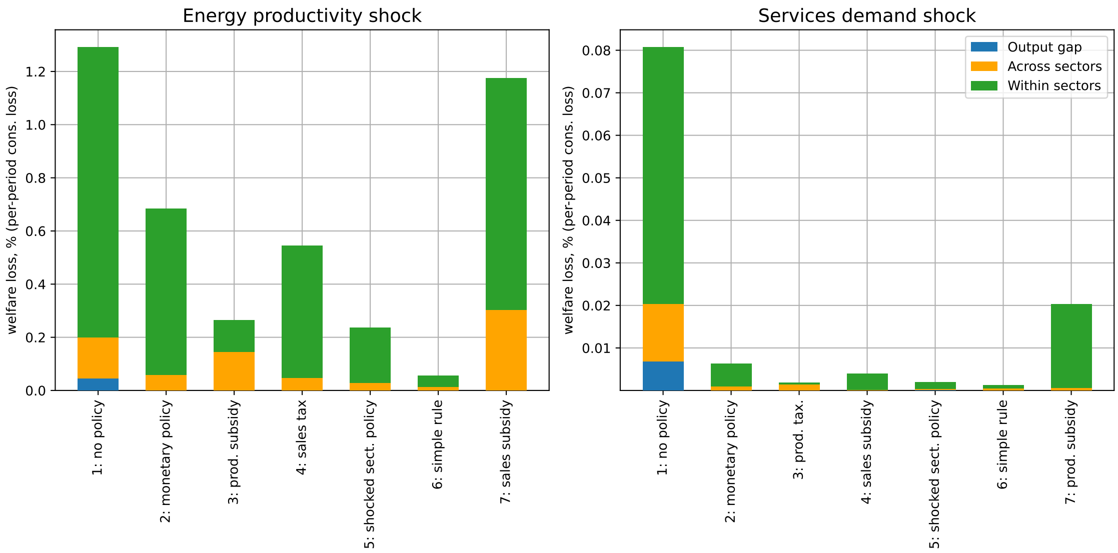
Last, Figure 4 also shows the response of the simple-rule policy to the shocks. Its sectoral distribution is indicated by the circles in both panels. Earlier, we have established that the simple-rule policy is the weighted transformation of the optimal policy. Considering the results in Figure 4, we note that it implies a policy response that across sectors is reasonably close to the optimal subsidy. Yet we also observe that it accounts less systematically for network proximity than the response under the optimal policy.

4.3 Sub-optimal policies

We now turn to alternative policies that are not (fully) optimal. We compute the welfare loss (19) associated with these policies for the two shock scenarios devised above, decomposing it into the contributions of the output gap, the cross-sector misallocation, and the within-sector misallocation. To set the stage, we first compute and decompose the welfare loss in the absence of any policy response. The leftmost bars in each panel of Figure 4.3 show the results. We find that in both cases price dispersion and the resulting allocative inefficiency within sectors (green) make the largest contribution to the welfare loss. Yet sector-specific output gaps (yellow) and, to a lesser extent, the aggregate output gap (blue) also contribute to the welfare loss. The latter disappears once we assume that monetary policy adjusts the money supply according to eq. (20).[15] This case is represented by the second bar in the panels of the same figure. Importantly, in this case, the welfare loss also declines along the other two dimensions. In particular, in the case of the demand shock, monetary policy goes a long way towards eliminating the welfare loss, even though the adverse

[15]Hence, even in the dynamic economy this rule nearly closes the aggregate output gap, see footnote 5.

Figure 5: Welfare loss for specific shock scenarios



Notes: Shock is an exogenous decline of productivity in "energy" (left) and of demand in "services" (right) by 1% (with sectors broadly defined, see Section 4.2). "no policy" (bar 1) assumes constant money supply; bar 3-7 assume monetary policy adjusts according to eq. (20) w/o fiscal adjustment; bar 2: no fiscal adjustment; Results for simple rule policy assume $\phi = 50$.

demand shock originates in "services" only.

The next bars, from left to right, represent alternative policies, each (except for the very last) being a constrained version of the optimal policy. First, we consider the effect of adjusting either only taxes or subsidies (bars 3 and 4, respectively), rather than deploying both policies jointly, as the optimal policy would have it. We obtain the intuitive result that the production subsidy reduces the within-sector inefficiency at the expense of raising the inefficiency across sectors. For the sales-tax-only policy, it is the opposite. We also consider the restricted two-instrument policy such that only shocked sectors are subjected to subsidies and taxes, while taxes and subsidies in the other sectors do not adjust (bar 5). Such a policy does somewhat better than the optimal single-instrument policies. However, the optimal policy requires adjustments not only in the shocked sector but also in other sectors connected to the shocked sector by the production network. To see why, recall that the fully optimal policy effectively replicates flexible prices. And the flexible-price adjustment in a shocked sector also triggers the desired price adjustment in other sectors. If this adjustment is subject to pricing friction, the distortion in other sectors will persist unless taxes and subsidies are adjusted. We also show results for the simple rule which adjusts taxes and subsidies in response to seller price inflation (bar 6). For a reaction coefficient of $\phi = 50$, the welfare loss is much reduced, in line with the results in Section 3.

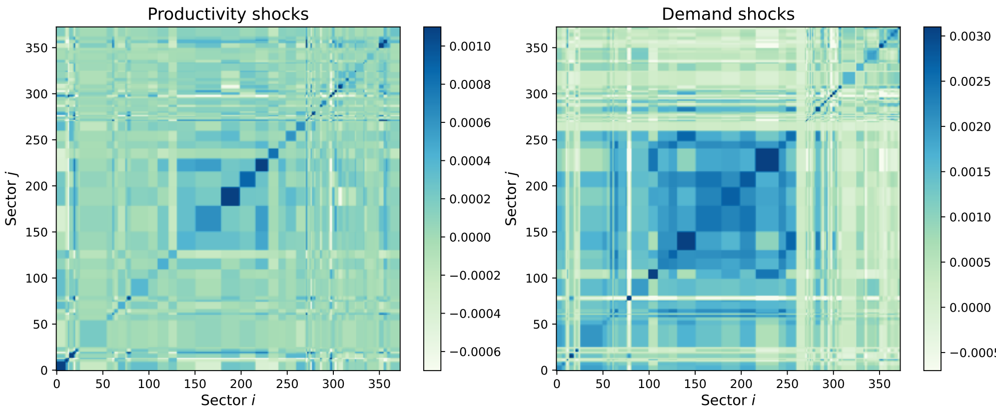
Finally, the rightmost bars in both panels show the welfare implications of "non-optimal" policies, which are meant to represent, although in a stylized manner, actual policies. Specifically, we consider policies in response to the supply (left) and the demand (right) shock that rely on one instrument only and—what's more—happen to have the "wrong sign," if bench-marked against the optimal policy derived above. To see why it makes sense to analyze such a policy response, recall that in response to the energy price shock following the Russian invasion of Ukraine, sales subsidies have—at times—been put in place in several countries (Langot et al., 2023). Similarly, in the face of falling sectoral demand policymakers have, at times, resorted to subsiding producers. The right bars in the panels of Figure show that both policies exacerbate the welfare loss relative to a scenario where only monetary policy is deployed. Subsizding energy use (rather than taxing it) in response to an adverse productivity shock in the energy sector causes welfare losses as it further increases distortions across sectors. Instead, subsiding production in the face of faltering demand (rather than taxing it) increases the within-sector distortions.

4.4 Unconditional welfare loss

The results presented so far concern the ex-post welfare loss in the face of specific shocks. We now compute the welfare loss ex-ante based on a measure of the actual distribution of productivity and demand shocks across sectors. For this purpose, we specify the shock distribution in order to capture the salient features of U.S. data for the period 1987–2021. Specifically, we compute the variance-covariance matrix of productivity shocks using the Integrated TFP index in Production Account Tables from BEA. This dataset contains yearly productivity measures for 63 sectors. Following Rubbo (2023), we construct sectoral productivity shocks as growth rates of sectoral productivity indices.[16] As we lack observations at a more granular level, we map the results to the 373 sectors in our model. This approach yields perfectly correlated productivity shocks for fairly narrowly defined groups of disaggregated sectors.

[16] Alternatively, we apply an HP-filter to detrend sectoral time series and compute shock innovations based on an AR(1) process with persistence parameter ρ set in accordance with the model. Results are very similar to those reported in the main text below, as we show in the Online Appendix.

Figure 6: Variance-covariance matrices



Notes: Variance-covariance matrices of productivity and demand shock innovations, as measured based on BEA data (see Section 4.4 for details). Darker entries indicate higher values for the (co-)variance.

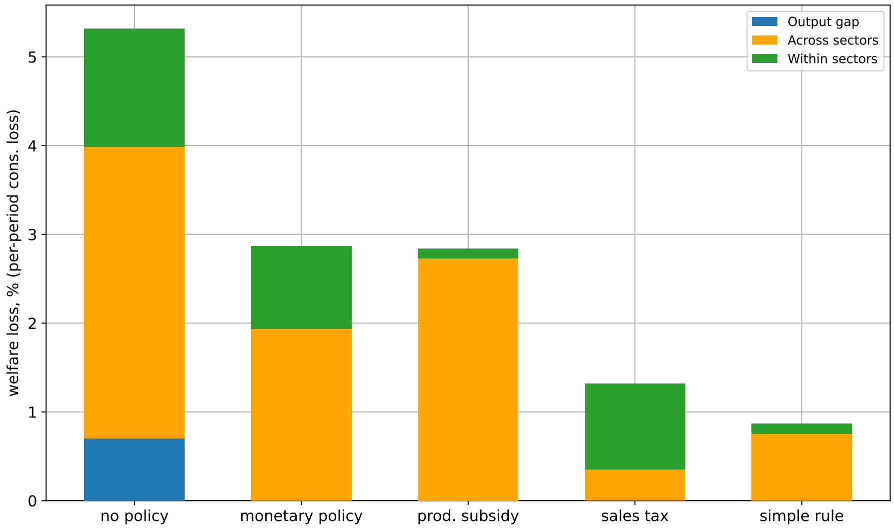
Next, to specify the variance-covariance matrix of demand shocks we first compute $b_{t,i}$ for each sector as the wedge in the first order condition for sectoral consumption $P_{t,i}C_{t,i} = b_{t,i}P_tC_t$ (see Appendix A).[17] Then we measure sectoral demand shocks as the growth rate of these demand shifters. We use the same level of disaggregation, sample period, and frequency for demand shocks as for productivity shocks.[18] The resulting variance-covariance matrices for the computed innovations are visualized in Figure 6, for productivity (left panel) and for demand (right panel) shocks. Darker entries indicate a higher value for the (co-)variance. The cross-sectoral covariance is generally lower than the within-sector variance for both types of shocks. Demand shocks, however, are more broadly correlated across sectors than productivity shocks.

Given the variance-covariance matrices, we compute the unconditional expected welfare loss for alternative suboptimal policies: 1) no policy, 2) monetary policy, 3) only production subsidy, 4) only sales tax, and 5) a simple tax rule. To this end, we simulate in each case the economy for T=1000 periods drawing realizations of sectoral productivity and demand shock innovations from the variance-covariance matrices. For each policy, we compute the welfare loss and decompose it into the contributions of the output gap, and the miscallocation

[17]We use data for sectoral consumption and price index series from the BEA National Income and Product Account (Underlying detail tables, section 2 table 2.4.4U and 2.4.5U).

[18]The resulting variance-covariance matrices capture volatility at a yearly frequency. For model simulations at a monthly frequency, we scale them down by a factor of 12.

Figure 7: Expected welfare loss under alternative policies



Notes: Model simulations for T=1000 periods, drawing realizations of TFP and demand shocks. Simple-rule policy for $\phi = 50$.

within and across sectors.

Figure 7 shows the results. Monetary policy (second bar) improves welfare compared to the no-policy case (leftmost bar). In particular, monetary policy, as with the specific shock scenarios analyzed above, eliminates the welfare loss due to the aggregate output gap. In addition, it also contributes towards limiting the welfare loss associated with misallocation within and across sectors, although substantial losses remain. Among the single-instrument policies, the sales tax yields a significant improvement relative to the monetary policy case, while the production subsidy does not. Note moreover that while the production subsidies mostly eliminate distortion within sectors, sales taxes mostly reduce the distortion across sectors. This is intuitive, as the production subsidy aims to stabilize seller prices while the sales tax aims to correct relative prices across sectors. Finally, the simple rule which deploys both instruments further improves welfare. In our simulation, we use a fairly strong response to sectoral inflation rates ($\phi = 50$). As established in Proposition 4, taking this response to the limit ($\lim_\phi = \infty$) completely eliminates the welfare loss.

We further investigate the drivers of the welfare loss and report results in the Online Appendix. Specifically, we consider productivity and demand shocks in isolation and compare the importance of sector-specific shocks and aggregate shocks. We find that sector-specific productivity shocks contribute more strongly to the welfare loss across policy scenarios and that sectoral (as opposed to aggregate) shocks account for the bulk of the losses. Finally, we show that the welfare loss generally increases as labor becomes less mobile across sectors.

5 Conclusion

The New Keynesian Network (NKN) model offers important insights into the transmission of sectoral shocks and the allocative role of prices. It also carries important lessons for policy. In the one-sector NK model, goods are intrinsically homogeneous and should therefore trade at the same price. Instead, sectoral shocks in the NKN model call for some non-trivial adjustment of relative prices. We put the spotlight on the tension between the adjustment of relative prices within and across sectors. As prices are adjusted infrequently, sectoral shocks cause price dispersion within sectors but also cause the response of relative prices across sectors to be muted.

This, in turn, results in inefficiencies that cannot be fully undone by monetary policy. However, in theory, it is possible to restore efficiency through an appropriate choice of tax instruments—by deploying simultaneously a production subsidy and a sales tax in all sectors. Implementing such a policy, while budgetary neutral, is clearly demanding in various ways. And while we show that a simple-rule policy can approximate the optimal policy well, we think of the optimal policy more like a benchmark which may be used to evaluate actual policies rather than a concrete policy proposal.

We conclude with a remark on the history of economic thought. As discussed in the introduction, the socialist calculation debate was concerned with how to achieve an efficient allocation in the absence of market prices (Von Mises, 1953; Lerner, 1934; Lange, 1936). Almost a century has passed since that debate and meanwhile the New Keynesian framework has emerged as the leading paradigm for business cycle analysis, sometimes also labeled as the "New Neoclassical Synthesis" (Goodfriend and King, 1997). A central theme of our analysis is that within this paradigm prices fail to do their job as foreseen in classic general equilibrium theory, suggesting a role for policy interventions in stabilizing the business cycle beyond monetary policy—very much in the spirit of a perhaps even more encompassing synthesis.

Appendix

A Model derivations

A.1 First-order conditions

Optimal allocation of consumption across sectors yields sectoral consumption demand

$$P_{t,i}C_{t,i} = \beta_{t,i}P_tC_t \tag{A.1}$$

Optimal consumption-leisure trade-off for sector-specific labor yields sectoral labor supply

$$W_{t,i} = N_{t,i}^{\gamma} \cdot P_tC_t \tag{A.2}$$

and for non-sector-specific labor

$$W_t = N_t^{\gamma} \cdot P_tC_t \tag{A.3}$$

The intertemporal consumption trade-off yields

$$Q_t = \delta E_t \left(\frac{C_{t+1}^{-1}}{C_t^{-1}} \cdot \frac{P_t}{P_{t+1}} \right) \tag{A.4}$$

Cost-minimizing input allocation yields sectoral labor demand and intermediate input demand

$$W_{t,i}L_{t,i}^{I} = \alpha_i^{I} MC_{t,i}Y_{t,i} \tag{A.5}$$

$$W_tL_{t,i}^{M} = \alpha_i^{M} MC_{t,i}Y_{t,i} \tag{A.6}$$

$$P_{t,j}X_{t,ij} = (1-\alpha_i)\omega_{ij}MC_{t,i}Y_{t,i} \tag{A.7}$$

The optimal price set by the firms is

$$P_{t,i}^{s,\star} = \frac{E_t \sum\limits_{s=t}^{\infty} Q_{t,s}\lambda_i^{s-t}(P_{s,i}^{s})^{\epsilon}Y_{s,i}(1-s_{s,i}^{p})MC_{s,i}}{E_t \sum\limits_{s=t}^{\infty} Q_{t,s}\lambda_i^{s-t}(P_{s,i}^{s})^{\epsilon}Y_{s,i}} \tag{A.8}$$

The selling evolution in sector i is

$$(P_{t,i}^{s})^{1-\epsilon} = \lambda_i \cdot (P_{t-1,i}^{s})^{1-\epsilon} + (1-\lambda_i) \cdot (P_{t,i}^{s,\star})^{1-\epsilon}. \tag{A.9}$$

A.2 Log-linearization

This appendix provides steps to log-linearize the model.

A.2.1 Product market clearing

Product market clearing implies that $Y_i = C_i + \sum_j X_{ji}$. Multiplying all by P_i and noting that $(1-\alpha_i)\omega_{ji} = \frac{P_i X_{ji}}{MC_j Y_j}$ and $P_i = \mathcal{M}_i MC_i$ we get

$$P_i Y_i = P_i C_i + \sum_j (1-\alpha_j)\omega_{ji} \frac{P_j Y_j}{\mathcal{M}_j} \tag{A.10}$$

Log-linearizing and dividing by steady state nominal final expenditure PC we get

$$\xi_i(p_i + y_i - \mu_i) = \beta_i(p_i + c_i) - \xi_i \mu_i + \sum_j (1-\alpha_j)\omega_{ji} \cdot \xi_j(p_j + y_j - \mu_j) \tag{A.11}$$

where $\xi_i = \frac{P_i Y_i}{PC}$ is the steady-state sales share of sector i also known as Domar weight and $g_i = \frac{P_i G_i}{PC}$ are nominal sectoral government spending as a share of steady state nominal final expenditure. Let us introduce the input-output matrix Ω such that $\Omega_{ij} = (1-\alpha_i)\omega_{ij}$. Log-linearizing sector-specific consumption demand we get

$$p_i + c_i = b_i + p^c + c \tag{A.12}$$

where $b_i = log(\beta_{t,i}) - log(\beta_i)$ is the log-deviation of sector i consumption share from steady state and p^c is consumption index ($p^c \equiv log(P_t) - log(P)$). Substituting (A.12) into (A.11) and solving the system of equations in sectoral form with respect to $p_i + y_i - \mu_i$ we get

$$\boldsymbol{p_t} + \boldsymbol{y_t} - \boldsymbol{\mu_t} = I_\xi^{-1} L' \boldsymbol{\beta}(p_t^c + c_t) + I_\xi^{-1} L' I_\beta \boldsymbol{b_t} - I_\xi^{-1} L' I_\xi \boldsymbol{\mu_t} \tag{A.13}$$

where $L = (I - W)^{-1}$ is the Leontief inverse matrix, W is input-output matrix.
Remark. From the market clearing condition (A.10) evaluated at steady state we see that $\xi_i = \beta_i + \sum_j (1-\alpha_j)\omega_{ji}\xi_j$ which gives $\boldsymbol{\xi} = L'\boldsymbol{\beta}$ and consequently $I_\xi^{-1} L' \boldsymbol{\beta} = \mathbf{1}$.

A.2.2 Sector-specific wage

Log-linearized demand for sector-specific labor is $w_i + l_i^I = p_i + y_i - \mu_i$ and log-linearized labor supply is $w_i = p^c + c + \gamma l_i^I$. Combining these two expressions we get $p_i + y_i - \mu_i = w_i(1 + \frac{1}{\gamma}) - \frac{1}{\gamma}(p^c + c)$ which, combined with (A.13) yields expression for sectoral wages

$$\boldsymbol{w_t} = \mathbf{1}(p_t^c + c_t) + \frac{\gamma}{1+\gamma} I_\xi^{-1} L' I_\beta \boldsymbol{b_t} - \frac{\gamma}{1+\gamma} I_\xi^{-1} L' I_\xi \boldsymbol{\mu_t} \tag{A.14}$$

A.2.3 Non-sector-specific wage

Log-linearized demand for non-sector-specific labor is $w + l_i^M = p_i + y_i - \mu_i$ and log-linearized labor supply is $w = p^c + c + \gamma l^M$. Non-sector-specific labor is used by all sectors $L^M = \sum_i L_i^M$ which yields $l^M = \sum_i \frac{L_i^M}{L^M} l_i^M$. In steady state $\frac{L_i^M}{L^M} = \frac{WL_i^M}{WL^M} = \frac{\alpha_i^M P_i Y_i}{\sum_i \alpha_i^M \xi_i PC} = \frac{\alpha_i^M \xi_i}{\sum_i \alpha_i^M \xi_i} = \frac{\alpha_i \xi_i}{\sum_i \alpha_i \xi_i}$ where the last equality uses the assumption that $\alpha_i^M = (1-k)\alpha_i$. Then, since $\sum_i \alpha_i \xi_i = \boldsymbol{\alpha}'\boldsymbol{\xi} = \boldsymbol{\alpha}' L' \boldsymbol{\beta} = 1$ we have $l^M = \sum_i \alpha_i \xi_i l_i^M$.

Remark. $L\boldsymbol{\alpha} = \mathbf{1}$. To see this, multiply both sides by L^{-1} which yields $\boldsymbol{\alpha} = (I - \Omega)\mathbf{1}$.

We have $w_t = p^c + c + \gamma \sum_i \alpha_i \xi_i (p_i + y_i - \mu_i - w)$ which yields $(1+\gamma)w = p^c + c + \gamma \sum_i \alpha_i \xi_i (p_i + y_i - \mu_i)$. Combining with (A.13) yields $(1+\gamma)w = p^c + c + \gamma \boldsymbol{\alpha}' I_\xi (I_\xi^{-1} L' \boldsymbol{\beta}(p_t^c + c_t) + I_\xi^{-1} L' I_\beta \boldsymbol{b_t} - I_\xi^{-1} L' I_\xi \boldsymbol{\mu_t})$. The resulting expression for non-sector-specific wages is

$$w_t = p_t^c + c_t - \frac{\gamma}{1+\gamma} \boldsymbol{\xi}' \boldsymbol{\mu_t} \tag{A.15}$$

A.2.4 Marginal costs and prices (Sectoral IS curves)

Log-linearizing marginal cost yields $mc_i = -a_i + \alpha_i^I l_i^I + \alpha_i^M l_i^M + \sum_j (1-\alpha_i)\omega_{ij} p_j$ and sectoral markup definition $\mu_i = p_i - mc_i$. Substituting for marginal cost and solving the system with respect to price vector yields

$$\boldsymbol{p_t} = L(\boldsymbol{\mu_t} - \boldsymbol{a_t} + I_{\alpha^M} \mathbf{1} w_t + I_{\alpha^I} \boldsymbol{w_t}) \tag{A.16}$$

Substituting expressions for sector-specific and non-sector-specific wages into (A.16) and using the fact that $\alpha_i^I = k\alpha_i$ we get a vector of sectoral prices expressed in terms of shocks, policy, and markups

$$\begin{aligned} \boldsymbol{p_t} = (p_t^c + c_t)\mathbf{1} + L(\boldsymbol{\mu_t} - \boldsymbol{a_t}) - \frac{\gamma}{1+\gamma} \boldsymbol{\xi}' \boldsymbol{\mu_t} + \\ + k \frac{\gamma}{1+\gamma} \left[L I_\alpha I_\xi^{-1} L' (I_\beta \boldsymbol{b_t} - I_\xi \boldsymbol{\mu_t}) + \mathbf{1} \cdot \boldsymbol{\xi}' \boldsymbol{\mu_t} \right] \end{aligned} \tag{A.17}$$

Note, that $\boldsymbol{\beta}' L I_\alpha I_\xi^{-1} L' = \mathbf{1}'$. Rearranging the terms in the previous equation gives

$$\boldsymbol{p_t} = (p_t^c + c_t)\mathbf{1} - \hat{L}(I_\beta \boldsymbol{b_t} - I_\xi \boldsymbol{\mu_t}) + L(\boldsymbol{\mu_t} - \boldsymbol{a_t})$$

where $\hat{L} = \frac{\gamma}{1+\gamma} \cdot [\mathbf{1}\boldsymbol{\beta'} + k(I - \mathbf{1}\boldsymbol{\beta'})] \cdot LI_\alpha I_\xi^{-1} L'$. Then, given that sales prices are related to market prices as $\boldsymbol{p_t} = \boldsymbol{p_t^s} + \boldsymbol{\tau_t^s}$ (where $\boldsymbol{\tau_t^s}$ is a vector of sales taxes), sectoral markups can be expressed as

$$(L - \hat{L}I_\xi)\boldsymbol{\mu_t} = \boldsymbol{p_t^s} - [(p_t^c + c_t)\mathbf{1} + \boldsymbol{\tau_t^s}] - [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}] \tag{A.18}$$

where the first square bracket combines policy ($p_t^c + c_t = m_t$ controlled by monetary authority) variables, and the second one combines exogenous quantities.

Link between interest rate and money supply growth. To see why $R_t = E_t[\Delta m_{t+1}]$, consider the log-linear Euler equation, $c_t = Ec_{t+1} - (R_t - E_t\pi_{t+1})$, and substitute money market equilibrium $m_t = p_t + c_t$ into it.

A.2.5 Sectoral and final consumption gaps

Sectoral consumption demand is $c_i + p_i = b_i + p_i^c + c_t$. Together with (A.18) this gives $\boldsymbol{c_t} = L\boldsymbol{a_t} - (\hat{L}I_\xi - I)\boldsymbol{b_t} - (L - \hat{L}I_\xi)\boldsymbol{\mu_t}$. Then, sectoral final sectoral output gaps (consumption gaps) are $\boldsymbol{\tilde{c}_t} = -(L - \hat{L}I_\xi)\boldsymbol{\mu_t}$. Multiplying (A.17) by $\boldsymbol{\beta'}$ we have final output (consumption)

$$c_t = \boldsymbol{\xi'}\boldsymbol{a_t} - \frac{1}{1+\gamma}\boldsymbol{\xi'}\boldsymbol{\mu_t} + \frac{\gamma}{1+\gamma}\boldsymbol{\beta}'\boldsymbol{b_t}$$

Note that $\boldsymbol{\beta}'\boldsymbol{b_t} = 0$. The the final output gap is $\tilde{y}_t = -\frac{1}{1+\gamma}\boldsymbol{\xi'}\boldsymbol{\mu_t}$.

A.2.6 Price-markup link (Sectoral Phillips curves)

In log-linear terms, the optimal price in sector i is

$$p_{t,i}^{\star,s} = (1 - \delta\lambda_i)E_t \sum_{s=t}^{\infty} (\delta\lambda_i)^{s-t}(mc_{s,i} - s_{s,i}^p) \tag{A.19}$$

Rewriting this recursively yields

$$p_{t,i}^{\star,s} = (1 - \delta\lambda_i)(mc_{t,i} - s_{t,i}^p) + \delta\lambda_i E_t p_{t+1,i}^{\star,s}$$

Log-linear sectoral price dynamics in sector i is given by

$$p_{t,i}^s = (1 - \lambda_i)p_{t,i}^{\star,s} + \lambda_i p_{t-1,i}^s \tag{A.20}$$

which yields

$$(1 + \delta\lambda_i^2)p_{t,i}^s = (1 - \lambda_i)(1 - \delta\lambda_i)(mc_{t,i} - s_{t,i}^p) + \lambda_i p_{t-1,i}^s + \delta\lambda_i E_t p_{t+1,i}^s$$

Taking into account that $mc_{t,i} = p_{t,i} - \mu_{t,i} = p^s_{t,i} + \tau^s_{t,i} - \mu_{t,i}$, we get from the expression above that

$$\lambda_i(1+\delta)p^s_{t,i} = (1-\lambda_i)(1-\delta\lambda_i)(-\mu_{t,i} - s^p_{t,i} + \tau^s_{t,i}) + \lambda_i p^s_{t-1,i} + \delta\lambda_i E_t p^s_{t+1,i}$$

In vector form

$$(1+\delta)I_\lambda \boldsymbol{p}^s_t = (I-I_\lambda)(I-\delta I_\lambda)(-\boldsymbol{\mu}_t - \boldsymbol{s}^p_t + \boldsymbol{\tau}^s_t) + I_\lambda \boldsymbol{p}^s_{t-1} + \delta I_\lambda E_t \boldsymbol{p}^s_{t+1} \quad \text{(A.21)}$$

Expressing the previous system in terms of sectoral inflation yields sectoral Phillips curves

$$\boldsymbol{\pi}^s_t = \tilde{I}_\lambda(-\boldsymbol{\mu}_t - \boldsymbol{s}^p_t + \boldsymbol{\tau}^s_t) + \delta E_t \boldsymbol{\pi}^s_{t+1} \quad \text{(A.22)}$$

where $\tilde{I}_\lambda = I^{-1}_\lambda(I-I_\lambda)(I-\delta I_\lambda)$.

A.2.7 Sellers price dynamics

Substituting the expression for markups A.18 into A.21, we obtain

$$\begin{aligned}
&[(1+\delta)I_\lambda + (I-I_\lambda)(I-\delta I_\lambda)(L-\hat{L}I_\xi)^{-1}]p^s_t = \\
&\qquad = (I-I_\lambda)(I-\delta I_\lambda)(L-\hat{L}I_\xi)^{-1} \cdot [[(p^c_t + c_t)\mathbf{1} + \\
&\qquad\qquad + (L-\hat{L}I_\xi)\cdot(-\boldsymbol{s}^p_t + \boldsymbol{\tau}^s_t) - \boldsymbol{\tau}^s_t] + \\
&\qquad\qquad\qquad + [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]] + \\
&\qquad\qquad\qquad\qquad + I_\lambda \boldsymbol{p}^s_{t-1} + \delta I_\lambda E_t \boldsymbol{p}^s_{t+1}
\end{aligned}$$

Let $\tilde{I}_\lambda = I^{-1}_\lambda(I-I_\lambda)(I-\delta I_\lambda)$. Then, we can write seller prices as

$$[(1+\delta)I + \tilde{I}_\lambda(L-\hat{L}I_\xi)^{-1}]\boldsymbol{p}^s_t = \tilde{I}_\lambda(L-\hat{L}I_\xi)^{-1} \cdot \tilde{X}\boldsymbol{x_t} + \boldsymbol{p}^s_{t-1} + \delta E_t \boldsymbol{p}^s_{t+1} \quad \text{(A.23)}$$

where

$$\tilde{X}\boldsymbol{x_t} = [(p^c_t + c_t)\mathbf{1} + (L-\hat{L}I_\xi)\cdot(-\boldsymbol{s}^p_t + \boldsymbol{\tau}^s_t) - \boldsymbol{\tau}^s_t] + [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$$

which combines all policy and exogenous variables at time t. Finally, the dynamic expectations equation for sellers' prices is

$$\boldsymbol{p}^s_t = Z\boldsymbol{p}^s_{t-1} + X\boldsymbol{x_t} + \delta Z E_t \boldsymbol{p}^s_{t+1} \quad \text{(A.24)}$$

where $Z = [(1+\delta)I + \tilde{I}_\lambda(L-\hat{L}I_\xi)^{-1}]^{-1}$ and $X = Z \cdot \tilde{I}_\lambda(L-\hat{L}I_\xi)^{-1}\tilde{X}$.

B Proofs

B.1 Proposition: First bast tax/subsidy policy

Proposition. *The first best sector tax policy implies that output gap, sector markups, and sector sellers' price inflation are stabilized* $\tilde{y} = 0$*,* $\boldsymbol{\mu} = 0$*,* $\boldsymbol{\pi^s} = 0$*. Let the initial sellers' prices be at steady state* $\boldsymbol{p^s_{-1}} = 0$*. Then, the sector policies achieving this outcome is*

$$\boldsymbol{s^p_t} = m_t \cdot \mathbf{1} + [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$$
$$\boldsymbol{\tau^s_t} = \boldsymbol{s^p_t}$$

Proof. First best policy insures that $\pi^s_{t,i} = 0$ and $\mu_{t,i} = 0$ for all i and t. $\pi^s_{t,i} = 0$ implies that $p^s_t = 0$ as long as $p^s_{-1} = 0$. Then the system of equations (A.18) gives $\boldsymbol{\tau^s_t} = m_t\mathbf{1} + [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$ and the system (A.22) gives $\boldsymbol{s^p_t} = \boldsymbol{\tau^s_t}$. □

From the above proposition, it follows that

Corollary (Optimal policy budget)**.** *To the first order, the ratio of government revenue from sales tax and production subsidies to output equals* $T_\tau = \boldsymbol{\xi'}\boldsymbol{\tau^s_t} - \boldsymbol{\xi'}\boldsymbol{s^p_t}$*. The optimal policy is budget-neutral, that is* $T_\tau = 0$*.*

Proof. The tax revenue is $T_\tau = \sum \tau^s_{t,i} P^s_{t,i} Y_{t,i} - \sum s^p_{t,i} MC_{t,i} Y_{t,i}$. Linearizing around the efficient steady state gives $T_\tau = \sum \tau^s_{t,i} P_i Y_i - s^p_{t,i} P_i Y_i$. Steady-state sales shares are $\frac{P_i Y_i}{PY} = \xi_i$. Then, in terms of final output, tax revenue is $\frac{T_\tau}{PY} = \sum \xi_i (\tau^s_{t,i} - s^p_{t,i})$ which gives 0 under optimal policy. □

B.2 Proposition: Monetary policy output gap stabilization

Rubbo (2023) shows that stabilizing the output gap is optimal monetary policy in a static network economy. Next, we construct the corresponding monetary policy in our model

Proposition. *Consider a static economy (*$\delta = 0$*) such that* $\boldsymbol{p^s_{-1}} = 0$*. Monetary policy stabilizing the output gap with no fiscal instruments is*

$$\tilde{y}_t = 0 \rightarrow m_t = -\frac{1}{\boldsymbol{\beta'}(X - I)\mathbf{1}} \cdot \boldsymbol{\beta'}(X - I) \cdot [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$$

Proof. The output gap is $\tilde{y} = -\boldsymbol{\beta'}(L - \hat{L}I_\xi)\boldsymbol{\mu_t}$. Using the system (A.18) under zero taxes we get $\tilde{y} = -\boldsymbol{\beta'}\boldsymbol{p^s_t} - m_t - \boldsymbol{\beta'}[\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$. In a static zero tax economy the system describing seller price dynamics (A.24) gives $\boldsymbol{p^s_t} = X \cdot [m_t \cdot \mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$. Substituting producers' prices into the output gap expression and equating the output gap to zero yields the result. □

B.3 Proposition: Simple policy rule

We can construct a simple tax rule such that taxes respond to sellers' price inflation.

Proposition. *Consider the following rule for adjusting subsidies and taxes:*

$$\boldsymbol{s_t^p} = \phi \cdot \boldsymbol{\pi_t^s},$$
$$\boldsymbol{\tau_t^s} = \boldsymbol{s_t^p},$$

where ϕ governs the strength of adjustment to sectoral producer price inflation. Then, the resulting allocation becomes first-best for $\phi \to \infty$.

For the static economy, $\delta = 0$ and $\boldsymbol{p_{t-1}^s} = 0$, the simple rule implies the solution:

$$\boldsymbol{s_t^p} = (I + \phi X)^{-1} \cdot \phi X \cdot \underbrace{(m_t \cdot \mathbf{1} + [\hat{L} I_\beta \boldsymbol{b_t} - L \boldsymbol{a_t}])}_{\text{optimal tax}}$$

Proof. Substituting the rule instead of taxes in the price dynamics equations (A.24) and considering the static case, we arrive at the result. □

B.4 Illustrative diagram

Let us consider a special case with $\delta = 0$ and $\boldsymbol{p_{t-1}} = 0$. Then from (10) we can write the deviation of market prices from the corresponding natural prices as

$$\boldsymbol{p_t} - \boldsymbol{p_t^n} = (L - \hat{L} I_\xi) \cdot \boldsymbol{\mu_t}$$

Then using the Phillips curve system, we obtain that sectoral sellers' price inflations and sectoral price deviations from the natural level are linked as

$$\boldsymbol{p_t} - \boldsymbol{p_t^n} = -(\tilde{I}_\lambda (L - \hat{L} I_\xi)^{-1})^{-1} \cdot \boldsymbol{\pi_t^s} + (L - \hat{L} I_\xi) \cdot (\boldsymbol{\tau_t^s} - \boldsymbol{s_t^p})$$

Further, let us assume that there is no input-output network $\Omega = \mathbf{0}$. In this case, sectors become decoupled from each other and welfare loss can be written as

$$\Delta_t = \frac{1}{2} \sum_{t=0}^{\infty} \delta^t \sum_i \{f_\mu^i \mu_{t,i}^2 + f_\pi^i \pi_{t,i}^2\} = \frac{1}{2} \sum_{t=0}^{\infty} \delta^t \sum_i \{f_\mu^i (p_{t,i} - p_{t,i}^n)^2 + f_\pi^i \pi_{t,i}^2\}$$

That is welfare loss in sector i is given by the weighted sum of squares of sectoral market price deviation from its natural level and suppliers' inflation. Let us denote this loss as $L_{t,i}$. Then we have

$$L_{t,i} = f_\mu^i (p_{t,i} - p_{t,i}^n)^2 + f_\pi^i \pi_{t,i}^2$$

And the link between sector i market price and sellers' inflation is

$$p_{t,i} - p^n_{t,i} = -b^i \cdot \pi^s_{t,i} + a^i \cdot (\tau^s_{t,i} - s^p_{t,i})$$

Combining the definition $p_{t,i} = \pi^s_{t,i} + \tau^s_{t,i}$ the above equation we obtain the market price and the sellers' inflation expressed as functions of policy variables

$$p_i = \frac{1}{1+b^i} \cdot [p^n_i + (a^i - b^i) \cdot \tau_i - a^i \cdot s_i]$$
$$\pi^s_i = \frac{1}{1+b^i} \cdot [p^n_i - (1 - a^i) \cdot \tau_i - a^i \cdot s_i]$$

This system pins down the pair p_i, π^s_i for a given tax/subsidy pair τ_i, s_i. Under the reasonable assumption that $1 > a^i > b^i > 0$ market price increases with sales tax, while seller price inflation is decreasing. Both market price and sellers' price inflation decrease with subsidy.

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Distorted Prices and Targeted Taxes in the New Keynesian Network Model Online Appendix

Anastasiia Antonova and Gernot J. Müller[1]

OA.1 Welfare loss function

We show that per-period welfare loss can be expressed as a quadratic form

$$\Delta_t \approx \frac{1}{2} \cdot E_0 \sum_{t=0}^{\infty} \delta^t \cdot \{ \underbrace{f_y \cdot \tilde{y}_t^2}_{\text{due to output gap}} + \underbrace{\boldsymbol{\mu}_t' \cdot F_\mu \cdot \boldsymbol{\mu}_t}_{\text{due to cross-sector}} + \underbrace{\boldsymbol{\pi}_t^{s\prime} \cdot F_p \cdot \boldsymbol{\pi}_t^s}_{\text{due to within-sector}} \} + t.i.p.$$

$\tilde{y}$ - output gap, $\boldsymbol{\mu}$ - sector markups, $\boldsymbol{\pi}^s$ - sector producer price inflation. The weights are

$$\begin{aligned} f_y &= 1 + \gamma \\ F_\mu &= (I_\xi - \boldsymbol{\xi}\boldsymbol{\xi}') + 2 \cdot N - k\frac{\gamma}{1+\gamma} M \\ F_p &= \epsilon \cdot I_\xi I_\lambda (I - I_\lambda)^{-1} (I - \delta I_\lambda)^{-1} \end{aligned}$$

where $N_{ij} = \xi_i(\Omega(I-\Omega)^{-1})_{ij}$ and $M = I_\xi(I_\xi^{-1}L' - \mathbf{1}\mathbf{1}')' I_\xi I_\alpha (I_\xi^{-1}L' - \mathbf{1}\mathbf{1}')I_\xi$. Welfare loss depends on the output gap $\tilde{c}$, vector of markups $\boldsymbol{\mu}$, and sellers price inflations $\boldsymbol{\pi}^s$. Hence, the optimal policy should set all these quantities to zero. Next, we derive this welfare loss.

General form of welfare loss. Let us denote one-period utility by $U(C,\ L)$. Let Δ denote one-period welfare loss. Consumption and labor can be written as $C = C^\star e^{\tilde{c}}$ and $L = L^\star e^{\tilde{l}}$ where $C^\star$, $L^\star$ are the efficient consumption and labor levels and $\tilde{c}$, $\tilde{l}$ are the log-deviations from the efficient levels. The second-order approximations are

$$\begin{aligned} C &= C^\star + C^\star \cdot \tilde{c} + \frac{1}{2} C^\star \cdot \tilde{c}^2 \\ L &= L^\star + L^\star \cdot \tilde{l} + \frac{1}{2} L^\star \cdot \tilde{l}^2 \end{aligned}$$

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Then, the second-order approximation of the welfare function is

$$\Delta = \frac{U - U^\star}{U_c' C^\star} = \tilde{c} + \frac{1}{2} \cdot \tilde{c}^2 + \frac{1}{2} \frac{U_{cc}''}{U_c'} \cdot \tilde{c}^2 + \frac{U_l' L^\star}{U_c' C^\star} \cdot (\tilde{l} + \frac{1}{2} \cdot \tilde{l}^2) + \frac{1}{2} \cdot \frac{U_{ll}''(L^\star)^2}{U_c' C^\star} \cdot \tilde{l}^2$$

With CRRA utility from consumption $\frac{U_{cc}''}{U_c'} C = -\sigma$; the inverse Frisch labor supply elasticity is $\frac{U_{ll}''(L^\star)}{U_l'} = \gamma$; from the optimal consumption-labor allocation in efficient equilibrium $\frac{U_l' L^\star}{U_c' C^\star} = -\frac{WL^\star}{PC^\star} = -1$. Then, we have

$$\Delta = \left[\frac{1-\sigma}{2} \tilde{c}^2 - \frac{1+\gamma}{2} \tilde{l}^2 \right] + \left[\tilde{c} - \tilde{l} \right]$$

Labor index. Given our functional forms, the labor index can be written as

$$(L)^{1+\gamma} = (L^M)^{1+\gamma} + \sum (L_i^I)^{1+\gamma}$$

First order $\tilde{c}$ and $\tilde{l}$ link. First-order approximation of labor index is

$$v'(L) L l = v'(L^M) L^M l^M + \sum v'(L^I) L^I l^I$$

where $v(L) = \frac{L^{1+\gamma}}{1+\gamma}$. Let W^X be the wage index corresponding to the aggregate labor index. The first-order conditions for labor supply at steady state give $v'(L)L = \frac{W^X L}{PC} = 1$, $v'(L^M)L^M = \frac{W^M L^M}{PC} = \sum \frac{W^M L_i^M}{P_i Y_i} \frac{P_i Y_i}{PC} = (1-k) \sum \alpha_i \xi_i$ and $v'(L_i^I) L_i^I = \frac{W_i L_i^I}{PC} = k \alpha_i \xi_i$. Combining these with the above equation yields

$$l = (1-k) l^M + k \sum \alpha_i \xi_i l_i^I \tag{OA.1.1}$$

where the property $\sum \alpha_i \xi_i = 1$ is used.

Now, log-linearized first-order conditions for labor supply are $\gamma l^M = w - p^c - c$ and $\gamma l_i^I = w_i - p^c - c$. Hence, $\gamma l = (1-k) w + k \sum \alpha_i \xi_i w_i - (p^c + c)$. Using the expressions for sectors-specific and non-sector-specific wages (see log-linearization section) we obtain $\sum \alpha_i \xi_i w_i = w = p_t^c + c_t - \frac{\gamma}{1+\gamma} - \boldsymbol{\xi}' \boldsymbol{\mu}_t$. Then $\gamma l = w - (p^c + c)$ and we have $l = -\frac{1}{\gamma} \frac{\gamma}{1+\gamma} \boldsymbol{\xi}' \boldsymbol{\mu}_t$, which yields $\tilde{l} = \tilde{c}$ to first order.

Welfare loss in terms of output gap and "productivity" wedge. Using the first order link $\tilde{l} = \tilde{c}$, the welfare can be written as

$$\Delta = -\frac{\sigma + \gamma}{2} \tilde{c}^2 - d \tag{OA.1.2}$$

where $d = \tilde{l} - \tilde{c}$, which is a second-order "productivity" wedge.

Resource constraint. Resource constraint: $Y_i = C_i + \sum_j X_{ji}$ which gives $P_i Y_i = P_i C_i + \sum_j P_i X_{ji} = P_i C_i + \sum_j (1-\alpha_j)\omega_{ji}\frac{P_j Y_j}{\mathcal{M}_j}$. In vector form $\boldsymbol{PY} = \boldsymbol{\beta} \cdot PC + W' I_{\mathcal{M}}^{-1} \cdot \boldsymbol{PY}$ which yields $\boldsymbol{PY} = (I - W' I_{\mathcal{M}}^{-1})^{-1} \boldsymbol{\beta} \cdot PC$.

Labor income. Since W^X is the nominal wage associated with labor index L, the labor income consists of mobile labor income and immobile sector-specific labor income

$$W^X L = WL^M + \sum W_i L_i^I = \sum WL_i^M + \sum W_i L_i^I = \sum \alpha_i MC_i Y_i = \sum \alpha_i \frac{P_i Y_i}{\mathcal{M}_i} = $$
$$= \boldsymbol{\alpha}' I_{\mathcal{M}}^{-1} \boldsymbol{PY}$$

where $I_{\mathcal{M}} = diag\{\mathcal{M}_i\}$ and $\boldsymbol{PY}$ is the vector of $P_i Y_i$ elements.

Labor income and final output link. Then, using the resource constraint and the previous equation, labor income is

$$W^X L = \boldsymbol{\alpha'} \cdot I_{\mathcal{M}}^{-1} (I - W' I_{\mathcal{M}}^{-1})^{-1} \boldsymbol{\beta} \cdot PC = \Gamma \cdot PC \qquad \text{(OA.1.3)}$$

Now, we take logarithms from both sides and express the deviations from the efficient state, which yields

$$d = \tilde{l} - \tilde{c} = \tilde{\Gamma} - (\tilde{w}^X - \tilde{p})$$

where $\tilde{\Gamma} = log(\Gamma) - log(1)$ (since Γ is 1 in the efficient state) and $\tilde{w}^X - \tilde{p} = log(W^X/P) - log(W^{X\star}/P^\star)$ is log-deviation of real effective wage from its efficient counterpart. Next, we find second-order approximations of $\tilde{\Gamma}$ and $\tilde{w}^X - \tilde{p}$.

Approximation of labor share $\tilde{\Gamma}$. Let us express Γ through markup deviation from its efficient level. $\Gamma = \boldsymbol{\alpha'} \cdot I_{\mathcal{M}}^{-1} (I - W' I_{\mathcal{M}}^{-1})^{-1} \boldsymbol{\beta} = \boldsymbol{\alpha'} \cdot (I_{\mathcal{M}} - W')^{-1} \boldsymbol{\beta} = \boldsymbol{\alpha'} \cdot (I_{\mathcal{M}} - I + I - W')^{-1} \boldsymbol{\beta}$. Let $\hat{I}_{\mathcal{M}} = I_{\mathcal{M}} - I$.

Then $\Gamma = \boldsymbol{\alpha'} \cdot \left[(I - W')^{-1} - (I - W')^{-1} \cdot \hat{I}_{\mathcal{M}} \cdot (I_{\mathcal{M}} - W')^{-1} \right] \cdot \boldsymbol{\beta}$.

Remark. The above equation is obtained using the following property: for two square matrices A, B we have $(A + B)^{-1} = B^{-1} - B^{-1} A (A + B)^{-1}$.

Now, $\boldsymbol{\alpha'} (I - W')^{-1} = ((I - W)\mathbf{1})' (I - W')^{-1} = \mathbf{1'}$ which yields

$$\Gamma = 1 - \mathbf{1'} \cdot \hat{I}_{\mathcal{M}} \cdot (I_{\mathcal{M}} - W')^{-1} \cdot \boldsymbol{\beta} = 1 - \mathbf{1'} \hat{I}_{\mathcal{M}} I_{\mathcal{M}}^{-1} (I - W' I_{\mathcal{M}}^{-1})^{-1} \boldsymbol{\beta} =$$
$$= 1 - \mathbf{1'} (I - I_{\mathcal{M}}^{-1}) \cdot \left[I - W' + W'(I - I_{\mathcal{M}}^{-1}) \right]^{-1} \boldsymbol{\beta}$$

Let $I_{\hat{\mathcal{M}}} = I - I_{\mathcal{M}}^{-1}$. Applying the property for the inverse of the sum of two matrices gives

$$\left[I - W' + W' I_{\hat{\mathcal{M}}}\right]^{-1} = (I - W')^{-1} - (I - W')^{-1} \cdot \left[W' I_{\hat{\mathcal{M}}}\right] \cdot \left[I - W' + W' I_{\hat{\mathcal{M}}}\right]^{-1}$$

Then we have $\Gamma = 1 - \mathbf{1'} I_{\hat{\mathcal{M}}} \boldsymbol{\xi} + \mathbf{1'} I_{\hat{\mathcal{M}}} L' W' I_{\hat{\mathcal{M}}} \cdot \left[I - W' + W' I_{\hat{\mathcal{M}}}\right]^{-1} \boldsymbol{\beta}$ where we use the property that $L' \boldsymbol{\beta} = \boldsymbol{\xi}$. To the second order approximation around $\mathcal{M} = 1$, this expression becomes

$$\Gamma = 1 - \mathbf{1'} I_{\hat{\mathcal{M}}} \boldsymbol{\xi} + \mathbf{1'} I_{\hat{\mathcal{M}}} L' W' I_{\hat{\mathcal{M}}} \boldsymbol{\xi}$$

Remember that $\tilde{\Gamma} = log(\Gamma) - log(1)$. Up to second order we have $\tilde{\Gamma} = \Delta\Gamma - \frac{1}{2}(\Delta\Gamma)^2$ where $\Delta\Gamma = \Gamma - 1$. At the same time matrix $I_{\hat{\mathcal{M}}} = diag\{1 - \frac{1}{\mathcal{M}}\} = diag\{1 - e^{-\mu}\}$ where $\mathcal{M} = e^{\mu}$. Then, to second order $I_{\hat{\mathcal{M}}} = diag\{\mu - \frac{1}{2}\mu^2\} = I_\mu - \frac{1}{2} I_\mu^2$. Then, to the second order

$$\begin{aligned} \Gamma - 1 &= -\mathbf{1'} I_\mu \boldsymbol{\xi} + \frac{1}{2} \mathbf{1'} I_\mu^2 \boldsymbol{\xi} + \mathbf{1'} I_\mu L' W' I_\mu \boldsymbol{\xi} \\ (\Gamma - 1)^2 &= (\mathbf{1'} I_\mu \boldsymbol{\xi})^2 \end{aligned}$$

which yields

$$\tilde{\Gamma} = -\mathbf{1'} I_\mu \boldsymbol{\xi} + \frac{1}{2} \mathbf{1'} I_\mu^2 \boldsymbol{\xi} + \mathbf{1'} I_\mu L' W' I_\mu \boldsymbol{\xi} - \frac{1}{2} (\mathbf{1'} I_\mu \boldsymbol{\xi})^2$$

Since $\mathbf{1'} I_\mu L' W' I_\mu \boldsymbol{\xi} = \boldsymbol{\xi'} I_\mu W L \boldsymbol{\mu} = -\boldsymbol{\xi'} I_\mu (I - W) L \boldsymbol{\mu} + \boldsymbol{\xi'} I_\mu L \boldsymbol{\mu} = -\boldsymbol{\xi'} \boldsymbol{\mu^2} + \boldsymbol{\xi'} I_\mu L \boldsymbol{\mu}$ where the first equality is obtained by transposing the scalar. Then

$$\tilde{\Gamma} = -\boldsymbol{\xi'} \boldsymbol{\mu} - \frac{1}{2} \boldsymbol{\xi'} \boldsymbol{\mu^2} - \frac{1}{2} (\boldsymbol{\xi'} \boldsymbol{\mu})^2 + \boldsymbol{\xi'} I_\mu L \boldsymbol{\mu}$$

Approximation of real wage index $\tilde{w}^X - \tilde{p}$. Wage index W^X is obtained from $W^X L = W L^M + \sum W_i L_i^I$. The corresponding optimization problem is to maximize $W L^M + \sum W_i L_i^I$ subject to $L^{1+\gamma} = (L^M)^{1+\gamma} + \sum (L_i^I)^{1+\gamma}$. This gives the following index

$$W^X = (W^{\frac{1+\gamma}{\gamma}} + \sum W_i^{\frac{1+\gamma}{\gamma}})^{\frac{\gamma}{1+\gamma}}$$

In the efficient state we have $W_i L_i^I = k \alpha_i P_i Y_i = k \alpha_i \xi_i PC$ and $W L^M = \sum W L_i^M = (1-k) \sum \alpha_i \xi_i PC = (1-k) PC$. Moreover, labor supply first order conditions give

$(L_i^I)^{1+\gamma} = \frac{W_i L_i^I}{PC} = k\alpha_i\xi_i$ and $(L^M)^{1+\gamma} = \frac{WL^M}{PC} = (1-k)$. Then we have

$$\left(\frac{W_i}{P}\right)^{\frac{1+\gamma}{\gamma}} = ((L_i^I)^\gamma C)^{\frac{1+\gamma}{\gamma}} = C^{\frac{1+\gamma}{\gamma}} k\alpha_i\xi_i$$
$$\left(\frac{W}{P}\right)^{\frac{1+\gamma}{\gamma}} = ((L^M)^\gamma C)^{\frac{1+\gamma}{\gamma}} = C^{\frac{1+\gamma}{\gamma}}(1-k)$$
$$\left(\frac{W^X}{P}\right)^{\frac{1+\gamma}{\gamma}} = C^{\frac{1+\gamma}{\gamma}}((1-k) + k\sum \alpha_i\xi_i) = C^{\frac{1+\gamma}{\gamma}}$$

Then, we can write

$$\left(\frac{W^X/P}{W^{X\star}/P^\star}\right)^{\frac{1+\gamma}{\gamma}} = \left(\frac{W/P}{W^\star/P^\star}\frac{W^\star/P^\star}{W^{X\star}/P^\star}\right)^{\frac{1+\gamma}{\gamma}} + \sum\left(\frac{W_i/P}{W_i^\star/P^\star}\frac{W_i^\star/P^\star}{W^{X\star}/P^\star}\right)^{\frac{1+\gamma}{\gamma}}$$

Now, let $\frac{W/P}{W^\star/P^\star} = e^{\tilde{w}-\tilde{p}}$ and $\frac{W_i/P}{W_i^\star/P^\star} = e^{\tilde{w}_i-\tilde{p}}$. Remember also that $\frac{W^X/P}{W^{X\star}/P^\star} = e^{\tilde{w}^X-\tilde{p}}$. Substituting gives the following expression for wage index expression approximated to the second order

$$\begin{aligned}\left(\frac{W^X/P}{W^{X\star}/P^\star}\right)^{\frac{1+\gamma}{\gamma}} &= (1-k)\left(e^{\tilde{w}-\tilde{p}}\right)^{\frac{1+\gamma}{\gamma}} + k\sum\alpha_i\xi_i\left(e^{\tilde{w}_i-\tilde{p}}\right)^{\frac{1+\gamma}{\gamma}} \approx \\ &\approx 1 + \frac{1+\gamma}{\gamma}\left((1-k)(\tilde{w}-\tilde{p}) + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})\right) + \\ &\quad + \frac{1}{2}\left(\frac{1+\gamma}{\gamma}\right)^2 \cdot \left((1-k)(\tilde{w}-\tilde{p})^2 + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2\right)\end{aligned}$$

Next we compute the first-order terms and second-order terms in the previous equation.

First-order terms. Sectoral prices can be expressed as

$$\boldsymbol{p} = \boldsymbol{\mu} - \boldsymbol{a} + W\boldsymbol{p} + (1-k)I_\alpha\mathbf{1}w + kI_\alpha\boldsymbol{w}$$

Solving for $\boldsymbol{p}$ and then multiplying this by $\boldsymbol{\beta}'$ yields $p = \boldsymbol{\xi}'\boldsymbol{\mu} - \boldsymbol{\xi}'\boldsymbol{a} + (1-k)w + k\sum\alpha_i\xi_i w_i$. Hence, the first-order terms are

$$(1-k)(\tilde{w}-\tilde{p}) + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p}) = -\boldsymbol{\xi}'\boldsymbol{\mu} + \boldsymbol{\xi}'\tilde{\boldsymbol{a}}$$

where $\tilde{\boldsymbol{a}}$ are second-order changes in sectoral productivity due to inter-sectoral misallocation.

Second-order terms. To compute second-order terms, we need to compute first-order approximations of $\tilde{w}_i - \tilde{p}$ and $\tilde{w} - \tilde{p}$. We do so by taking log-linear expressions for wages. We assume $\tau^c = 0$ and $g = 0$ and compute these log-linear expressions in terms of gaps from the efficient state, which yields

$$\tilde{w}_i - \tilde{p} = \tilde{c} - \frac{\gamma}{1+\gamma}\frac{1}{\xi_i}\sum_j l_{ji}\xi_j\mu_j$$
$$\tilde{w} - \tilde{p} = \tilde{c} - \frac{\gamma}{1+\gamma}\sum_j \xi_j\mu_j$$

moreover, note that output gap is $\tilde{c} = -\frac{1}{1+\gamma}\sum\limits_j \xi_j\mu_j$. These expressions give second-order terms expressed through markups.

Now, we need to express second-order approximation of $log(\frac{W^X/P}{W^{X\star}/P^\star})$ given that we have an approximation of $\left(\frac{W^X/P}{W^{X\star}/P^\star}\right)^{\frac{1+\gamma}{\gamma}}$. Note that $log(\frac{W^X/P}{W^{X\star}/P^\star}) = \frac{\gamma}{1+\gamma}log\left(\left(\frac{W^X/P}{W^{X\star}/P^\star}\right)^{\frac{1+\gamma}{\gamma}}\right) = \frac{\gamma}{1+\gamma}log(F)$ where $log(F) \approx (F-1) - \frac{1}{2}(F-1)^2$. Where

$$F - 1 \approx \frac{1+\gamma}{\gamma}\left((1-k)(\tilde{w}-\tilde{p}) + k\sum\alpha_i\xi_i(\tilde{w}_i - \tilde{p})\right) + \\ + \frac{1}{2}\left(\frac{1+\gamma}{\gamma}\right)^2 \cdot \left((1-k)(\tilde{w}-\tilde{p})^2 + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2\right)$$

$$(F-1)^2 \approx \left(\frac{1+\gamma}{\gamma}\right)^2 \left((1-k)(\tilde{w}-\tilde{p}) + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})\right)^2$$

Hence, we have

$$\begin{aligned} log(\frac{W^X/P}{W^{X\star}/P^\star}) &\approx \frac{\gamma}{1+\gamma}(F-1) - \frac{\gamma}{1+\gamma}\frac{1}{2}(F-1)^2 = \\ &= \left((1-k)(\tilde{w}-\tilde{p}) + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})\right) + \\ &+ \frac{1}{2}\left(\frac{1+\gamma}{\gamma}\right)\cdot\left((1-k)(\tilde{w}-\tilde{p})^2 + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2\right) - \\ &- \frac{1}{2}\left(\frac{1+\gamma}{\gamma}\right)\cdot\left((1-k)(\tilde{w}-\tilde{p}) + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})\right)^2 = \\ = -\boldsymbol{\xi}'\boldsymbol{\mu} + \boldsymbol{\xi}'\tilde{\boldsymbol{a}} &+ \frac{1}{2}\left(\frac{1+\gamma}{\gamma}\right)\cdot\left(\left((1-k)(\tilde{w}-\tilde{p})^2 + k\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2\right) - (\boldsymbol{\xi}'\boldsymbol{\mu})^2\right) = \\ &= -\boldsymbol{\xi}'\boldsymbol{\mu} + \boldsymbol{\xi}'\tilde{\boldsymbol{a}} + \frac{1}{2}\cdot\frac{1+\gamma}{\gamma}\cdot k\left(\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2 - (\boldsymbol{\xi}'\boldsymbol{\mu})^2\right) \end{aligned}$$

where the last step uses the fact that $(\tilde{w}-\tilde{p})^2 = (\boldsymbol{\xi}'\boldsymbol{\mu})^2$.

Next, we express $\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2 - (\boldsymbol{\xi}'\boldsymbol{\mu})^2$ as a quadratic form of sectoral markup vector $\boldsymbol{\mu}$. Note that $\tilde{w}_i - \tilde{p} = \tilde{c} - \frac{\gamma}{1+\gamma}\frac{1}{\xi_i}\sum\limits_j l_{ji}\xi_j\mu_j$ and taking the average with weights $\alpha_i\xi_i$ yields $\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p}) = \tilde{w}-\tilde{p} = -\boldsymbol{\xi}'\boldsymbol{\mu}$. Then we have

$$\sum\alpha_i\xi_i(\tilde{w}_i-\tilde{p})^2 - (\boldsymbol{\xi}'\boldsymbol{\mu})^2 = \sum\alpha_i\xi_i(\tilde{w}_i-\tilde{w})^2$$

At the same time $\tilde{w}_i - \tilde{w} = -\frac{\gamma}{1+\gamma}\left(\frac{1}{\xi_i}\sum_j l_{ji}\xi_j\mu_j - \boldsymbol{\xi}'\boldsymbol{\mu}\right)$, which written in vector form becomes

$$\tilde{\boldsymbol{w}} - \mathbf{1}\tilde{w} = -\frac{\gamma}{1+\gamma}\left(I_\xi^{-1}L'I_\xi\boldsymbol{\mu} - \mathbf{11}'I_\xi\boldsymbol{\mu}\right) = -\frac{\gamma}{1+\gamma}(I_\xi^{-1}L' - \mathbf{11}')I_\xi\boldsymbol{\mu}$$

and yields

$$\sum \alpha_i\xi_i(\tilde{w}_i - \tilde{p})^2 - (\boldsymbol{\xi}'\boldsymbol{\mu})^2 = \left(\frac{\gamma}{1+\gamma}\right)^2 \boldsymbol{\mu}'I_\xi(I_\xi^{-1}L' - \mathbf{11}')'I_\xi I_\alpha(I_\xi^{-1}L' - \mathbf{11}')I_\xi\boldsymbol{\mu}$$

Collecting all the results together, we have the second-order approximation

$$\tilde{w}^X - \tilde{p} = log(\frac{W^X/P}{W^{X\star}/P^\star}) \approx -\boldsymbol{\xi}'\boldsymbol{\mu} + \boldsymbol{\xi}'\tilde{\boldsymbol{a}} + \frac{1}{2}\cdot\frac{\gamma}{1+\gamma}\cdot k\cdot\boldsymbol{\mu}'M\boldsymbol{\mu}$$

where $M = I_\xi(I_\xi^{-1}L' - \mathbf{11}')'I_\xi I_\alpha(I_\xi^{-1}L' - \mathbf{11}')I_\xi$

Productivity loss due to within-sector misallocation. Finally, we express the second-order productivity change as a function of price dispersion within a sector which in turn depends on seller price inflation. The within-sector productivity loss is derived in a standard way, see Woodford and Walsh (2005); Galí (2015). Consider a two-stage production in a sector: 1) competitive intermediate output Y_i^I sold at price $P_i^I = MC_i$ and 2) final good, produced from Y_i^I by a set of monopolistically competitive producers. For a firm k the "technology" is $Y_i(k) = Y_i^I(k)$. Then $Y_i^I = \int Y_i^I(k) = \int Y_i(k) = \int\left(\frac{P_i(k)}{P_i}\right)^{-\epsilon} Y_i$ which gives $Y_i = \frac{P_i^{-\epsilon}}{\int P_i(k)^{-\epsilon}}Y_i^I$. Let $\tilde{A}_i = \frac{P_i^{-\epsilon}}{\int P_i(k)^{-\epsilon}}$ then $\tilde{a}_i + log(\int e^{-\epsilon(p_i(k)-p_i)}) = 0$ or, to the second order,

$$\tilde{a}_i = \epsilon\int(p_i(k) - p_i) - \frac{\epsilon^2}{2}\int(p_i(k) - p_i)^2 \tag{OA.1.4}$$

On the other hand, $\frac{\int P_i(k)^{1-\epsilon}}{P_i^{1-\epsilon}} = \int e^{(1-\epsilon)\cdot(p_i(k)-p_i)} = 1$, for which the second order approximation gives $0 = (1-\epsilon)\int(p_i(k) - p_i) + \frac{(1-\epsilon)^2}{2}\int(p_i(k) - p_i)^2$ or $0 = \epsilon\int(p_i(k) - p_i) + \frac{\epsilon(1-\epsilon)}{2}\int(p_i(k) - p_i)^2$. Combining with (OA.1.4) we get productivity loss from within-sector price distortion

$$\tilde{a}_i = -\frac{\epsilon}{2}\int(p_i(k) - p_i)^2 \equiv -\frac{\epsilon}{2}Var(p_i^s) \tag{OA.1.5}$$

Finally, we express within-sector seller price variance in terms of sectoral seller price inflation. Seller price is $p_i^s = p_i - \tau_i^b$. Sectoral seller price is $p_i^s = (1-\lambda_i)p_i^{s,\star} + \lambda_i p_{i,-1}^s$ which gives seller price inflation $\lambda_i\pi_i^s = (1-\lambda_i)(p_i^{s,\star} - p_i^s)$

with $\pi_i^s = p_i^s - p_{i,-1}^s$. Price variance is $Var(p_i^s) = (1-\lambda_i)\int(p_i^{s,\star} - p_i^s)^2 + \lambda_i \int (p_{i,-1}^s(k) - p_{i,-1}^s - \pi_i^s)^2 = \frac{\lambda_i^2}{(1-\lambda_i)}(\pi_i^s)^2 + \lambda_i(p_{i,-1}^s) + \lambda_i(\pi_i^s)^2$ which gives

$$Var(p_i^s) = \frac{\lambda_i}{1-\lambda_i}(\pi_i^s)^2 + \lambda_i Var(p_{i,-1}^s) = \frac{\lambda_i}{1-\lambda_i}\sum_{s=0}^{t}\lambda_i^s \pi_{i,t-s}^2$$

Note that lifetime welfare loss due to within-sector misallocation in sector i is

$$\xi_i \cdot \frac{\epsilon}{2}\sum_{t=0}^{\infty}\delta^t Var(p_i^t) = \xi_i \cdot \frac{\epsilon}{2}\cdot\frac{\lambda_i}{1-\lambda_i}\sum_{s=0}^{t}\delta^s \pi_{i,s}^2 \sum_{t=s}^{\infty}(\delta\lambda_i)^{t-s} = \xi_i \cdot \frac{\epsilon}{2}\cdot\frac{\lambda_i}{1-\lambda_i}\cdot\frac{1}{1-\delta\lambda_i}\sum_t \delta^t \pi_{i,t}^2$$

Hence, total welfare loss over time due to productivity loss caused by within-sector misallocation is

$$\sum_t \delta^t \boldsymbol{\xi}'\tilde{\boldsymbol{a}} = -\frac{\epsilon}{2}\sum_i \xi_i \sum_t \delta^t Var(p_{i,t}^s) = -\frac{\epsilon}{2}\sum_t \delta^t \sum_i \frac{\xi_i\lambda_i}{(1-\lambda_i)(1-\delta\lambda_i)}\pi_{i,t}^2 \quad \text{(OA.1.6)}$$

Final expression for welfare loss. To summarize the results, welfare loss is

$$\Delta = -\frac{\sigma+\gamma}{2}\tilde{c}^2 - d$$

where $d = \tilde{l} - \tilde{c}$.

The distortion d is

$$d = \tilde{l} - \tilde{c} = \tilde{\Gamma} - (\tilde{w}^X - \tilde{p})$$

The labor share $\tilde{\Gamma}$ is

$$\tilde{\Gamma} = -\boldsymbol{\xi}'\boldsymbol{\mu} - \frac{1}{2}\boldsymbol{\xi}'\boldsymbol{\mu}^{\mathbf{2}} - \frac{1}{2}(\boldsymbol{\xi}'\boldsymbol{\mu})^2 + \boldsymbol{\xi}' I_\mu L\boldsymbol{\mu}$$

The real wage index is

$$\tilde{w}^X - \tilde{p} = -\boldsymbol{\xi}'\boldsymbol{\mu} + \boldsymbol{\xi}'\tilde{\boldsymbol{a}} + \frac{1}{2}\cdot\frac{\gamma}{1+\gamma}\cdot k \cdot \boldsymbol{\mu}' M\boldsymbol{\mu}$$

where $M = I_\xi(I_\xi^{-1}L' - \mathbf{1}\mathbf{1}')' I_\xi I_\alpha (I_\xi^{-1}L' - \mathbf{1}\mathbf{1}')I_\xi$.

The productivity loss gives

$$\boldsymbol{\xi}'\tilde{\boldsymbol{a}} = -\frac{\epsilon}{2}(\sum \xi_i \frac{\lambda_i}{1-\lambda_i}(\pi_i^s)^2 + \sum \xi_i\lambda_i Var(p_{i,-1}^s))$$

Now, we combine these results together and write the final expression for welfare loss:

$$\Delta = -\frac{\sigma+\gamma}{2}\tilde{c}^2 + \frac{1}{2}\cdot\frac{\gamma}{1+\gamma}\cdot k\cdot \boldsymbol{\mu}' M \boldsymbol{\mu} + \frac{1}{2}\boldsymbol{\xi}'\boldsymbol{\mu}^2 + \frac{1}{2}(\boldsymbol{\xi}'\boldsymbol{\mu})^2 - \boldsymbol{\xi}' I_\mu L \boldsymbol{\mu} -$$
$$-\frac{\epsilon}{2}\left(\sum \xi_i \frac{\lambda_i}{1-\lambda_i}(\pi_i^s)^2 + \sum \xi_i \lambda_i Var(p_{i,-1}^s)\right)$$

Rearrange the terms using $\boldsymbol{\xi}' I_\mu L\boldsymbol{\mu} = \boldsymbol{\xi}'\boldsymbol{\mu}^2 + \boldsymbol{\xi}' I_\mu \Omega L \boldsymbol{\mu}$ and that $\boldsymbol{\xi}' I_\mu \Omega L \boldsymbol{\mu} = \boldsymbol{\mu}' N \boldsymbol{\mu}$ where $N_{ij} = \xi_i(\Omega(1-\Omega)^{-1})_{ij}$ we obtain welfare loss

$$\Delta_t = \frac{\sigma+\gamma}{2}\tilde{c}^2 + \frac{1}{2}\cdot(\boldsymbol{\xi}'\boldsymbol{\mu}^2 - (\boldsymbol{\xi}'\boldsymbol{\mu})^2) + \boldsymbol{\mu}'(N - \frac{k}{2}\frac{\gamma}{1+\gamma}M)\boldsymbol{\mu} +$$
$$+\frac{\epsilon}{2}\left(\sum \xi_i \frac{\lambda_i}{1-\lambda_i}(\pi_i^s)^2 + \sum \xi_i \lambda_i Var(p_{i,-1}^s)\right)$$

where $N_{ij} = \xi_i(\Omega(I-\Omega)^{-1})_{ij}$ and $M = I_\xi(I_\xi^{-1}L' - \mathbf{11}')' I_\xi I_\alpha (I_\xi^{-1}L' - \mathbf{11}') I_\xi$.

Note that in the absence of a production network ($N = 0$ and $M = 0$) the third term disappears and welfare loss depends on the output gap, markup variance, and sellers' price inflation.

Finally, summing up the welfare loss over time and noting that the last term sums to OA.1.6 we have the final expression as $Loss_t = \sum_t \delta^t \Delta_t$. Note that one-period welfare loss Δ_t puts less weight on inflation in each period, than the lifetime welfare loss ($\frac{\lambda_i}{1-\lambda_i}$ versus $\frac{\lambda_i}{(1-\lambda_i)(1-\delta\lambda_i)}$, see result OA.1.6). This reflects the result known as gains from commitment in the literature.

OA.2 Additional results and proofs

OA.2.1 Upstream and downstream effect of shocks

Matrices L and $\hat{L}$ are the measures of shock-specific proximity. The ij-th element of each matrix captures the effect of shock in j on tax in i.

- For productivity shocks, the Leontief inverse matrix $L = (I-\Omega)^{-1}$, where $\Omega_{ij} = \frac{X_{ij}P_j}{MC_i Y_i}$ is share of input j in production of i. Hence, productivity shocks propagate downstream.

- For demand shocks, the matrix $\hat{L} I_\beta$ consists of an upstream and downstream parts

$$\hat{L} I_\beta = \frac{\gamma}{1+\gamma}\cdot \underbrace{[\mathbf{1}\boldsymbol{\beta}' + k(I - \mathbf{1}\boldsymbol{\beta}')]\cdot L I_\alpha}_{=D} \cdot \underbrace{I_\xi^{-1} L' I_\xi}_{=U} \cdot I_\xi^{-1} I_\beta$$

where $I_\xi^{-1} I_\beta$ is diagonal matrix with elements $\frac{P_i C_i}{P_i Y_i}$ (final sales in total sales) on diagonal. Upstream matrix $U = I_\xi^{-1} L' I_\xi = (I - \tilde{\Omega})^{-1}$ is the Leontief inverse such that $\tilde{\Omega}_{ij} = \frac{X_{ji} P_i}{MC_i Y_i}$ is the share of i sold to j in total sales of i. Hence, U captures the upstream propagation of demand shocks. The downstream matrix $D = [\mathbf{1}\boldsymbol{\beta}' + k(I - \mathbf{1}\boldsymbol{\beta}')] \cdot L I_\alpha$ captures the downstream propagation of demand shocks.

Proof. The fact that $\Omega_{ij} = \frac{X_{ij} P_j}{MC_i Y_i}$ follows from our definition of input-output matrix. Now, let us prove that $\tilde{\Omega}_{ij} = \frac{X_{ji} P_i}{MC_i Y_i}$. We have $U^{-1} = I_\xi^{-1}(I - \Omega)' I_\xi = I - (I_\xi^{-1} \Omega I_\xi)'$. The ij-th element of matrix in brackets is $\frac{1}{\xi_j} \Omega_{ij} \xi_i = \frac{X_{ij} P_j}{MC_j Y_j}$. Let $\tilde{\Omega} = (I_\xi^{-1} \Omega I_\xi)'$. Then $\tilde{\Omega}_{ij} = \frac{X_{ji} P_i}{MC_i Y_i} = \frac{X_{ji}}{Y_i}$ in efficient steady state. □

OA.2.2 Labor and input-specific subsidies

Instead of production subsidies $\boldsymbol{s}_t^p$ we may have a set of sector-specific labor subsidies $\boldsymbol{s}_t^L$ and/or input-specific subsidies $\boldsymbol{s}_t^I$ such that $\boldsymbol{s}_t^p = I_\alpha \cdot \boldsymbol{s}_t^L + \Omega \cdot \boldsymbol{s}_t^I$. As a result, the appropriate sector-specific labor subsidy or input-specific subsidy might be constructed to substitute for the optimal production subsidy.

OA.2.3 Three-sector example

Let energy be used in both the services and manufacturing sectors as in Figure OA.2.1 The Leontieff inverse matrix capturing the effect of productivity is

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 - \alpha^M & 1 & 0 \\ 1 - \alpha^S & 0 & 1 \end{pmatrix}$$

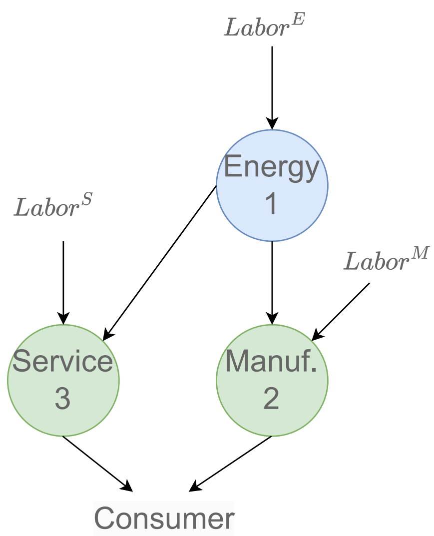
Productivity shocks propagate downstream as before. For instance, productivity shock in the energy sector affects natural prices in energy, manufacturing, and services (first column of matrix L).

Let the Domar weights be ξ^E, ξ^M, ξ^S. Then we can compute the upstream propagation matrix as

$$U = I_\xi^{-1} L' I_\xi = \begin{pmatrix} 1 & \frac{\xi^M}{\xi^E}(1 - \alpha^M) & \frac{\xi^S}{\xi^E}(1 - \alpha^S) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The upstream matrix captures the upstream direction of propagation of demand shocks. For instance, a demand shock in manufacturing also affects natural

Figure OA.2.1: Three-sector economy example: energy in services



prices in the energy sector (second column in the matrix U). The upstream propagation of manufacturing demand shock is then complemented by its further downstream propagation downstream. This further downstream propagation is captured by matrix LI_α which is

$$LI_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 1-\alpha^M & \alpha^M & 0 \\ 1-\alpha^S & 0 & \alpha^S \end{pmatrix}$$

As long as energy is used in services ($\alpha^S \neq 1$), the price change in the energy sector (caused by manufacturing demand shock) then propagates downstream to services. To see this note that the third element in the first column is non-zero, suggesting the downstream propagation of the energy sector changes. The overall propagation of the demand shocks with immobile labor is given by $\hat{L}I_\beta = \frac{\gamma}{1+\gamma} \cdot L \cdot I_\alpha \underbrace{I_\xi^{-1} L' I_\xi}_{=U} \cdot I_\xi^{-1} I_\beta$.

OA.2.4 Irrelevance of sectoral demand shocks with mobile labor

Proposition. *As long as* ***labor is fully mobile*** *across sectors ($k = 0$), the sectoral demand shifters ($\mathbf{b}_t$)* ***do not affect*** *the distribution of markups and sellers' prices across* *sectors.*

Proof. With $k = 0$ we have $\hat{L} = \frac{\gamma}{1+\gamma}\mathbf{1}\mathbf{1}'$. Then we have $\hat{L}I_\beta \boldsymbol{b_t} = \frac{\gamma}{1+\gamma}\mathbf{1}\sum \beta_t b_{t,i}$ does not depend on the distribution of sectoral demand shifts but only on the aggregate demand. As a result, they do not affect the shape of the distribution of markups and seller prices. □

OA.2.5 Optimal tax policy with one tax instrument

Next, we consider a static economy and a situation where only one type of tax is available: either production subsidy or sales tax. We split the proposition of the main text into two underlying propositions: production subsidy and sales tax.

Proposition (Optimal production subsidy policy)**.** *Consider a static economy $\delta = 0$ such that $\boldsymbol{p^s_{-1}} = 0$ and production subsidy policy $\boldsymbol{s^p}$ that minimizes welfare loss subject to constraints. The first-order conditions are*

$$\hat{F}_\mu \cdot (L - \hat{L}I_\xi) \cdot \boldsymbol{\mu_t} = -\hat{F}_p \cdot \boldsymbol{\pi^s_t}$$

reflect trade-off between minimizing markups and sellers' price inflation. The vector of sellers' taxes achieving this trade-off is

$$\boldsymbol{s^p_t} = (L - \hat{L}I_\xi)^{-1} \cdot (I - X^{-1}(\hat{F}_\mu + \hat{F}_p)^{-1}\hat{F}_\mu) \cdot \underbrace{(m_t \cdot \mathbf{1} + [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}])}_{\textit{optimal sellers tax}}$$

where $\hat{F}_\mu = \tilde{F}_\mu + \tilde{F}'_\mu$ and $\tilde{F}_\mu = (L' - I_\xi \hat{L}')^{-1} \cdot (\frac{f_y}{(1+\gamma)^2}\boldsymbol{\xi}\boldsymbol{\xi}' + F_\mu) \cdot (L - \hat{L}I_\xi)^{-1}$ and $\tilde{F}_p = F_p$

Proof. One period welfare loss is

$$\Delta_t = \frac{1}{2}\{\underbrace{f_y \cdot \tilde{y}_t^2}_{\text{due to output gap}} + \underbrace{\boldsymbol{\mu'_t} \cdot F_\mu \cdot \boldsymbol{\mu_t}}_{\text{due to cross-sector}} + \underbrace{\boldsymbol{p^{s\prime}_t} \cdot F_p \cdot \boldsymbol{p^s_t}}_{\text{due to within-sector}}\}$$

Let $\boldsymbol{\tilde{\mu}_t} = (L - \hat{L}I_\xi)\boldsymbol{\mu_t}$. Given the link between output gap and markups $\tilde{y} = -\frac{1}{1+\gamma}\boldsymbol{\xi'}\boldsymbol{\mu_t}$, the welfare loss can be rewritten as

$$\Delta_t = \frac{1}{2}\{\boldsymbol{\tilde{\mu}'_t} \cdot \tilde{F}_\mu \cdot \boldsymbol{\tilde{\mu}_t} + \boldsymbol{p^{s\prime}_t} \cdot \tilde{F}_p \cdot \boldsymbol{p^s_t}\}$$

The first order condition with respect to sellers' (or buyers' tax) is

$$\boldsymbol{\tilde{\mu}'_t}\hat{F}_\mu \frac{d\boldsymbol{\tilde{\mu}_t}}{d\boldsymbol{s^p_t}} + \boldsymbol{p^{s\prime}_t}\hat{F}_p \frac{d\boldsymbol{p^s_t}}{d\boldsymbol{s^p_t}} = 0 \qquad \text{(OA.2.1)}$$

According to the system (A.18) we have $\tilde{\boldsymbol{\mu}}_{\boldsymbol{t}} = \boldsymbol{p}^{s}_{\boldsymbol{t}} + \boldsymbol{\tau}^{s}_{\boldsymbol{t}} - [m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$. Then for sellers tax we have $\frac{d\boldsymbol{p}^s_t}{d\boldsymbol{s}^p_t} = \frac{d\tilde{\boldsymbol{\mu}}_t}{d\boldsymbol{s}^p_t}$. The first-order condition becomes $\tilde{\boldsymbol{\mu}}'_{\boldsymbol{t}}\hat{F}_\mu + \boldsymbol{p}^{s\prime}_{\boldsymbol{t}}\hat{F}_p = 0$. The corresponding optimal sellers' price is

$$\boldsymbol{p}^s_{\boldsymbol{t}} = (\hat{F}_\mu + \hat{F}_p)^{-1} \cdot \hat{F}_\mu \cdot [m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$$

The sellers' tax achieving this price is found from the price dynamics equations (A.24)

$$\boldsymbol{s}^p_{\boldsymbol{t}} = -(L - \hat{L}I_\xi)^{-1} \cdot (X^{-1}(\hat{F}_\mu + \hat{F}_p)^{-1} \cdot \hat{F}_\mu - I) \cdot [m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$$

□

Next, we find optimal buyer's tax if only this tax instrument is available.

Proposition (Optimal sales tax policy)**.** *Consider a static economy $\delta = 0$ such that $\boldsymbol{p}^s_{-\mathbf{1}} = 0$ and sales tax policy $\boldsymbol{\tau}^s$ that minimizes welfare loss subject to constraints. The welfare maximizing vector of buyers' taxes is*

$$\boldsymbol{\tau}^s_{\boldsymbol{t}} = [X(L - \hat{L}I_\xi - I) + A]^{-1} \cdot (A - X) \cdot \underbrace{([m_t \cdot \mathbf{1} + [\hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}])}_{\text{optimal buyers tax}}$$

where $\hat{F}_\mu = \tilde{F}_\mu + \tilde{F}'_\mu$ *and* $\tilde{F}_\mu = (L' - I_\xi\hat{L}')^{-1} \cdot (\frac{f_y}{(1+\gamma)^2}\boldsymbol{\xi}\boldsymbol{\xi}' + F_\mu) \cdot (L - \hat{L}I_\xi)^{-1}$ *and* $\tilde{F}_p = F_p$ *and* $A = (((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu + (L - \hat{L}I_\xi - I)'X'\hat{F}_p)^{-1} \cdot ((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu$

Proof. The beginning of the proof is identical to the previous proposition/ According to the system (A.18) we have $\tilde{\boldsymbol{\mu}}_{\boldsymbol{t}} = \boldsymbol{p}^{s}_{\boldsymbol{t}} + \boldsymbol{\tau}^{s}_{\boldsymbol{t}} - [m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$. Then for buyers' tax we have $\frac{d\boldsymbol{p}^s_t}{d\boldsymbol{\tau}^s_t} + I = \frac{d\tilde{\boldsymbol{\mu}}_t}{d\boldsymbol{\tau}^s_t}$. From the system (A.24) we have $\frac{d\boldsymbol{p}^s_t}{d\boldsymbol{\tau}^s_t} = X \cdot [L - \hat{L}I_\xi - I]$. Then, the first-order condition (OA.2.1) becomes $((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu \cdot \tilde{\boldsymbol{\mu}}_{\boldsymbol{t}} + (L - \hat{L}I_\xi - I)'X'\hat{F}_p\boldsymbol{p}^s_{\boldsymbol{t}} = 0$. Then, we can express prices as

$$\boldsymbol{p}^s_{\boldsymbol{t}} = A \cdot [m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t} - \boldsymbol{\tau}^s_{\boldsymbol{t}}]$$

where $A = (((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu + (L - \hat{L}I_\xi - I)'X'\hat{F}_p)^{-1} \cdot ((L - \hat{L}I_\xi - I)'X' + I) \cdot \hat{F}_\mu$. Combining this with price dynamic equations (A.24) we get the expression for optimal buyers' taxes

$$\boldsymbol{\tau}^s_{\boldsymbol{t}} = [X(L - \hat{L}I_\xi - I) + A]^{-1} \cdot (A - X) \cdot [m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}]$$

□

OA.2.6 Properties of aggregate consumption under optimal policy

Now, we derive the properties of aggregate response notably of aggregate consumption and consumer price index to shocks under optimal policy in a static case. Again, consider static economy $\delta = 0$ initially at steady state $\boldsymbol{p}^s_{\boldsymbol{t-1}} = 0$.

Property 1 (Aggregate consumption response). *Aggregate consumption has the same response to shocks under **monetary policy along** and a **combination of monetary policy with optimal tax policy**. In both cases*

$$\boldsymbol{\beta}'\boldsymbol{c_t} = \frac{1}{1+\gamma} \cdot \boldsymbol{\beta}'\boldsymbol{b_t} + \boldsymbol{\xi}'\boldsymbol{a_t}$$

Moreover, aggregate consumption is not affected by relative demand shocks such that $\boldsymbol{\beta}'\boldsymbol{b_t} = 0$.

Also, if the sector is the final good sector $\beta_i = \xi_i$, the effect of productivity shock on final cons. is fully attributed to a shocked sector under opt. policy

Proof. Sectoral consumption is $\boldsymbol{c} = m_t\mathbf{1} + \boldsymbol{b_t} - \boldsymbol{p^s_t} - \boldsymbol{\tau^s_t}$. Then, the aggregate final consumption is $\boldsymbol{\beta}'\boldsymbol{c_t} = m_t + \boldsymbol{\beta}'\boldsymbol{b_t} - \boldsymbol{\beta}'\boldsymbol{p^s_t} - \boldsymbol{\beta}'\boldsymbol{\tau^s_t}$. The aggregate price without taxes is $\boldsymbol{\beta}'\boldsymbol{p^s_t} = \boldsymbol{\beta}'X\mathbf{1}m_t + \boldsymbol{\beta}'X[\hat{L}I_\beta\boldsymbol{b_t} - L\boldsymbol{a_t}]$. With suggested monetary policy and zero tax the final consumption is

$$\boldsymbol{\beta}'\boldsymbol{c_t} = \boldsymbol{\beta}'[I - \hat{L}I_\beta]\boldsymbol{b_t} + \boldsymbol{\xi}'\boldsymbol{a_t} = \frac{1}{1+\gamma}\boldsymbol{\beta}'\boldsymbol{b_t} + \boldsymbol{\xi}'\boldsymbol{a_t}$$

Now, under optimal tax policy $\boldsymbol{p^s_t} = 0$, which yields $\boldsymbol{\beta}'\boldsymbol{c_t} = m_t + \boldsymbol{\beta}'\boldsymbol{b_t} - \boldsymbol{\beta}'\boldsymbol{\tau^s_t}$. Substituting the optimal tax policy we get again that $\frac{1}{1+\gamma}\boldsymbol{\beta}'\boldsymbol{b_t} + \boldsymbol{\xi}'\boldsymbol{a_t}$. That is final consumption response is not altered by the presence of optimal taxes.

Under optimal policy sectoral consumptions are $\boldsymbol{c_t} = [I - \hat{L}I_\beta]\boldsymbol{b_t} + L\boldsymbol{a_t}$. If sector i is final, the i-th column of L has zeros everywhere except the intersection with the main diagonal. In this case, productivity shock changes only consumption in the shocked sector and does not affect other sectoral consumptions. □

Next, we look into the behavior of the consumer price index under optimal policy

Property 2 (Consumer price index). *Consumer price index under a combination of monetary policy with optimal tax policy is*

$$\boldsymbol{\beta}'\boldsymbol{p_t} = \boldsymbol{\beta}'\boldsymbol{\tau^s_t} = \boldsymbol{\beta}'X\boldsymbol{\tau^s_t}$$

Then, from the properties of matrix X, it follows that

- *when prices are fully rigid ($\lambda_i \to 1$ for all i) we have $X \to 0$. Then shocks do not have any effect on the consumer price index, that is $\boldsymbol{\beta' p_t} = 0$.*
- *when no network $L = I$, the matrix X is diagonal with each element decreasing in the respective λ_i. In this case only shocks in flex. price sectors affect CPI*

Proof. Optimal monetary policy yields $\boldsymbol{\beta}'(X - I)\tau_t^s = 0$ where $\tau_t^s = m_t\mathbf{1} + \hat{L}I_\beta \boldsymbol{b_t} - L\boldsymbol{a_t}$. Then, $\boldsymbol{\beta}'\boldsymbol{\tau_t^s} = \boldsymbol{\beta}' X \boldsymbol{\tau_t^s}$. When all prices are rigid $\tilde{I}_\lambda = 0$ and hence $X = 0$.

□

OA.3 Quantitative appendix

OA.3.1 Solving the model

The set of policy and exogenous variables is $x_t = [m_t,\ s_t^p,\ \tau_t^s,\ b_t,\ a_t]$. Also, let us assume that x_t follows the autoregressive process

$$x_t = W x_{t-1} + \tilde{\epsilon}_t,\ E_t \tilde{\epsilon}_{t+1} = 0$$

Search for the solution of the dynamic equation (A.24) in the form

$$p_t^s = A p_{t-1}^s + B \cdot x_t$$

By substituting into the equation (A.24) and simplifying, we get the expressions for unknown matrices

$$A = (I - \delta Z A)^{-1} Z$$
$$B = (I - \delta Z A)^{-1}(X + \delta Z B W)$$

Matrices A and B can be found recursively iterating on the above expressions.

Once we have a dynamic path for sectoral selling prices p_t^s, we can compute other objects in the model, such as market prices, markups, and aggregate and sectoral production/consumption. This allows us to build responses to shocks and simulate the model.

OA.3.2 Impulse responses to sectoral shocks

Next, we examine the response to sectoral shocks in the calibrated economy under three policies: no monetary or tax policy, optimal monetary policy, and optimal monetary and tax policy. In addition to aggregate impulse response, we compare the impulse responses of the shocked sectors to the corresponding response of the rest of the sectors. While the shocked sectors are directly affected by the shock, the rest of the sectors are affected only indirectly - through the production network. We also examine the economy's response to some real-world tax policies, which, nevertheless, deviate from our optimal policy result.

Our decomposition of impulse response into the shocked sectors and the rest of the economy requires defining the sales price and quantity indices, used to aggregate sectors within each of these two groups. Consider total sales in K sectors ($K \leq N$). Let $\hat{P}$ be sales price index and $\hat{Y}$ be the sales quantity index for these K sectors such that

$$\hat{P}_t \hat{Y}_t = \sum_K P_{t,i} Y_{t,i}$$

Let P be the consumer price and Y be the final output. In the efficient steady-state $\frac{P_i Y_i}{PY} = \xi_i$ and $\frac{\hat{P}\hat{Y}}{PY} = \sum_K \xi_i$ where ξ_i is the ratio of sales in sector i to aggregate final output. Log-linearizing around this steady state, we get

$$\hat{p}_t + \hat{y}_t = \sum_K \frac{\xi_i}{\sum_K \xi_i} \cdot p_{t,i} + \sum_K \frac{\xi_i}{\sum_K \xi_i} \cdot y_{t,i}$$

Given the above expression, we define sales price and quantity indices $\hat{p}_t$ and $\hat{y}_t$ as

$$\hat{p}_t = \sum_K \frac{\xi_i}{\sum_K \xi_i} \cdot p_{t,i} \tag{OA.3.1}$$

$$\hat{y}_t = \sum_K \frac{\xi_i}{\sum_K \xi_i} \cdot y_{t,i} \tag{OA.3.2}$$

Note that we are using sales indices rather than consumption indices since sales indices also reflect changes in the intermediate goods sectors. Above we investigated the properties of the impulse response of aggregate consumption price and quantity indices and demonstrated that as long as monetary policy stabilizes the aggregate consumption gap (output gap), sectoral tax policy has a very limited effect on the aggregate consumption index, as long as monetary policy is taken into account.

OA.3.2.1 Productivity shock in energy sectors

Figure OA.3.1 depicts the impulse response of sales in shocked sectors and the rest of the economy. The second row illustrates the optimal monetary and tax response to the shock. A negative productivity shock in the energy sector is contractionary; however, the presence of sticky prices renders the economic contraction insufficient, resulting in an overheated economy. This is a standard feature in New Keynesian models, where price stickiness prevents prices from increasing in response to a negative productivity shock, leading to an inefficiently high output. The optimal monetary policy induces more contraction in all sectors, and optimal tax instruments further contribute to the contraction. The fact that a negative productivity shock creates an overheated economy is also reflected in the optimal monetary policy response—monetary policy stabilizing the output gap is contractionary.

Figure OA.3.1: Negative 1% productivity shock in energy sectors

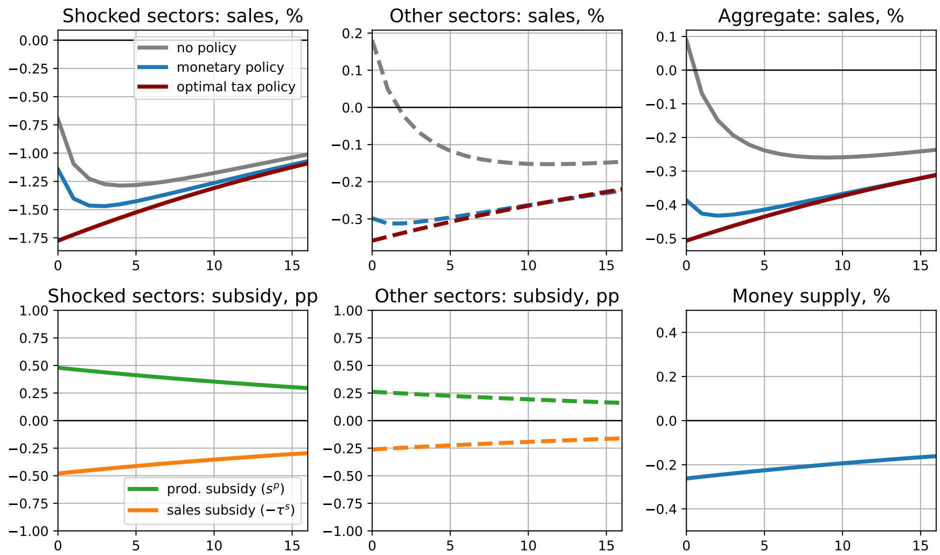
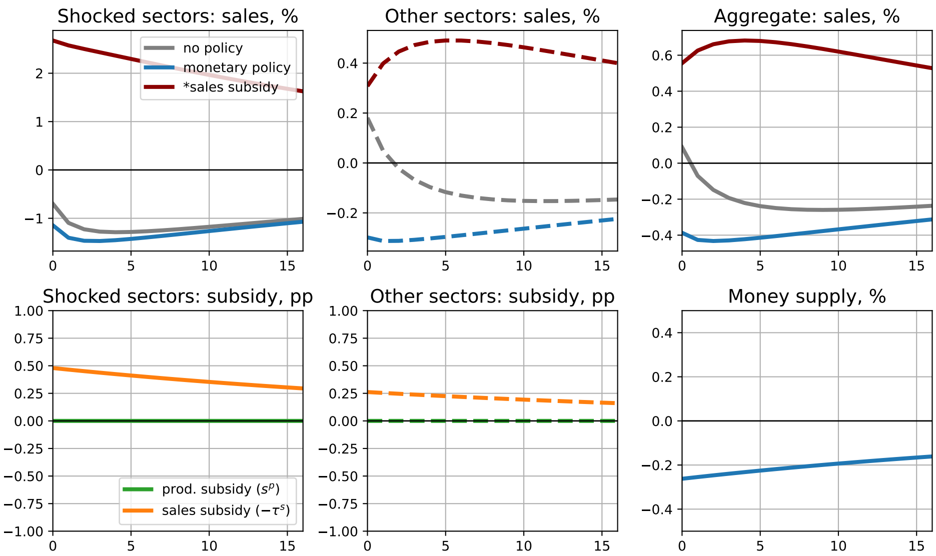


Figure OA.3.2 illustrates the economy's response to an energy shock under real-world response to the shock, that is, sales subsidy; the absolute size of the subsidy corresponds to the optimal subsidy (but the sign is inverse). Although this policy stimulates output, it leads to inefficiently high production of energy and energy-intensive goods, resulting in a welfare loss.

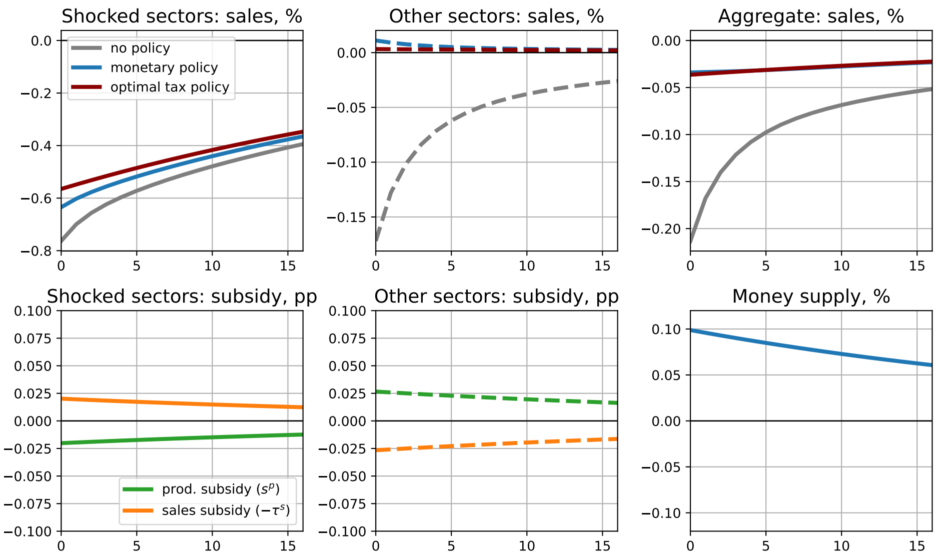
Figure OA.3.2: Negative 1% productivity shock in energy sectors (sales subsidy)



OA.3.2.2 Demand shock in services sectors

Figure OA.3.3 illustrates the response to a negative services demand shock. The negative demand shock is contractionary and leads to suppressed aggregate demand. Hence, the optimal monetary policy is expansionary and somewhat mitigates the contraction of the economy. The optimal sectoral tax policy rebalances sectoral demand by stimulating output in shocked sectors and reducing output in the rest of the economy. This rebalancing of sectoral demand necessitates the introduction of sales subsidies in shocked sectors and sales taxes in the rest of the sectors. The optimal policy also requires the introduction of offsetting production taxes.

Figure OA.3.3: Negative 1% demand shock in services sectors



Finally, we examine the effect of a real-world policy, that is, subsidizing production. Figure OA.3.4 illustrates the response to a services demand shock under this sub-optimal policy. We observe that the production subsidy fails to sufficiently redistribute demand to the services sectors from the rest of the economy.

Figure OA.3.4: Negative 1% demand shock in services sectors (production subsidy)

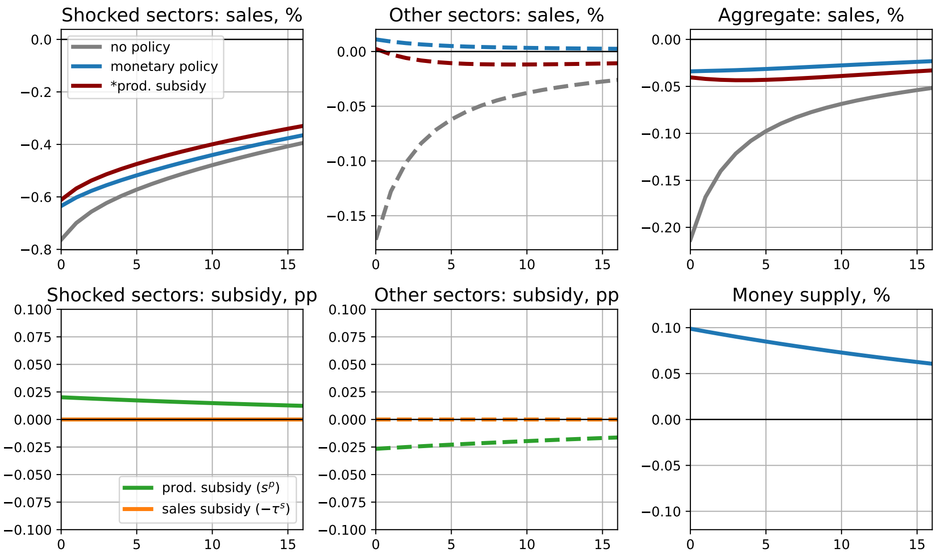
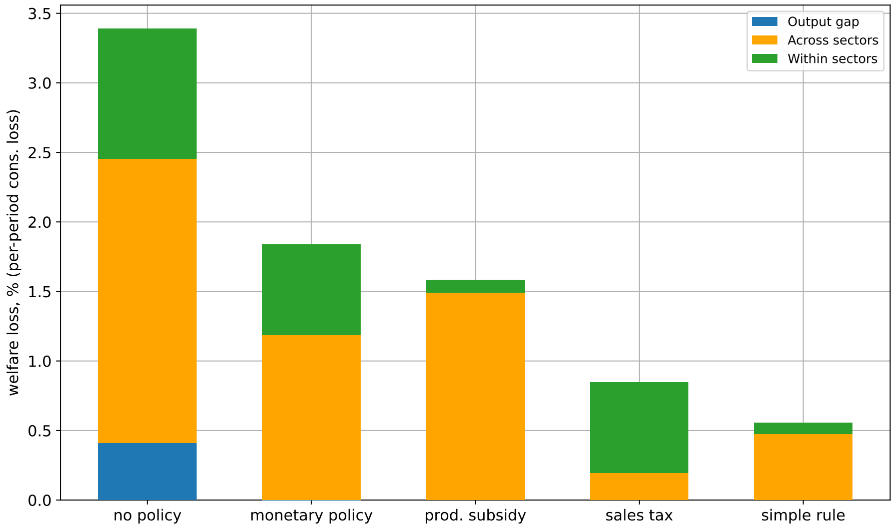


Figure OA.3.5: Expected welfare loss (alternative shock calibration)



Notes: Model simulations for T=1000 periods, drawing realizations of TFP and demand shocks. Simple-rule policy for $\phi = 50$.

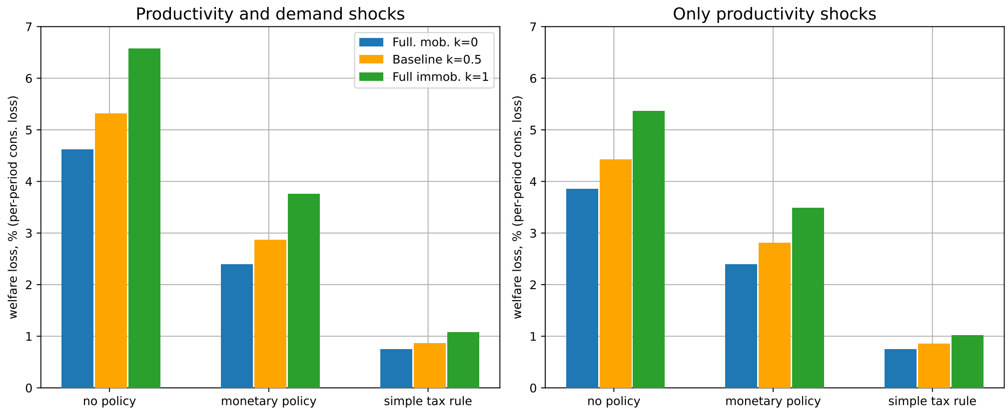
OA.3.3 Welfare loss with alternative shock construction

We consider an alternative calibration of variance-covariance matrices of productivity and demand shocks which relies on HP-filter. Instead of taking the growth rates of the non-stationary sectoral demand and productivity indices to construct sectoral shocks, we apply the HP filter and extract the cyclical component of productivity and demand in each sector. Since our data is yearly frequency, we use $\lambda = 100$ as an HP-filter parameter. Then we employ these cyclical components to compute the sectoral shocks from sector-specific AR(1) processes with a persistence ρ which corresponds to the yearly counterpart of our model persistence. This allows us to construct the alternative measure of the variance-covariance matrices of shocks. Figure OA.3.5 plots the welfare loss in the model under the alternative shock distribution. We see that the alternative distribution yields somewhat lower welfare loss across policies, but the general pattern is similar to our baseline calibration.

OA.3.4 Welfare loss determinants

Additionally, we quantitatively look into the possible determinants of welfare loss in our calibrated economy. First, we consider welfare loss under three alternative policies (no policy, monetary policy, and simple rule) for the alternative

Figure OA.3.6: Expected welfare loss

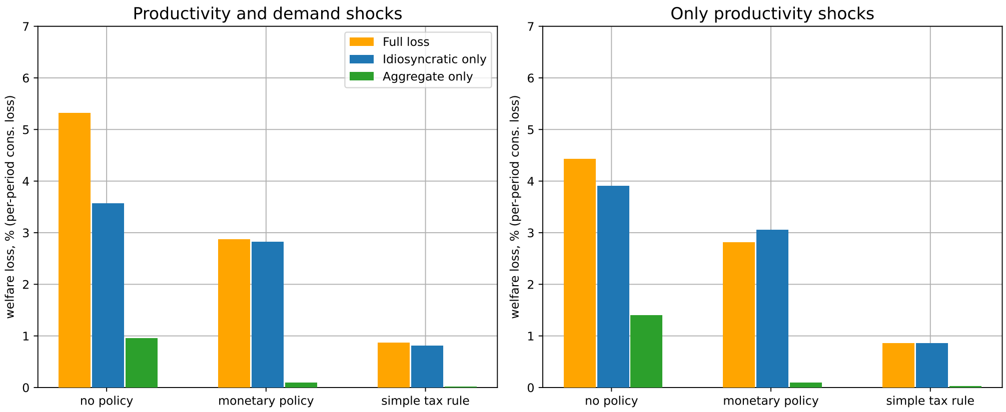


Notes: for simple rule policy $\phi = 50$

degrees of labor mobility. We perform a simulation featuring both productivity and demand shocks, as well as an alternative simulation with productivity shocks only (as previous literature relies on productivity shocks, Rubbo (2021); La'O and Tahbaz-Salehi (2022) as a sole source of fluctuations). The resulting welfare losses are presented in Figure OA.3.6. Welfare loss is generally higher under fully sector-specific labor. Regardless of the share of sector-specific labor, the bulk of the welfare loss is attributed to productivity shocks; the additional welfare loss induced by the demand shocks is increasing with the share of sector-specific labor.

Second, we look into the welfare loss generated by purely sector-specific and purely aggregate fluctuations. To compute welfare loss due to sector-specific fluctuations we set the off-diagonal elements of the correlation matrix to zero for both shocks. To compute the welfare loss due to aggregate fluctuations we set the off-diagonal elements of the correlation matrix to one for both shocks. In both cases, we scale corresponding shocks so that the aggregate standard deviation is the same as in the baseline case. In Figure OA.3.7 we see that the sector-specific fluctuations account for most of the welfare loss, while loss due to aggregate fluctuations is much smaller. Note, that monetary policy is well-suited to reduce the welfare loss induced by aggregate fluctuations. The presence of sector-specific fluctuations requires sectoral tax policy to achieve significant improvement in welfare.

Figure OA.3.7: Expected welfare loss: due to sector-specific vs. aggregate fluctuations



Notes: labor mobility is $k = 0.5$; for simple rule policy $\phi = 50$