Spouses with benefits: on match quality and consumption inside households*

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Abstract

We model the interaction between the marriage market and the intrahousehold allocation of resources. We do this within a setting that accounts for both economic gains to marriage (through public consumption) and unobserved non-material match quality, without relying on the transferable utility assumption. We adopt an axiomatic approach that leads to the empirically tractable "Additive Quantity Shifting" (AQS) model. We develop a revealed preference methodology that is able to identify individuals' heterogeneous match qualities and to quantify them in money metric terms. The methodology can include both preference factors, affecting individuals' preferences over private and public goods, and match quality factors, driving differences in unobserved match quality. We demonstrate the practical usefulness of our methodology through an application to the Belgian MEqIn data. Our results reveal intuitive patterns of match quality that allow us to rationalise both the observed matches and the within-household allocations of time and money.

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1 Introduction

This is a paper about how material and non-material benefits determine who marries whom and who gets what within formed households. Following Becker (1973), a large literature has developed on these issues. This literature is largely split into two separate strands (see Browning, Chiappori, and Weiss, 2014, for a survey). One strand focuses on who marries whom and the associated gains to marriage. The other focuses mainly on the intrahousehold allocations of material aspects such as time and money within existing unions. Although the two strands often intersect in the theoretical literature, there is a paucity of empirical analyses that take into account the interactions between the two. There has always been a perception that it would be desirable to develop such an over-arching framework for empirical work but it is only very recently that progress has been made (see, for example, Cherchye, De Rock, Surana, and Vermeulen, 2020a; Cherchye, Demuynck, De Rock, and Vermeulen, 2017; Goussé, Jacquemet, and Robin, 2017; Weber, 2018).

Gains to marriage. A main distinguishing feature of our approach is that we rationalise observed marital patterns in terms of both material consumption gains and non-material "match quality" benefits that are associated with marriage. This allows us to identify individuals' unobserved match qualities and to quantify them in money metric terms. We can do so while allowing for consumption transfers between spouses but without making use of the usual transferable utility assumption.

A simple example will motivate our approach. Suppose we have a survey of married couples with information on time use such as market work, housework and leisure of each partner in the household. The same survey also collects information on expenditures for private goods for each individual and for intrahousehold public goods. In one of these households we observe that the woman does a lot of market work relative to her partner but also spends less on private goods than her partner. Moreover, she has a higher wage than her spouse. This is a puzzle if we consider only material welfare, since it looks like this woman could do much better by finding an alternative match which entails less market work, at least as much public goods and more private expenditure for her. Clearly, there is something other than material considerations that is keeping her in the marriage. If the marriage is to be stable, her partner must have some attributes that she evaluates positively or she must have some negative attributes, or they must have attributes which are highly complementary. In short, as Becker (1973) emphasised, this requires us to consider matching and intrahousehold allocations simultaneously.

In this paper we present estimates of a structural empirical model that simultaneously takes into account detailed information on within-household allocations and the stability of the observed matching in a marriage market. When we have rich data that include time use decisions and private and public expenditures within the same household, we see matches which seem unstable from a material point of view, albeit few are as extreme as the example in the previous paragraph (see, for example, Browning and Gørtz, 2012; Cherchye, De Rock, and Vermeulen, 2012)).[1] Our model allows us to identify from the observed marriage and household consumption patterns how individuals trade off material and non-material benefits to marriage.

[1]Notice that it is only when we have information on both time use *and* expenditures that a puzzle arises. If we do not observe expenditures we could rationalise the observation that she works more by allowing that she receives more private expenditures (for example, the data of Goussé, Jacquemet, and Robin (2017) has time use information but no information on private or public expenditures). Conversely, having only consumption information, we can rationalise any matching allocation by appealing to unobserved differences in leisure.

Empirical analyses of within-household allocations routinely consider how outside options in the marriage market impact the intrahousehold allocation, but this is usually done through "reduced form" approaches, accounting for the impact of "distribution factors" on the distribution of power within the household (for example, Pareto weights). These distribution factors include within-household variables such as the relative wages of the two partners, individual education levels, individual attitudes to family values and societal factors such as divorce settlement legislation or sex ratios in the local marriage market. See, for example, Browning, Chiappori, and Weiss (2014, Table 5.1), for a listing of 17 such factors that have been used in the empirical literature. In the other strand of the literature, empirical analyses of marriage matching take limited account of observed within-household allocation of time and money (see, for example, Chiappori, 2017).

Our structural model. We develop an axiomatic approach to structurally model matching with transfers within the household that incorporates unobserved match quality. Our axiomatic approach leads to an empirically tractable model, which we term the "Additive Quantity Shifting" model (AQS).[2] Our model yields a money metric measure of unobservable match quality for a given match. It includes some preference structures that are sufficient for transferable utility (TU), but it is not nested within the class of Affine Conditional Indirect Utility (ACIU) preferences that are also necessary for TU (see Chiappori and Gugl, 2020).

Formally, our model is an imperfectly transferable utility model (ITU) (see, for example, Chiappori, 2017; Galichon, Kominers, and Weber, 2019). Compared to the TU case, relatively little is known theoretically about the ITU case. Consequently, in our theoretical discussion, we will also address issues such as existence and uniqueness of a stable matching equilibrium for our model. However, the primary focus of our paper is to establish the conditions under which the observed matching patterns and within-household allocations in a given sample of the population are stable. We will use these conditions to empirically assess the trade-off between the material and non-material match surpluses associated with different marital matchings.

We focus on cross-sectional conditions for marital stability in a frictionless marriage market with within-household transfers (Becker, 1973; Shapley and Shubik, 1972). We do not explicitly model intertemporal considerations and frictions that

[2]What we call AQS is actually similar in spirit to the notion of "Absolute Equivalence Scale Exactness" (AESE) that is discussed by Blackorby and Donaldson (1994). See also Pendakur (2005) for an empirical application.

drive marital choice behaviour. Admittedly, this implies a substantial simplification of a very complex reality. However, the notion of marital stability that we consider here is a natural equilibrium concept to start from when studying marriage and consumption allocations, which is our core research question.[3] Evidently, intertemporal aspects and frictions on the marriage market do become particularly relevant when focusing on household decisions with a long-term impact (for example, related to fertility) and/or dynamic aspects of observed marriage and divorce patterns (which may rather require a search model to explain the matching allocations). Allowing for these features in our structural framework falls beyond the scope of the current study.

At this point, we emphasise that adopting a static perspective (ignoring intertemporal aspects of household decisions) is not necessarily in contradiction with the widespread observation that households divorce. It simply implies that myopic individuals do not take into account future shocks (for example, related to individual preferences, labour productivity or remarriage opportunities) that may change their current (and future) choices. Static models are popular in the literature (see, for example, Browning, Chiappori, and Weiss, 2014) and can be considered as a building block for more advanced models that focus on the intertemporal aspects of household decisions (see, for example, Chiappori and Mazzocco, 2017, for a review). We actually envisage the dynamic extension of our structural methodology as a promising avenue for follow-up research.

Bringing our model to the data. Our approach requires a cross-section of households (married couples and singles) and their intrahousehold allocation. We specify observable preference factors, which affect individuals' preferences over such intrahousehold allocations. In addition, we specify observable "match quality factors", that is, spousal characteristics that drive differences in unobserved match quality. These preference and match quality factors allow us to stratify individuals into observable "preference types" and "match quality types". Female and male individuals of the same preference type have homogeneous preferences over goods and time use. Individuals of the same marital quality type experience the same type-specific systematic match quality (besides individual-specific idiosyncratic match quality).

The basic ingredient of our methodology is a revealed preference characterisa-

[3]This also explains why this stability concept is usually considered in the related empirical literature. See, for example, Choo and Siow (2006) and, more recently, Galichon and Salanié (2022).

tion of marital stability in terms of intrahousehold allocation patterns that include unobserved match quality. Our results extend work of Cherchye, Demuynck, De Rock, and Vermeulen (2017) by incorporating unobserved match quality in a structural manner, in addition to considering preference types and match quality types.[4] Our characterisation provides necessary and sufficient conditions for the observed marriage allocations to be rationalisable in terms of stable marital matchings. The characterisation is nonparametric in the tradition of Afriat (1967), Diewert (1973) and Varian (1982), meaning that it does not require a prior functional specification of individual utilities. The conditions are linear in unknowns, which makes them suitable for practical applications. As we will explain, they also provide a productive basis for the nonparametric set identification of unobserved aspects of spouses' individual preferences and intrahousehold allocation patterns.

Our application uses the Belgian MEqIn data set, which provides information on marital status for a cross-section of Belgian households. The survey also provides measures of individuals' leisure, domestic work and the consumption of Hicksian aggregate private and public commodities. We use age, education level and the presence of children as preference factors to define 12 female and male preference types. We use marital status and spouses' education levels to define six match quality types. Our application focusses on three empirical questions. First, we analyse whether using preference factors to define preference types enhances the set identification analysis. Second, we set identify the unobserved match quality experienced by married individuals in alternative household types, allowing us to study the individuals' trade-offs between material and non-material gains to marriage. Finally, we consider singles and document their match quality (i.e., the "quality of singlehood") needed to rationalise female and male singlehood as a stable situation through the lens of our structural model.

Outline. Section 2 motivates our AQS model to include unobserved match quality in individual utility functions. Section 3 presents our empirical set-up. Section 4 introduces our notion of rationalisable household consumption behaviour under the assumption of a stable marriage market. Section 5 discusses practical

[4] Cherchye, Demuynck, De Rock, and Vermeulen (2017) used so-called "stability indices" to represent income losses associated with exiting the current marriage, and they argued that these stability indices can be interpreted as (reduced form) measures of unobserved match quality (see in particular their Table 4). Essentially, these stability indices are "unexplained residuals" in terms of their structural model of household consumption. By contrast, a main novel feature of the current paper is that we explicitly include match quality in our model, and we will show that this allows us to structurally identify the unobserved match quality from the observed household behaviour.

issues that relate to bringing our theoretical characterisation to empirical data, and introduces our concept of match quality types. Section 6 presents the set-up of our empirical application to the Belgian MEqIn data. Section 7 considers the identification of unobserved match quality for the Belgian households, and documents the corresponding intrahousehold allocation patterns. Section 8 concludes. Appendix A discusses methodological aspects that relate to the practical application of our characterisation of marital stability. Appendices B and C provide additional information on the sample of households that we study in our empirical application. The Online Appendix contains the proofs of our main theoretical results.

2 Match quality and individual utility

If a man and a woman form a couple, they consume within their household a set of n private goods, $q \in \mathbb{R}^n_+$, and a set of N (household level) public goods, $Q \in \mathbb{R}^N_+$. We denote by $q^m \in \mathbb{R}^n_+$ the private consumption of the man and by $q^w \in \mathbb{R}^n_+$ the private consumption of the woman, with $q^m + q^w = q$. These private and public quantities represent the individuals' material consumption within marriage. As stated in the introduction, we adopt an axiomatic approach to account for unobserved match quality in our structural analysis of household consumption and marriage behaviour. As we will demonstrate, this approach will lead to the "Additive Quantity Shifting" (AQS) model. We will start by providing the two main axioms that underlie a formal characterisation of the AQS structure in terms of individual preferences. Subsequently, we will define our money metric measure for assessing match quality.

Compensation and independence. Let M be the set of men and W the set of women on the marriage market, with typical elements $r \in W$ and $i, j \in M$. In what follows, we focus on the preferences of a woman r; but a directly similar reasoning holds for the preferences of men. Specifically, we assume that every woman r has a preference ordering R^r over $\mathbb{R}^n \times \mathbb{R}^N_+ \times M$.[5] Typical elements of this last set are denoted by (q, Q, i), which we call allocations. An allocation (q, Q, i) represents the situation where woman r consumes the bundle (q, Q) and is matched with man i.

[5]For technical reasons, we define preferences of private goods to take values in $\mathbb{R}^n$ instead of the usual non-negative Cartesian orthant $\mathbb{R}^n_+$. See the Online Appendix for more details.

We write:

$$(q, Q, i)\, R^r (q', Q', j)$$

if woman r finds that consuming the bundle (q, Q) and being married to i is at least as good as consuming the bundle (q', Q') while being married to j. We assume that R^r is transitive and complete. We denote by P^r the asymmetric part of R^r (strict preference) and by I^r the symmetric part (indifference).

Our AQS model crucially assumes that individual preferences satisfy so-called "compensation" and "independence" axioms. Intuitively, compensation requires that it is always possible to compensate for a particular match by either providing or removing a sufficient amount of private goods. This condition implies that a less favourable match compared to another can always be compensated by a finite transfer of private goods to the woman. More specifically, for any two males i and j there exists such a transfer that makes woman r prefer living with i over living with j, and vice versa.

Axiom 1 (Compensation)**.** *For all allocations (q, Q, i) and all $j \in M$, there exists a value $\Delta \in \mathbb{R}^n$ such that:*

$$(q + \Delta, Q, j)\, P^r (q, Q, i) \text{ and } (q, Q, i)\, P^r (q - \Delta, Q, j).$$

As shown in the Online Appendix, if preferences are continuous, then Axiom 1 guarantees that we can find a vector $\theta \in \mathbb{R}^n$ such that $(q - \theta, Q, i)\, I^r (q, Q, j)$ for all allocations (q, Q, i) and all $j \in M$.[6] This effectively quantifies the value for woman r of living with man i compared to living with man j through a bundle θ of private goods. In particular, r deems the bundle $(q - \theta, Q)$ when living with i to be equally good as receiving the bundle (q, Q) when living with j. In this sense θ gives the amount of private consumption she is willing to give up to stay with i compared to the situation where she is matched to j. Of course θ can be negative, in which case r needs to be compensated to live with i instead of living with j. In this sense, one could think of the bundle θ as the amount of "non-material benefit", "match quality" or "love" that r experiences when living together with i (compared to living with j). From now on, we will simply use "match quality" as an umbrella term capturing these aspects.

Our second axiom requires that the amount of match quality that one has for another person is independent of the level of consumption (q, Q). This captures the idea that "love is blind", transcending material wealth and remaining unaffected by

[6]Note that solely based on this condition, theoretically, the value θ may also vary with r, i, j, q and Q, which, for the sake of simplicity, we exclude from the notation.

it. Note that this does not mean that there is no trade-off between match quality and material consumption. Woman r might indeed prefer to be in a relationship with high consumption (q, Q) and a low level of θ compared to a relationship where one cares a lot about each other (high θ) but is materially very poor (low (q, Q)). It also does not imply that θ might not depend on characteristics of the potential partner i that are correlated with household income (like education). Independence simply requires that, while keeping all other factors constant, the level of match quality does not vary with the level of consumption experienced within the relationship.

Axiom 2 (Independence)**.** *For all $q, q' \in \mathbb{R}^n$, $Q, Q' \in \mathbb{R}^N_+$, $i, j \in M \cup \{\emptyset\}$ and $r \in W$,*

$$\text{if } (q - \theta, Q, i)\, I^r (q, Q, j),$$

then

$$(q' - \theta, Q', i)\, I^r (q', Q', j).$$

Axioms 1 and 2 impose the main structure on our model. Both conditions are intuitive in our context but will, of course, restrict individual preferences, which we discuss next.

Additive Quantity Shifting. In the Online Appendix, we show that the axioms of compensation and independence lead to the so-called AQS utility structure if preferences are continuous and strictly convex in private consumption. Specifically, for each individual i and r we have utility functions u^i and u^r and vectors $\theta^m_{i,r} \in \mathbb{R}^n$ and $\theta^w_{i,r} \in \mathbb{R}^n$, such that man i receives utility $u^i\left(q^m + \theta^m_{i,r}, Q\right)$, and woman r receives utility $u^r\left(q^w + \theta^w_{i,r}, Q\right)$ when r and i are matched and consume the bundle (q^m, q^w, Q).

The vector $\theta^m_{i,r}$ represents the match quality that man i experiences when matched with woman r, and $\theta^w_{i,r}$ represents the match quality that woman r experiences when matched with man i. In the Online Appendix, we show that these vectors are unique up to normalisation. Specifically, only the differences $\theta^w_{i,r} - \theta^w_{j,r}$ (over all men i and j) matter for woman r when deciding upon her marriage allocation, and likewise, only the differences $\theta^m_{i,r} - \theta^w_{i,s}$ (over all men r and s) matter for man i. Therefore, we can always normalise the match qualities such that $\theta^w_{i^*,r}$ equals zero for all women r and some man i^*, and similarly, θ^m_{i,r^*} equals zero for all men r and some woman r^*. In this case, i considers consuming (q^m, Q) and being matched with r to be equivalent to consuming $(q^m + \theta^m_{i,r}, Q)$ while being matched with r^* (i.e., $(q, Q, r)\, I^i(q + \theta^m_{i,r}, Q, r^*)$).

So far, we have not addressed singles. However, the framework can easily be extended to include singlehood for women by adding an additional element to M, say i_0, to represent singlehood (and similarly for men). In this case, (q, Q, i_0) represents the state where r receives the bundle (q, Q) while single. The vector $\theta^w_{i_0,r}$ then represents the value r places on being single compared to being matched with i^*. Since being in a couple always provides positive material benefits (through public consumption), it follows that staying single can only be considered optimal if, for the individual, $\theta^w_{i_0,r}$ is positive. This refers to so-called "happy singles", who consider being single preferable to being married despite the higher material benefits of marriage. It suggests a "quality of singlehood" or a "cost of marriage", which we will also document in our empirical application (see Section 7).

Throughout this section we will assume that the functions u^i and u^r are differentiable, strictly increasing and quasi-concave in private and public quantities.[7] We will also find it convenient to define the "conditional" utility function for man i for a given level of public quantities Q as:

$$u^i_Q(q^m + \theta^m_{i,r}) = u^i\left(q^m + \theta^m_{i,r}, Q\right),$$

and similarly for woman r. Given the assumed properties of the direct utility function, we have that man i's conditional utility is strictly increasing and quasi-concave in $\theta^m_{i,r}$ (for fixed levels of public goods), and similarly for woman r. In the interest of notational clarity, we will often drop the superscripts m and w and subscripts i and r in the remainder of this section.

Money metric measure of match quality. An important feature of the AQS specification is that it allows us to define a money metric measure of unobserved match quality. Let $p \in \mathbb{R}^n_{++}$ represent the price vector for private consumption and let us denote by $e_Q(p, \theta, u)$ the expenditure function (conditional on the level of public goods and the match quality vector θ). Given an interior solution of the expenditure minimisation problem, we have (using the change of variables

[7] We assume differentiability in the current section for expositional convenience. To be precise, the utility functions that we construct in the sufficiency arguments of Theorem 1 below are subdifferentiable. This, however, does not affect the core of our following argument.

$\tilde{q} = q + \theta$):

$$\begin{aligned} e_Q(p,\theta,u) &= \min_q\{p'q \text{ subject to } u_Q\,(q+\theta) \geq u\} \\ &= \min_{\tilde{q}}\{p'\,(\tilde{q}-\theta)\,\text{subject to } u_Q(\tilde{q}) \geq u\} \\ &= \min_{\tilde{q}}\{p'\tilde{q} \text{ subject to } u_Q(\tilde{q}) \geq u\} - p'\theta \\ &= e_Q(p,0_n,u) - p'\theta. \end{aligned}$$

The crucial feature here is that the expenditure function is additively separable in u and θ. The monetary value (willingness to pay) of a match relative to the zero match quality case is then given by:

$$e_Q\,(p,0_n,u) - e_Q(p,\theta,u) = p'\theta.$$

This is a difference between two expenditure functions, which inherits linear homogeneity (in p). With AQS, the money metric measure of match quality is bilinear in prices and the match quality vector θ. The measure can be positive or negative and is zero if $\theta = 0_n$.

Summarising, we define the "individual" (money metric) match quality for man i matched to woman r as $p'\theta^m_{i,r}$ and the individual (money metric) match quality for woman r married to man i as $p'\theta^w_{i,r}$. Suppose the man has two potential partners, r and s. Holding constant the amount of public and private consumption, he will strictly prefer r to s if and only if $p'\theta^m_{i,r} > p'\theta^m_{i,s}$. The AQS match quality measures are directly operationalised if we can identify the unobserved vectors $\theta^m_{i,r}$ and $\theta^w_{i,r}$. This identification will be addressed in Section 5.

Match quality and individual demand. Match quality affects individuals' Hicksian and Marshallian demands for private goods. An increase in any component of the vector θ decreases the cost of attaining a given utility level, since $\frac{\partial e_Q(p,\theta,u)}{\partial \theta_k} = -p_k < 0$. By Shephard's lemma, the Hicksian (compensated) conditional demand for good k is:

$$\begin{aligned} h^k_Q(p,\theta,u) &= \frac{\partial e_Q(p,\theta,u)}{\partial p_k} = \frac{\partial e_Q(p,0_n,u)}{\partial p_k} - \theta_k \\ &= h^k_Q(p,0_n,u) - \theta_k, \end{aligned}$$

so that match quality shifts the compensated demands up or down relative to the zero match quality demands. Taking second order derivatives further shows that

AQS requires substitution effects to be independent from match quality. As such, match quality can be seen as mainly generating income effects.

Let x denote "total expenditure". To obtain the Marshallian (uncompensated) demands, we start from the conditional indirect utility function $V_Q(p, \theta, x)$, which satisfies the identity:

$$e_Q(p, \theta, V_Q(p, \theta, x)) = e_Q(p, 0_n, V_Q(p, 0_n, x)) - p'\theta = x,$$

This implies:

$$V_Q(p, \theta, x) = V_Q(p, 0_n, x + p'\theta),$$

and confirms that $p'\theta$ acts like an income shifter. Using Roy's identity, the conditional Marshallian demand for good k is given by:

$$\begin{aligned} q_Q^k(p, \theta, x) &= -\frac{\frac{\partial V_Q(p,\theta,x)}{\partial p_k}}{\frac{\partial V_Q(p,\theta,x)}{\partial x}} = -\frac{\frac{\partial V_Q(p,0_n,x+p'\theta)}{\partial p_k} + \frac{\partial V_Q(p,0_n,x+p'\theta)}{\partial x}\theta_k}{\frac{\partial V_Q(p,0_n,x+p'\theta)}{\partial x}} \\ &= q_Q^k(p, 0_n, x + p'\theta) - \theta_k. \end{aligned}$$

This shows that match quality impacts the conditional Marshallian demand in two ways. First it shifts the demand curve up or down. This is due to the fact that match quality and private consumption act as perfect substitutes. Next, match quality also generates an income effect, as for a given match quality θ one needs $p'\theta$ less income to reach the same level of utility than someone without any match quality. Consequently, ordinal preferences also depend on match quality. Allowing that match quality changes ordinal preferences is unusual. It is similar in spirit though to the widely accepted idea that preferences change when going from being single to being married or when there are children in the household.

Further discussion. An often used specification in theoretical and empirical analyses of matching is transferable utility (TU). A necessary condition for TU is that individual expenditure functions (conditional on public goods) take a quasi-homothetic form with the marginal cost of utility being independent of match quality and the utility level. The latter implies that the marginal cost of utility is the same across all potential matches. This is Affine Conditional Indirect Utility (ACIU) in the terminology of Chiappori and Gugl (2020):

$$e_Q(p, \theta, u) = \beta_Q(p)\, u + \alpha_Q(p, \theta).$$

Here, $\beta_Q(p)$ and $\alpha_Q(p,\theta)$ are strictly increasing, linear homogeneous and concave in prices. Although both AQS and ACIU have specifications that display additive separability between the utility level and the match quality, neither specification is nested in the other.[8] For example, the match-quality component in AQS is bilinear in prices and match quality, whereas ACIU allows the less restrictive form $\alpha_Q(p,\theta)$. On the other hand, AQS imposes no restrictions on the utility component, whereas ACIU imposes the strong restriction that Engel curves with zero match quality are linear in u. An important corollary of this is that our AQS specification does not impose TU, even though it admits consumption transfers. The AQS model is therefore an imperfectly transferable utility (ITU) model.

3 Empirical set-up

Like before, we consider a marriage market with a finite set of men M and a finite set of women W. Married couples are defined by a matching function $\sigma: M \cup W \to M \cup W$, such that:

- for all men $i \in M, \sigma(i) \in W$,
- for all women $r \in W, \sigma(r) \in M$,
- and $\sigma(i) = r$ if and only if $\sigma(r) = i$.

To simplify the notation, we will not explicitly address singles in the following formal exposition; we will model all observed individuals as "married" and, thus, $|M| = |W|$. Importantly, the analysis does include the possibility that some males or females in the data set are actually singles. Specifically, single females (males) correspond to (virtual) couples with the male (female) consuming nothing. As a matter of fact, we will include singles in our empirical application in Sections 6 and 7.

We assume that the empirical analyst observes the public consumption Q as well as the individuals' private consumption q^m and q^w for the matched couples, but not for other potential (unmatched) couples. We do observe individuals' private consumption for the married couples in our empirical application (see Section 6). In principle, it is not required to observe the within-household allocation of private consumption. If such information were not available, the unknown individual quantities q^m and q^w can be treated similarly to the unknown individual

[8] Forms that are stronger than ACIU and that are sufficient for TU are, however, nested within AQS. An example is the quasi-linear utility specification.

prices P^m and P^w in our nonparametric characterisation of marital stability in Definition 2 below.

Further, we define discrete preference factors to stratify female and male individuals as observable preference types, with common preferences within a type. This boils down to partitioning the male and female sets M and W into subsets, with each subset characterised by a type-specific utility function. Formally, let $\tau : M \cup W \to T_M \cup T_W$ be a type function that associates with each man i a type $\tau(i) \in T_M$ and with each women r a type $\tau(r) \in T_W$, where T_M and T_W are finite sets of men and women types. A typical element of T_M will be denoted by ψ and a typical element of T_W will be denoted by ω.

Budget constraints are specific to both married and potential couples $(i, r) \in M \times W$. First, $p_{i,r} \in \mathbb{R}^n_{++}$ denotes the prices for private consumption and $P_{i,r} \in \mathbb{R}^N_{++}$ the prices for public consumption. Next, a potential couple (i, r) can spend the income $y_{i,r}$.[9] The couple's consumption possibilities are determined by the associated budget set:

$$B_{i,r} = \left\{(q^m, q^w, Q)|p'_{i,r}(q^m + q^w) + P'_{i,r}Q \leq y_{i,r}\right\}.$$

Summarising, for a given marriage market we assume the data set:

$$S = \left\{\sigma, \tau, \{q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)}\}_{i\in M}, \{p_{i,r}, P_{i,r}, y_{i,r}\}_{i\in M, r\in W}\right\},$$

which consists of a matching function σ, a type function τ, observed intrahousehold allocations:

$$(q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)}),$$

for all married couples $(i, \sigma(i))$, and couple-specific prices $(p_{i,r}, P_{i,r})$ and incomes $y_{i,r}$ for all potential couples (i, r), such that:

$$p'_{i,\sigma(i)}(q^m_{i,\sigma(i)} + q^w_{i,\sigma(i)}) + P'_{i,\sigma(i)}Q_{i,\sigma(i)} = y_{i,\sigma(i)},$$

for every married couple $(i, \sigma(i))$. We assume that the quantities $q^m_{i,\sigma(i)}$ and $q^w_{i,\sigma(i)}$ are strictly positive for all matches.

[9]Couple-specific budget sets are relevant, for example, when the modelled consumption includes spouses' leisure, as in our application in Sections 6 and 7. In this case, the price of an individual's leisure equals that individual's wage, and the couple's income equals full potential (labour and non-labour) income.

4 Rationalisable household consumption

We begin this section by defining our concept of rationalisable household consumption behaviour, which states that the observed behaviour (captured by the data set S) can be represented in terms of a stable allocation on the marriage market. Next, we introduce our revealed preference characterisation of rationalisable behaviour, which defines conditions that can be used to empirically analyse the observed behaviour under the assumption of marital stability.

Rationalisability. A data set is said to be rationalisable if there exist type-specific utility functions for which the observed intrahousehold allocation is utility maximising, such that the observed matching is stable. Stability of the marriage market requires both "individual rationality" and "no blocking pairs". Individual rationality means that no matched individual wants to become single, while no blocking pairs means that no two currently unmatched individuals prefer to marry each other.

In our theoretical analysis, we will focus on the no blocking pairs condition. However, our following arguments actually also include the individual rationality condition implicitly. More specifically, the individual rationality requirement coincides with the no blocking pairs requirement when using "individuals pairing with nobody" as potentially blocking pairs. Our empirical application in Sections 6 and 7 will use both the no blocking pair condition and this individual rationality requirement for marital stability.

Definition 1. *The data set S is rationalisable by a stable matching if, for all male types $\psi \in T_M$ and female types $\omega \in T_W$, there exist strictly monotone, continuous and quasi-concave utility functions $u^\psi : \mathbb{R}^{n+N} \to \mathbb{R}$ and $u^\omega : \mathbb{R}^{n+N} \to \mathbb{R}$ and, for all males $i \in M$ and females $r \in W$, there exist match quality vectors $\theta^m_{i,r} \in \mathbb{R}^n$ and $\theta^w_{i,r} \in \mathbb{R}^n$ such that, for all couples $(i, r) \in M \times W$, with $\tau(i) = \psi$ and $\tau(r) = \omega$, and all allocations (q^m, q^w, Q), if:*

$$u^\psi(q^m + \theta^m_{i,r}, Q) \geq u^\psi(q^m_{i,\sigma(i)} + \theta^m_{i,\sigma(i)}, Q_{i,\sigma(i)}) \text{ and}$$
$$u^\omega(q^w + \theta^w_{i,r}, Q) \geq u^\omega(q^w_{\sigma(r),r} + \theta^w_{\sigma(r),r}, Q_{\sigma(r),r}),$$

with at least one strict inequality, then $(q^m, q^w, Q) \notin B_{i,r}$.

In words, any consumption allocation (q^m, q^w, Q) that gives greater utility to both individuals than in their current match, with at least one strict inequality, must be infeasible for the given budget set. If this last condition were not met, then both individuals would be better off by exiting their current marriage and remarrying each other, which would make the given matching allocation unstable.

We remark that our rationalisability condition in Definition 1 automatically implies that within-household consumption allocations are Pareto efficient. For each married couple, the condition imposes that there cannot exist a consumption allocation that makes both spouses better off (and at least one spouse strictly better off) than the given allocation $(q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)})$, which effectively excludes the possibility of Pareto improvements. This is a convenient implication, as the implicit assumption of Pareto efficiency fits within the collective model of household consumption (Chiappori, 1988, 1992), which has become the workhorse model in the household economics literature (see Browning, Chiappori, and Weiss, 2014, for a review).

Relying on a general result of Alkan and Gale (1990), we show in the Online Appendix that a stable allocation always exists under a mild set of assumptions.[10] We remark that this existence result does not necessarily imply a unique stable marriage matching.[11] Importantly, however, non-uniqueness does not interfere with the validity (and, thus, applicability) of the ARSM characterisation and associated (set) identification results that we derive below.

Characterisation. Our main theoretical result shows that a data set S is rationalisable by a stable matching if and only if it satisfies the following Axiom of Revealed Stable Matchings (ARSM). We say that an observed matching allocation that is consistent with the ARSM is *"revealed stable"*.

Definition 2 (ARSM)**.** *A data set S satisfies the Axiom of Revealed Stable Matchings (ARSM) if, for all couples $(i, r) \in M \times W$, with $\tau(i) = \psi$ and $\tau(r) = \omega$, there exist:*

- *a utility value $U^{\psi}(i)$ for man i of type ψ,*
- *a utility value $U^{\omega}(r)$ for women r of type ω,*
- *price vectors $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$ with $P^m_{i,r} + P^w_{i,r} = P_{i,r}$,*

[10]More specifically, we show existence if one of the following regularity conditions holds: (1) for each potential couple (i, r) any point on the Pareto frontier corresponds to an allocation with the private consumption of woman r (or man i) strictly positive, or (2) if woman r (or man i) gets zero private consumption in the couple (i, r) then she (he) would prefer to be single rather than match with i (r). We refer to the Online Appendix for precise formal statements of these conditions.

[11]See, for example, Eeckhout (2000), Clark (2006), and Legros and Newman (2010) for conditions that guarantee uniqueness in a nontransferable utility setting that is similar to ours.

- *match quality vectors* $\theta^m_{i,r}, \theta^w_{i,r} \in \mathbb{R}^n$,

such that, for all types $\psi \in T_M$ *and* $\omega \in T_W$*, all men* i, k *of type* ψ *and all women* r, s *of type* ω*:*

$$U^\psi(k) \geq U^\psi(i) \text{ and } U^\omega(s) \geq U^\omega(r),$$

implies:

$$y_{i,r} + p'_{i,r}(\theta^m_{i,r} + \theta^w_{i,r}) \leq p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s}, \\ + p'_{i,r}(\theta^m_{k,\sigma(k)} + \theta^w_{\sigma(s),s}), \tag{BP}$$

with a strict inequality if $U^\psi(k) > U^\psi(i)$ *or* $U^\omega(s) > U^\omega(r)$*.*

To explain the intuition of this ARSM condition, let us first regard the simplified setting without match quality (i.e., $\theta^m_{i,r} = \theta^w_{i,r} = 0_n$). The condition first attaches a utility value $U^\psi(i)$ to the consumption bundle $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ for male i of type ψ and, similarly, a utility value $U^\omega(r)$ to the bundle $(q^w_{\sigma(r),r}, Q_{\sigma(r),r})$ for female r of type ω. Next, it defines individual prices $P^m_{i,r}$ and $P^w_{i,r}$ reflecting the willingness-to-pay of, respectively, male i and female r for the public consumption in the allocation $(q^m_{i,r}, q^w_{i,r}, Q_{i,r})$. Pareto efficiency implies $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, that is, the individual prices $P^m_{i,r}$ and $P^w_{i,r}$ must add up to the actual price $P_{i,r}$ and can be interpreted as "Lindahl prices" associated with the efficient consumption of public goods.

The ARSM condition imposes that there must exist at least one specification of these individual utility values $U^\psi(i), U^\omega(r)$ and individual prices $P^m_{i,r}, P^w_{i,r}$ that represents the observed data set S as a stable matching allocation. This specification must satisfy the no blocking pair requirement of Definition 1, in the following sense: if (i) male type ψ is better off with the consumption bundle of individual k than with the bundle of individual i (i.e., $U^\psi(k) \geq U^\psi(i)$) and (ii) female type ω is better off with the bundle of individual s than with the bundle of individual r (i.e., $U^\omega(s) \geq U^\omega(r)$), then we must have:

$$y_{i,r} \leq \ p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) + P^{m\prime}_{i,r} Q_{k,\sigma(k)} + P^{w\prime}_{i,r} Q_{\sigma(s),s}, \tag{BP'}$$

which states that the "income" $y_{i,r}$ available to the potentially blocking pair (i, r) does not allow for buying more than the "preferred" bundles $(q^m_{k,\sigma(k)}, Q_{k,\sigma(k)})$ and $(q^w_{\sigma(s),s}, Q_{\sigma(s),s})$ under the prevailing prices $p_{i,r}$, $P^m_{i,r}$ and $P^w_{i,r}$. If this inequality did not hold, then the pair (i, r) would block the observed matching allocation, which would violate marital stability.

So far, we have assumed $\theta^m_{i,r} = \theta^w_{i,r} = 0_n$. In case the unobserved match quality can be non-zero, we additionally need to correct for a potential difference in match quality. Under AQS, this difference can be expressed in money metric terms as the difference between $p'_{i,r}(\theta^m_{i,r} + \theta^w_{i,r})$ and $p'_{i,r}(\theta^m_{k,\sigma(k)} + \theta^w_{\sigma(s),s})$. Plugging this into (BP') effectively yields (BP).

We can show that the ARSM condition in Definition 2 is both necessary and sufficient for an observed matching allocation to be revealed stable. In other words, it exhausts all implications of marital stability for the empirical setting under study.[12]

Theorem 1. *A data set S is rationalisable by a stable matching if and only if it satisfies the ARSM.*

In Appendix A we show that the ARSM condition can be reformulated in terms of inequality constraints that are linear in unknowns and characterised by (binary) integer variables. These linear inequality constraints are easily operationalised, which is convenient from an application point of view.[13]

Importantly, when we impose no further structure on the match quality vectors $\theta^m_{i,r}$ and $\theta^w_{i,r}$, our ARSM characterisation as such does not have empirical bite, meaning that any data set S will satisfy the testable conditions. For example, rationalisability is trivially obtained by setting the values $\theta^m_{i,r}$ and $\theta^w_{i,r}$ low enough for the unmatched couples and high enough for the matched couples. In the next section, we will address this by making a distinction between systematic match quality, which will depend on match quality types, and idiosyncratic match quality, which will capture the remaining match quality after accounting for the variation in systematic match quality.

Further discussion. We can sharpen the intuition of our ARSM characterisation by showing that it naturally encompasses Varian (1982)'s GARP (Generalised Axiom of Revealed Preference) condition for the existence of a rationalising utility function. Specifically, this GARP condition is imposed through the within-type rationality implications of our ARSM requirement.

Towards this end, let us consider all possible (re)matches between male individuals of type ψ and some given female individual (for example, female r of type

[12]See the Online Appendix for the proof of Theorem 1.

[13]We use the software package IBM ILOG CPLEX Optimisation Studio for our empirical application in Sections 6 and 7. Our CPLEX codes are available upon request.

ω).[14] When abstracting from differences in unobserved match quality (for example, by setting $\theta^m_{i,r} = \theta^w_{i,r} = 0$ for all i and r), the no blocking pair requirement in Definition 2 imposes, for all male individuals i and k of type ψ,[15]

$$U^\psi(k) \geq U^\psi(i) \text{ implies } p'_{i,\sigma(i)} q^m_{i,\sigma(i)} + P^{m\prime}_{i,\sigma(i)} Q_{i,\sigma(i)} \leq p'_{i,\sigma(i)} q^m_{k,\sigma(k)} + P^{m\prime}_{i,\sigma(i)} Q_{k,\sigma(k)},$$

with a strict inequality if $U^\psi(k) > U^\psi(i)$. It follows from Varian (1982) (Theorem 2) that this requirement is exactly equivalent to the GARP condition imposed on the set of quantities $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ and prices $(p_{i,\sigma(i)}, P^m_{i,\sigma(i)})$ defined over all males i of type ψ. This shows that the within-type rationalisability implications of our ASRM condition neatly nests Varian's GARP condition for the existence of a type-specific rationalising utility function.

Building on this connection between our ARSM condition and Varian's GARP condition, one may choose to further restrict the form of individual utilities (as in Varian, 1983) or the behavioural responses to price and/or income changes (e.g. impose normality on the individual demand; see Cherchye, Demuynck, De Rock, and Surana, 2020b). Such extensions can draw on the existing literature. For compactness, we will not explore this further in the current paper.

Finally, it is interesting to consider the limiting case of full preference heterogeneity. In our set-up, this formally corresponds to $|T_M| = |M|$ and $|T_W| = |W|$, that is, there is a single observed individual per preference type. It is fairly easy to show that, in this case, our ARSM condition reduces to the requirement of Cherchye, Demuynck, De Rock, and Vermeulen (2017) for consistency of a given data set with marital stability. The only difference being that our ARSM requirement also includes unobserved match quality, which is not modelled by Cherchye et al. (2017). These authors established that their requirement is necessary for an observed marriage allocation to be revealed stable under full preference heterogeneity. We thus complete their result by showing that the condition is not only necessary but also sufficient for such rationalisability.

5 Match quality types and identification

When bringing our theory to data, our prime interest will be in identifying the match qualities. To this end, we will use match quality factors to define match

[14]A directly analogous argument applies to the case that considers all possible (re)matches between female individuals of type ω and a given male individual.

[15]This expression is a simplified version of the requirement in Definition 2 that $U^\psi(k) \geq U^\psi(i)$ and $U^\omega(r) \geq U^\omega(r)$ implies $y_{i,r} \leq p_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(r),r}) + P^m_{i,r} Q_{k,\sigma(k)} + P^w_{i,r} Q_{\sigma(r),r}$.

quality types (for example, education types). Individuals of the same match quality type are then assumed to experience the same systematic match quality, in addition to individual-specific idiosyncratic match quality. After formalising these concepts, we will discuss how to use our ARSM condition to (set) identify these systematic match qualities.

Match quality types. We denote a typical male match quality type by κ and a typical female match quality type by λ. Assuming that male i is of type κ and female r of type λ, we can decompose the match quality in a systematic and an idiosyncratic component, as follows:[16]

$$\theta^m_{i,r} = \theta^m_{\kappa,\lambda} + \tilde{\theta}^m_{i,r} \text{ and } \theta^w_{i,r} = \theta^w_{\kappa,\lambda} + \tilde{\theta}^w_{i,r},$$

where $\theta^m_{\kappa,\lambda}, \theta^w_{\kappa,\lambda}$ represent the systematic components and $\tilde{\theta}^m_{i,r}, \tilde{\theta}^w_{i,r}$ the idiosyncratic components. In our empirical application, we will choose values for the idiosyncratic components that solve:

$$\min \sum_i \sum_r (|\tilde{\theta}^m_{i,r}| + |\tilde{\theta}^w_{i,r}|),$$

subject to the rationalisability restrictions given by the ARSM.[17] By minimising the absolute values of the idiosyncratic match qualities, we make the match quality that cannot be ascribed to the match quality factors as small as possible. Intuitively, we maximally load the explanation of the observed matching and consumption behaviour on the systematic match quality components.[18]

[16]Notably, our dealing with systematic versus idiosyncratic match quality is more flexible than the usual practice in the empirical literature, which invariably assumes that the idiosyncratic components $\tilde{\theta}^m_{i,r}$ and $\tilde{\theta}^w_{i,r}$ depend on the type but not the identity of the partner (following Choo and Siow, 2006; see also Galichon and Salanié, 2022, for detailed discussion). Galichon, Kominers, and Weber (2019) make the same assumption in a setting of imperfectly transferable utility that is similar to ours.

[17]To obtain a linear objective in our practical application, we replace the objective that minimises the sum of absolute values by the equivalent objective:

$$\min \sum_i \sum_r (b^m_{i,r} + b^w_{i,r}),$$

where we add the constraints

$$\tilde{\theta}^m_{i,r} \leq b^m_{i,r}, -\tilde{\theta}^m_{i,r} \leq b^m_{i,r} \text{ and } \tilde{\theta}^w_{i,r} \leq b^w_{i,r}, -\tilde{\theta}^w_{i,r} \leq b^w_{i,r}$$

for the extra variables $b^m_{i,r}$ and $b^w_{i,r}$.

[18]In our empirical application, the above minimisation problem obtains that the idiosyncratic match qualities equal zero in about 99.5 % of the potentially blocking pairs. Their average values (and standard deviations) amount to 0.297 (6.234) and 0.262 (6.073) for females and males,

Evidently, one may well choose to impose further structure on the idiosyncratic components (for example, assume a Type I Extreme Value distribution as in Choo and Siow, 2006). While we do see this as an interesting avenue for follow-up research, we will not explore this further in the current paper. Our justification for this simplification is that our principal interest is in identifying type-specific match quality.

Set identification. When determining the idiosyncratic match qualities $\tilde{\theta}^m_{i,r}$ and $\tilde{\theta}^w_{i,r}$ by the values that solve the minimisation problem presented above, the given data set satisfies our ARSM rationalisability condition for at least one specification of the remaining unknowns (including the systematic match qualities $\theta^m_{\kappa,\lambda}$ and $\theta^w_{\kappa,\lambda}$). Given this, we can address alternative identification questions under the maintained assumption of marital stability. Particularly, we can set identify the unknowns in the ARSM condition (such as individual utilities and individual "Lindahl" prices) in Definition 2.

Our main focus in the following empirical analysis will be on identifying the systematic match qualities $\theta^m_{\kappa,\lambda}$ and $\theta^w_{\kappa,\lambda}$. When expressed in money metric terms, these match qualities equal $p'_{i,r}\theta^m_{\kappa,\lambda}$ for the male and $p'_{i,r}\theta^w_{\kappa,\lambda}$ for the female. These expressions are linear in the unknown match quality vectors and, thus, we can define upper/lower bounds by maximising/minimising these linear functions subject to our linear rationalisability restrictions defined by the ARSM. These bounds provide set identification of the unobserved systematic match qualities.

6 Belgian household data

We apply our method to a sample of Belgian households drawn from the MEqIn data set, which contains a rich set of economic and socio-demographic variables.[19] In what follows we first discuss how the MEqIn data were collected and we motivate

respectively. These values are very small when compared to the systematic match qualities that we report in Section 7. At this point, however, it is worth remarking that the idiosyncratic match qualities are also quite substantial in some cases: the lowest values equal -85.395 for females and -64.956 for males, while the highest values amount to 449.080 for females and 469.870 for males. For compactness, we will not explicitly discuss our results on idiosyncratic match quality in what follows.

[19]The MEqIn dataset is collected by a team of researchers from the Université catholique de Louvain, the University of Leuven, the Université libre de Bruxelles, and the University of Antwerp. The collection of the MEqIn data was enabled by the financial support of the Belgian Science Policy Office (BELSPO) through the grant BR/121/A5/MEQIN (BRAIN MEqIn). The MEqIn data is available upon request for researchers and students. For detailed information on the data set, we refer to Capéau et al. (2020) and the following website (which also includes a codebook): https://sites.google.com/view/meqin/data.

our sample selection criteria. Next, we explain how we define our observable types and provide summarising descriptives for the basic variables that we will use in our empirical analysis. Finally, we introduce a subsampling procedure that we use to apply our nonparametric method to the MEqIN data.

Data. The MEqIn survey contains household information gathered in 2015-2016. The original data set comprises a random sample of 3404 respondents, belonging to 2098 households that were selected on the basis of the Belgian National Register. The sample is representative of all people living in Belgium. It provides detailed information on various aspects of the individual well-being of all adults living in the interviewed households, as well as information on the relative importance of the different life dimensions according to the respondents. For each surveyed household, some additional data on children could be sent back by the respondents through a drop-off questionnaire. In total, 371 families provided information on 618 children.

Of special importance for our study is the time use and expenditure information. The time use questionnaire in the MEqIn survey asks for the average amount of time that individuals spend on a number of broad and exhaustive categories in a typical week. As such, these data are retrospective and not diary based. The same applies to the expenditure information. Contrary to standard budget surveys, which focus on households' expenditures on many hundreds goods and services, the MEqIn survey concentrates on an exhaustive set of broad categories of non-durable goods and services (like food, clothing, housing, expenditures on children, etc.). At the same time, the MEqIn data set provides information that is richer than that contained in typical budget surveys, in the sense that it also gives information about who is consuming what within the surveyed households. The data set makes a distinction between privately and publicly consumed goods and services, while it provides information on the adult household members' shares in the household's total private expenditures.

The set of households used for this study was subject to the following sample selection rules. First, because we need wage information, we only consider households with adults working at least 10 hours per week, with or without children. Next, we excluded the self-employed to avoid issues regarding the imputation of wages and the separation of consumption from work-related expenditures. After deleting the households with important missing information (mostly, incomplete information on one of the spouses), we obtained a sample containing 581 individuals: 194 females and males in couples, 124 single females and 69 single males.

We observe the privately consumed quantities of the two spouses. In our set-up, private consumption is a Hicksian good with price normalised to one. It includes individual expenditures on food (at home and outdoors), transport, tobacco, clothing, personal care and products, schooling and other personal expenditures. Further, we will assume that leisure is privately consumed. We also observe the publicly consumed quantities of the household, which is again a Hicksian good with price normalised to one; it includes joint food consumption at home, joint transport, mortgage and rent, utilities and insurances, holidays, restaurant visits, child expenditures and other public expenditures. Finally, we will treat time spent on domestic work (including child care) by the two individuals as public consumption.[20]

Our method requires prices and incomes that apply to the exit options from marriage (i.e., becoming single or remarrying). For our labour supply application, prices correspond to individual wages. We assume that wages outside marriage are the same as inside marriage (i.e., exiting marriage does not affect labour productivity). This may seem to be a rather strong assumption in light of the literature on marriage premiums and penalties. However, we emphasise that, in principle, the wages and incomes in the counterfactual situations of being single or with a different partner can also be imputed. Moreover, it can be argued that the wage rate inside marriage is probably a good benchmark when individuals compare their opportunity sets inside their current marriage and outside marriage as a single or with a different partner.

For the observed couples, we use a consumption-based measure of total non-labour income, that is, non-labour income equals reported consumption expenditures minus full income. Then, we treat individual non-labour incomes as unknowns that are subject to the restriction that they must add up to the observed (consumption-based) total non-labour income. As compared to the alternative that fixes the intrahousehold distribution of non-labour income (for example, at 50% for each individual), this procedure to endogenously define the individual non-labour incomes effectively puts minimal non-verifiable structure on these unobserved variables. However, to exclude unrealistic scenarios, in our application we will impose that individual non-labour incomes after divorce must lie between 40% and 60% of the total non-labour income under marriage. The same procedure

[20]In this respect, each individual's time spent on household production actually represents an input and not an output that is consumed inside the household (see Becker, 1965). Under the assumption that each individual produces a different household good by means of an efficient one-input technology characterised by constant returns-to-scale, the individual's input value can serve as the output value.

was adopted by Cherchye, Demuynck, De Rock, and Vermeulen (2017).

Table 1 provides summary statistics for the couples in our sample. Wages are net hourly wages. Leisure is measured in hours per week. To compute leisure hours, we assume that an individual needs 8 hours per day for sleeping and personal care (i.e., leisure = 168−56−hours worked in the labour market and at home). Full income and (Hicksian) consumption are measured in euros per week. Table 1 also reports on the presence of children, and the age and education levels of the individuals in our sample. Individuals are deemed to be highly educated if they hold a degree beyond secondary education.

Table 1: sample summary statistics

	mean	*st.dev.*
male wage (euro/hour)	10.552	3.631
female wage (euro/hour)	10.145	3.505
full income (euro/week)	1638.845	734.051
male private consumption (euro/week)	126.213	59.271
female private consumption (euro/week)	116.139	57.308
public consumption (euro/week)	371.293	188.936
male leisure (hours/week)	50.550	15.333
female leisure (hours/week)	47.724	16.810
male domestic production (hours/week)	14.316	11.627
female domestic production (hours/week)	24.490	15.146
presence of children (1 = yes/0 = no)	0.553	0.498
number of children	0.920	0.969
male age (years)	41.684	9.776
female age (years)	39.918	9.397
male higher education (1 = yes/0 = no)	0.426	0.495
female higher education (1 = yes/0 = no)	0.513	0.501
dummy for couple	0.501	0.501
dummy for single male	0.178	0.383
dummy for single female	0.320	0.467

Notes: there are 194 couples, 124 female singles and 69 male singles; full income and consumption are in euros per week, wages in euros per hour, and leisure and domestic production in hours per week.

Preference types and match quality types. Our methodology uses preference and match quality factors to define preference and match quality types. For both types of factors we use variables that are popular in the empirical literature on consumption and time use behaviour and marital matching decisions. We define preference types based on age, education and the presence of children. For each gender we consider three age classes (below 35, between 35 and 50 and above 50) and two education classes (higher educated or not), and we assume that parents of children can have different preferences than other individuals. In total, this defines 12 (= 3 × 2 × 2) male and 12 female preference types. Further, we use the individuals' education level (high or low) as a match quality factor.[21] This implies six male and six female match quality types. More specifically, we have two education types of each gender (high/low educated). Each of these two types is married to one of the two types of the other gender or to nobody (when single), which defines six (= 2 × (2 + 1)) possible matchings among these types.

We highlight that we use education as both a preference factor and a match quality factor. By doing so, we effectively consider the match quality channel as well as the economic channel (i.e., budget conditions and individual preferences) through which education shapes marital matching and household consumption patterns. For each couple type, we will quantify the unobserved match quality in terms of privately consumed quantities (but not privately consumed leisure). This implies that $\theta^m_{\kappa,\lambda}$ and $\theta^w_{\kappa,\lambda}$ are scalars. Moreover, as we include private consumption as a Hicksian good in our empirical set-up, its price is normalised at unity for every household in our sample, which makes it easy to compare our (money metric) match quality estimates across different types of couples.

Table 2 reports on the marriage allocations for different education (EDU) types in our sample of households. Some interesting observations emerge. First, our data clearly reveal assortative matching in education: 66% of all observed couples consist of a male and a female of the same education type. However, even though "same-type" couples are clearly prevalent, the fraction of "mixed" couples is rather substantial. Relatedly, we see singles of every type. When compared to married individuals, single females and males are mostly lower educated.

Tables 3 and 4 document the consumption allocations for our different match quality types. We report on the private and public consumption shares as well as total consumption (expressed in monetary value). Not surprisingly, total consumption increases with the level of education. Next, for married couples we observe quite some heterogeneity in consumption allocations across match quality types: both the individual private shares and the public shares vary considerably with the education levels. The same applies to singles. Here, a notable feature is that a single males generally spend lower budget shares on public consumption than single females. Finally, we also observe considerable variation in expenditure

[21] In principle, we may well consider more than one match quality factor (similar to our use of multiple preference factors). For example, this would enable the analysis of interaction effects between different match quality factors. However, our sample size is too small to meaningfully analyse systematic match qualities involving multiple factors. We choose to focus on education because this factor has received by far the most attention in the relevant literature (starting with Choo and Siow, 2006). Other factors driving match quality will thus be captured by the idiosyncratic match quality component in our analysis.

patterns within match quality types.

In general, there is a lot of cross-type and within-type heterogeneity in consumption and marriage behaviour. We also observe considerable variation in characteristics across household types; see Tables 12 and 13 in Appendix B. For example, single females have substantially more children than single males. The question is how we can rationalise these patterns of marriage and consumption behaviour. Part of the explanation may be heterogeneity in budget conditions (prices and incomes) and preferences (for example, related to education, age and the presence of children). Another part may be match quality that is specific to partners' education types, which will be the main focus of our empirical analysis.

Table 2: percentage shares of education (EDU) types in our sample

	couples		
	low EDU female	*high EDU female*	*all*
low EDU male	31.443%	22.680%	54.124%
high EDU male	10.825%	35.052%	45.876%
all	42.268%	57.732%	
	singles		
	low EDU	*high EDU*	
males	66.667%	33.333%	
females	58.871%	41.129%	

Table 3: consumption shares and total consumption per match quality type - couples

female EDU	*male EDU*	*total* *mean*	*total* *st.dev*	*public* *mean*	*public* *st.dev*	*private female* *mean*	*private female* *st.dev*	*private male* *mean*	*private male* *st.dev*
low	low	1842.952	413.571	736.938	268.154	517.839	178.686	588.175	226.990
	high	2262.657	891.805	868.275	284.085	724.357	708.282	670.025	243.407
high	low	2100.250	413.515	918.819	330.879	547.071	254.699	634.360	215.804
	high	2437.128	735.040	1017.675	437.479	677.601	394.090	741.852	443.425

Notes: individual (male/female) private consumption includes leisure in monetary value, and total consumption equals individual (male/female) private consumption plus household public consumption (including domestic work in monetary value).

Subsampling. Because our structural model explicitly includes match quality, our revealed preference methodology does not require a prior specification of the marriage market that is relevant for each different individual under study (contrary to Cherchye, Demuynck, De Rock, and Vermeulen, 2017). Intuitively, individuals may not be considered as potential partners when the associated match quality is sufficiently negative. Thus, every possible combination (i, r) consisting of a male i

Table 4: consumption shares and total consumption per match quality type - singles

female	*total consumption*		*public*		*private*	
EDU	*mean*	*st.dev*	*mean*	*st.dev*	*mean*	*st.dev*
low	1047.810	283.408	449.745	229.901	598.065	210.014
high	1262.447	359.679	601.843	259.191	660.604	273.488
male	*total consumption*		*public*		*private*	
EDU	*mean*	*st.dev*	*mean*	*st.dev*	*mean*	*st.dev*
low	1025.543	313.661	368.031	220.796	657.511	236.174
high	1222.247	434.592	437.617	184.925	784.630	456.480

Notes: individual (male/female) private consumption includes leisure in monetary value, and total consumption equals individual (male/female) private consumption plus household public consumption (including domestic work in monetary value).

and a female r in our sample may in principle be treated as a potentially blocking pair when bringing our ARSM condition to the data. However, considering the whole data set at once makes our empirical analysis vulnerable to sampling error and outlier behaviour.

To mitigate this concern, we take a pragmatic approach that makes use of subsampling.[22] Specifically, we randomly draw 200 subsamples of 40 households (i.e., about 10 percent) from our original sample. We apply our revealed preference methodology to every subsample separately, and we will report summary results defined over these 200 subsamples in what follows. In our subsampling procedure, we draw every observed household 20.341 times on average (st. dev. 4.242), with a minimum of 10 times and a maximum of 35 times.

This subsampling procedure yields multiple values of the lower and upper bounds on education-based match quality for every household in our sample. We use the average of these values as our household-specific estimates for the lower and upper bounds of the systematic match quality component. We will use these household-specific bounds as the basis for our following empirical analysis. Appendix C reports the mean values and standard deviations of the household-specific bounds for every match quality type that we consider in our analysis.

[22]Subsampling has a long history in statistics (see Politis, Romano, and Wolf, 2012). It can be used to build a valid inferential procedure under weaker assumptions than resampling methods such as the bootstrap (Politis and Romano, 1994). Subsampling has also received renewed attention in economics recently, where the properties of estimators pooled across subsamples have been studied (Lee and Ng, 2020). Exploring the statistical properties of subsampling procedures in our specific context falls beyond the scope of the current paper, but we do see this as an interesting avenue for follow-up research.

7 Systematic match quality

As we explained in Section 2, for an individual (male or female) of a given match quality type only differences in unobserved match quality matter when comparing marriage allocations. Therefore, we can select a benchmark marriage for which we normalise the individual (systematic) match quality to zero. Then, the identified values of match quality capture how much more (if positive) or less (if negative) private consumption the individual would need in this benchmark marriage to be equally well off as in the evaluated marriage. In our following analysis, we choose "same-type" marriages as our benchmark marriages, meaning that an individual's systematic match quality is set to zero when married to an individual of the same education type.

Bound tightness. We compute the difference between lower and upper bounds on the systematic match quality for two model specifications. The first specification uses the three preference factors introduced above (age, education and presence of children) to capture observed heterogeneity in preferences. Our second model assumes fully heterogeneous preferences (which fits as a limiting case in our general set-up, as we discussed at the end of Section 4). By comparing these bounds, we can assess whether the use of preference factors adds empirical bite to the analysis, in terms of generating tighter bounds.

Table 5 presents our results.[23] Interestingly, the differences between the lower and upper bounds turn out to be generally small for both models, indicating tight set identification. For example, the difference values reported in Table 5 represent only a very small fraction of the households' full incomes (which amount to 1638 euros on average, as shown in Table 1). The bounds are somewhat tighter for females than males but, more importantly, they turn out to be informatively tight for both genders. Finally, and most notably, we find that using preference factors obtains a quite substantial tightening of the bounds. Specifically, in the last column of Table 5 (labelled "% difference") we express the difference in bound tightness between our two model specifications in percentage terms. The average improvement following the use of preference factors equals as much as 33% for females and 29% for males, and the median improvements amount to no less than

[23] As explained at the end of the previous section, our subsampling procedure yields household-specific bounds on systematic match quality. Table 5 reports on the differences between these bounds for the households in our sample. We exclude same-type couples, for which the individual match qualities are normalised to zero and, thus, we (only) take up households with members of different education types. Self-evidently, by doing so we avoid that the average and median differences in Table 5 are artificially downward biased.

53% and 42%, respectively. This shows that assuming preference types effectively enhances the (set) identification.

In our remaining analysis, we will focus exclusively on the match quality bounds for the model with three preference factors. Comfortingly, our main qualitative conclusions also hold for the model that assumes fully heterogeneous individual preferences. These last results are summarised in Appendix D.

Table 5: Bounds on systematic match quality - tightness

females: difference between lower and upper bounds			
	3 preference factors	*full heterogeneity*	*% difference*
mean	3.840	5.738	33.070%
median	2.114	4.514	53.175%
st.dev.	3.718	5.480	
males: difference between lower and upper bounds			
	3 preference factors	*full heterogeneity*	*% difference*
mean	6.948	9.851	29.470%
median	4.006	6.997	42.757%
st.dev.	7.488	8.587	

Married individuals. We next investigate the systematic match qualities for married females and males. For every female and male education type we compare the material consumption with the match quality that is associated with alternative marital outcomes. To facilitate our exposition, we will only consider mean consumption levels and match qualities in our following discussion. It is important to keep in mind, however, that these mean levels may hide substantial variation across individual households (see Table 1 and Appendix C).

Table 6 presents the public and private consumption levels of married females, together with our bound estimates of their systematic match qualities. As expected, we find that both low educated and high educated females are better off in material terms when married to a high educated husband; public and private consumption increases with the male education level. Interestingly, we also observe that both low and high educated females associate a higher systematic match quality with a low educated husband than with a high educated husband. In money metric terms, the match quality experienced by a low educated female is between 70 and 78 euros higher for a low educated husband than for a high educated husband; and, similarly, for a high educated female it is between 37 and 43 euros higher when the husband is low educated than when he is high educated.

Thus, the material and match quality gains to marriage go in opposite directions: low educated husbands are less attractive in economic terms, but this is (partly) offset through higher match qualities.

Table 7 shows the "economic significance" of these findings. For each female match quality type, it expresses the systematic match quality as a fraction of the female's private consumption (with and without leisure, respectively). We observe that the differences in match quality between high educated and low educated husbands are quite substantial. For example, a high educated female is willing to give up no less than 35% to 40% of her private consumption (excluding leisure) when married to a low educated male instead of a high educated male. The difference is even more pronounced for low educated females, who are willing to give up between 61% and 69% of their private consumption (excluding leisure) when married to a low educated instead of a high educated husband.

Table 6: Material consumption and systematic match quality per match quality type (mean levels) - married females

Material consumption				
	low EDU female		*high EDU female*	
	public	*private female*	*public*	*private female*
low EDU male	736.938	517.839	918.819	547.071
high EDU male	868.275	724.357	1017.675	677.601
Systematic match quality				
	low EDU female		*high EDU female*	
low EDU male	0.000		(37.289; 42.897)	
high EDU male	(-78.008; -69.551)		0.000	

Notes: for same-type marriages the match qualities are normalised to zero; for mixed-type marriages we report lower and upper bounds on the match qualities between brackets.

Tables 8 and 9 yield the same qualitative conclusions for married males as for their female counterparts. For both high and low educated males, a high educated wife is more attractive than a low educated wife in economic terms but less attractive in terms of education-based match quality. The economic significance related to the match qualities is again substantial, and is similar in magnitude to that for married females.

In our opinion, these results clearly reveal the relevance of match quality to rationalise the observed marital matching and household consumption patterns. Non-material gains to marriage make low educated spouses more attractive relative to high educated spouses. Matching patterns would turn out to be quite different if only material gains to marriage mattered and match qualities were ir-

Table 7: Systematic match quality as fraction of private consumption - married females

As fraction of private consumption female with leisure		
	low EDU female	*high EDU female*
low EDU male	0.000	(6.816%; 7.841%)
high EDU male	(-10.108%; -9.096%)	0.000
As fraction of private consumption female without leisure		
	low EDU female	*high EDU female*
low EDU male	0.000	(34.785%; 40.017%)
high EDU male	(-68.507%; -61.080%)	0.000

Notes: for same-type marriages the match qualities are normalised to zero; for mixed-type marriages we report lower and upper bounds on the match qualities between brackets.

relevant. For example, high educated individuals would be even more attracted to high educated individuals of the other gender; and, thus, we should see even less mixed marriages with low and high educated spouses. Generally, match quality gains associated with low educated spouses (partly) compensate for the economic attractiveness of high educated spouses. This suggests that other considerations than merely human capital play a role in defining gains to marriage. For example, in a recent paper Low (2024) highlighted the importance of reproductive capital (i.e., fertility), emphasising a possible trade-off between human capital and reproductive capital through career investments.

Table 8: Material consumption and systematic match quality per match quality type (mean levels) - married males

Material consumption				
	low EDU female		*high EDU female*	
	public	*private male*	*public*	*private male*
low EDU male	736.938	588.175	918.819	634.360
high EDU male	868.275	670.025	1017.675	741.852
Systematic match quality				
	low EDU female		*high EDU female*	
low EDU male	0.000		(-37.162; -32.236)	
high EDU male	(50.203; 61.919)		0.000	

Notes: for same-type marriages the match qualities are normalised to zero; for mixed-type marriages we report lower and upper bounds on the match qualities between brackets.

Table 9: Systematic match quality as fraction of private consumption - married males

As fraction of private consumption male with leisure		
	low EDU female	*high EDU female*
low EDU male	0.000	(-5.858%; -5.082%)
high EDU male	(7.493%; 9.241%)	0.0000
As fraction of private consumption male without leisure		
	low EDU female	*high EDU female*
low EDU male	0,000	(-32.047%; -23.988%)
high EDU male	(47.479%; 58.560%)	0.0000

Notes: for same-type marriages the match qualities are normalised to zero; for mixed-type marriages we report lower and upper bounds on the match qualities between brackets.

Singles. As a final exercise, Tables 10 and 11 report on the material consumption patterns and systematic match qualities for single females and males. Because singles do not benefit from scale economies related to public consumption, our structural model requires a positive unobserved match quality to rationalise singlehood as a stable situation (relative to a same-type marriage, of which the match quality is normalised to zero). Given that we do observe single males and females of both education types, this can be interpreted through the lens of our structural model as a "quality of singlehood" (for example, reflecting the value of independence) or, alternatively, as a "cost of marriage". We find that this match quality of singles is very substantial, particularly if we express it as a fraction of the singles' private consumption (with or without leisure). Similar to before, these results highlight the relevance of match quality if we are to rationalise singlehood as a stable marital outcome.

A notable observation is that particularly the match quality of single females is very high and considerably above that of single males. This can be related to the fact that single females have different preferences over public and private consumption than single males. In particular, single females are much more likely to have children than single males (see Table 13 in Appendix C). We need a high quality of singlehood to offset the absence of scale economies associated with public consumption in marriage. Given that children's preferences are internalised in adults' preferences, it can be expected that this implies a stronger preference for public consumption by the average single female (see also the private and public consumption shares in Table 10). As public consumption is "cheaper" for individuals in couples (because of scale economies), we thus need a substantially higher match quality for single females than for single males to rationalise their singlehood. Moreover, it is plausible that a single woman with children is more reluctant to enter a new relationship than a single man, because it is much more likely that her preferences internalise her children's preferences. This, on its turn, is translated in a quality of singlehood that is higher for females than for males.

Table 10: Material consumption and systematic match quality per match quality type (mean levels) - single females

Material consumption		
EDU	*public*	*private*
low	449.745	598.065
high	601.843	660.604
Systematic match quality		
EDU	*match quality*	
low	(202.926; 205.520)	
high	(231.642; 233.840)	
As fraction of private consumption		
EDU	*with leisure*	*without leisure*
low	(33.930%; 34.364%)	(224.897%; 227.715%)
high	(35.065%; 35.398%)	(179.936%; 181.641%)

Notes: we report lower and upper bounds on the match qualities between brackets.

8 Conclusion

We introduced a novel methodology to empirically analyse rational household consumption under the assumption of marriage market stability. Our method allows us to (set) identify individuals' matching surplus as capturing both observed (material) public consumption and unobserved (non-material) match quality. Using our axiomatically-based Additive Quantity Shifting (AQS) model, we can quantify match quality in money metric terms. We consider a setting in which the empirical analysis can use preference and match quality factors to divide agents into observable preference and match quality types. The methodology builds on a revealed preference characterisation of rationalisable household behaviour that is intrinsically nonparametric, making it robust to functional specification error.

We have demonstrated the practical usefulness of our methodology through an

Table 11: Material consumption and systematic match quality per match quality type (mean levels) - single males

Material consumption		
EDU	*public*	*private*
low	368.031	657.511
high	437.617	784.630
Systematic match quality		
EDU	*match quality*	
low	(88.432; 91.735)	
high	(103.542; 117.295)	
As fraction of private consumption		
EDU	*with leisure*	*without leisure*
low	(13.450%; 13.952%)	(66.887%; 69.385%)
high	(13.196%; 14.949%)	(66.262%; 75.064%)

Notes: we report lower and upper bounds on the match qualities between brackets.

application to the Belgian MEqIn data. Our application showed that our nonparametric method has substantial (set) identifying power, even when imposing little prior structure on the setting at hand. We can identify bounds on the systematic match quality (which varies with education) that are informatively tight, and we can meaningfully analyse the intrahousehold allocation of consumption and match quality. We have shown that using preference factors to define preference types enhances the identification analysis by tightening the match quality bounds.

Our results reveal that match quality gains associated with low educated spouses (partly) compensate for the economic attractiveness of high educated spouses. This explains that we see more mixed marriages with low and high educated spouses than if only material gains to marriage mattered and non-material gains were irrelevant. In addition, we identify a positive match quality of singles, which is substantially higher for females than for males. These patterns provide an intuitive explanation of the observed marriage and consumption allocations through the lens of our structural model. Specifically, higher unobserved match quality can compensate for (material) losses that follow from lower consumption. This holds in particular for individuals with a strong preference for public consumption in the household, such as single females with children.

Our empirical application has mainly concentrated on the identification of unobserved match quality, which is the main novelty of our newly proposed methodology. Importantly, our method is versatile in that it can also be used to identify other unobserved aspects of household consumption decisions (related to individual utilities and intrahousehold sharing). In addition, it can be usefully combined with other revealed preference methods, so further enriching the empirical analysis. For example, we can explicitly include a model of household production as in Cherchye, De Rock, Walther, and Vermeulen (2021). Next, we can empirically identify the degree of publicness of household consumption (defining the intrahousehold scale economies) by using the toolkit of Cherchye, De Rock, Surana, and Vermeulen (2020a). Finally, as our method allows us to identify the individual (Lindahl) prices for public consumption, we can integrate the methodology of Cherchye, Cosaert, De Rock, Kerstens, and Vermeulen (2018) to evaluate individual welfare in money metric terms for households that consume public goods, along the lines of Chiappori and Meghir (2014) and Chiappori (2016).

References

Afriat, S. N., 1967. The construction of utility functions from expenditure data. International Economic Review 8, 67–77.

Alkan, A., Gale, D., 1990. The core of the matching game. Games and economic behavior 2 (3), 203–212.

Becker, G. S., 1965. A theory of the allocation of time. Economic Journal 75, 493–517.

Becker, G. S., 1973. A theory of marriage: Part i. Journal of Political Economy 81 (4), 813–846.

Blackorby, C., Donaldson, D., 1994. Measuring the cost of children: a theoretical framework. The measurement of household welfare, 51–69.

Browning, M., Chiappori, P.-A., Weiss, Y., 2014. Family Economics. Cambridge University Press.

Browning, M., Gørtz, M., 2012. Spending time and money within the household. The Scandinavian Journal of Economics 114 (3), 681–704.

Capéau, B., Cherchye, L., Decancq, K., Decoster, A., De Rock, B., Maniquet, F., Nys, A., Périlleux, G., Ramaekers, E., Rongé, Z., Schokkaert, E., Vermeulen, F., 2020. Well-being in Belgium: beyond happiness and income. Springer.

Cherchye, L., Cosaert, S., De Rock, B., Kerstens, P. J., Vermeulen, F., 2018. Individual welfare analysis for collective households. Journal of Public Economics 166, 98–114.

Cherchye, L., De Rock, B., Surana, K., Vermeulen, F., 2020a. Marital matching, economies of scale and intrahousehold allocations. The Review of Economics and Statistics 102, 823–837.

Cherchye, L., De Rock, B., Vermeulen, F., 2012. Married with children: A collective labor supply model with detailed time use and intrahousehold expenditure information. American Economic Review 102 (7), 3377–3405.

Cherchye, L., De Rock, B., Walther, S., Vermeulen, F., 2021. Where did it go wrong? marriage and divorce in malawi. Quantitative Economics 12, 505–545.

Cherchye, L., Demuynck, T., De Rock, B., Surana, K., 2020b. Revealed preference analysis with normal goods: Application to cost-of-living indices. American Economic Journal: Microeconomics 12 (3), 165–188.

Cherchye, L., Demuynck, T., De Rock, B., Vermeulen, F., 2017. Household consumption when the marriage is stable. American Economic Review 107, 1507–1534.

Chiappori, P., 1988. Rational household labor supply. Econometrica 56, 63–90.

Chiappori, P., 1992. Collective labor supply and welfare. Journal of Political Economy 100, 437–467.

Chiappori, P., 2016. Equivalence versus indifference scales. Economic Journal 126, 523–545.

Chiappori, P., Gugl, E., 2020. Transferable utility and demand functions. Theoretical Economics 15, 1307–1333.

Chiappori, P., Meghir, C., 2014. Intra-household welfare. NBER working paper 20189.

Chiappori, P.-A., 2017. Matching with transfers. Princeton University Press.

Chiappori, P.-A., Mazzocco, M., 2017. Static and intertemporal household decisions. Journal of Economic Literature 55 (3), 985–1045.

Choo, E., Siow, A., 2006. Who marries whom and why. Journal of Political Economy 114, 175–201.

Clark, S., 2006. The uniqueness of stable matchings. Contributions in Theoretical Economics 6 (1).

Diewert, W. E., 1973. Afriat and revealed preference theory. The Review of Economic Studies 40, 419–425.

Eeckhout, J., 2000. On the uniqueness of stable marriage matchings. Economics Letters 69 (1), 1–8.

Galichon, A., Kominers, S. D., Weber, S., 2019. Costly concessions: An empirical framework for matching with imperfectly transferable utility. Journal of Political Economy 127 (6), 2875–2925.

Galichon, A., Salanié, B., 2022. Cupid's invisible hand: Social surplus and identification in matching models. The Review of Economic Studies 89 (5), 2600–2629.

Goussé, M., Jacquemet, N., Robin, J.-M., 2017. Marriage, labor supply, and home production. Econometrica 85, 1873–1919.

Lee, S., Ng, S., 2020. An econometric perspective on algorithmic subsampling. Annual Review of Economics 12, 45–80.

Legros, P., Newman, A., 2010. Co-ranking mates: Assortative matching in marriage markets. Economics Letters 106 (3), 177–179.

Low, C., 2024. The human capital–reproductive capital trade-off in marriage market matching. Journal of Political Economy 132 (2), 000–000.

Pendakur, K., 2005. Semiparametric estimation of lifetime equivalence scales. Journal of Applied Econometrics 20 (4), 487–507.

Politis, D. N., Romano, J. P., 1994. Large sample confidence regions based on subsamples under minimal assumptions. The Annals of Statistics, 2031–2050.

Politis, D. N., Romano, J. P., Wolf, M., 2012. Subsampling. Springer Science & Business Media.

Shapley, L., Shubik, M., 1972. The assignment game I: The core. International Journal of Game Theory 1, 111–130.

Varian, H., 1982. The nonparametric approach to demand analysis. Econometrica 50, 945–974.

Varian, H., 1983. Non-parametric tests of consumer behavior. The Review of Economic Studies 50, 99–110.

Weber, S., 2018. Collective models and the marriage market. Mimeo.

Appendix

A Mixed integer linear programming formulation

We ease the notational burden by focusing on the specific case without unobserved match quality (i.e., all $\theta^m_{i,r}$ and $\theta^w_{i,r}$ are set to zero). The characterisation below extends to the case with unobserved match quality by using the same transformations as in our proofs of Theorem 1 in the Online Appendix.

We can reformulate the ARSM condition in Definition 2 in terms of inequality constraints that are linear in unknowns and characterised by binary integer variables, which makes them easy to operationalise. For convenience, let us focus on the ARSM condition expressed in terms of weak inequalities (the argument for the case of strict inequalities is directly similar). Particularly, we use the binary variables $Z^{\psi}_{i,k} \in \{0,1\}$ and $Z^{\omega}_{r,s} \in \{0,1\}$ to represent the utility orderings of male type ψ and female type ω, in the following sense:

$$Z^{\psi}_{i,k} = 1 \text{ if } U^{\psi}(k) \geq U^{\psi}(i), \tag{1}$$

$$Z^{\omega}_{r,s} = 1 \text{ if } U^{\omega}(s) \geq U^{\omega}(r). \tag{2}$$

Then, we can state the following result (using G to denote a sufficiently large number; for example, $G \geq y_{i,r}$ for all i and r).[24]

Proposition 1. *A data set S satisfies the ARSM if and only if, for all couples $(i,r) \in M \times W$, with $\tau(i) = \psi$ and $\tau(r) = \omega$, there exist*

[24] We note that the strict inequalities $U^{\psi}(k) - U^{\psi}(i) < Z^{\psi}_{i,k}$ and $U^{\omega}(s) - U^{\omega}(r) < Z^{\omega}_{r,s}$ are difficult to use in mixed integer linear programming analysis. Therefore, in practice we can replace them with $U^{\psi}(k) - U^{\psi}(i) + \epsilon \leq Z^{\psi}_{i,k}$ and $U^{\omega}(s) - U^{\omega}(r) + \epsilon \leq Z^{\omega}_{r,s}$ for ϵ (> 0) arbitrarily small.

- *a utility value* $U^\psi(i) \in [0,1]$ *for man* i *of type* ψ,
- *a utility value* $U^\omega(r) \in [0,1]$ *for women* r *of type* ω,
- *price vectors* $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$ *with* $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, *and*
- *binary variables* $Z^\psi_{i,k}, Z^\omega_{r,s} \in \{0,1\}$,

such that:

$$U^\psi(k) - U^\psi(i) < Z^\psi_{i,k}, \tag{3}$$

$$U^\omega(s) - U^\omega(r) < Z^\omega_{r,s}, \tag{4}$$

and:

$$y_{i,r} - p'_{i,r}(q^m_{k,\sigma(k)} + q^w_{\sigma(s),s}) - P^{m\prime}_{i,r} Q_{k,\sigma(k)} - P^{w\prime}_{i,r} Q_{\sigma(s),s} \leq (2 - Z^m_{i,k} - Z^w_{r,s})G. \tag{5}$$

For this result, we normalise without loss of generality the utility values $U^\psi(i)$ and $U^\omega(r)$ so that they can only take values between 0 and 1. Then, (3) implements (1), which ensures that the binary variables $Z^\psi_{i,k} \in \{0,1\}$ represent the utility orderings of male type ψ. Similarly, (4) implements (2), which pertains to the utility orderings of female type ω. Also, (5) will be non-void only if both $Z^m_{i,k}$ and $Z^w_{r,s}$ are equal to 1. Given all this, the equivalence between the ARSM specification in Definition 2 and the one in Proposition 1 follows readily.

B Belgian household data: additional information

Table 12: preference factors per type - couples (mean values)

female EDU	*male EDU*	*female age*	*male age*	*presence of children (1 = yes/0 = no)*	*number of children*
low	low	37.148	39.721	0.639	1.131
	high	38.714	44.619	0.667	0.952
high	low	37.886	39.727	0.750	1.159
	high	39.441	40.985	0.632	1.176

Table 13: preference factors per type - singles (mean values)

female EDU	*age*	*presence of children (1 = yes/0 = no)*	*number of children*
low	43.329	0.521	0.849
high	41.235	0.667	1.059
male EDU	*age*	*presence of children (1 = yes/0 = no)*	*number of children*
low	45.174	0.130	0.152
high	43.043	0.304	0.565

C Bounds on match quality: additional results

Table 14: Systematic match quality - married female types

female	*male*	*lower bounds*		*upper bounds*	
EDU	*EDU*	*mean*	*st.dev.*	*mean*	*st.dev.*
low	low	0.000	0.000	0.000	0.000
	high	-78.008	30.925	-69.551	32.215
high	low	37.289	16.897	42.897	17,309
	high	0.000	0.000	0.000	0.000

Table 15: Systematic match quality - married male types

male	*female*	*lower bounds*		*upper bounds*	
EDU	*EDU*	*mean*	*st.dev.*	*mean*	*st.dev.*
low	low	0.000	0.000	0.000	0.000
	high	-37.162	17.291	-32.236	17.956
high	low	50.203	38.442	61.919	38.378
	high	0.000	0.000	0.000	0.000

Table 16: Systematic match quality - single female types

female	*lower bounds*		*upper bounds*	
EDU	*mean*	*st.dev.*	*mean*	*st.dev.*
low	202.926	9.561	205.520	9.223
high	231.642	18.746	233.840	18.233

Table 17: Systematic match quality - single male types

male	*lower bounds*		*upper bounds*	
EDU	*mean*	*st.dev.*	*mean*	*st.dev.*
low	88.432	13.484	91.735	12.854
high	103.542	27.686	117.295	21.140

D Fully heterogeneous preferences: results

Table 18: Systematic match quality - married female types

female	*male*	*lower bounds*		*upper bounds*	
EDU	*EDU*	*mean*	*st.dev.*	*mean*	*st.dev.*
low	low	0.000	0.000	0.000	0.000
	high	-88.314	34.900	-73.467	37.554
high	low	38.430	17.536	46.789	18.805
	high	0.000	0.000	0.000	0.000

Table 19: Systematic match quality - married male types

male	*female*	*lower bounds*		*upper bounds*	
EDU	*EDU*	*mean*	*st.dev.*	*mean*	*st.dev.*
low	low	0.000	0.000	0.000	0.000
	high	-42.636	19.343	-34.354	20.073
high	low	46.824	42.905	65.928	40.946
	high	0.000	0.000	0.000	0.000

Table 20: Systematic match quality - single female types

female	*lower bounds*		*upper bounds*	
EDU	*mean*	*st.dev.*	*mean*	*st.dev.*
low	194.146	9.384	197.448	9.225
high	220.932	16.663	224.146	16.414

Table 21: Systematic match quality - single male types

male	*lower bounds*		*upper bounds*	
EDU	*mean*	*st.dev.*	*mean*	*st.dev.*
low	81.773	13.432	86.317	13.537
high	94.067	29.071	109.085	22.059